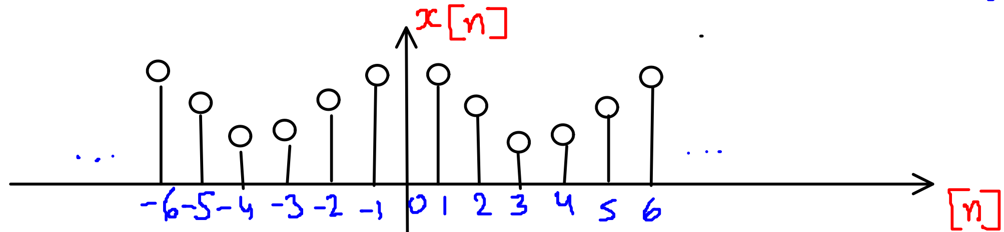
Chapter 4

Sampling and Aliasing

$$x(t) = A\cos(\omega t + \phi)$$

Continuous Signal (Analog signal)

Both value and time are continuous (not discrete)

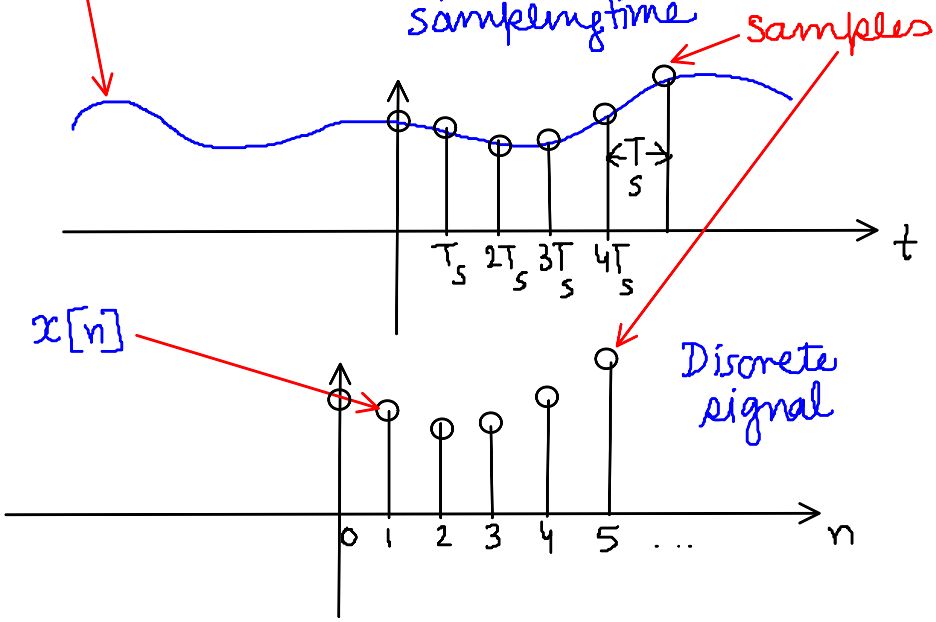


Discrete signal

$x[n]$ $n = 0, \pm 1, \pm 2 \cdots$ (sequence)

