$$\omega_0 = \frac{2\pi}{T_0} \qquad f_0 = \frac{1}{T_0}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt} = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 kt}$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(2\pi f_0 kt + \phi_k)$$

$$A_0 = a_0 \qquad a_k = \tfrac{1}{2} A_k e^{j\phi_k}$$

$$A_k = 2|a_k|$$

$$\phi_k = \angle a_k$$

Fourier Series: Analysis

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\, dt$$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt \quad \text{average value of } x(t)$$

dc component

$k=0$

Fourier Analysis Equation

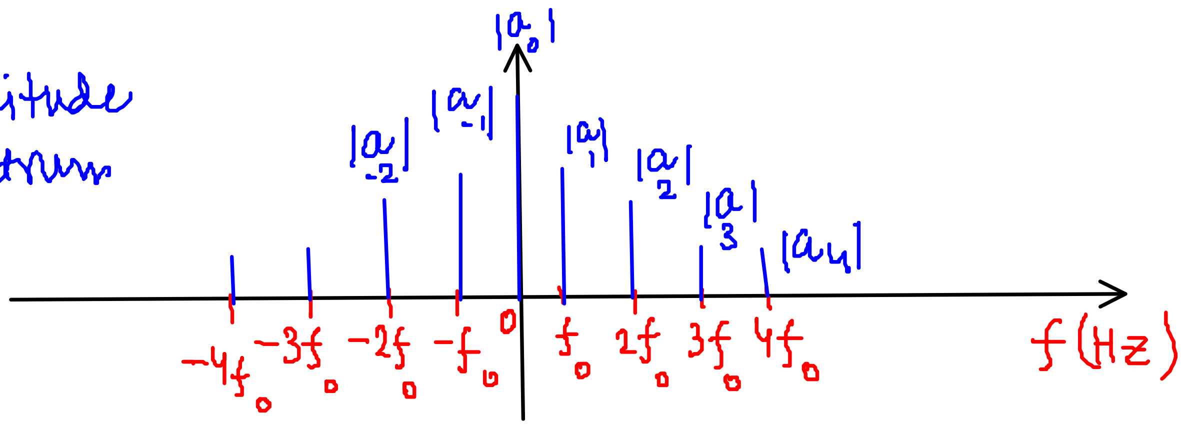
$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\, dt$$

Fourier Synthesis Equation

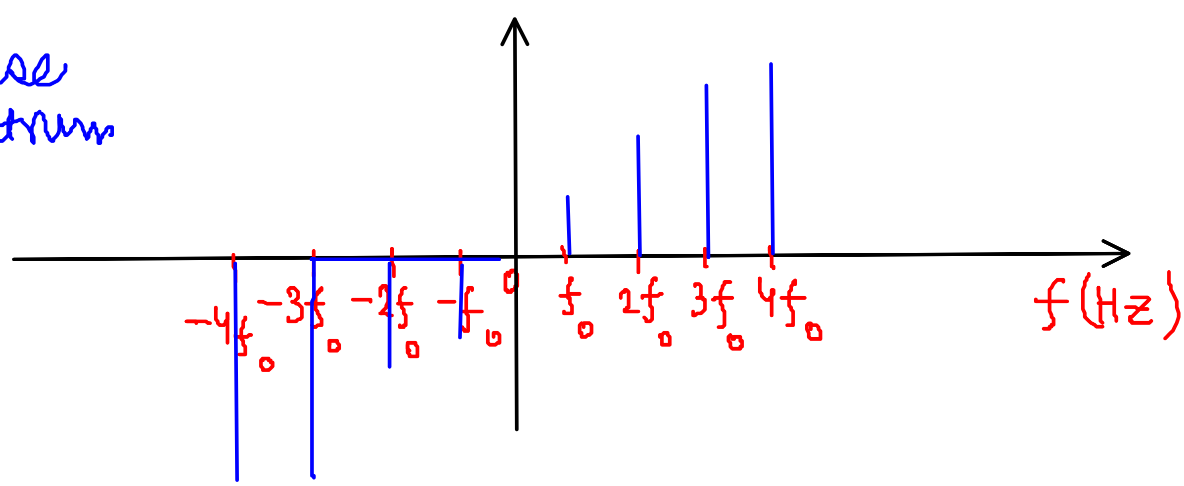
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\left(\frac{2\pi}{T_0}\right)kt}$$

Spectrum $(0, a_0)(f_0, a_1)(2f_0, a_2)\ (3f_0, a_3)\ \cdots$
$(-f_0, a_1^*)(-2f_0, a_2^*)(-3f_0, a_3^*)$

magnitude spectrum



phase spectrum



Example

$$x(t) = \sin^3(3\pi t)$$

Two ways to get a_k (fourier coefficients)

i) Use Fourier Analysis Equation

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\omega_0 k t}\, dt$$

ii) Use the inverse Euler formula

$$x(t) = \left(\frac{e^{j3\pi t} - e^{-j3\pi t}}{2j}\right)^3$$

$$= \frac{j}{8} e^{j9\pi t} + \frac{-3j}{8} e^{j3\pi t} + \frac{3j}{8} e^{-j3\pi t} + \frac{-j}{8} e^{-j9\pi t}$$

