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Seeing the Math in the Story: On How Abstraction Promotes Performance on Mathematical Word Problems

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Abstract

The negative social, health, financial, and other life outcomes associated with mathematical proficiency deficits highlight the need to understand the underlying cognitive operations entailed in solving math problems. We focus specifically on mathematical word problems and propose that abstraction can enhance performance by helping people see beyond the incidental details described in word problems and to recognize instead the underlying mathematical relationships. Three studies manipulated abstraction as a procedural mind-set (i.e., inducing abstraction in one task and observing its "carry-over" effect in subsequent unrelated tasks) and observed performance on both numeric and word problems. Participants in the abstract, relative to concrete, mind-set condition were more successful in translating word problems into their analogous numeric forms, resulting in improved performance. We discuss implications of these findings for understanding individual and group differences in mathematics proficiencies, which may stem from both chronic and situational factors, and for the development of novel interventions.

Keywords

academic achievement, social cognition, educational psychology, motivation and performance, achievement

Deficits in mathematical proficiency have deleterious effects on individuals' decision-making abilities (Peters, 2012; Schley & Peters, 2014) and impact health (Lipkus & Peters, 2009; Reyna, Nelson, Han, & Dieckmann, 2009) and financial outcomes (Lusardi & Mitchell, 2007). Research has also documented troubling group differences in math proficiency, which may play a key role in maintaining gender and racial disparities (e.g., Steele, 1997; Walton & Cohen, 2011). Understanding the causes of mathematical deficiencies may provide insight into why some struggle with math, highlight situational conditions that are likely to facilitate versus impair performance, and suggest novel educational interventions for improving proficiency.

To address this issue, we examine some of the underlying cognitive processes entailed in solving math problems. Research suggests that one critical cognitive operation in solving math problems may be abstraction (Kaminski, Sloutsky, & Heckler, 2008; but see Koedinger, Alibali, & Nathan, 2008). Abstraction is a reductive process that entails the extraction of essential and core (i.e., gist) aspects of objects or events while ignoring salient surface-level or verbatim features (Reyna, 2012; Reyna & Brainerd, 2011; Trope & Liberman, 2010). In categorization, for example, abstraction facilitates looking beyond the unique and idiosyncratic features that distinguish one individual from another to the defining and gist features that are common across individuals (Medin & Ortony, 1989; Rosch & Mervis, 1975). Abstraction may promote math performance by simplifying the representations and computations required for correct solutions and by facilitating knowledge transfer from the initial learning context to other novel domains (Goldstone & Sakamoto, 2003; Kaminski et al., 2008; Son, Smith, & Goldstone, 2008; Sloutsky, Kaminski, & Heckler, 2005).

Yet much of this past work has examined the effect of abstract versus concrete stimuli on math performance, rather than the effect of abstraction per se. That is, participants are presented with math problems in what are purportedly abstract (e.g., "2 + 2") or concrete formats (e.g., "add two plus two"), which are believed to facilitate abstract versus concrete information processing, respectively (Kaminski et al., 2008). Although these differences in materials appear to differentially impact performance, it is still unclear whether these effects reflect the operation of abstraction per se or some other processing that is more stimulus-specific (e.g., fluency of encoding). To demonstrate that such effects are not stimulus bound and instead reveal the cognitive operation of abstraction more generally, in the present studies, we capitalized on the ability to

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manipulate abstraction as a procedural mind-set. Previous research has demonstrated that having individuals engage in abstraction in one task enhances the tendency to engage in abstraction in subsequent unrelated tasks (e.g., Förster, Friedman, & Liberman, 2004; Freitas, Gollwitzer, & Trope, 2004; Fujita, Trope, Liberman, & Levin-Sagi, 2006). By manipulating it independently of the stimuli (i.e., all participants were presented with identical mathematical materials), we can make more definitive conclusions about the role of abstraction in mathematical problem solving.