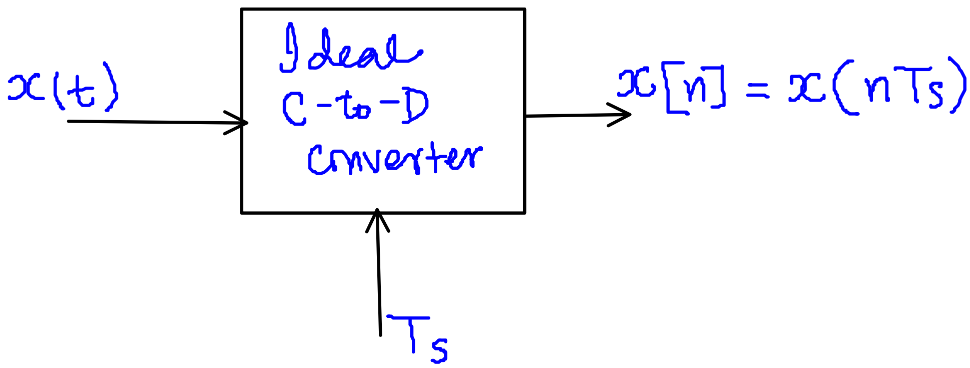
$$x(t) \longrightarrow x(nT_s) = x[n]$$

sampling time



Sampling rate → $f_s = \frac{1}{T_s}$ → sec

samples/sec



Another way to obtain a discrete signal (directly from a formula)

$$x[n] = n^2 - 5n + 3$$

Sampling Sinusoidal Signals

$$x(t) = A\cos(\omega t + \phi)$$

$$x[n] = x(nT_s) = A\cos(\underbrace{\omega T_s}_{\hat{\omega}} n + \phi)$$

$\hat{\omega}$ ← normalized freq.

$$x[n] = A\cos(\hat{\omega} n + \phi)$$ discrete time sinusoid [fig 4-3]

$\hat{\omega}$ → normalized freq. units radians/sample

$$x[n] = A\cos(2\pi \hat{f} n + \phi) = A\cos(\hat{\omega} n + \phi)$$

$$\hat{f} = \frac{\hat{\omega}}{2\pi}$$

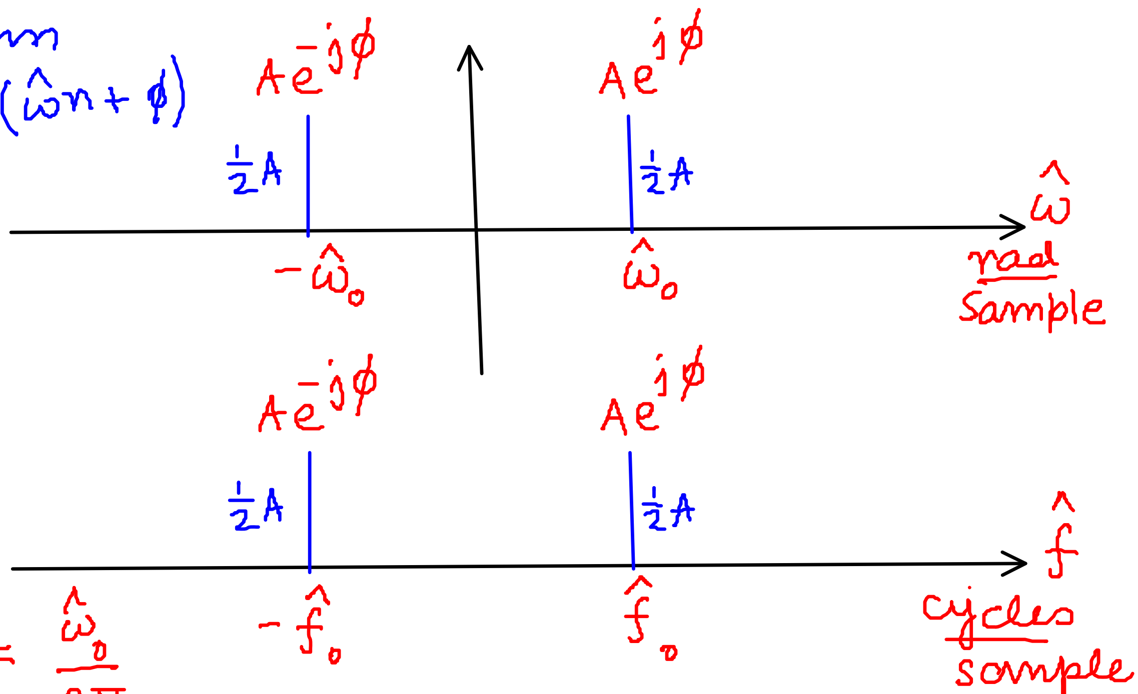
$\hat{\omega}$: rad / sample

$\hat{f}$: cycles/sample

$$A\cos(\hat{\omega}_0 n + \phi) = \frac{1}{2}A e^{j\hat{\omega}_0 n} e^{j\phi} + \frac{1}{2}A e^{-j\hat{\omega}_0 n} e^{-j\phi}$$