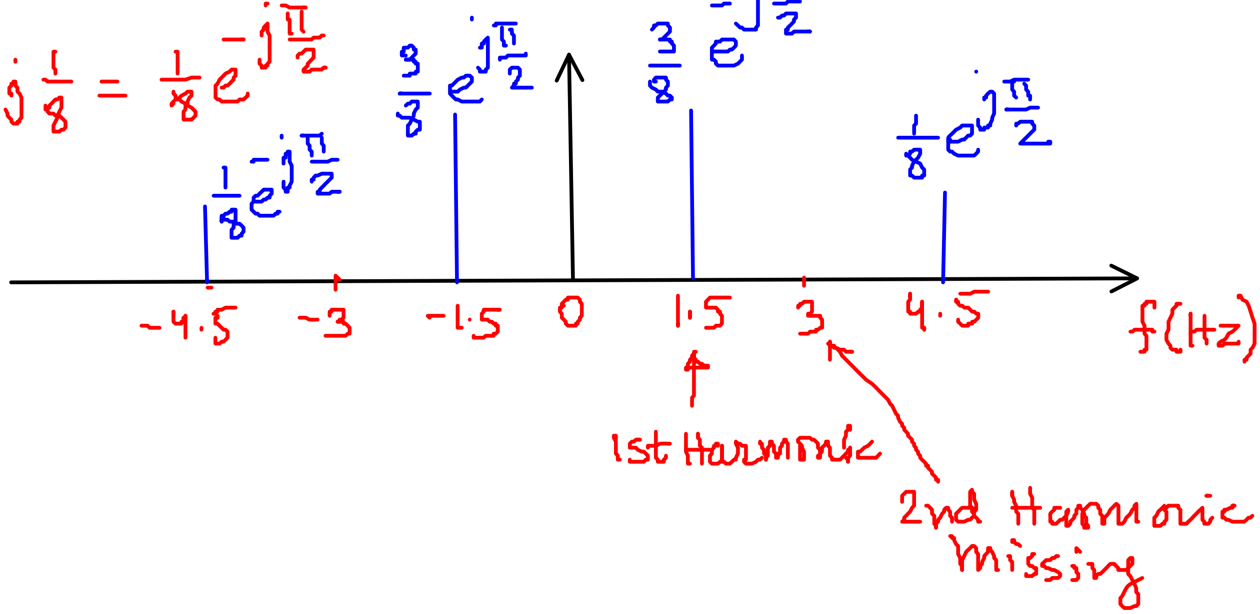
frequencies $\omega = \pm 9\pi$ rad/s , $\omega = \pm 3\pi$ rad/s
fundamental frequency $\omega_0 = 3\pi$ rad/s

$$a_k = \begin{cases} 0 & \text{for } k = 0 \\ \pm j\frac{3}{8} & \text{for } k = \pm 1 \\ 0 & \text{for } k = \pm 2 \\ \pm j\frac{1}{8} & \text{for } k = \pm 3 \\ 0 & \text{for } k = \pm 4, \pm 5, \ldots \end{cases}$$

$$j\frac{3}{8} = \frac{3}{8}e^{j\frac{\pi}{2}} \qquad -j\frac{3}{8} = \frac{3}{8}e^{-j\frac{\pi}{2}}$$

