$\boxed{\text{Recall from Lecture 35}}$

$$x[n] \longrightarrow \boxed{\text{FIR}} \longrightarrow y[n]$$

Difference Equation

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

To evaluate impulse response $x[n] \rightarrow \delta[n]$

$$\delta[n] \longrightarrow \boxed{\text{FIR}} \longrightarrow h[n]$$

Difference equation

$$h[n] = \sum_{k=0}^{M} b_k \, \delta[n-k]$$

impulse response

coefficients of the difference equation

$$h[n] = \{ \underset{\uparrow \atop 0}{b_0} \; b_1 \; b_2 \; b_3 \; \cdots \; b_M \}$$

$$h[n] = b[n]$$

Example

An FIR filter is described by the difference equation

$$y[n] = 6x[n] - 5x[n-1] + x[n-2]$$

Determine the system function $H(z)$

$$h[n] = \{ \underset{\uparrow}{6} \;\; -5 \;\; 1 \;\; 0 \;\; 0 \;\; \cdots \}$$

$$= 6\delta[n] - 5\delta[n-1] + \delta[n-2]$$

$$H(z) = 6 - 5z^{-1} + z^{-2}$$

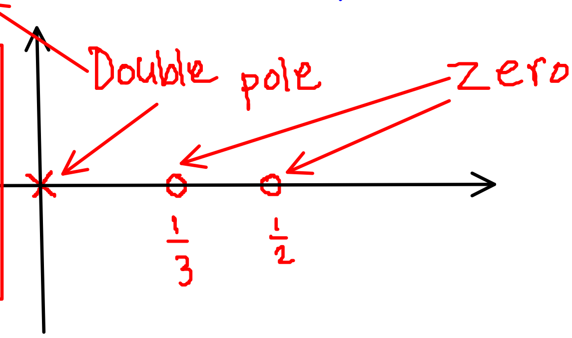
$z \neq 0$
ROC

Zeros and poles of $H(z)$?

$$H(z) = 6 - \frac{5}{z} + \frac{1}{z^2} = \frac{1}{z^2}\left(6z^2 - 5z + 1\right)$$

$$H(z) = \frac{6}{z^2}\left(z - \frac{1}{3}\right)\left(z - \frac{1}{2}\right)$$

z_0 is a pole if $H(z_0) \to \infty$
z_0 is a zero if $H(z_0) \to 0$



Properties of z-transforms

Superposition property

$$a x_1[n] + b x_2[n] \leftrightarrow a X_1(z) + b X_2(z)$$

Time Delay property

$$\underset{\text{causal}}{x[n]} \longleftrightarrow \sum_{k=0}^{\infty} x[k] z^{-k}$$

$$x[n-1] \longleftrightarrow \sum_{k=0}^{\infty} x[k-1] z^{-k} \qquad k-1 = k'$$

$$= \sum_{k'=-1}^{\infty} x[k'] z^{-(k'+1)}$$

$$= \underbrace{x[-1]}_{\substack{0 \\ \text{(causal)}}} + \sum_{k'=0}^{\infty} x[k'] z^{-k'} z^{-1}$$

$$x[n-1] \longleftrightarrow z^{-1} \underbrace{\sum_{k=0}^{\infty} x[k] z^{-k}}_{X(z)}$$

$$x[n-1] \longleftrightarrow z^{-1} X(z)$$

$$x[n-1] \longleftrightarrow z^{-1}X(z)$$

unity delay in n-domain

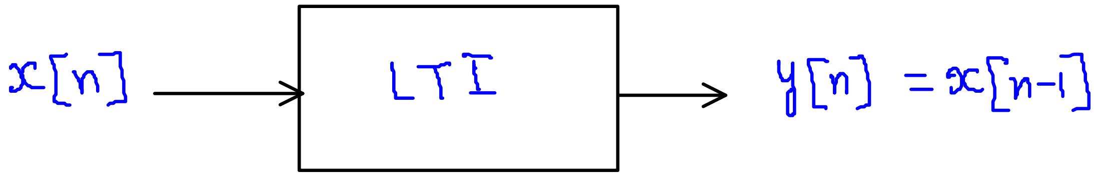
unity delay in z-domain

In general

$$\boxed{x[n-n_0] \longleftrightarrow z^{-n_0}X(z)}$$

delay by n_0 samples

$n_0 > 0$



what is the system function?

