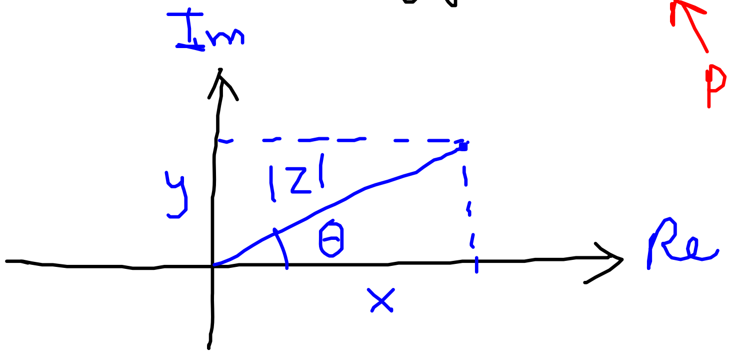
rectangular

$$z = x + jy = |z|\,e^{j\theta} = |z|\angle\theta$$

polar



rectangular to polar

$$|z| = \sqrt{x^2 + y^2}$$

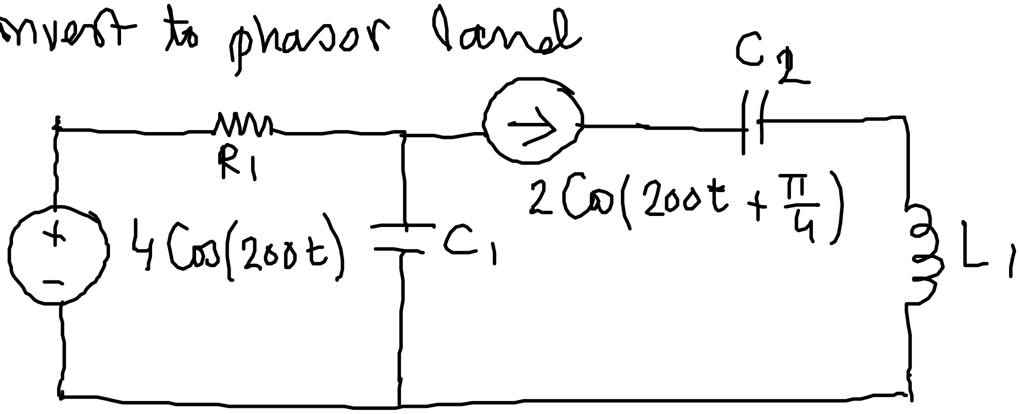
$$\theta = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & \text{if } (x \geq 0 \;\&\; y > 0) \text{ or } (x > 0 \;\&\; y \leq 0) \\ \tan^{-1}\left(\frac{y}{x}\right) - \pi & \text{if } (x \leq 0 \;\&\; y < 0) \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi & \text{if } (x < 0 \;\&\; y \geq 0) \end{cases}$$

