Example

Design a one pole High Pass (HP) digital filter.

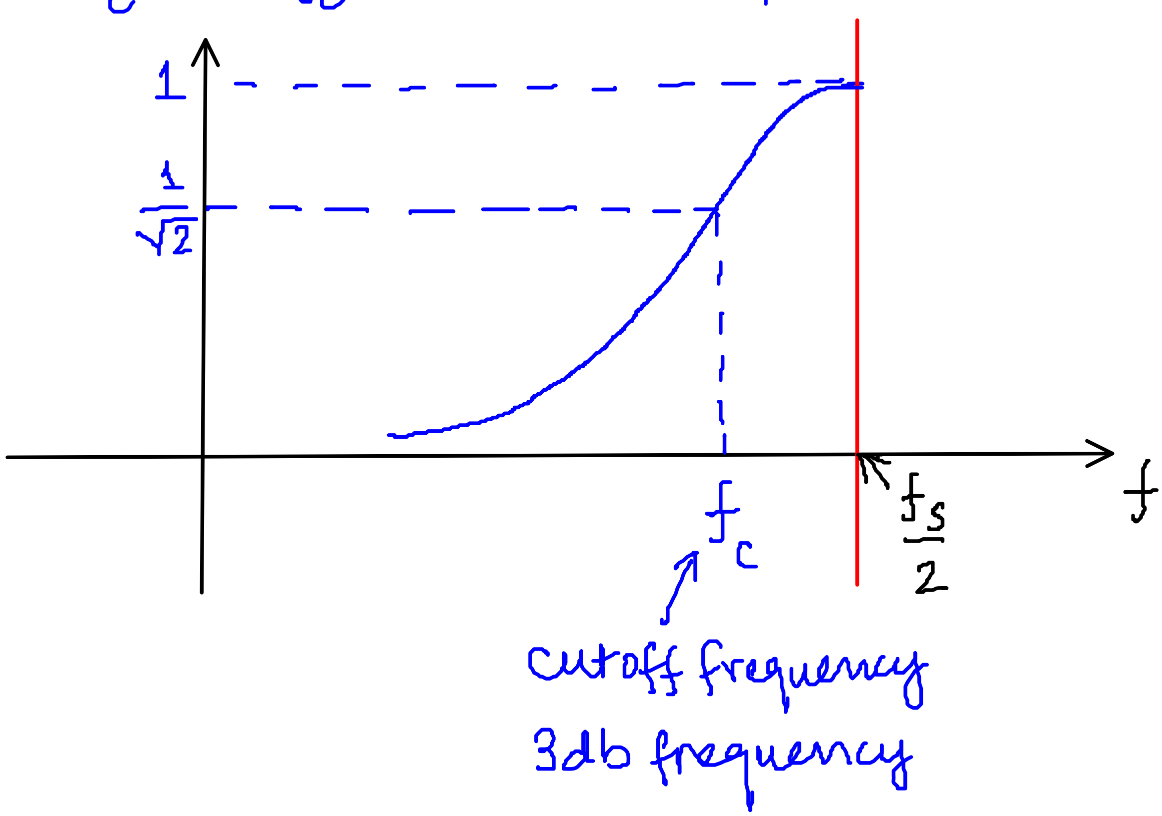
Cutoff frequency $f_c = 20\ \text{KHz}$

Sampling rate used by the ADC and DAC $f_s = 46.675\ \text{KHz}$

$$\left| H(f) \right|_{f = \frac{f_s}{2}} = 1$$

$$\hat{f}_c = \frac{f_c}{f_s} = 0.428 \text{ cycles/sample}$$

$$\hat{\omega}_c = 2\pi \hat{f}_c = 2.69 \text{ radians/sample}$$



$$H(z) = \frac{K}{1 + a z^{-1}} \qquad K, a > 0$$

pole at $z = -a$

$$H(\hat{\omega}) = H(e^{j\hat{\omega}}) = \frac{K}{1 + a e^{-j\hat{\omega}}}$$

$$|H(\hat{\omega})| = \frac{K}{|1 + a\cos\hat{\omega} - ja\sin\hat{\omega}|}$$

$$|H(\hat{\omega})| = \frac{K}{\sqrt{(1 + a\cos\hat{\omega})^2 + (a\sin\hat{\omega})^2}}$$