Translating Word Problems Into Numbers: Extracting Gist via Abstraction

In the present studies, we focus specifically on the role of abstraction in solving word problems. Research suggests that although individuals are capable of calculating solutions correctly when presented with problems in numeric format (e.g., "2 + 2"), they appear to struggle when presented with analogous problems in word format (e.g., "add two to two"; Cummins, Kintsch, Reusser, & Weimer, 1988; Lewis, 1989; but see Koedinger et al., 2008). Solving word problems requires distinguishing the narrative component of the problem from the numeric information (Kintsch & Greeno, 1985). Failure to do so can interfere with the translation of the narrative format of word problems into their correct numeric equivalents (for review, see Lewis, 1989). Indeed, recent functional magnetic resonance imaging data suggest a neural dissociation between algebraic operations and language (Monti, Parsons, & Osherson, 2012), suggesting a structural basis for the difficulty in recognizing mathematical relationships in word problems. Research suggests that interventions that facilitate translating word problems into their numeric equivalents, such as simpler question wording (Cummins et al., 1988), placing the question before the presentation of the word problem (Thevenot, Devidal, Barrouillet, & Fayol, 2007), employing graphical illustrations (Berends & van Lieshout, 2009), and explicitly teaching mathematics translation skills (Lewis, 1989), can improve performance on word problems.

We propose that key to this translation process is extracting gist. Specifically, we suggest that the incidental secondary information that word problems present distracts people from recognizing the core of problem: the underlying mathematical relations being assessed. Consider an example: "Robert enjoys collecting baseball cards. Robert has 34 baseball cards. If he gets 17 more cards, he will have exactly half as many cards as his friend Greg. How many cards does Greg have?" This word problem presents a number of details that are superfluous for solving the problem: the names Robert and Greg, that they are collectors, and that they are exchanging baseball cards. Conversational norms may suggest to problem solvers that this information is noteworthy and important (Grice, 1975). One needs to ignore these details, however, to understand the essence of the problem: Greg has $(17 + 34) \times 2$ cards. Thus, at the heart of the translation problem may be difficulties in understanding the gist of word problems.

Findings from two research traditions, namely construal level theory (CLT; Trope & Liberman, 2003, 2010) and fuzzy trace theory (FTT; Reyna, 2004; Reyna & Brainerd, 1995), suggest that cognitive abstraction may facilitate this extraction of gist. Both theories distinguish two general processing styles: one that focuses on concrete, situation-specific details versus one that focuses on more abstract, global information (CLT: low-level vs. high-level construal; FTT: verbatim vs. gist processing). Each theory proposes different functional properties for these two processing styles. More germane to this article, however, is that both CLT and FTT research has demonstrated the benefits of abstraction for judgment and decision making. A key characteristic of abstraction is determining what is or is not relevant to a given representation. Appreciating that a chihuahua is more generally a dog, for example, requires ignoring size and shape and instead focusing on its essential canine characteristics (Medin & Ortony, 1989). Research suggests that this abstraction-induced parsing of relevant versus irrelevant information can enhance performance in numerous judgment and decision-making tasks (for review, see Fujita, Trope, & Liberman, in press; Reyna, 2004). Consider, for example, findings reported by Fukukura, Ferguson, and Fujita (2013). Research suggests that people are often overwhelmed by the amount of information (Jacoby, 1977; Malhotra, 1982; Wilkie, 1974) and by the number of choice options they have (Iyengar & Lepper, 2000; Schwartz et al., 2002). This information and choice overload induce confusion as to what information to attend to and what to ignore. This in turn leads to suboptimal decisions. Fukukura and colleagues (2013) demonstrated that experimentally inducing abstract over concrete processing facilitated the extraction of gist, promoting participants' ability to ignore salient yet irrelevant information and to instead focus on more global and essential features of the decision problem. This in turn enhanced identification of superior choice options. Thus, abstraction appears to facilitate the extraction of gist, which in turn can enhance performance.

