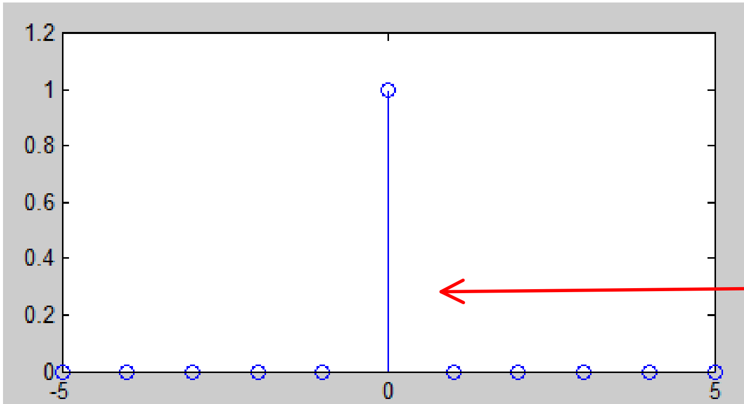
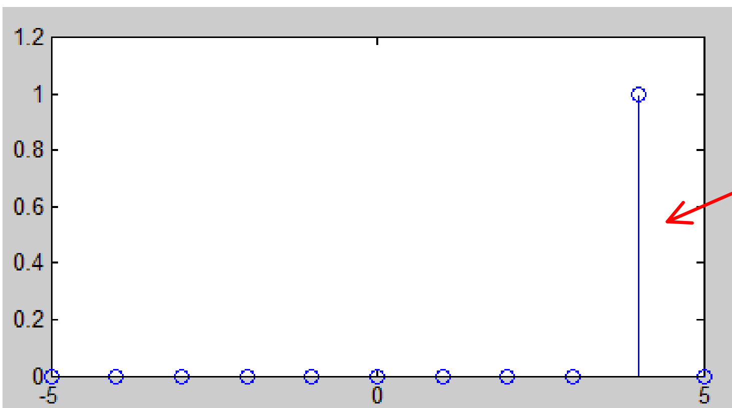
5-3.2 Impulse Response