$$|H(\hat{\omega})| = \frac{K}{\sqrt{1 + 2a\cos\hat{\omega} + a^2}}$$

$$\underbrace{|H(f)|_{f=\frac{f_s}{2}}}_{\text{continuous time freq. response}} = \underbrace{|H(\hat{\omega})|_{\hat{\omega}=\pi}}_{\text{discrete time freq. response}} = 1$$

$$\frac{K}{\sqrt{1 - 2a + a^2}} = \frac{K}{1-a} = 1$$

$$\Rightarrow \boxed{K = 1 - a}$$

Cornerfrequency $\hat{\omega}_c$

$$\frac{K}{\sqrt{1 + 2a\cos\hat{\omega}_c + a^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{K}{\sqrt{1 + 2a\cos\hat{\omega}_c + a^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{K^2}{1 + 2a\cos\hat{\omega}_c + a^2} = \frac{1}{2}$$

$$1 + 2a\cos\hat{\omega}_c + a^2 = 2K^2 \qquad K = 1 - a$$

$$1 + 2a\cos\hat{\omega}_c + a^2 = 2(1-a)^2$$

$$1 + 2a\cos\hat{\omega}_c + a^2 = 2(1 - 2a + a^2)$$

$$a^2 - a2[2 + \cos\hat{\omega}_c] + 1 = 0$$

$$a = [2 + \cos\hat{\omega}_o] \pm \sqrt{[2 + \cos\hat{\omega}_o]^2 - 1}$$

choose − sign for $a < 1$

$$\boxed{a = [2 + \cos\hat{\omega}_c] - \sqrt{[2 + \cos\hat{\omega}_c]^2 - 1}}$$

$$K = (1 - a)$$

$$H(z) = \frac{K}{1 + az^{-1}}$$

```
num =[K 0];
den=[1 a];

 zplane(num,den);
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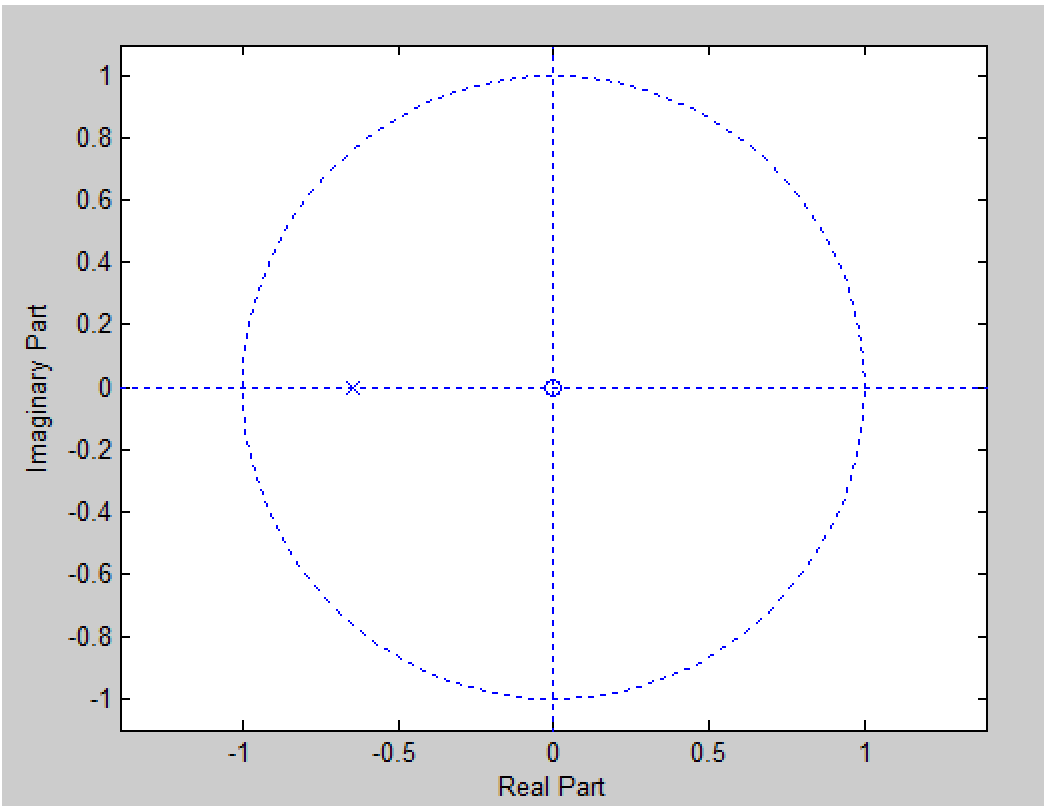
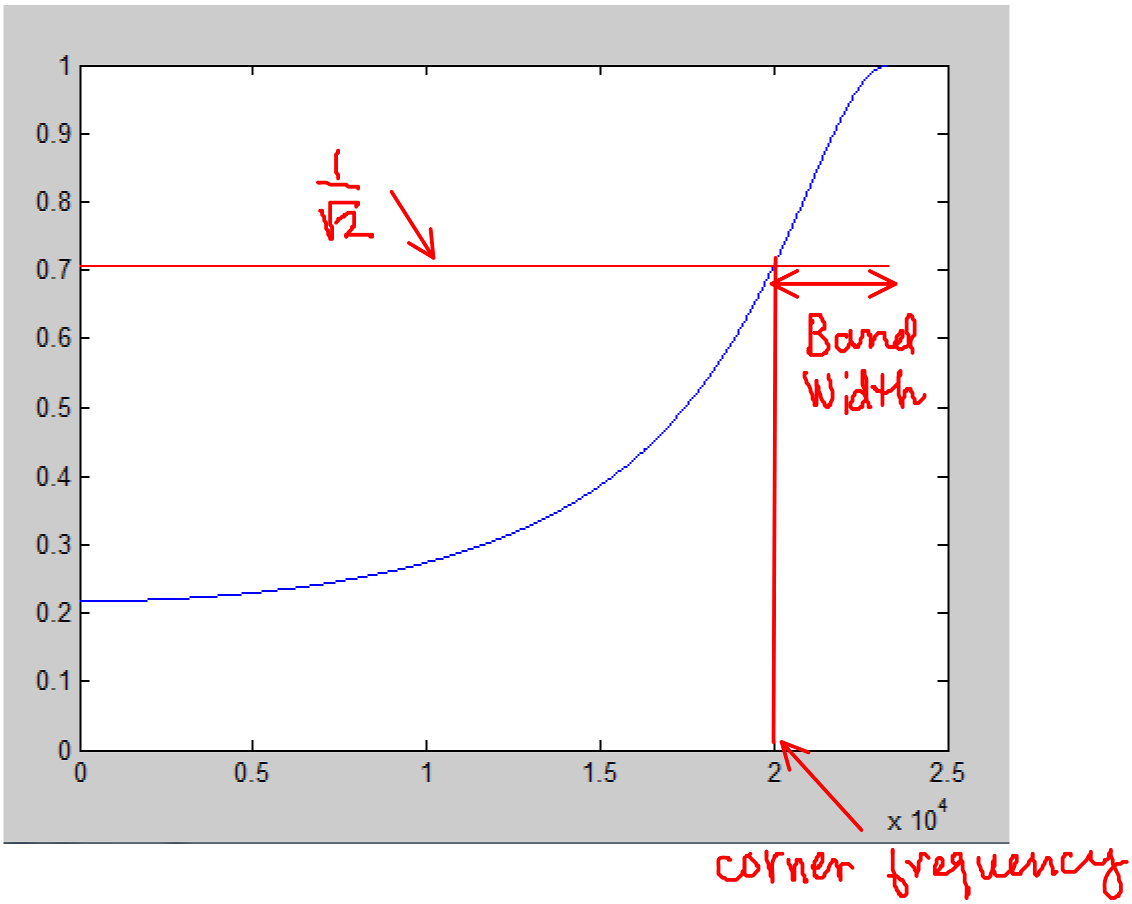


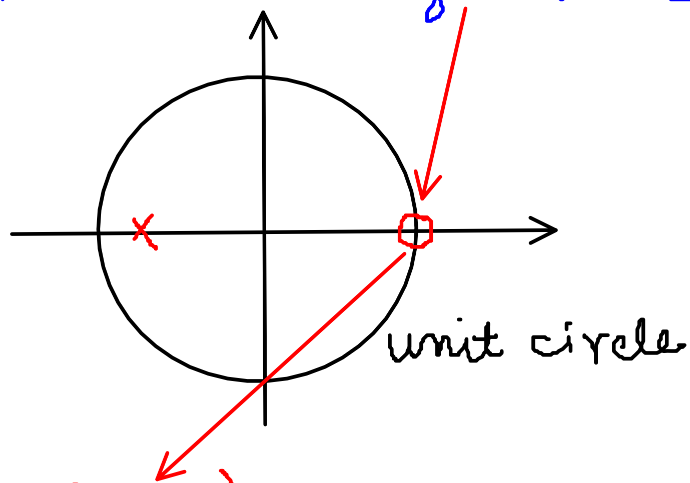
pole
zero
plot