In the present research, we apply these insights to understand math performance. Specifically, we propose that to the extent that translating word problems into their numerical equivalents requires extraction of gist, inducing more abstract (vs. concrete) processing should promote math proficiency. Thus, in the present studies, we manipulated abstraction as a procedural mind-set, using methods developed in past research (e.g., Förster et al., 2004; Freitas et al., 2004; Fujita et al., 2006). We then asked participants to complete both word and numerical math problems. We predicted that manipulating abstraction as a procedural mind-set should improve individuals' ability to ignore incidental surface features of word problems and to extract the essential gist (i.e., the underlying mathematical relationships). This improved ability to extract gist, in turn, should mitigate the translation problem and enhance performance. We predicted no differences among performance of numeric problems, as no abstraction is necessary as the gist of the problems are explicit in their numeric form.

Studies 1 and 2

The purpose of Studies 1 and 2 was to investigate the influence of a mind-set manipulation on numeric and word mathematics problem performance. Furthermore, Studies 1 and 2 allowed us to test our proposed mechanism—that abstract mind-sets enhance the translation of word problems into their numeric equivalents by facilitating the extraction of gist.

Method

Participants for Studies 1 ($N = 193$; 58% female; $M_{age} = 32.2$; range 18–68 years) and 2 ($N = 201$; 51% female; $M_{age} = 31.5$; range 18–77 years) were recruited online from independent Mechanical Turk samples and paid US$0.50; no participants were excluded from the current or subsequent studies. Although participants should not vary in numeracy by condition, given random assignment, we nevertheless measured participants' numeracy on the subjective numeracy scale (SNS; Fagerlin et al., 2007) to control for the potential confound of mathematics ability. We used the SNS rather than objective numeracy measures, as the latter typically employs word problems (see Weller et al., 2013) which is the dependent measure in the current study.

Mind-Set Manipulation

Participants in Study 1 completed a "double-barreled" abstraction manipulation. They first completed a 10-item categorization task, taken from a common 40-item mind-set manipulation task (see Fujita et al., 2006). In the categorization task, participants in the abstract mind-set condition were presented with a stimulus (e.g., car) and asked to provide a superordinate category (e.g., vehicle), whereas participants in the concrete mind-set condition were asked to provide an exemplar (e.g., BMW). To strengthen the manipulation, participants in Study 1 completed an additional mind-set manipulation with condition held constant. Research suggests that thinking about the distant versus near future promotes abstraction (e.g., Trope & Liberman, 2010). As such, participants in the abstract mind-set condition wrote a statement about their lives a year from now, whereas those in the concrete mind-set condition wrote about their lives tomorrow (Förster et al., 2004).

One concern about Study 1 was that the two mind-set manipulations may not behave additively. To address this concern, Study-2 participants completed only the categorization task, employing 30 of the 40 items used by Fujita, Trope, Liberman, and Levin-Sagi (2006).

Mathematics Problems

After the mind-set manipulation, participants completed three numeric problems (e.g., $x = [55 - 23] \div 4$) and three word problems (e.g., baseball problem mentioned earlier), order counterbalanced (see Online Supplementary Materials for all numeric and word problems). The word problems had the same mathematical relationships as the numeric problems but employed different numbers. There was no time limit for the problems.

Test of Mechanism

As a test of our proposed mechanism that abstraction facilitates extraction of the mathematical relationships (i.e., gist) in word problems, participants were presented with three novel word problems (see Online Supplementary Materials). In this "translation" task, participants were asked to indicate which of the four possible mathematical expressions (randomly ordered) represented the mathematical relationship in the word problem (i.e., participants were asked to translate the word problem into its equivalent numeric form). Each translation problem also included a "none of the above" option.

Procedural Differences

In Study 1, participants began with the SNS, followed by the mind-set manipulation, mathematics problems, and the translation task. We began Study 1 with the SNS to avoid possible influences of mind-set on participants' SNS responses. It is possible, however, that having participants self-rate their proficiency with mathematics on the SNS might interact with the mind-set manipulation. As such, in Study 2, the SNS was instead presented at the end of the study.