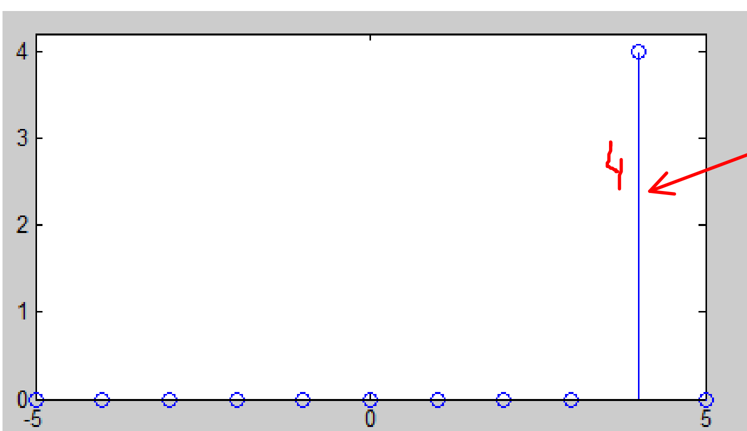


Unit Impulse signal

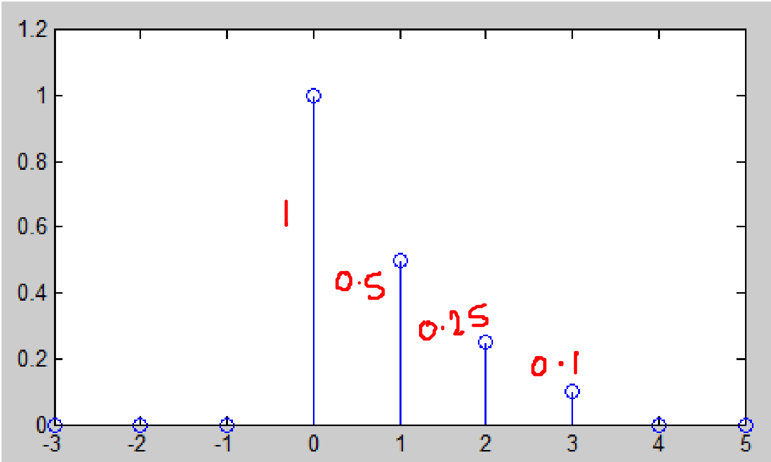
$\delta(n)$



$\delta(n-4)$



$4\delta(n-4)$

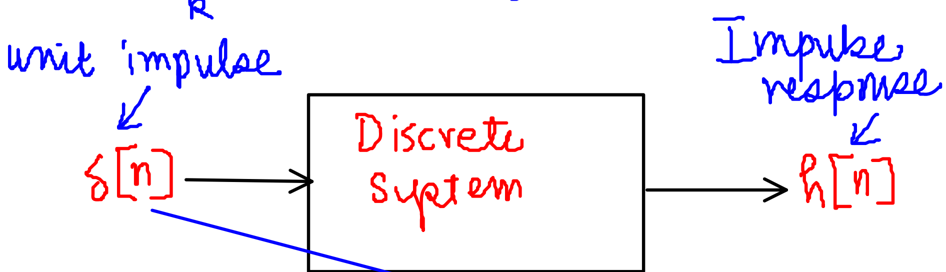


$x[n]$

$$x[n] = 1\delta[n] + 0.5\delta[n-1] + 0.25\,\delta[n-2] + 0.1\delta[n-3]$$

Any sequence can be represented in this way

$$x[n] = \sum_{k} x[k]\,\delta[n-k]$$



$$h[n] = \underbrace{-\sum_{k=1}^{N} a_k h[n-k]}_{\text{recursive}} + \sum_{k=0}^{M} b_k\,\delta[n-k]$$

In general

$$h[n] = \sum_{k=0}^{M} c_k \delta[n-k]$$

Systems can be classified into two catagories

IIR [Infinite Impulse Response] if $M \to \infty$

FIR [Finite Impulse Response] if M is finite

If recursive part is present:

$$h[n] = \underbrace{-\sum_{k=1}^{N} a_k h[n-k]}_{\text{recursive}} + \sum_{k=0}^{M} b_k \delta[n-k]$$

then the system is IIR

If recursive part is absent:

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

then the system is FIR

for a FIR system the difference equation actually gives the impulse response

$$h[n] = b[n]$$

Convolution

convolution

$$y[n] = x_1[n] * x_2[n]$$

$$= \sum_{\ell=-\infty}^{\infty} x_1[\ell]\, x_2[n-\ell]$$

sequence $x_1[n]$ is convolved with the sequence $x_2[n]$ to produce $y[n]$

$$x[n] * \delta[n-n_0] = \sum_{\ell=-\infty}^{\infty} x[\ell]\, \delta[n-\ell-n_0]$$

non zero when $\ell = n - n_0$

$$\boxed{x[n] * \delta[n-n_0] = x[n-n_0]}$$

Properties of convolution:

commutative property $\quad x_1[n] * x_2[n] = x_2[n] * x_1[n]$

Associative Property

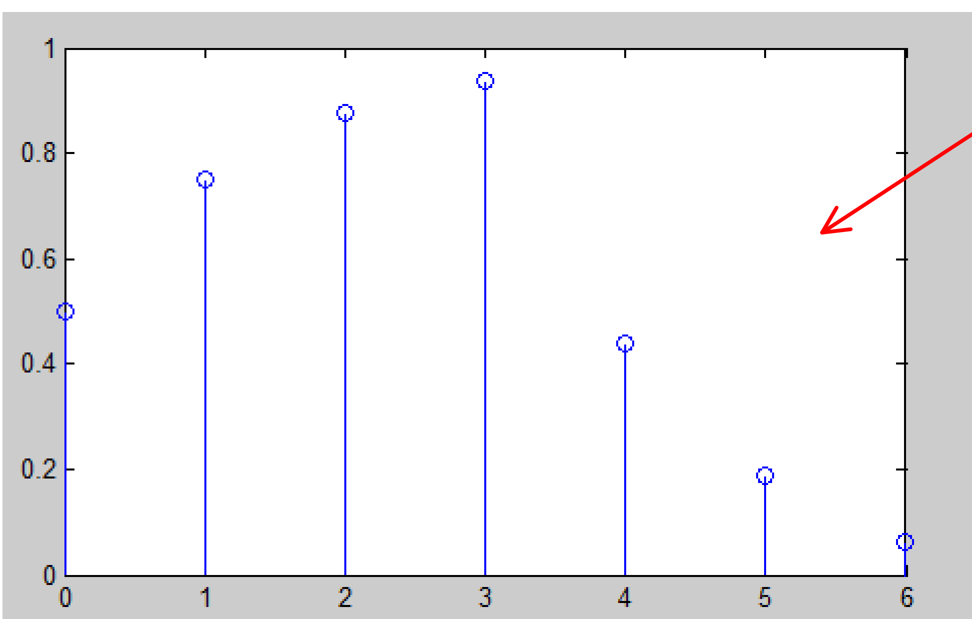
$$(x_1[n] * x_2[n]) * x_3[n]$$
$$= x_1[n] * (x_2[n] * x_3[n])$$