```
h3 = [1/sqrt(2) 1/sqrt(2)];
f3 = [0 fs/2];

[h,f]=freqz(num,den,1001,fs);
plot(f,abs(h), f3, h3, 'r');
axis([0 2.5e4 0 1]);
```

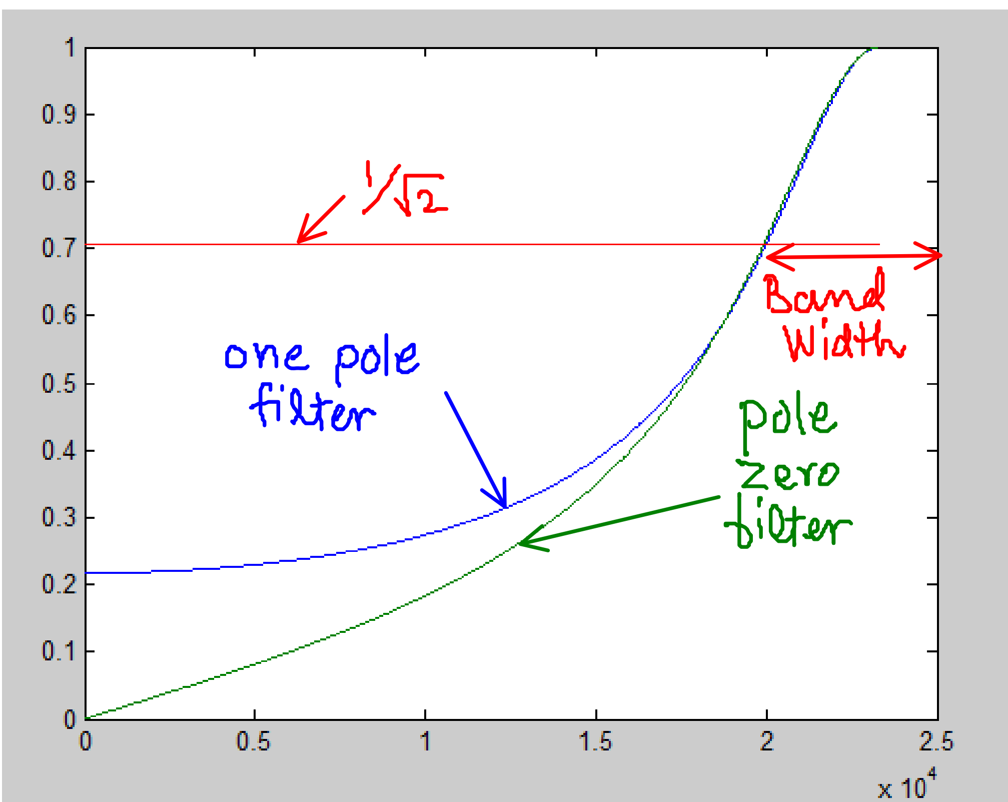


To improve the tail of the frequency response let us add a zero at $z = 1$

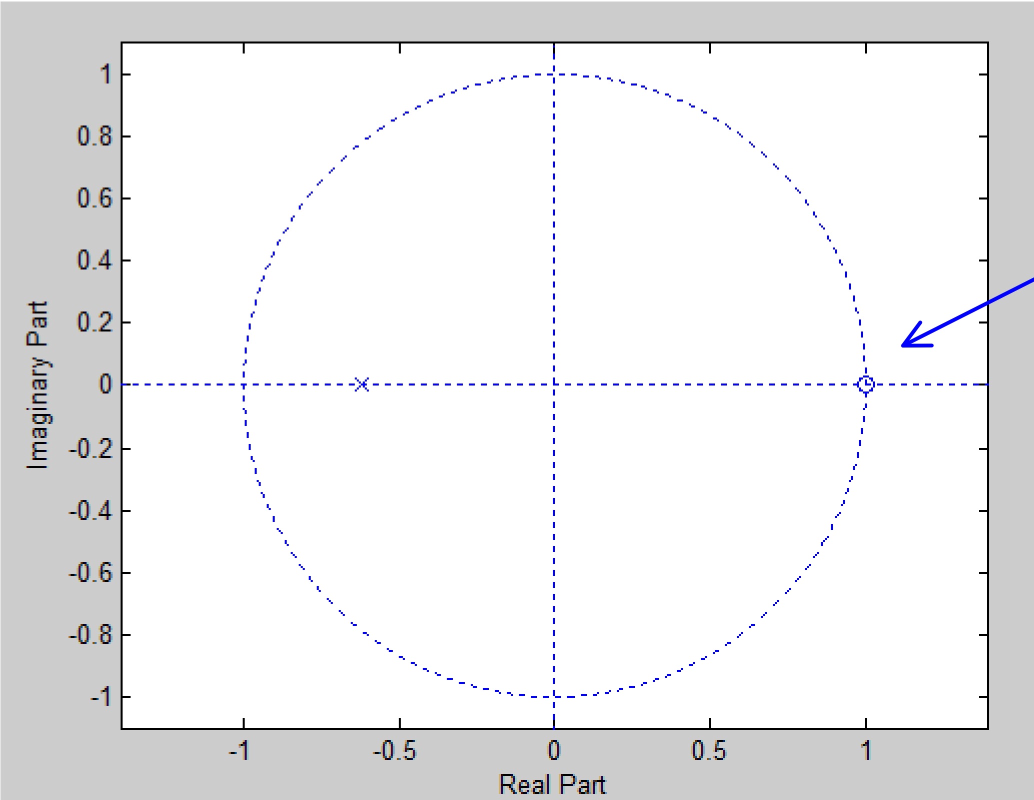


$$H(z) = \frac{K(1 - z^{-1})}{1 + a z^{-1}}$$

slightly tweak the pole $a = 0.966 \times$ old pole



Both filters have the same cutoff frequency



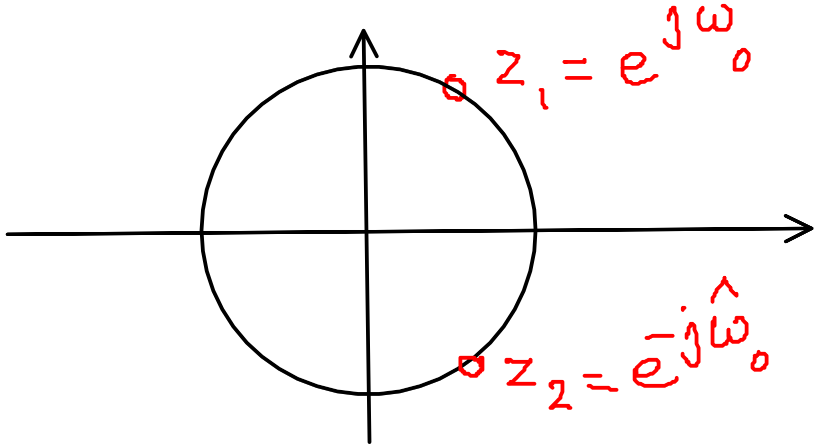
pole zero plot

Example

Design a digital nulling filter that will null the 60Hz frequency. The ADC & DAC sample at the rate $f_s = 46.675\ \text{kHz}$

$\downarrow$ digital land

$$\hat{\omega}_0 = 2\pi \hat{f}_0 = 2\pi \frac{\overset{60\text{Hz}}{f_0}}{f_s} = 0.0081 \text{ rad/samp}$$



$$H(z) = (1 - z_1 z^{-1})(1 - z_2 z^{-1})$$

$$H(z) = (1 - e^{j\hat{\omega}_0} z^{-1})(1 - e^{-j\hat{\omega}_0} z^{-1})$$

$$= 1 - e^{-j\hat{\omega}_0} z^{-1} - e^{j\hat{\omega}_0} z^{-1} + z^{-2}$$

$$H(z) = 1 - 2\cos\hat{\omega}_0\, z^{-1} + z^{-2}$$

```
w0_hat =  0.0081;

num = [1 -2*cos(w0_hat) 1];
den = [1 0 0];
zplane(num, den)
```