Transfer Function

Transfer function of a circuit or system describes the output response to an input excitation as a function of the angular frequency ω.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_{\text{out}}(\omega)}{\mathbf{V}_{\text{in}}(\omega)}$$ Voltage Gain

$$\mathbf{H}(\omega) = M(\omega)\, e^{j\phi(\omega)},$$

where by definition,

$$M(\omega) = |\mathbf{H}(\omega)|, \qquad \phi(\omega) = \tan^{-1}\left\{\frac{\Im\mathrm{m}[\mathbf{H}(\omega)]}{\Re\mathrm{e}[\mathbf{H}(\omega)]}\right\}$$

Magnitude Phase

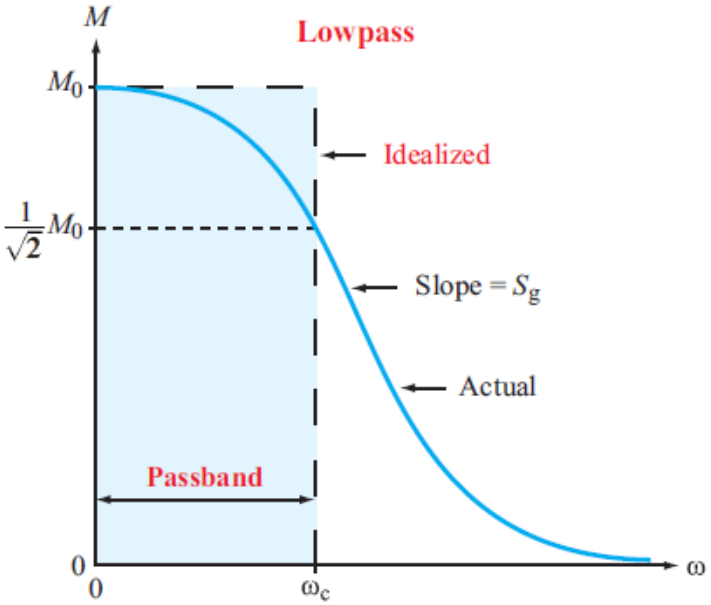
Other Transfer Functions

Current gain: $\mathbf{H}_I(\omega) = \dfrac{\mathbf{I}_{\text{out}}(\omega)}{\mathbf{I}_{\text{in}}(\omega)},$

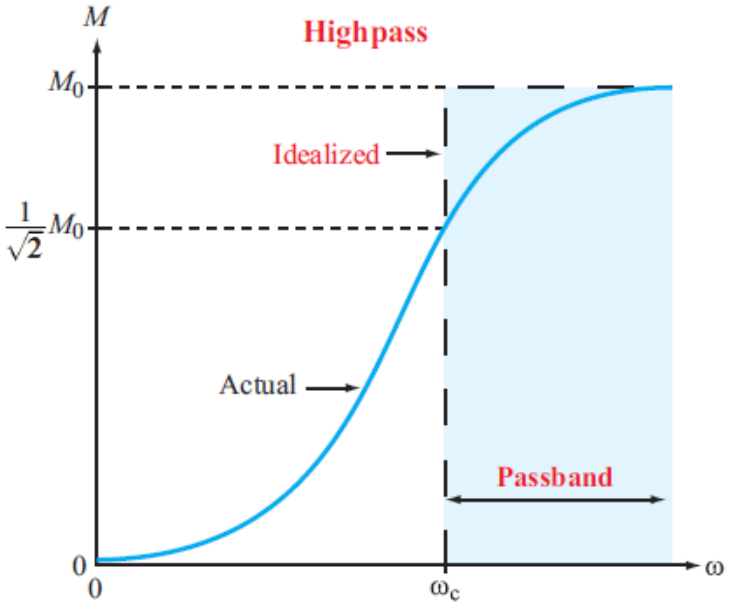
Transfer impedance: $\mathbf{H}_{\mathbf{Z}}(\omega) = \dfrac{\mathbf{V}_{\text{out}}(\omega)}{\mathbf{I}_{\text{in}}(\omega)},$

Transfer admittance: $\mathbf{H}_{\mathbf{Y}}(\omega) = \dfrac{\mathbf{I}_{\text{out}}(\omega)}{\mathbf{V}_{\text{in}}(\omega)}.$

