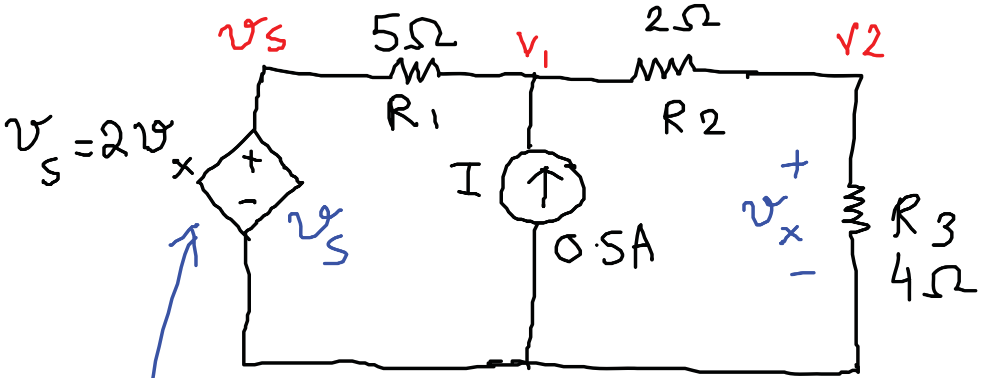
Node & Mesh Analysis with Dependent sources