Results and Discussion

Participants' average problem completion times in Studies 1 and 2 were 50.2 and 46.7 s, respectively. Completion times did not differ by mind-set condition in Study 1, $p = .95$, nor Study 2, $p = .80$. Participants' results are displayed by item in Table 1 and averaged across items in Figure 1. Question order had no significant influence on participants' performance and is not discussed further. Scores on the SNS did not vary by condition; yet, to control for any potential individuals differences in mathematical proficiency, we nevertheless controlled for it in all subsequent analyses. We calculated the percentage of correct answers for each set of problems (i.e., numeric, word, and translation problems). We present the results of analyses of covariance (ANCOVAs), although findings were similar with repeated measures logistic regression. Consistent with predictions, results indicated no influence of mind-set on participants' accuracy with numeric problems—Study 1: $M_{AbsAvg} = 57.2\%$ and $M_{ConAvg} = 55.4\%$ correct, $p > .60$; Study 2: $M_{AbsAvg} = 51.6\%$ and $M_{ConAvg} = 53.0\%$ correct, $p > .70$. In contrast, participants in the abstract mind-set condition, compared to the concrete mind-set condition, performed significantly better on the word problems—Study 1: $M_{AbsAvg} = 52.9\%$ correct and $M_{ConAvg} = 44.9\%$ correct, $F(1, 190) = 4.20$, $p = .04$, $\eta_p^2 = .02$; Study 2: $M_{AbsAvg} = 52.5\%$ correct, $M_{ConAvg} = 43.5\%$ correct, $F(1, 198) = 4.93$, $p = .03$, $\eta_p^2 = .02$.

Table 1. Percentage Correct per Question, for Studies 1 and 2.

		Study 1		Study 2	
		Mind-Set		Mind-Set	
	Problem	Concrete	Abstract	Concrete	Abstract
Numeric problems	1	84.2 (36.7)	79.3 (40.7)	80.0 (40.2)	84.9 (36.0)
	2	32.7 (47.1)	32.6 (47.1)	32.6 (47.1)	26.4 (44.3)
	3	49.5 (50.2)	59.8 (49.3)	46.3 (50.1)	43.4 (49.8)
Word problems	1	72.3 (45.0)	78.3 (41.5)	62.1 (49.0)	72.6 (44.8)
	2	25.7 (43.9)	29.3 (45.8)	27.3 (44.8)	34.9 (47.9)
	3	36.6 (48.4)	51.1 (50.3)	41.1 (49.5)	50.0 (50.2)
Translation problems	1	55.4 (50.0)	62.0 (48.8)	51.6 (50.2)	63.2 (48.5)
	2	28.7 (45.5)	39.1 (49.1)	26.3 (44.3)	34.9 (47.9)
	3	28.7 (45.5)	35.9 (48.2)	24.2 (43.1)	29.2 (45.7)

Note. Standard deviations are given in parentheses.

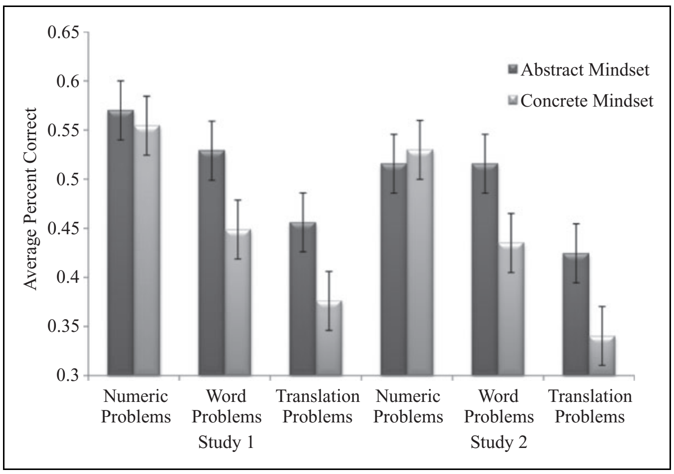

Figure 1. Average of participants' percentage correct across the 3 items within each problem set by condition in Studies 1 and 2; bars reflect standard errors.