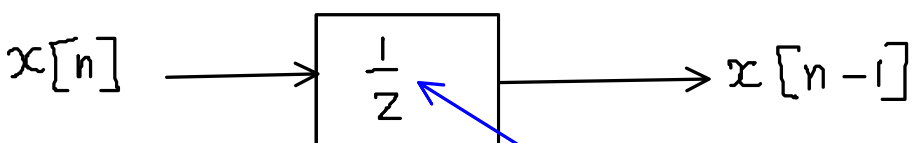
diff. equation

$$y[n] = x[n-1]$$

$$h[n] = \{\underset{\uparrow}{0}\ 1\ 0\ 0\ \cdots\}$$

$$H(z) = 0 \cdot (z^{-1})^0 + 1 \cdot (z^{-1})^1 + 0 \cdot (z^{-1})^2 + \cdots$$

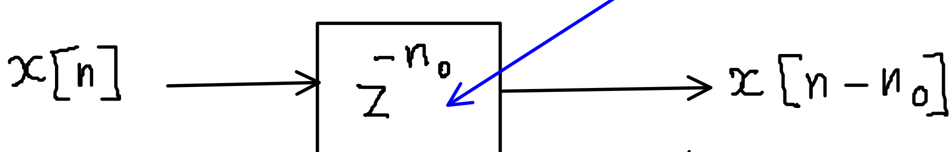
$$H(z) = z^{-1}$$



Unit Delay

In general

system function



$n_0 > 0$

n_0 sample delay

Convolution property

$x_1[n]$
$x_2[n]$
causal

$$y[n] = x_1[n] * x_2[n]$$

$$= \sum_{k=0}^{\infty} x_1[k]\, x_2[n-k]$$

$$Y(z) = \sum_{n=0}^{\infty} \left[\sum_{k=0}^{\infty} x_1[k]\, x_2[n-k] \right] z^{-n}$$

$$= \sum_{k=0}^{\infty} x_1[k] \underbrace{\sum_{n=0}^{\infty} x_2[n-k]\, z^{-n}}_{\text{z transform of } x_2[n] \text{ delayed by } k \text{ samples}}$$

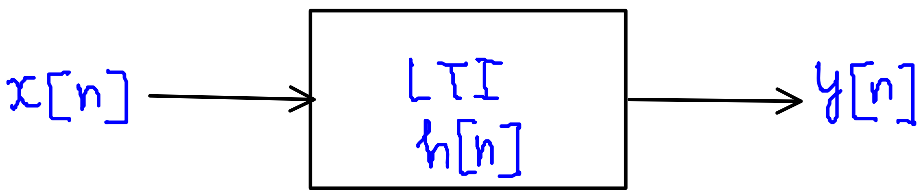
$$= \sum_{k=0}^{\infty} x_1[k]\, \underbrace{z^{-k} X_2(z)}$$

$$= X_2(z) \underbrace{\sum_{k=0}^{\infty} x_1[k]\, z^{-k}}_{X_1(z)} \quad \text{z transform of } x_1[n]$$

n-domain $\quad Y(z) = X_1(z)\, X_2(z) \quad$ Z-domain

$$\boxed{x_1[n] * x_2[n] \longleftrightarrow X_1(z)\, X_2(z)}$$

$\sum_k x_1[k]\, x_2[n-k]$ $\qquad$ simple product



$$y[n] = h[n] * x[n] = \sum_{k=0}^{\infty} h[k]\,x[n-k]$$

n - domain

convolution sum

$$Y(z) = H(z)X(z)$$

Simple multiplication

$$Y(z) = H(z)X(z)$$

z - domain

Convolution via z-transform

$$x[n] = \delta[n-1] - \delta[n-2] + \delta[n-3] - \delta[n-4]$$

$$= \{ \underset{\substack{\uparrow \\ 0}}{0} \quad 1 \quad -1 \quad 1 \quad -1 \quad 0 \quad 0 \}$$

$$y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$$

$$= \{ \underset{\substack{\uparrow \\ 0}}{1} \quad 2 \quad 3 \quad 4 \quad 0 \quad 0 \}$$