polar to rectangular

$$x = |z| \cos\theta$$

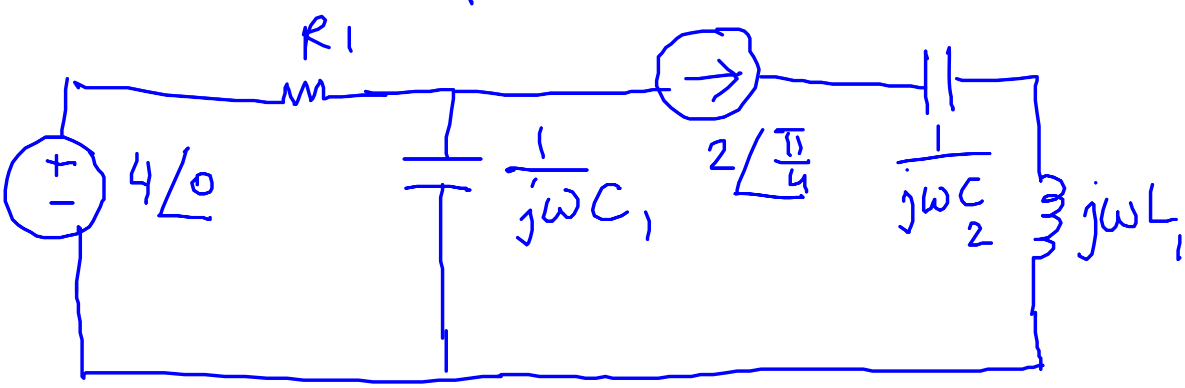
$$y = |z| \sin\theta$$

Convert to phasor land

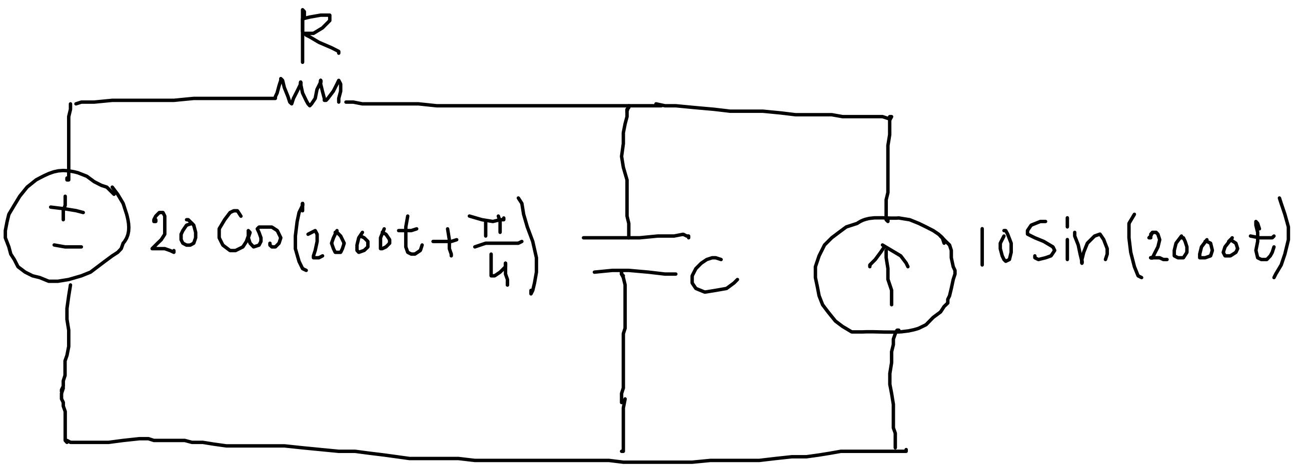


$$4\cos(200t) \rightarrow 4\angle 0$$

$$2\cos\left(200t + \frac{\pi}{4}\right) \rightarrow 2\angle\frac{\pi}{4} \qquad \omega = 200$$



Convert to frequency domain (phasor land)