$$= \frac{1}{2}A e^{j\phi} e^{j\hat{\omega}_0 n} + \frac{1}{2}A e^{-j\phi} e^{-j\hat{\omega}_0 n}$$

spectrum of $A\cos(\hat{\omega} n + \phi)$



$$\hat{f}_0 = \frac{\hat{\omega}_0}{2\pi}$$

4-1.2 Aliasing

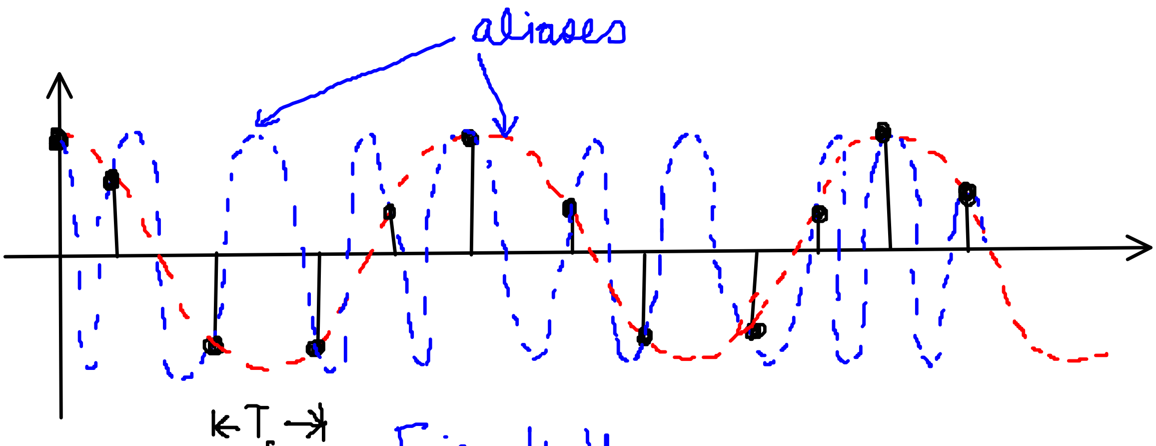


Fig. 4-4

$$A\cos(\omega t+\phi) \rightarrow A\cos(\omega T_s n+\phi)$$

$$A\cos\left[\left(\omega+\frac{2\pi}{T_s}\right)t+\phi\right] \rightarrow A\cos\left(\left(\omega+\frac{2\pi}{T_s}\right)T_s n+\phi\right)$$

$$= A\cos(\omega T_s n+2\pi n+\phi)$$

$$= A\cos(\omega T_s n+\phi)$$

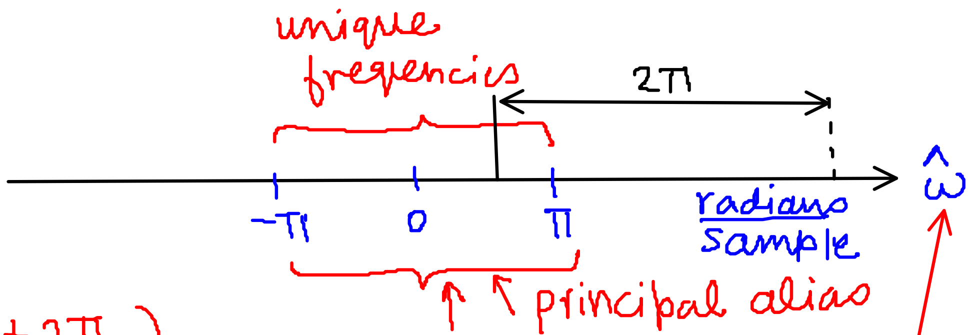
Both these continuous sinusoids produce the same discrete sinusoid when sampled at the rate $\frac{1}{T_s}$