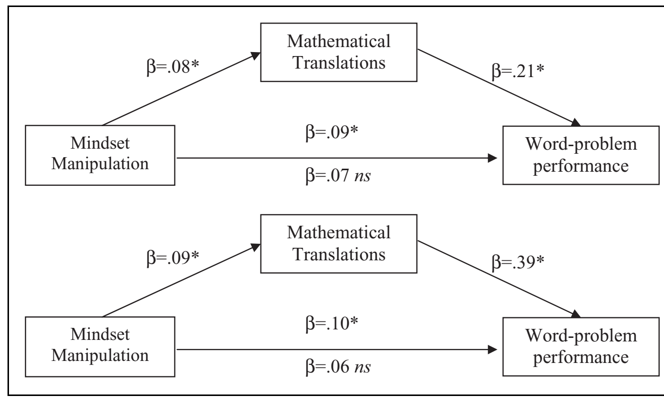

Figure 2. Mediation results from Studies 1 (top) and 2 (bottom). The β above the horizontal arrow represents the total effect of mind-set on word problem performance and the β below represents the direct effect (i.e., the residual effect of mind-set, controlling for the mediating/indirect effect of mathematic translations on word problem performance).

Mediation Analyses

Consistent with our proposed mechanism that abstraction facilitates translation of word problems into their numerical equivalents, participants in the abstract mind-set condition performed significantly better than participants in the concrete mind-set condition on the translation task—Study 1: $M_{\text{AbsAvg}} = 45.7\%$ and $M_{\text{ConAvg}} = 37.6\%$ correct, $F(1, 190) = 4.15$, $p = .04$, $\eta_p^2 = .02$; Study 2: $M_{\text{AbsAvg}} = 42.5\%$ and $M_{\text{ConAvg}} = 34.0\%$ correct, $F(1, 198) = 4.67$, $p = .03$, $\eta_p^2 = .02$. Furthermore, translation task performance was positively related to word problem performance—Study 1: $F(1, 190) = 8.64$, $p = .004$, $\eta_p^2 = .04$; Study 2: $F(1, 198) = 29.65$, $p < .0001$, $\eta_p^2 = .11$. When both mind-set and translation task performance were included in the ANCOVA predicting word problem performance, the effect of mind-set decreased—Study 1: $F(1, 189) = 2.78$, $p = .10$, $\eta_p^2 = .01$; Study 2: $F(1, 197) = 2.40$, $p > .10$, $\eta_p^2 = .01$ (see Figure 2). Indeed, the test of the indirect effect of mind-set on word problem performance through translation task performance using bias-corrected boot-strapping procedures ($N = 10{,}000$; Preacher & Hayes, 2004) to generate 95% confidence intervals (CIs) was statistically significant—Study 1: $\beta_{\text{Indirect}} = 0.02$, CI [0.002, 0.052]; Study 2: $\beta_{\text{Indirect}} = 0.04$, CI [0.005, 0.080]. Thus, the ability to extract essential mathematical relationships appears to mediate the effect of mind-set on word problem performance.

Study 3

Study 3 aims to generalize the previous results to a student population using a different mind-set manipulation. Study 3 also builds on the previous studies by including a baseline condition and increasing the number of total math items from 6 to 14 (i.e., 7 word problems and 7 numeric problems).

Method

Participants ($N = 229$; 48% female; $M_{\text{age}} = 18.8$; range 18–29 years) were recruited from a large university and given course credit in exchange for their participation in Study 3. To ensure that the observed results of the previous studies were not an idiosyncrasy of the categorization-based mind-set manipulation in the previous studies, Study 3 employed a different mind-set manipulation taken from Freitas, Gollwitzer, and Trope (2004) in which they answer *how*/*why* questions to induce a concrete/abstract mind-set. Participants were presented the focal action "maintain good personal relationships." To induce an abstract mind-set, participants were asked *why* they maintain good personal relationships. After providing an initial response (e.g., "Feel connected to others"), participants were asked *why* they engaged in their response (e.g., "Why feel connected to others?"). Participants completed four iterations of this process, each iteration prompting a more abstract response. Participants in the concrete mind-set condition were asked *how* they maintain good personal relationships. After providing an initial response (e.g., "Make time for friends"), participants were asked *how* they engaged in their response (e.g., "How do you make time for friends?"). Participants completed four iterations of this process, each iteration prompting a more concrete response. In the baseline condition, participants did not complete a mind-set manipulation. Given a sampling error, sample sizes were 83 in each of the experimental conditions and 63 in the baseline condition.