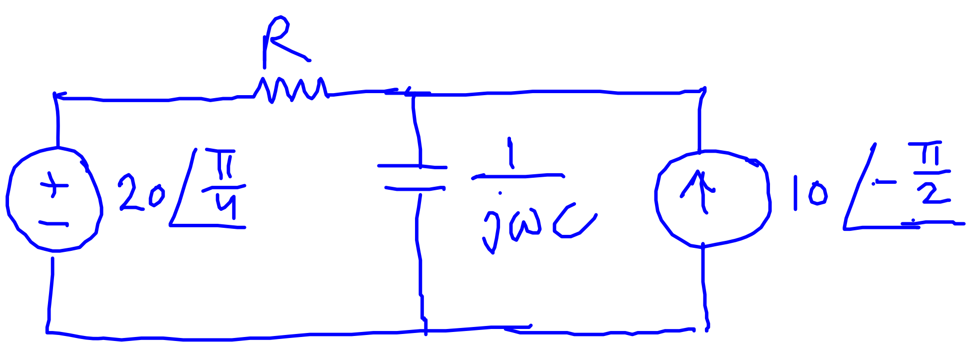
$\omega = 2000$ rad/sec

$20\cos(2000t + \frac{\pi}{4}) \rightarrow 20\angle\frac{\pi}{4}$

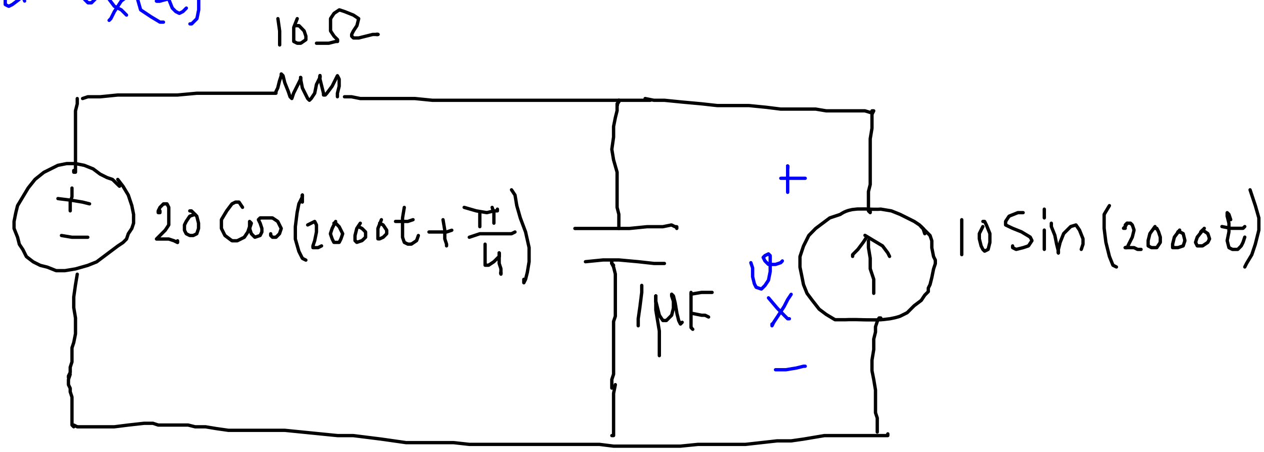
$10\sin(2000t) = 10\cos(2000t - \frac{\pi}{2}) \rightarrow 10\angle -\frac{\pi}{2}$

$-\frac{\pi}{2} = \frac{3\pi}{2}$ $\qquad\qquad \Downarrow$

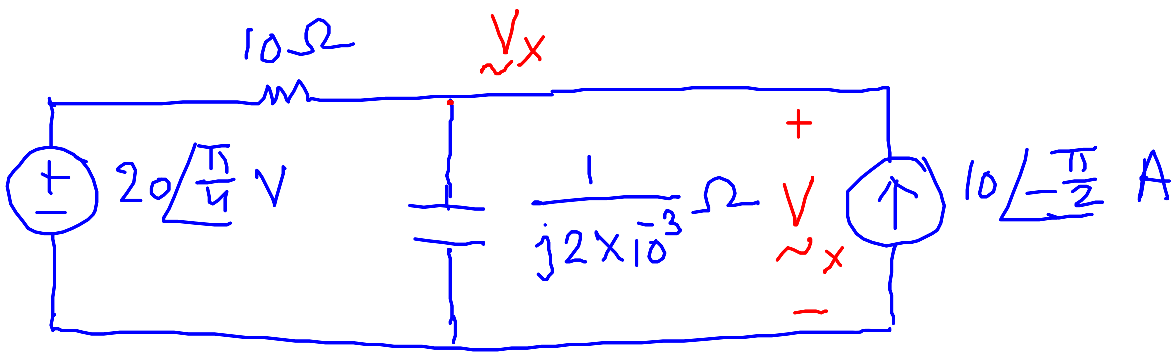
$10\angle\frac{3\pi}{2}$



Find $v_x(t)$



$\omega = 2000$ r/s ⇓ phasorland



node analysis

$$\frac{20\angle\frac{\pi}{4} - \underset{\sim}{V}_x}{10\,\Omega} + \frac{0 - \underset{\sim}{V}_x}{\frac{1}{j2\times10^{-3}}\,\Omega} + 10\angle-\frac{\pi}{2} = 0$$

$$2\angle\frac{\pi}{4} - 0.1\,\underset{\sim}{V}_x - j2\times10^{-3}\,\underset{\sim}{V}_x + 10\angle-\frac{\pi}{2} = 0$$

$$\underset{\sim}{V}_x\left(0.1 + j2\times10^{-3}\right) = 2\angle\frac{\pi}{4} + 10\angle-\frac{\pi}{2}$$

$$\underset{\sim}{V}_x\left(0.1 + j2\times10^{-3}\right) = 2\angle\frac{\pi}{4} + 10\angle-\frac{\pi}{2}$$

$$\underset{\sim}{V}_x = \frac{2\angle\frac{\pi}{4} + 10\angle-\frac{\pi}{2}}{0.1 + j2\times10^{-3}}$$

$$= \frac{2\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) + 10\left(\cos\frac{\pi}{2} + j\sin\left(-\frac{\pi}{2}\right)\right)}{0.1 + j2\times10^{-3}}$$