Convolution in MATLAB

$$s1 = [\tfrac{1}{2} \quad \tfrac{1}{4} \quad \tfrac{1}{8} \quad \tfrac{1}{16}]$$
$$s2 = [1 \quad 1 \quad 1 \quad 1]$$

```
c = conv(s1, s2)
```

```
s1 = 0.5000    0.2500    0.1250    0.0625
s2 = 1     1     1     1
c = 0.5000    0.7500    0.8750    0.9375    0.4375    0.1875    0.0625
```



c

Example: Manually compute

$$y[n] = x_1[n] * x_2[n]$$
$$= \sum_{\ell=-\infty}^{\infty} x_1[\ell]\, x_2[n-\ell]$$

$x_1[n] = \cdots 0, 0, 1, 2, 1, -1\ 0\ 0 \cdots$

$n \quad \cdots -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4 \cdots$

$x_2[n] = \cdots 0\ 0\ 1\ 2\ 3\ 1\ 0\ 0 \cdots$

$n \quad \cdots -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5 \cdots$

$$y[n] = \sum_{\ell=-\infty}^{\infty} x_1[\ell]\, x_2[-\underbrace{(\ell-n)}]$$

inversion shifting

$x_1[\ell] = \cdots 0, 0, 1, 2, 1, -1\ 0\ 0 \cdots$

$\ell \quad \cdots -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4 \cdots$

$x_2[\ell] = \cdots 0\ 0\ 1\ 2\ 3\ 1\ 0\ 0 \cdots$

$\ell \quad \cdots -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5 \cdots$

$x_2[-\ell] = 0\ 0\ 1\ 3\ 2\ 1\ 0\ 0$

$\quad -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2$

$$y[n] = \sum_{\ell=-\infty}^{\infty} x_1[\ell]\, x_2[-(\ell-n)]$$

$x_1[\ell] = \cdots 0, 0, 1, 2, 1, -1\ 0\ 0 \cdots$

$\ell \quad \cdots -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4 \cdots$

$x_2[-\ell] = \cdots 0\ 0\ 1\ 3\ 2\ 1\ 0\ 0 \cdots$

$\ell \quad -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2$

$y[-2]$?

$x_2[-(\ell+2)] = \cdots 1\ 3\ 2\ 1\ 0\ 0\ 0\ 0 \cdots$

$-5\ -4\ -3\ -2\ -1\ 0\ 1\ 2$

$y[-2] = 0\times 1 + 0\times 3 + 0\times 2 + 0\times 1 + 1\times 0 + 2\times 0$

$\ell \quad -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2$

$y[-2] = 0$

$y[-1]$?