Participants then completed 14 randomly ordered mathematics problems—7 word problems and 7 numeric problems (see Online Supplementary Materials). As Study 3 used considerably more mathematics problems than the previous studies, we did not include the translation task to avoid overburdening the participants. To make the study more representative of math tests and to ensure no differences in the amount of time devoted to the task, participants were presented a 60-s time limit and countdown timer for each mathematics problem. If participants did not provide a response in that time, the item was coded as incorrect. Participants then completed the SNS measure to control for general math ability.

Results and Discussion

Results of Study 3 are displayed by item in Table 2. Once again question order had no significant influence on participants' performance and is not discussed further. Similarly, scores on the SNS did not vary by condition and were controlled for in subsequent analyses. Data in Study 3 were analyzed in the same manner as in the previous studies. On average, participants ran out of time on 20% of the items; this did not vary by condition, $p = .93$. Consistent with our hypothesis and replicating the findings in Studies 1 and 2, results indicated no main effect of mind-set on participants' accuracy with numeric problems, $M_{\text{AbsAvg}} = 50.6\%$, $M_{\text{Baseline}} = 50.6\%$, and $M_{\text{ConAvg}} = 50.8\%$ correct, $p > .80$. In contrast, there was a significant main effect of condition of word problems, $F(2, 225) = 4.47, p = .01, \eta_p^2 = .04$. Specifically, participants in the abstract mind-set condition, $M_{\text{AbsAvg}} = 44.2\%$ correct, were significantly more accurate than participants in the baseline condition, $M_{\text{Baseline}} = 38.1\%$ correct, $F(1, 143) = 5.62, p = .02, \eta_p^2 = .04$. There were no apparent differences between the baseline and concrete mind-set conditions, $M_{\text{ConAvg}} = 36.8\%$ correct, $F(1, 143) = 0.02$, $p = .88$, $\eta_p^2 = .00$.

Table 2. Percentage Correct per Question, for Study 3.

	Item	Mind-Set		
		Concrete	Baseline	Abstract
Numeric problems	1	85.7 (35.2)	88.9 (31.7)	81.9 (38.7)
	2	13.1 (33.9)	15.9 (36.8)	14.5 (35.4)
	3	33.3 (47.4)	33.3 (47.5)	33.7 (47.6)
	4	77.4 (42.1)	71.4 (45.5)	75.9 (43.0)
	5	73.8 (44.2)	73.0 (44.7)	72.3 (45.0)
	6	15.5 (36.4)	15.9 (36.8)	16.9 (37.7)
	7	57.1 (49.8)	56.6 (50.1)	59.0 (49.5)
Word problems	1	75.0 (43.6)	77.8 (41.9)	88.0 (32.8)
	2	36.9 (48.5)	42.9 (49.9)	48.2 (50.3)
	3	32.1 (47.0)	34.9 (48.1)	42.2 (49.7)
	4	73.8 (44.2)	74.6 (43.9)	79.5 (40.6)
	5	09.5 (29.5)	06.3 (24.6)	10.8 (31.3)
	6	15.5 (36.4)	12.7 (33.6)	15.7 (36.6)
	7	17.9 (39.5)	17.5 (38.3)	25.3 (43.7)

Note. Standard deviations are given in parentheses.

We caution those who might overinterpret any comparisons of the two mind-set conditions to the "baseline" condition. Although the results of Study 3 may suggest that abstraction facilitates performance on word problems (rather than concretization impairing performance), observe that the mind-set manipulation is designed to produce relative, not absolute, differences in abstraction. What the "baseline" level of abstraction is for any sample may depend on a combination of population characteristics and situational factors (for further elaboration, see General Discussion section). As such, we prefer to limit our conclusions to reflect the relative differences in abstraction, rather than making strong assertions about the absolute effect of abstraction versus concretization on word problem performance.