$$\sin\left(-\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$$

$$= \frac{2(.707 + j.707) + 10(0 - j)}{0.1 + j2\times10^{-3}}$$

$$= \frac{1.414 + j1.414 - 10j}{0.1 + j2\times10^{-3}}$$

$$= \frac{1.414 - j8.586}{0.1 + j2\times10^{-3}}$$

$$= \frac{1.414 - j8.586}{0.1 + j2\times10^{-3}}$$

$$= \frac{\sqrt{(1.414)^2 + (8.586)^2}\ \tan^{-1}\left(-\frac{8.586}{1.414}\right)}{\sqrt{(0.1)^2 + (2\times10^{-3})^2}\ \tan^{-1}\left(\frac{2\times10^{-3}}{0.1}\right)}$$

$$= \frac{\sqrt{75.7}\ \angle -1.4}{\sqrt{0.010004}\ \angle 0.0199} = \sqrt{\frac{75.7}{0.01}}\ \angle -1.4 - 0.02$$

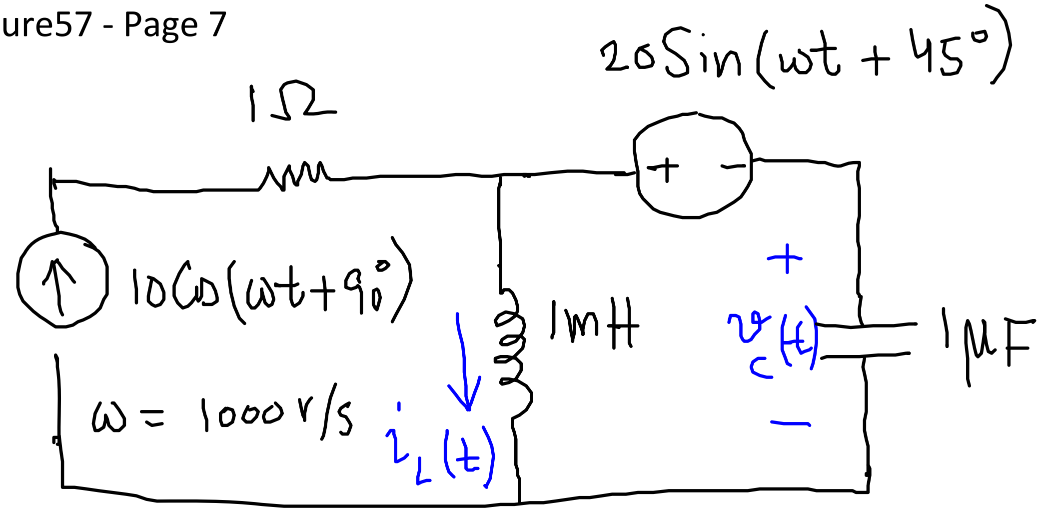
$$\tilde{V}_x = 87\angle -1.42\ V = 87\angle -81.36^\circ\ V$$

$$\tilde{V}_x = 87e^{-j1.42}\ V$$

$\downarrow$ real domain

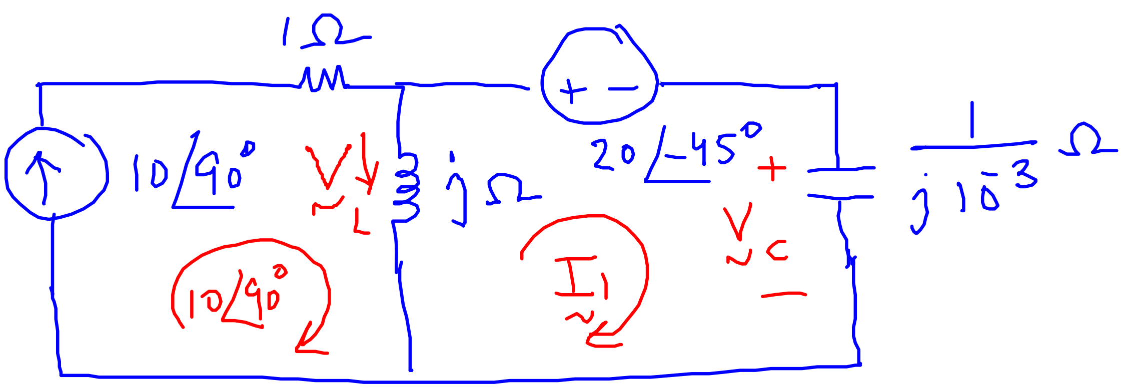
$$v_x(t) = \mathrm{Re}\left[e^{j2000t}\ 87e^{-j1.42}\right] V$$

$$v_x(t) = 87\cos(2000t - 81.36^\circ)\ V$$



determine $i_L(t)$ & $v_C(t)$

phasor domain $\quad \sin(\omega t + 45) = \cos(\omega t \overbrace{-45}^{45^\circ - 90^\circ})$