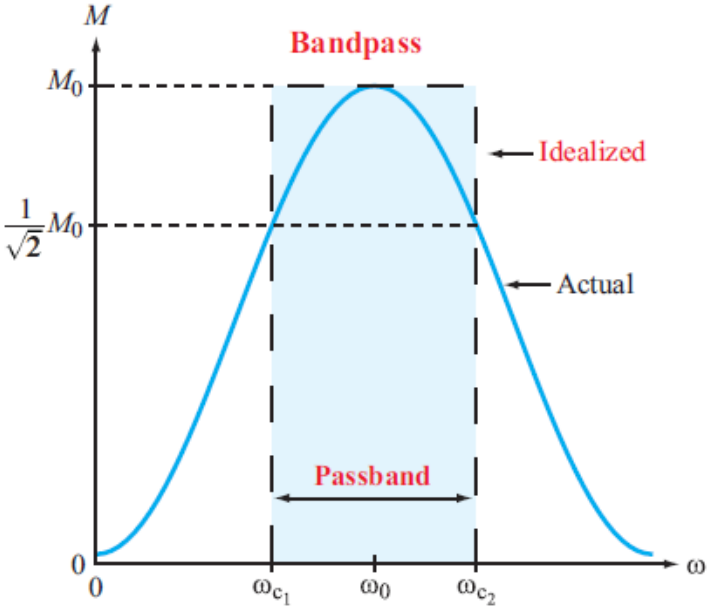
Filters



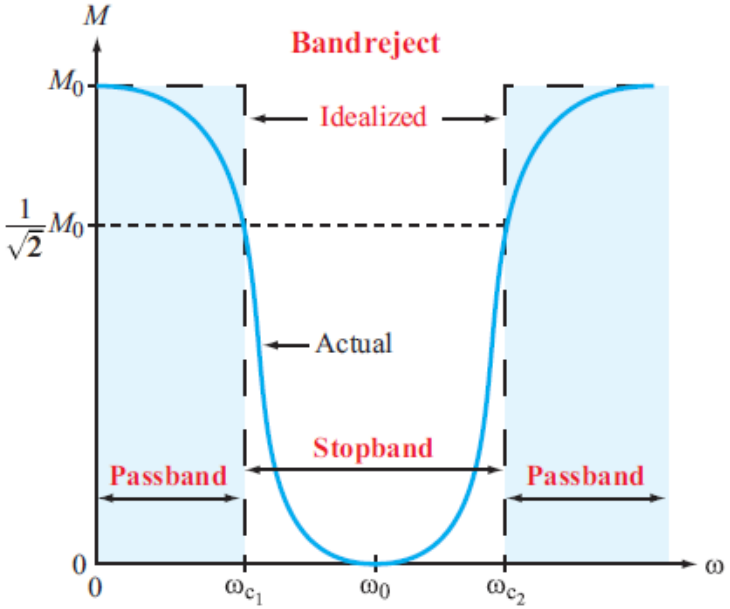
(a) Lowpass filter



(b) Highpass filter

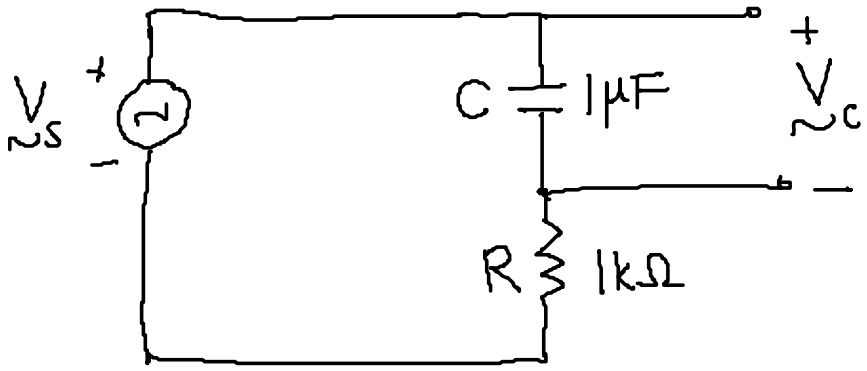


(c) Bandpass filter



(d) Bandreject filter

Low Pass Filter



$$V_c = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_s$$

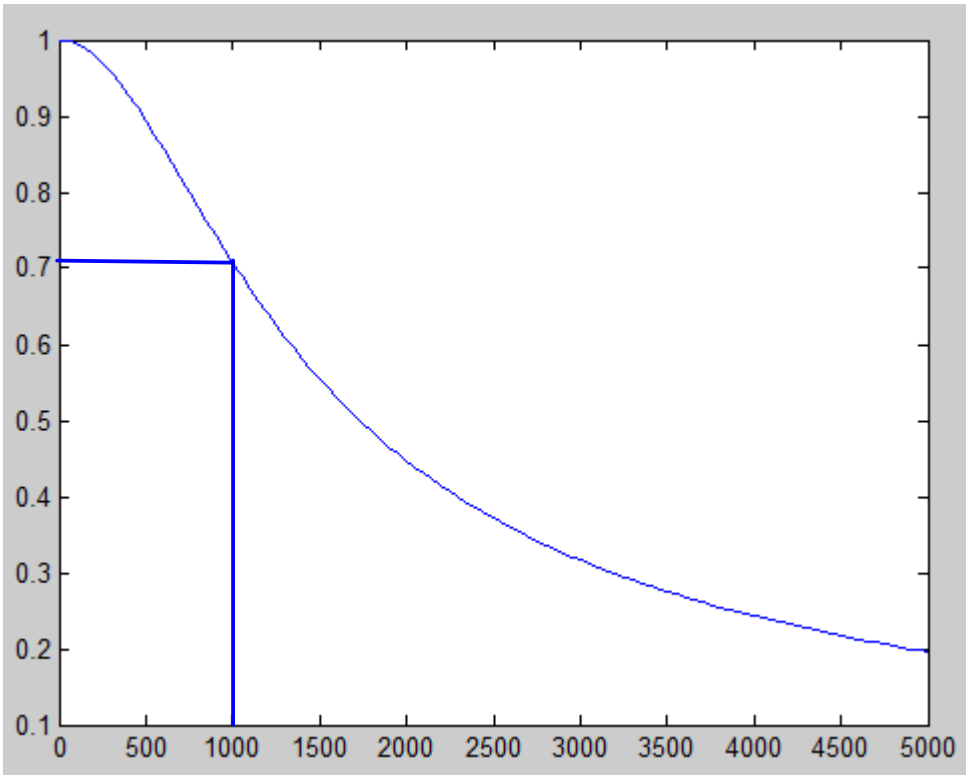
$$H_c(\omega) = \frac{V_c}{V_s} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$M_c(\omega) = |H_c(\omega)| = \frac{1}{|1 + j\omega RC|} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