General Discussion

Results of three experiments supported our hypothesis that promoting cognitive abstraction improves performance on word problems. The effect of abstraction appeared specific to word problems: No such effect was evident on performance on numeric problems. This is consistent with our proposal that abstraction enhances the ability to extract gist and facilitates recognizing the essential mathematical relationships described in word problems. Indeed, when we asked participants to translate word problems into their numerical equivalents, those in the abstract mind-set performed better relative to those in the concrete mind-set condition. Thus, abstraction appears to be a critical cognitive operation in solving word problems but not numeric problems.

That abstraction enhanced performance specifically on word problems is of particular note, as it helps to address several potential alternative interpretations of our findings. For example, one might have suggested that the abstract mind-set manipulation makes individuals more deliberative in their judgments, resulting in improved performance on word problems. Alternatively, abstract mind-sets may increase knowledge transfer from previously learned material (Kaminski et al., 2008). Although reasonable suppositions, both reinterpretations suggest that abstraction should promote math performance generally.

Still, that our abstraction manipulation had no impact on performance of numeric problems may have implications for understanding what cognitive operations are necessary to solve such problems. Solving math problems can involve retrieval processes (e.g., remembering that $3 \times 4 = 12$), in addition to procedural (e.g., rote computation of 3×4) and conceptual knowledge (i.e., understanding why a particular procedure would be used to solve a problem; Reyna et al., 2009; Siegler, 1988). Both numeric and word problems require retrieval and procedural knowledge. In the present studies, however, whereas the word problems required conceptual knowledge to assess which computational procedures to employ, the numeric problems did not (i.e., the procedures were explicitly provided). This may suggest that whereas accessing conceptual knowledge may require abstraction, retrieval and procedural knowledge may not. Thus, individual differences on numeric word problem performance may result from differences in retrieval processes and procedural knowledge, rather than conceptual knowledge.

One key advance of the present work is that we manipulated abstraction as a procedural mind-set rather than by presenting participants with abstract versus concrete test materials. This has two important implications. First, we were able to study the impact of abstraction on math performance while holding test questions constant. This addresses potential confounds that manipulating abstract versus concrete test materials might introduce, permitting stronger conclusions about the role of abstraction as a critical cognitive process in math performance. Note, however, that the present work focused exclusively on translation. Abstraction has been implicated in other aspects of math performance, such as the transfer of knowledge to novel domains (Kaminski et al., 2008). Future research might use the methods that we have introduced to confirm that these other phenomena are also not bound to the testing stimuli but rather reflect more generally changes in cognitive abstraction. Additional research might also examine to what extent abstract versus concrete math stimuli induces cognitive abstraction processes as a mind-set. To the extent that these materials facilitate abstract versus concrete processing, one should be able to observe a ‘‘carryover’’ effect on subsequent unrelated events (e.g., Förster et al., 2004; Freitas et al., 2004; Fujita et al., 2006). Such evidence would bolster the claim that abstract versus concrete test stimuli promote math performance via facilitating abstraction processes.

Second, these findings suggest novel methods that might be adopted to enhance math performance. Beyond the methods used in the current studies, existing research has highlighted numerous other situational factors that enhance abstraction (e.g., Trope & Liberman, 2010). For example, adopting a third-person rather than a first-person visual perspective can enhance abstraction and may be expected to systematically impact math performance (e.g., Kross, Ayduk, & Mischel, 2005; Libby, Shaeffer, & Eibach, 2009). Similarly, positive rather than negative moods appear to promote abstraction (e.g., Gasper & Clore, 2002) and should be expected to have similar effects. Mundane features of the test environment such as ceiling height (e.g., Meyers-Levy & Zhu, 2007) or ambient temperature (e.g., IJzerman & Semin, 2009) have also been shown to influence abstraction, might be expected to impact math performance. Beyond situational factors, the present research suggests that some individual and group differences in mathematics aptitude may be rooted in differences in the application of cognitive abstraction to solve word problems. Perhaps those who perform better have learned to engage in abstraction to solve such problems, whereas those with deficiencies have not. Future research is clearly warranted to test these possibilities.