Note!
this should have been $j10\angle 90^\circ$

$$j\Omega(\tilde{I}_1 - 10\angle 90^\circ) + 20\angle -45^\circ + \frac{1}{j10^{-3}}\tilde{I}_1 = 0$$

$$\tilde{I}_1(j - j10^3) = 10\angle 90^\circ - 20\angle -45^\circ$$

Note!
this should have been −999

$$j0.999\,\tilde{I}_1 = 10[\cos 90 + j\sin 90]$$
$$-20[\cos(-45) + j\sin(-45)]$$

$$j0.999\,\underset{\sim}{I}_1 = 10[\cos 90 + j\sin 90] - 20[\cos(-45) + j\sin(-45)]$$

$$= 10[0 + j] - 20[0.707 - j0.707]$$

$$= 10j - 14.14 + j14.14$$

$$j0.999\,\underset{\sim}{I}_1 = -14.14 + j24.14 \qquad x<0 \quad y>0$$

$$\underset{\sim}{I}_1 = \frac{\sqrt{(14.14)^2 + (24.14)^2}\ \angle \tan^{-1}\left(-\frac{24.14}{14.14}\right) + 180^\circ}{0.999\ \angle 90^\circ}$$

$$\frac{27.97\ \angle -\tan^{-1}(1.707) + 180^\circ}{0.999\ \angle 90^\circ}$$

$$\frac{27.97\ \angle -59.64^\circ + 180^\circ}{0.999\ \angle 90^\circ}$$

$$\frac{27.97\ \angle 120.36^\circ}{0.999\ \angle 90^\circ} = 27.99\ \angle 120.36^\circ - 90^\circ$$

$$\underset{\sim}{I}_1 = 28\ \angle 30.36^\circ$$

$$\underset{\sim}{V}_L = j\Omega\left(10\angle 90^\circ - \underset{\sim}{I}_1\right)$$

$$= j\Omega\left(10\angle 90 - 28\angle 30.36\right)$$

$$= 1\angle 90^\circ\left[10(\cos 90 + j\sin 90) - 28(\cos(30.36^\circ) + j\sin(30.36^\circ))\right]$$

$$= 1\angle 90^\circ\left[10j - 28(\cos(30.36^\circ) + j\sin(30.36^\circ))\right]$$

$$= 1\angle 90^\circ\left[10j - 28(0.863 + j0.505)\right]$$

$$= 1\angle 90^\circ\left[10j - 24.16 - j14.14\right] \text{ A}$$

$$= 1\angle 90^\circ\left[-24.16 + j4.14\right] \text{ A}$$

$x < 0$
$y > 0$

$$= 1\angle 90^\circ\left[\sqrt{(24.16)^2 + (4.14)^2}\angle \tan^{-1} - \frac{4.14}{24.16} + 180^\circ\right]$$

$$= (1\angle 90^\circ)(24.51\angle -9.72^\circ + 180^\circ)$$

$$= (1\angle 90^\circ)(24.51\angle 170.28)$$

$$\underset{\sim}{V}_L = 24.51\angle 90^\circ + 170.28 = 24.51\angle 260.28^\circ$$

$$\underset{\sim}{V}_L = 24.51\angle 260.28^\circ \text{ V}$$

$$\underset{\sim}{V}_C = \underset{\sim}{I}_1 \frac{1}{j10^{-3}} = \frac{28\angle 30.36}{10^{-3}\angle 90^\circ}$$

$$\underset{\sim}{V}_C = 28\times10^3 \angle 30.36^\circ - 90.64^\circ$$

$$\underset{\sim}{V}_C = 28\times10^3 \angle -59.64^\circ \quad V$$

$\downarrow$ real time domain

$$\underset{\sim}{V}_L = 24.51 \angle 260.28^\circ$$

$$\downarrow$$

$$v_L(t) = 24.51 \cos(1000t + 260.28^\circ)$$

$$v_C(t) = 28\times10^3 \cos(1000t - 59.64^\circ) V$$