Treat this as a normal voltage source

$$\frac{v_s - v_1}{5\Omega} + 0.5A + \frac{V_2 - V_1}{2\Omega} = 0$$

$$\frac{V_1 - V_2}{2\Omega} + \frac{0 - V_2}{4\Omega} = 0$$

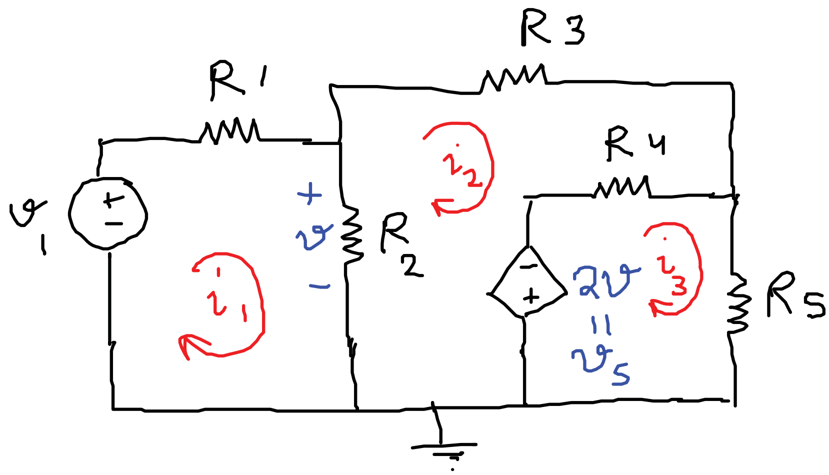
$$v_s = 2v_x = 2(V_2 - 0) = 2v_2$$

express v_x in terms of unknowns

$$\frac{2v_2 - v_1}{5\Omega} + 0.5A + \frac{V_2 - V_1}{2\Omega} = 0$$

$$\frac{V_1 - V_2}{2\Omega} + \frac{0 - V_2}{4\Omega} = 0$$

Two equations in two unknowns



$$-v_1 + R_1 i_1 + R_2(i_1 - i_2) = 0$$

$$R_3 i_2 + R_4(i_2 - i_3) - v_s - R_2(i_2 - i_1) = 0$$

$$R_4(i_3 - i_2) + R_5 i_3 + v_s = 0$$

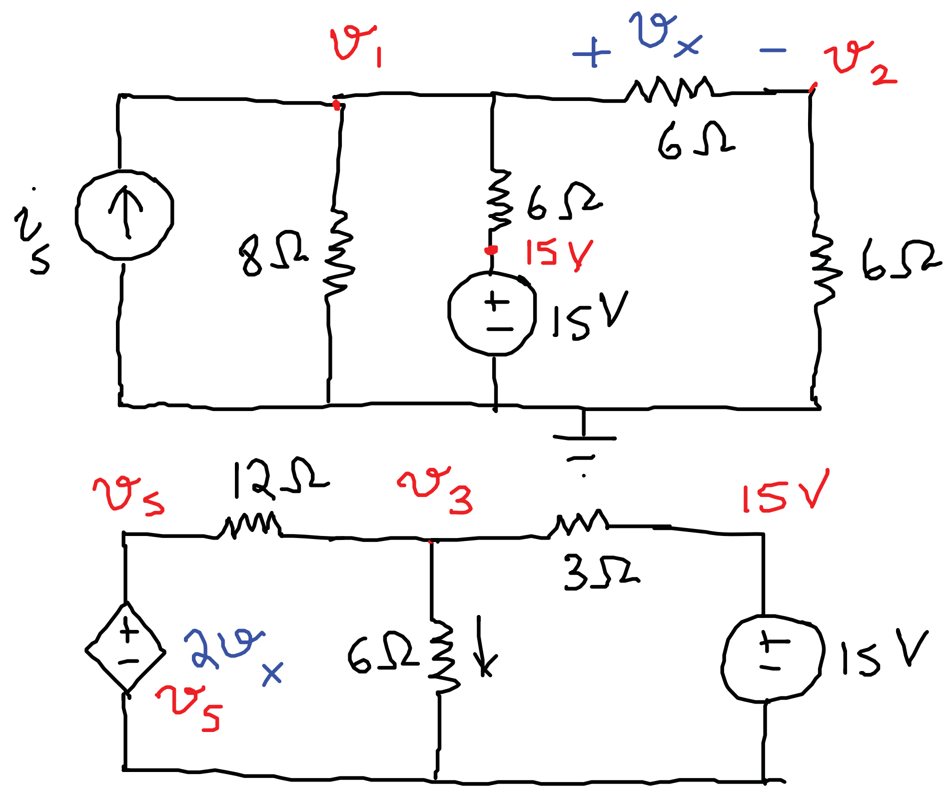
$$v_s = 2v$$

expresss in terms of unknowns

$$v = R(i_1 - i_2)$$

$$v_s = 2R(i_1 - i_2)$$

Three equations in three unknowns



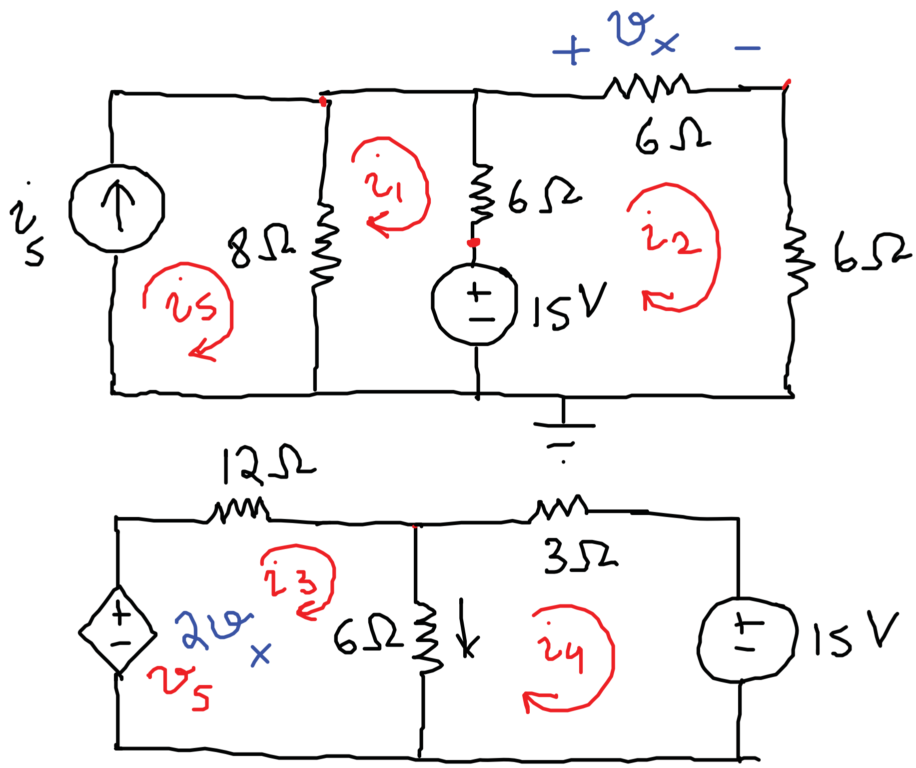
v_1: $\quad i_s + \dfrac{0 - v_1}{8\,\Omega} + \dfrac{15\text{V} - v_1}{6\,\Omega} + \dfrac{v_2 - v_1}{6\,\Omega} = 0$

v_2: $\quad \dfrac{v_1 - v_2}{6\,\Omega} + \dfrac{0 - v_2}{6\,\Omega} = 0$

v_3: $\quad \dfrac{v_s - v_3}{12} + \dfrac{0 - v_3}{6\,\Omega} + \dfrac{15 - v_s}{3\,\Omega}$

$v_s = 2v_x \quad = 2(v_1 - v_2)$

Three equations in 3 unknowns



i_1: $15V + 8\Omega(i_1 - i_5) + 6\Omega(i_1 - i_2) = 0$

i_2: $-15V + 6\Omega(i_2 - i_1) + 6\Omega\, i_2 + 6\Omega\, i_2 = 0$

i_3: $-v_5 + 12\Omega\, i_3 + 6\Omega(i_3 - i_4) = 0$

i_4: $15V + 6\Omega(i_4 - i_3) + 3\Omega\, i_4 = 0$

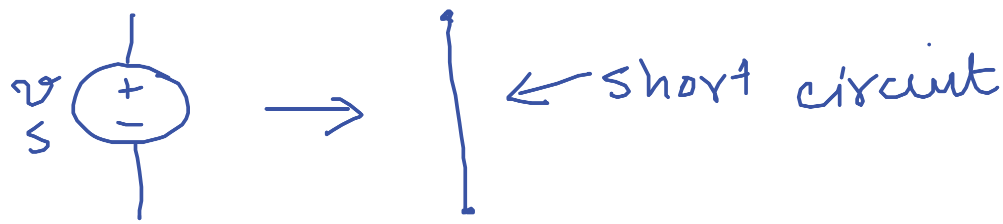
$v_5 = 2v_x = 2 \times 6\Omega\, i_2 = 12 i_2$