The present work may provide insight into intellectual underperformance by females (relative to males) and by racial and ethnic minorities. An extensive literature documents that gender and racial disparities on intellectual performance are exacerbated under conditions that make gender and race stereotypes salient, a phenomenon referred to as stereotype threat (e.g., Spencer, Steele, & Quinn, 1999; Steele, 1997; Steele & Aronson, 1995). Although this phenomenon has been widely replicated, the precise mechanisms for the effect are still unclear (e.g., Schmader, Johns, & Forbes, 2008; Steele, 1997). One particularly powerful intervention that appears to reduce gaps in performance is value affirmation (Cohen, Garcia, Apfel, & Master, 2006; Martens, Johns, Greenberg, & Schimel, 2006). Emerging research suggests that affirming one’s important values promotes cognitive abstraction (e.g., Schmeichel & Vohs, 2009; Wakslak & Trope, 2009) and that this enhanced cognitive abstraction may mediate the positive effect of value affirmation on underperformance due to stereotype threat (Sherman et al., 2013). The present work may advance this research literature by detailing more specifically the mechanisms by which cognitive abstraction impacts performance on intellectual tests, particularly those involving math proficiency. With respect to math, rather than enhancing performance broadly, the beneficial effects of value affirmation on ameliorating underperformance may be specific to word, and not numerical, math problems. This proposition has yet to be documented by the existing literature, and we know of no other approach that makes such a nuanced prediction. Another implication of the present work is that value affirmation may at times be detrimental to performance, particularly on tasks that require concretization (e.g., Smith & Trope, 2006; Wakslak, Trope, Liberman, & Alony, 2006). This too has yet to be documented, yet clearly represents an intriguing possibility.

More generally, the influence of abstraction on scholastic achievement is not likely limited to math, but likely influences performance on verbal reasoning measures as well.

For instance, reading comprehension tests are common verbal assessments that involve reading a bulk paragraph and attempting to uncover the underlying themes of the paragraph, a process that likely entails abstraction. The methodology we used to explore the role of abstraction in math proficiency can equally be applied to verbal proficiency in future research.

The present work contributes to a growing body of research highlighting abstraction as a critical cognitive process in a myriad of judgment, decision-making, and performance tasks (Reyna, 2004; Trope & Liberman, 2010). An intriguing future research avenue might take advantage of any discrepancies in word versus numeric problem performance at the individual level to predict differences in judgment and decision-making in nonmathematical domains. The present research suggests that discrepancies in performance result not necessarily from a failure of mathematical computation but rather can result from a failure to engage in cognitive abstraction in a functional manner to facilitate the translation of mathematics problems from word to numeric formats. One might consider using the mathematical translation task that we implemented in these studies to examine other judgments and decisions that critically depend on abstraction, such as choice under information overload (Fukukura, Ferguson, & Fujita, 2013), self-control (e.g., Fujita et al., 2006), value–behavior correspondence (e.g., Eyal, Sagristano, Trope, Liberman, & Chaiken, 2009), person perception (e.g., Eyal, Hoover, Fujita, & Nussbaum, 2011; Nussbaum, Trope, & Liberman, 2003), and perceptions of risk (e.g., Reyna, 2004). To the extent that a discrepancy in performance between word and numeric math problems reveals some domain general deficiency in the functional usage of abstraction, one might expect these discrepancies to predict individual-level differences in other abstraction-dependent judgments and decisions. To our knowledge, no such individual difference measure of domain general abstraction capability has yet to be investigated.

To some, examining the role of cognitive abstraction in solving mathematical word problems may appear to some to be a narrow research inquiry. We instead argue that given the importance of mathematics proficiency on important judgments and decisions (Lipkus & Peters, 2009; Lusardi & Mitchell, 2007; Peters, 2012; Reyna et al., 2009), research that enhances performance has the potential to improve life outcomes. The current research draws from cognitive and social psychological theories to provide an integrative theoretical framework and new empirical tools with which to understand mathematical proficiency and its consequences. We encourage and look forward to future research that explores the consequences of "seeing the forest" versus "the trees" on mathematics proficiency, intellectual performance more generally, and its implications for social, health, and financial judgment and decision making.

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Supplemental Material

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