$x_2[-(\ell+1)] = \cdots 0\ 0\ 1\ 3\ 2\ 1\ 0\ 0 \cdots$

$\ell \cdots -6\ -5\ -4\ -3\ -2\ -1\ 0\ 1 \cdots$

$y[-1] = 0\times 0 + 0\times 1 + 0\times 3 + 0\times 2 + 1\times 1 + 2\times 0$

$-5\ -4\ -3\ -2\ -1\ 0\ 1\ 2$

$y[-1] = 1$

$$y[n] = \sum_{\ell=-\infty}^{\infty} x_1[\ell]\, x_2[-(\ell-n)]$$

$$x_1[\ell] = \cdots 0,\ 0,\ 1,\ 2,\ 1,\ -1\ \ 0\ \ 0\ \cdots$$
$$\ell \quad \cdots -3\ \ -2\ \ -1\ \ \underset{\uparrow}{0}\ \ 1\ \ 2\ \ 3\ \ 4\ \cdots$$

$$x_2[-\ell] = \cdots 0\ \ 0\ \ 1\ \ 3\ \ 2\ \ 1\ \ 0\ \ 0\ \cdots$$
$$\ell \quad -5\ \ -4\ \ -3\ \ -2\ \ -1\ \ \underset{\uparrow}{0}\ \ 1\ \ 2$$

$y[2]$?

$$x_2[-(\ell-2)] = \quad 0\ \ 0\ \ 0\ \ 1\ \ 3\ \ 2\ \ 1\ \ 0$$
$$-4\ \ -3\ \ -2\ \ -1\ \ \underset{\uparrow}{0}\ \ 1\ \ 2$$

$$y[2] = 0\times 0 + 0\times 0 + 0\times 0 + 0\times 0 + 1\times 1 + 2\times 3 + 1\times 2 + (-1)(1)$$
$$\ell \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2$$

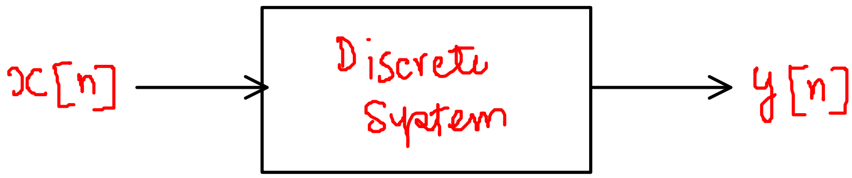
$$y[2] = 0+0+0+0+1+6+2-1 = 8$$

HW evaluate the rest of $y[n]$

Answer

$$y[n] = \cdots 0,\ 0,\ 1,\ 4,\ 8,\ 8,\ 3,\ -2,\ -1,\ 0,\ 0\ \cdots$$
$$\cdots\ -2\ \ -1\ \ \underset{\uparrow}{0}\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \cdots$$

5-5 Linear Time Invariant (LTI) Systems

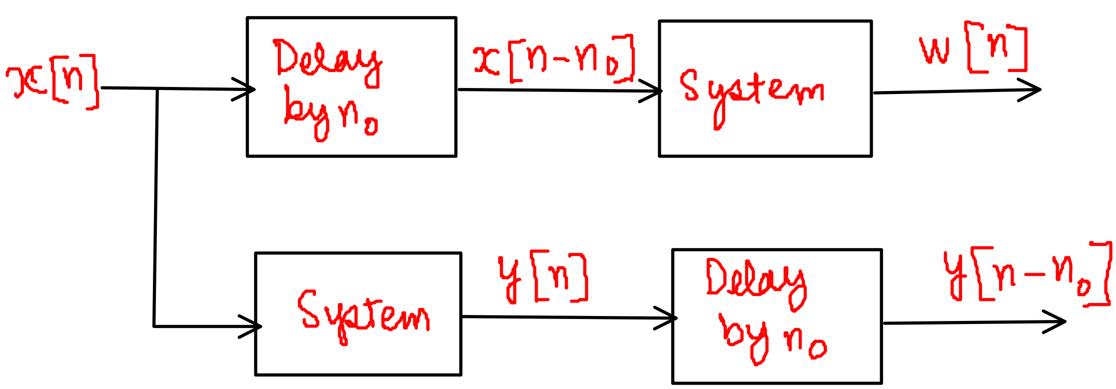


transformation

$$x[n] \mapsto y[n]$$

Time invarience :

