

Problem 8.5

determine trivial poles & zeros

$$y[n] = \frac{1}{2}y[n-1] - \frac{1}{3}y[n-2] - x[n]$$

$$H(z) = \frac{-1}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}}$$

$$0 : z^{-1} \rightarrow \infty$$

$$H(z) \rightarrow \frac{1}{(\infty)^2} \rightarrow (0)^2 \text{ double zero}$$

$$\infty : z^{-1} \rightarrow 0 \quad H(z) \rightarrow \frac{-1}{1} \text{ no pole or zero at } 0$$

$$y[n] = \frac{1}{2}y[n-1] - \frac{1}{3}y[n-2] - x[n-2]$$

$$H(z) = \frac{-z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{1}{3}z^{-2}}$$

$$0 : z^{-1} \rightarrow \infty \quad H(z) \rightarrow \frac{\infty^2}{\infty^2} \rightarrow \text{constant} \quad \text{no pole or zero at } 0$$

$$\infty : z^{-1} \rightarrow 0 \quad H(z) = \frac{-(0)^2}{1} \rightarrow (0)^2 \text{ double zero at } \infty$$

$$y[n] = \frac{1}{2}y[n-1] - \frac{1}{3}y[n-2] - x[n-4]$$

$$H(z) = \frac{-z^{-4}}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}}$$

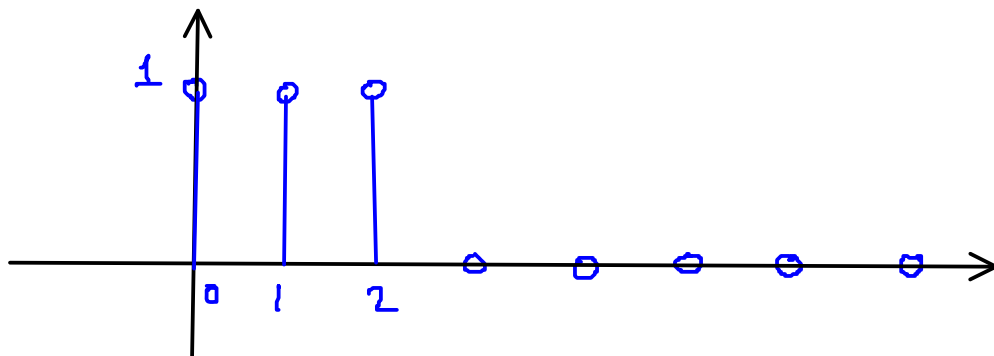
$$0: z^{-1} \rightarrow \infty \quad H(z) \rightarrow \frac{(\infty)^4}{(\infty)^2} \rightarrow \infty^2 \quad \text{Double pole at } 0$$

$$\infty: z^{-1} \rightarrow 0 \quad H(z) \rightarrow \frac{(0)^4}{1} \quad \text{4th order zero at } \infty$$

Problem 8.6

$$y[n] = -\frac{1}{2}y[n-1] + x[n]$$

$$H(z) = \frac{1}{1 + \frac{1}{2}z^{-1}}$$



$$h[n] = \left(-\frac{1}{2}\right)^n u[n] \quad \text{table lookup}$$

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

input	$\delta[n]$	output	$\left(-\frac{1}{2}\right)^n u[n]$
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	$\delta[n-1]$	output	$\left(-\frac{1}{2}\right)^{n-1} u[n-1]$
--	---------------	--------	--

	$\delta[n-2]$	output	$\left(-\frac{1}{2}\right)^{n-2} u[n-2]$
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input	$\delta[n] + \delta[n-1] + \delta[n-2]$	use superposition
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output	$\left(-\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^{n-1} u[n-1] + \left(-\frac{1}{2}\right)^{n-2} u[n-2]$
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Problem P-8.11

$$(a) \frac{1 - z^{-1}}{1 + 0.77z^{-1}}$$

```
num = [1 -1];
```

```
den = [1 0.77];
```

```
[r,p,k]=residuez(num,den);
```

```
r =
```

```
2.2987
```

```
p =
```

```
-0.7700
```

```
k =
```

```
-1.2987
```

$$-1.2987 + \frac{2.2987}{1 + 0.77z^{-1}}$$

↑
table lookup
↓

$$-1.2987\delta[n] + 2.2987(-0.77)^n u(n)$$

$$(c) \frac{z^{-2}}{1 - 0.9z^{-1}}$$

```
num = [0 0 1];
den = [1 -0.9];
[r,p,k]=residuez(num,den);
```

```
r =
    1.2346
p =
    0.9000
k =
   -1.2346  -1.1111
```

$$-1.2346 - 1.1111 z^{-1} + \frac{1.2346}{1 - 0.9z^{-1}}$$

↑
table lookup
↓

$$\delta[n] \leftrightarrow 1$$

$$\delta[n-1] \leftrightarrow z^{-1}$$

$$(0.9)^n u(n) \leftrightarrow \frac{1}{1 - 0.9z^{-1}}$$

$$-1.2346 \delta[n] - 1.1111 \delta[n-1] + (0.9)^n u(n)$$

another way

$$\frac{z^{-2}}{1 - 0.9z^{-1}}$$

$$z^{-2} \left(\frac{1}{1 - 0.9z^{-1}} \right)$$

$$(0.9)^n u(n) \longleftrightarrow \frac{1}{1 - 0.9z^{-1}}$$

$$(0.9)^{n-2} u[n-2] \longleftrightarrow z^{-2} \left(\frac{1}{1 - 0.9z^{-1}} \right)$$

answer 1

$$-1.2346\delta[n] - 1.1111\delta[n-1] + (0.9)^n u(n)$$

answer 2

$$(0.9)^{n-2} u(n-2)$$

answer 1 and answer 2 are the same!