For a given sampling rate there are an infinite number of aliases

$$A\cos\left[\left(\omega+\frac{2m\pi}{T_s}\right)t+\phi\right] \qquad m=0,\pm1,\pm2\ \cdots$$

$$A\cos([\hat{\omega}+2\pi m]n+\phi) = A\cos(\hat{\omega}n+\phi+2\pi m n)$$
$$= A\cos(\hat{\omega}n+\phi)$$

$\hat{\omega}$ and $\hat{\omega}+2\pi m$ give us the same discrete sinusoid!



$$\left.\begin{array}{l}\hat{\omega}+2\pi\\ \hat{\omega}+4\pi\\ \vdots\end{array}\right\}\text{ aliases of }\hat{\omega}$$

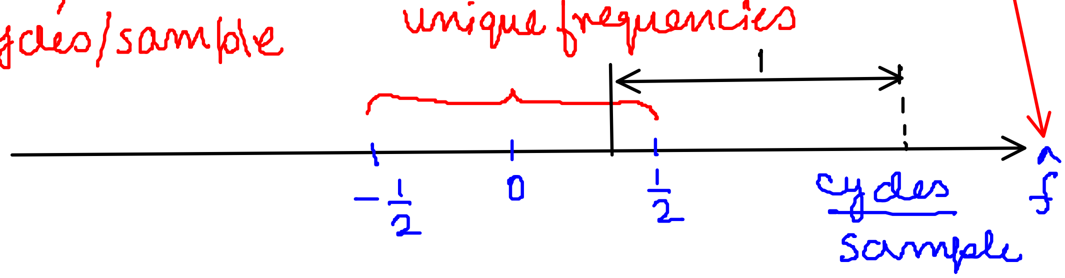
normalized frequency

$$A\cos(\hat{\omega}n+\phi) = A\cos(2\pi\hat{f}n+\phi)$$