$$+j\frac{1}{8} = \frac{1}{8}e^{j\frac{\pi}{2}}$$

$$-j\frac{1}{8} = \frac{1}{8}e^{-j\frac{\pi}{2}}$$



Useful property for integration

$$v_k(t) = e^{j\left(\frac{2\pi}{T_0}\right)kt} = e^{j\omega_0 kt}$$

$$v_k(t+T_0) = v_k(t)$$

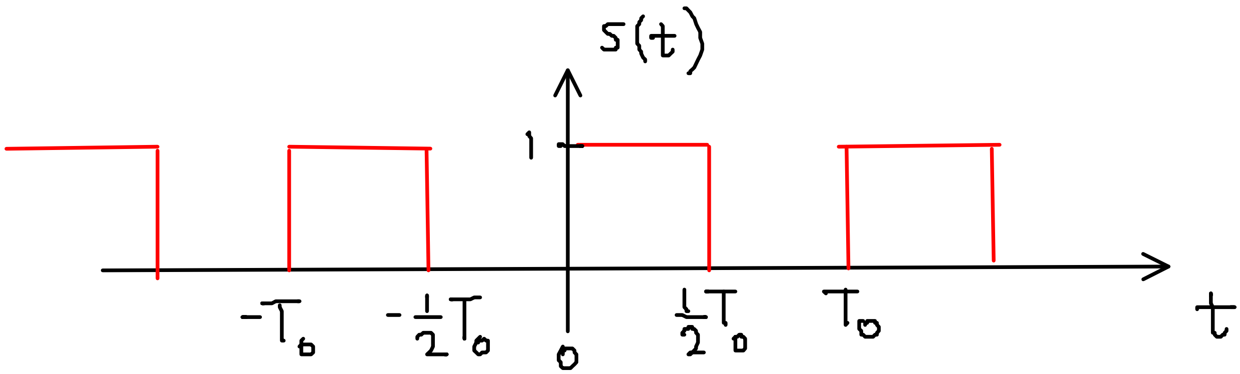
Orthoganility property

$$\boxed{\int_0^{T_0} v_k(t)\, v_l^*(t)\, dt = \begin{cases} 0 & \text{if } k \neq l \\ T_0 & \text{if } k = l \end{cases}} = T_0\,\delta_{kl}$$

$$\delta_{kl} = \begin{cases} 0 & \text{if } k \neq l \\ 1 & \text{if } k = l \end{cases}$$

3-6.1 The Square Wave

$$s(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2}T_0 \\ 0 & \frac{1}{2}T_0 \leq t \leq T_0 \end{cases}$$



$$a_k = \frac{1}{T_0}\int_0^{T_0/2} (1)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\, dt$$

(see derivation page 53)

$$a_k = \frac{1-(-1)^k}{j2\pi k} \quad \text{for } k \neq 0$$

$$a_0 = \frac{1}{T_0}\int_0^{\frac{T_0}{2}} (1)\, dt = \frac{1}{2}$$

$$a_k = \begin{cases} \frac{1}{j\pi k} & k = \pm 1, \pm 3, \ldots \\ 0 & k = \pm 2, \pm 4, \ldots \\ \frac{1}{2} & k = 0 \end{cases}$$

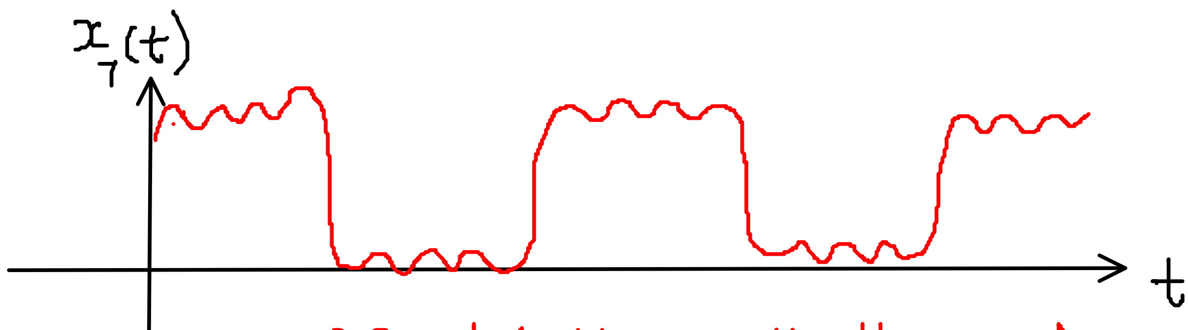
only odd harmonics are present

($s(t)$ is odd)

See plot of spectrum page 3-16, page 54

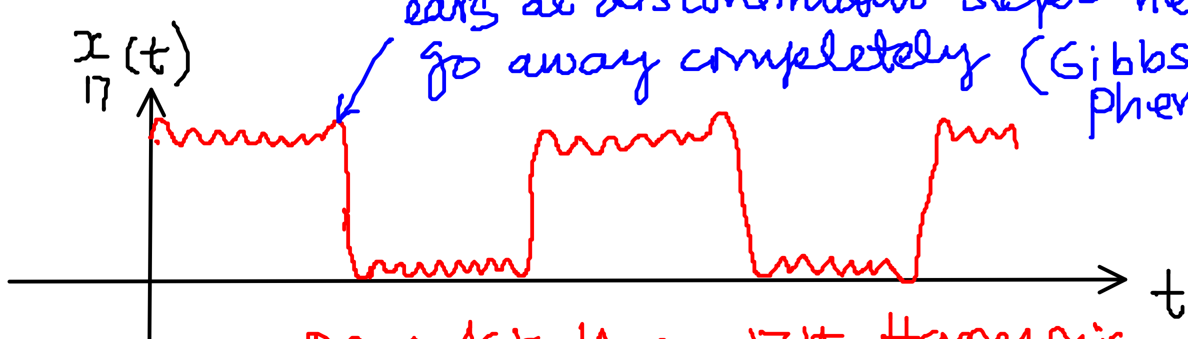
Synthesis of a Square Wave

DC+ 1st and 3rd Harmonic



DC+ 1st thru 7th Harmonic

"ears" at discontinuous steps never go away completely (Gibbs phenomenon)



DC + 1st thru 17th Harmonic

Figure 3-17

3-6.4 Triangular wave

Read Section 3-6.4 in the book