$$x[n-n_0] \mapsto y[n-n_0] \quad \text{for any } n_0$$

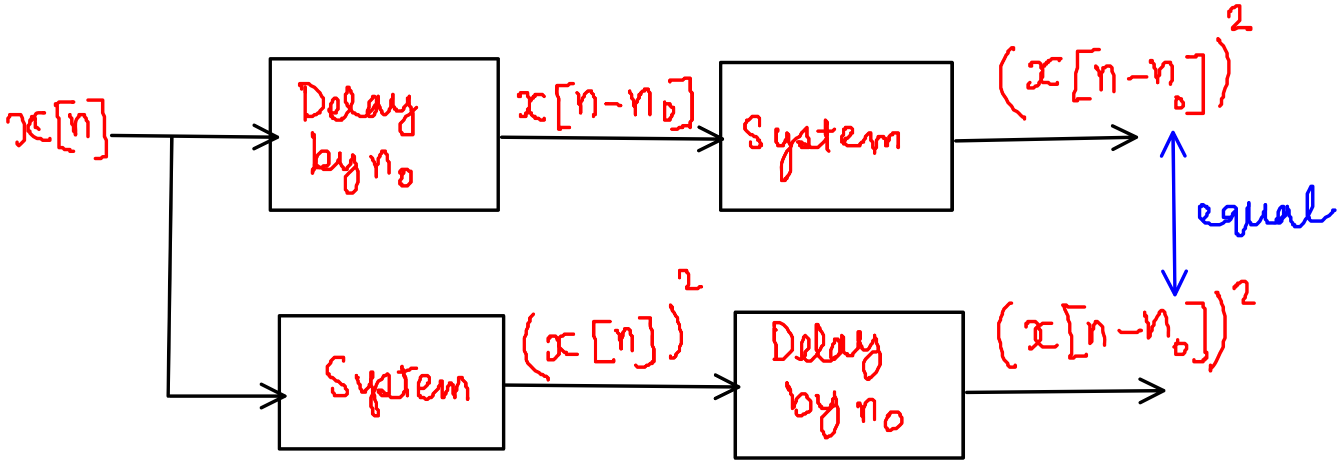


System is TI (time Invarient) if $w[n] = y[n-n_0]$

Example

$$x[n] \mapsto y[n]$$

$$y[n] = (x[n])^2$$