Use z-transform to convolve $x[n]$ & $y[n]$

$$x[n] * y[n] \longleftrightarrow X(z)Y(z)$$

$$X(z) = z^{-1} - z^{-2} + z^{-3} - z^{-4}$$

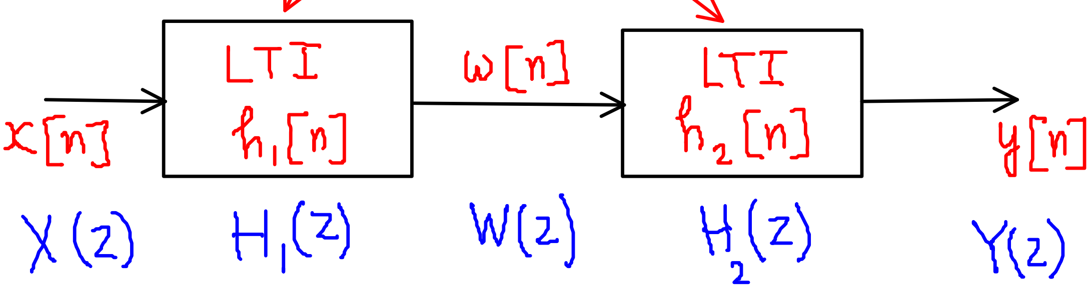
$$Y(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

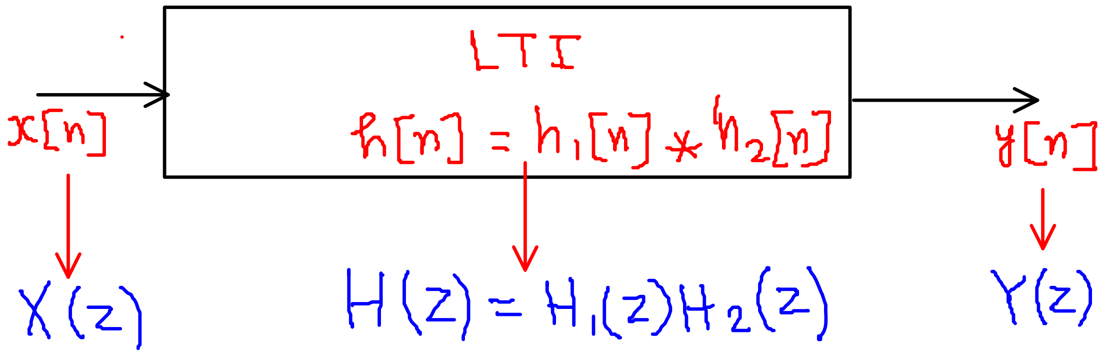
$$X(z)Y(z) = z^{-1} + z^{-2} + 2z^{-3} + 2z^{-4} - 3z^{-5} + z^{-6} - 4z^{-7}$$

$\downarrow$ inverse z-transform

$$x[n] * y[n] = \delta[n-1] + \delta[n-2] + 2\delta[n-3] + 2\delta[n-4]$$
$$- 3\delta[n-5] + \delta[n-6] - 4\delta[n-7]$$

Cascading systems





Example

$$w[n] = 3x[n] - x[n-1]$$

$$Y[n] = 2w[n] - w[n-1]$$

$$H(z) = \frac{Y(z)}{X(z)} \quad ?$$

$$H_1(z) = 3 - z^{-1} \qquad H_2(z) = 2 - z^{-1}$$

$$H(z) = (3 - z^{-1})(2 - z^{-1}) = 6 - 5z^{-1} + z^{-2}$$

$$h[n] = 6\delta[n-0] - 5\delta[n-1] + \delta[n-2]$$

$$\cdot\ h[n] = \underbrace{6\delta[n-0] - 5\delta[n-1] + \delta[n-2]}_{\text{finite size } \therefore \text{ FIR}}$$

Difference equation of FIR system

$$\downarrow$$

$$y[n] = 6x[n] - 5x[n-1] + x[n-2]$$