Problem 8.12

$$X_b(z) = \frac{1 + z^{-2}}{1 + 0.9z^{-1} + 0.81z^{-2}}$$

num = [1 0 1];

den = [1 0.9 0.81];

[r,p,k]=residuez(num,den);

r =

-0.1173 + 0.6451i

-0.1173 - 0.6451i

p =

-0.4500 + 0.7794i

-0.4500 - 0.7794i

k =

1.2346

$$= 1.2346$$

$$+ \frac{-0.1173 + j0.6451}{1 - (-0.45 + j0.7794)z^{-1}} + \frac{-0.1173 - j0.6451}{1 - (-0.45 - j0.7794)z^{-1}}$$

$$= 1.2346$$

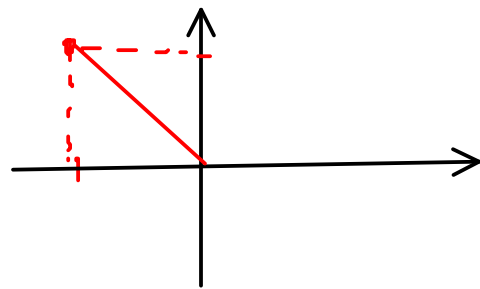
$$- \frac{0.1173}{0.5} \left[\frac{0.5}{1 - (-0.45 + j0.7794)z^{-1}} + \frac{0.5}{1 - (-0.45 - j0.7794)z^{-1}} \right]$$

$$- \frac{0.6451}{0.5} \left[\frac{-0.5j}{1 - (-0.45 + j0.7794)z^{-1}} + \frac{0.5j}{1 - (-0.45 - j0.7794)z^{-1}} \right]$$

$$a = -0.45 \quad b = 0.7794$$

$$a = -0.45 \quad b = 0.7794$$

$$c = \sqrt{(0.45)^2 + (0.7794)^2} = 0.9$$



$$\text{atan2}(\text{imag}(z), \text{real}(z))$$

$$\begin{aligned} \omega_0 &= \text{atan2}(0.7794, -0.45) \\ &= 2.0944 \text{ rad} = \frac{2\pi}{3} \text{ rad} \\ &= 120^\circ \end{aligned}$$

$$1 \longleftrightarrow \delta[n]$$

$$c^n \cos \omega_0 n \longleftrightarrow \left[\frac{0.5}{1 - (-0.45 + j0.7794)\bar{z}^{-1}} + \frac{0.5}{1 - (-0.45 - j0.7794)\bar{z}^{-1}} \right]$$

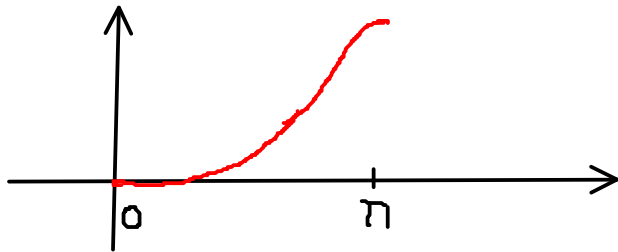
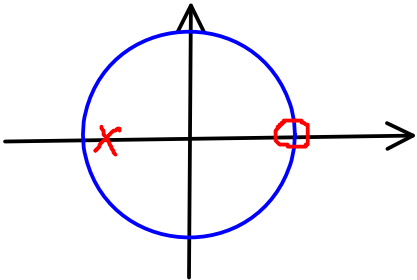
$$c^n \sin \omega_0 n \longleftrightarrow \left[\frac{-0.5j}{1 - (-0.45 + j0.7794)\bar{z}^{-1}} + \frac{0.5j}{1 - (-0.45 - j0.7794)\bar{z}^{-1}} \right]$$

$$1.2346 \delta[n] - 0.2436 (0.9)^n \cos\left(\frac{2\pi}{3}n\right) - 1.2902 (0.9)^n \sin\left(\frac{2\pi}{3}n\right)$$

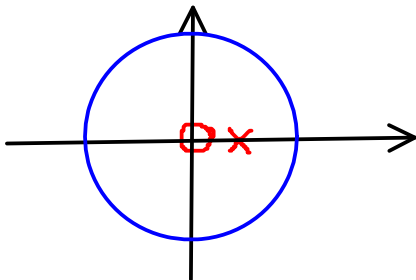
$$1.2346 \delta[n] - (0.9)^n \left[0.2436 \cos\left(\frac{2\pi}{3}n\right) + 1.2902 \sin\left(\frac{2\pi}{3}n\right) \right]$$

Problem 8-16

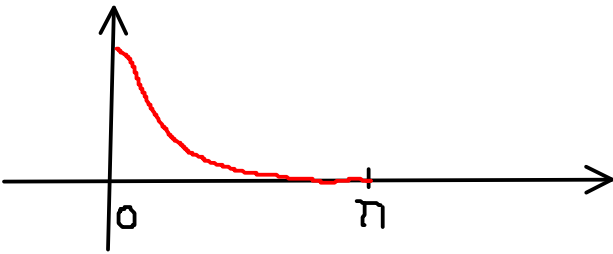
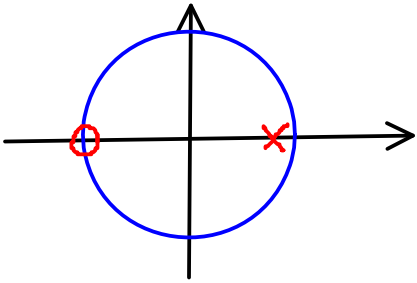
#1



#2



#3



#4