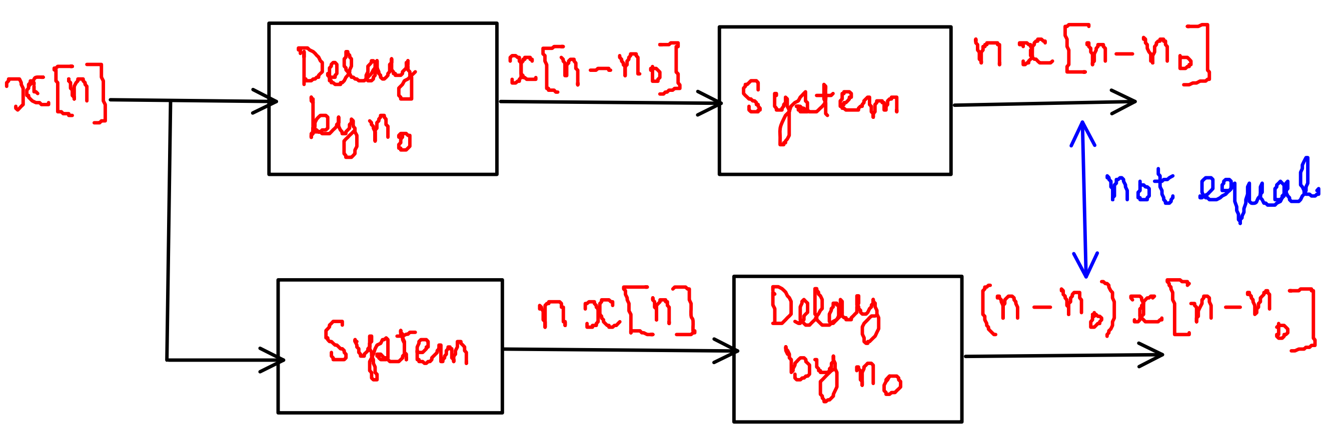


System is TI

Example

$$x[n] \mapsto y[n]$$

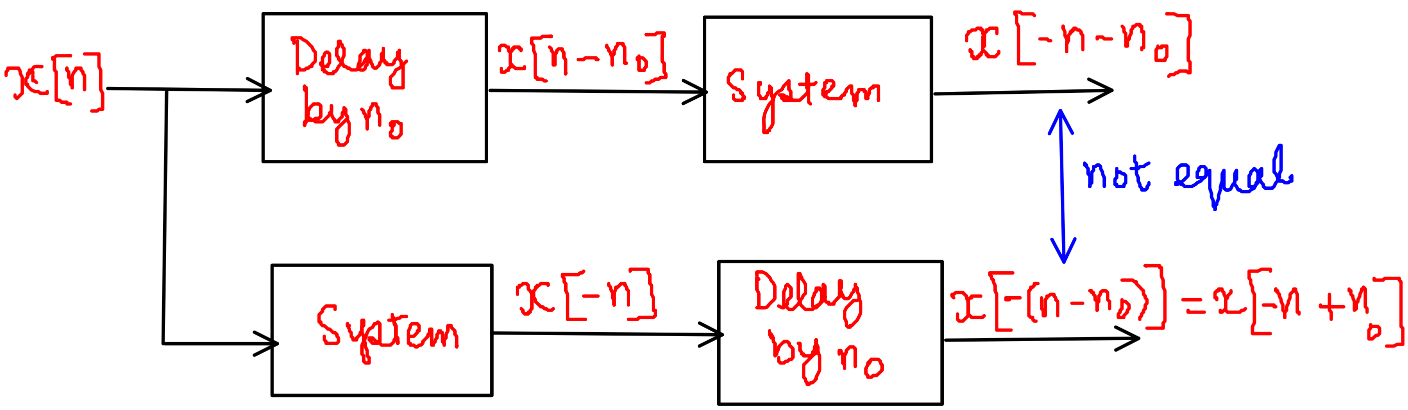
$$y[n] = n\,x[n]$$



System is not TI

Example $x[n] \mapsto y[n]$

$$y[n] = x[-n]$$



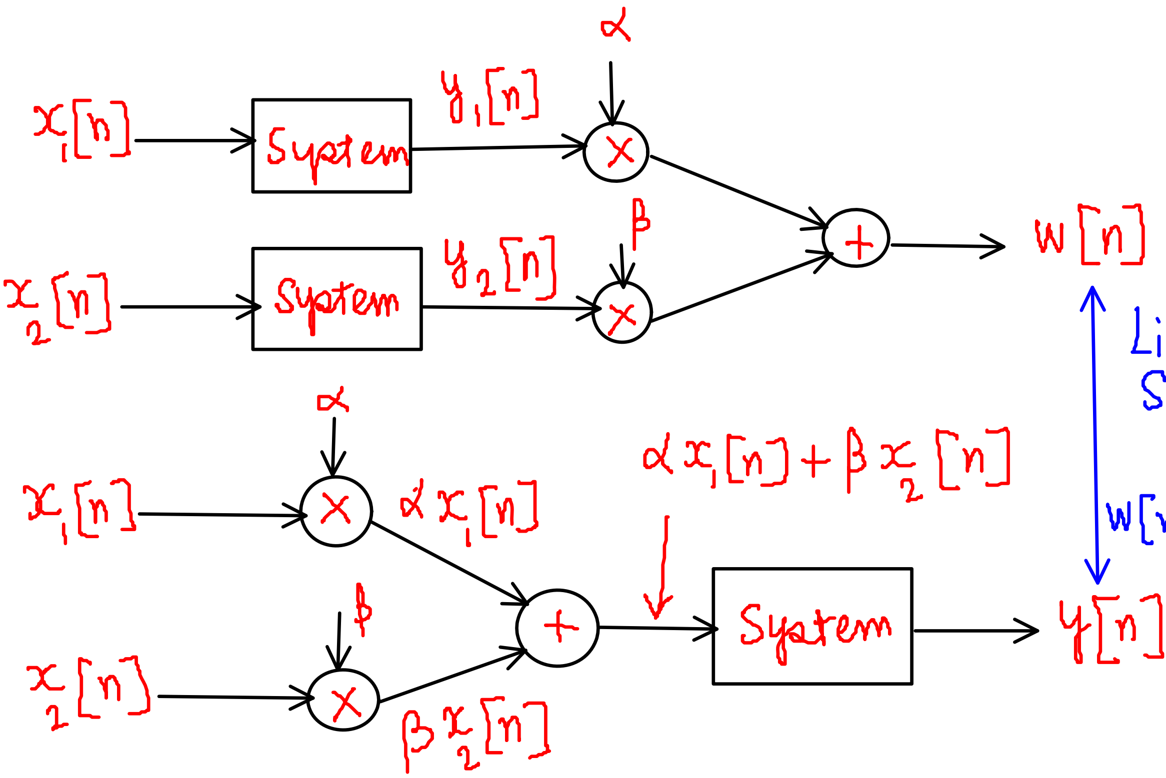