$$\hat{f} = \frac{\hat{\omega}}{2\pi}$$

radians/sample

cycles/sample



$$A\cos([\hat{\omega}+2\pi m]n+\phi)$$

$$A\cos([2\pi m-\hat{\omega}]n-\phi) = A\cos(2\pi mn-\hat{\omega}n-\phi)$$

$$= A\cos(-\hat{\omega}n-\phi)$$

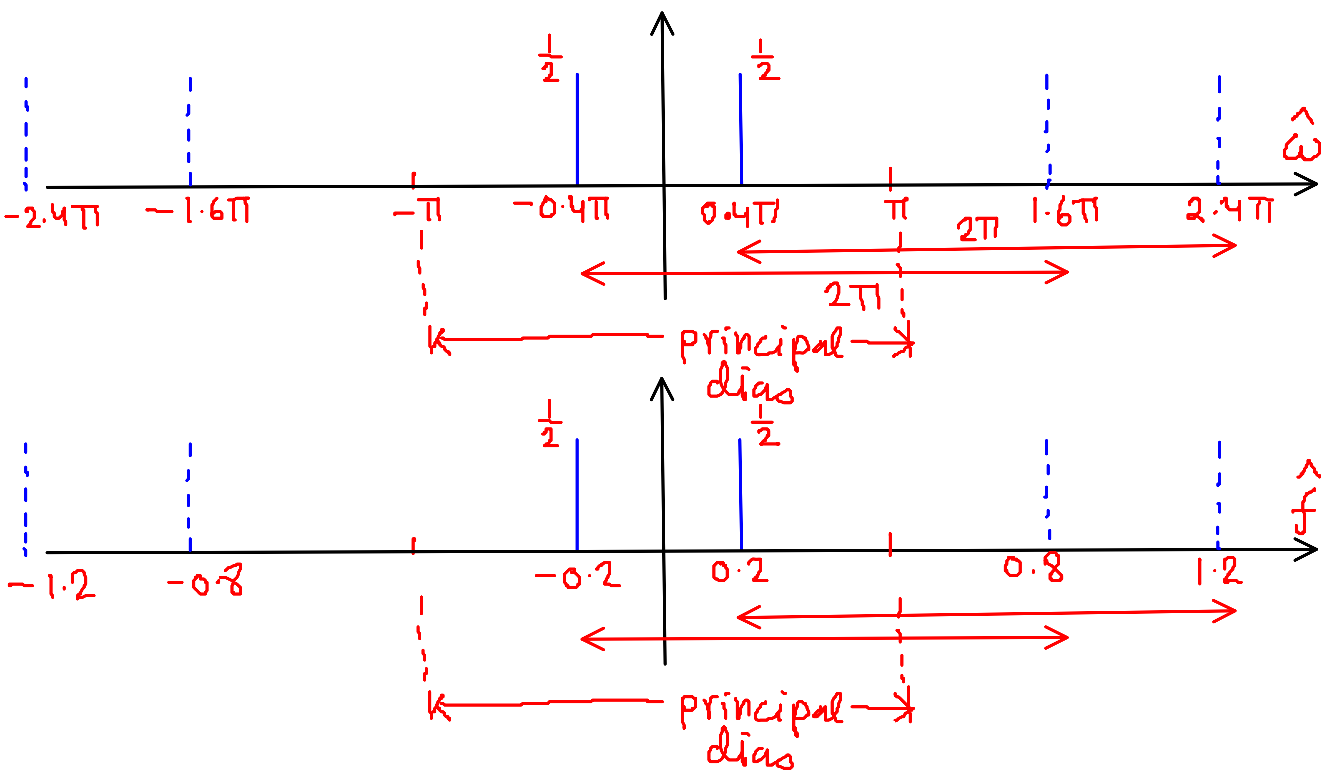
$$= A\cos(\hat{\omega}n+\phi)$$

folded alias

principal alias

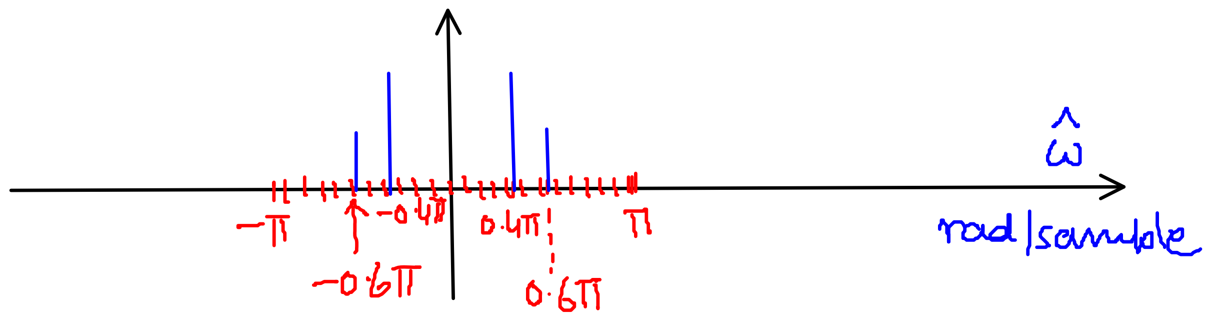
4-1.3 Spectrum of a discrete time signal

Spectrum of discrete signal $\cos(0.4\pi n)$



Examples

1) $x[n] = 2\cos(0.4\pi n) + \cos(0.6\pi n)$



2) $x[n] = 2\cos(0.5\pi n) + 2\cos(1.1\pi n)$