$v_5 = 12 i_2$

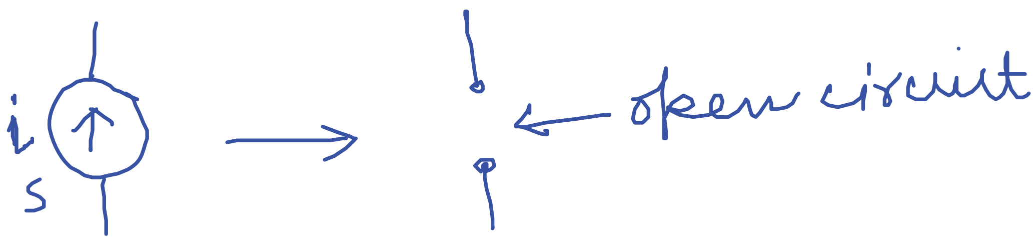
4 equations in 4 unknowns

3.5 Principle of Superposition

How to kill an independent voltage source :



How to kill an independent current source



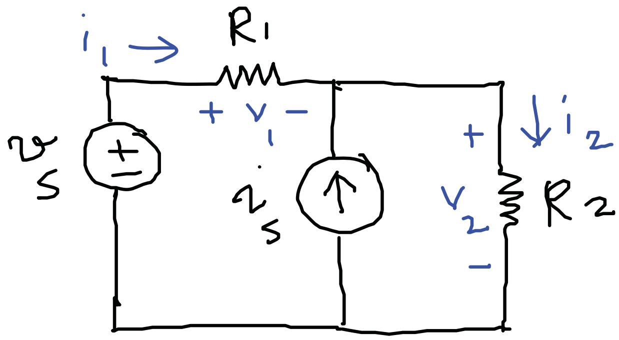
visit an independent source

kill all independent sources except for the visited source

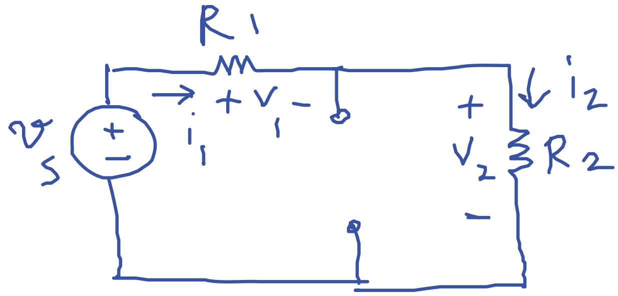
solve the problem and determine the voltages and currents

visit the next independent source

add the voltages and currents from each visited independent source

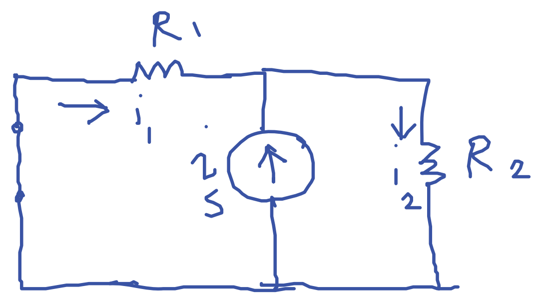


v_s:



$$i_1 = i_2 = \frac{v_s}{R_1+R_2} \qquad v_1 = \frac{R_1}{R_1+R_2} v_s \qquad v_2 = \frac{R_2 v_s}{R_1+R_2}$$

i_s:



$$i_1 = -\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}} i_s = -\frac{R_2}{R_1+R_2} i_s , \quad v_1 = -\frac{R_1 R_2}{R_1+R_2} i_s$$

$$i_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}} i_s = \frac{R_1}{R_1+R_2} i_s , \quad v_2 = \frac{R_1 R_2 i_s}{R_1+R_2}$$

use superposition to get the correct total answer

$$v_1 = \frac{R_1}{R_1+R_2} v_s - \frac{R_1 R_2}{R_1+R_2} i_s$$

$$v_2 = \frac{R_2}{R_1+R_2} v_s + \frac{R_1 R_2}{R_1+R_2} i_s$$

$$i_1 = \frac{v_s}{R_1+R_2} - \frac{R_2}{R_1+R_2} i_s$$

$$i_2 = \frac{v_s}{R_1+R_2} + \frac{R_1}{R_1+R_2} i_s$$