System is not TI

5-5.2 Linearity

$$x_1[n] \mapsto y_1[n]$$

$$x_2[n] \mapsto y_2[n]$$

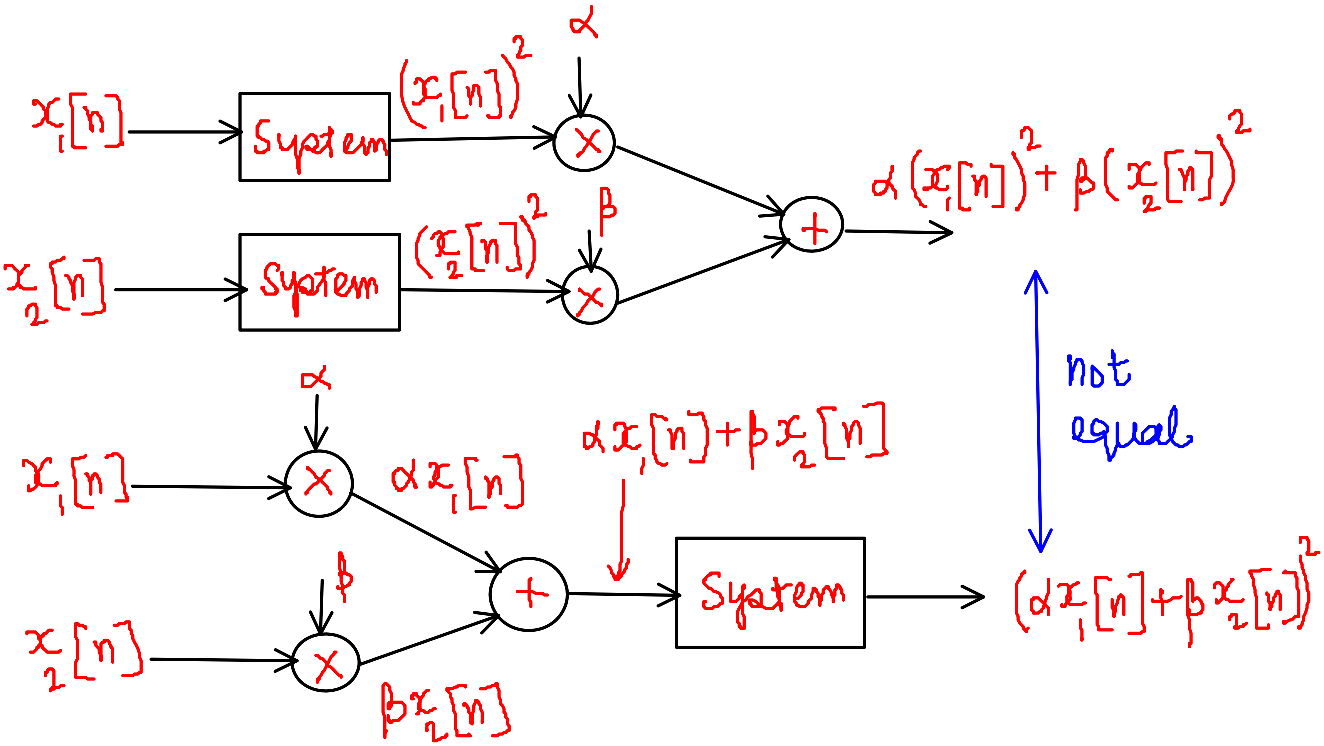
$$\alpha x_1[n] + \beta x_2[n] \mapsto \alpha y_1[n] + \beta y_2[n]$$



Linear System if $w[n] = y[n]$

Example

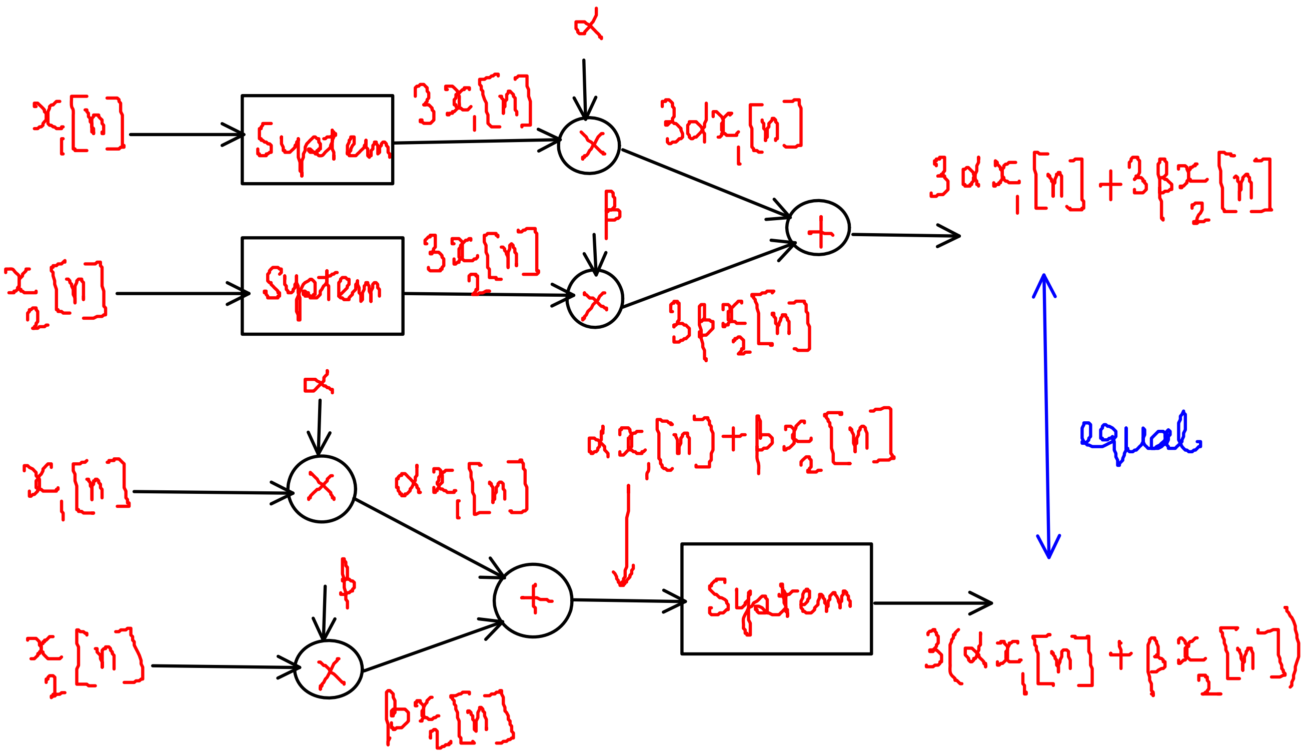
$$x[n] \mapsto y[n] = \left(x[n]\right)^2$$



System is not Linear

Example

$$x[n] \mapsto y[n] = 3x[n]$$



System is Linear

$$\boxed{\text{An LTI system is Linear and Time Invariant}}$$

$$x[n] \longrightarrow \boxed{\text{Discrete LTI System}} \longrightarrow y[n]$$

$$\boxed{\text{An LTI system is Linear and Time Invarient}}$$

A causal LTI system is described by a linear constant coefficient difference equation

$$y[n] = \underbrace{-\sum_{k=1}^{N} a_k\, y[n-k]}_{\text{recursive}} + \sum_{k=0}^{M} b_k\, x[n-k]$$

A system is causal if the output of the system at any time n [i.e y[n]] depends only on present and past inputs [x[n], x[n-1], x[n-2] ...]