MATLAB

```
R = 10^3;
C = 10^-6;
w_max = 5*10^3;
w = [0:w_max/100:w_max];
H_mag = 1./sqrt(1 + (w*R*C).^2);
plot(w, H_mag);
```

Low Pass Filter



$\frac{1}{\sqrt{2}}$

ω_c

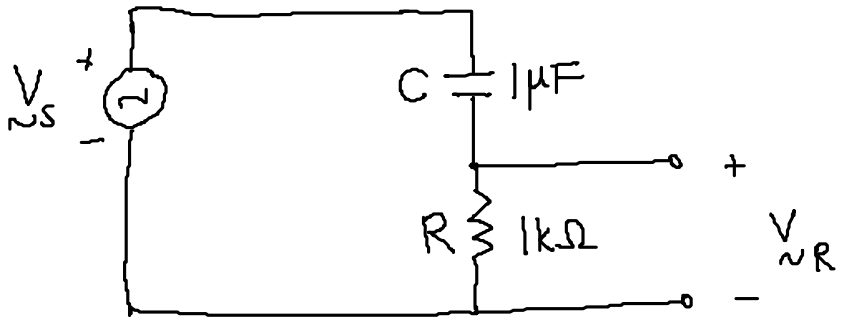
$$M_c(\omega_c) = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{1+\omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{1+\omega_c^2 R^2 C^2} = \frac{1}{2}$$

$$\omega_c = \frac{1}{RC} = 1000 \text{ rad/s}$$

High Pass Filter



$$V_R = \frac{R}{R + \frac{1}{j\omega C}} V_s$$

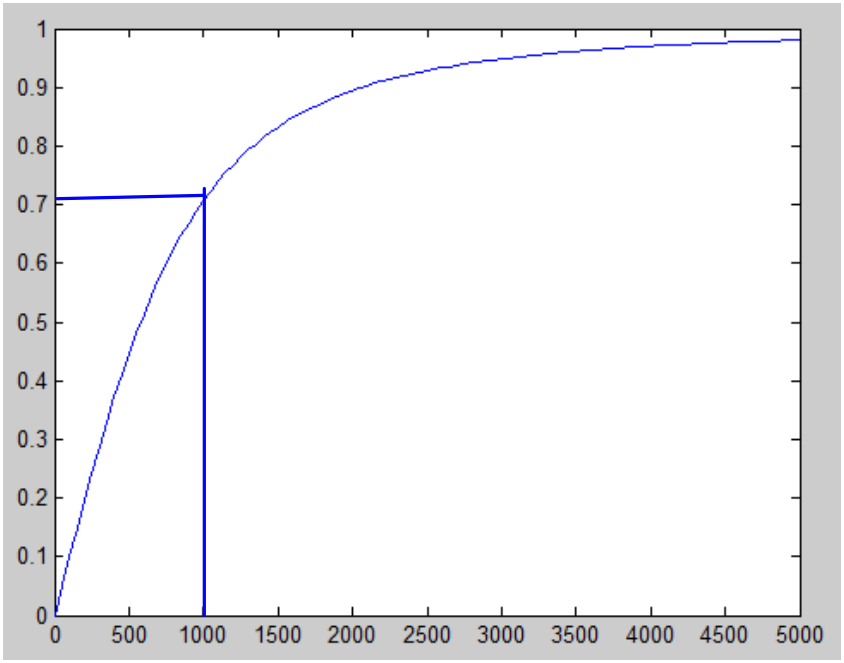
$$H_R(\omega) = \frac{V_R}{V_s} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

$$M_R(\omega) = |H_R(\omega)| = \frac{|j\omega RC|}{|1 + j\omega RC|} = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

MATLAB

```
R = 10^3;
C = 10^-6;
w_max = 5*10^3;
w = [0:w_max/100:w_max];
H_mag = (w*C*R)./sqrt(1 + (w*R*C).^2);
plot(w, H_mag);
```

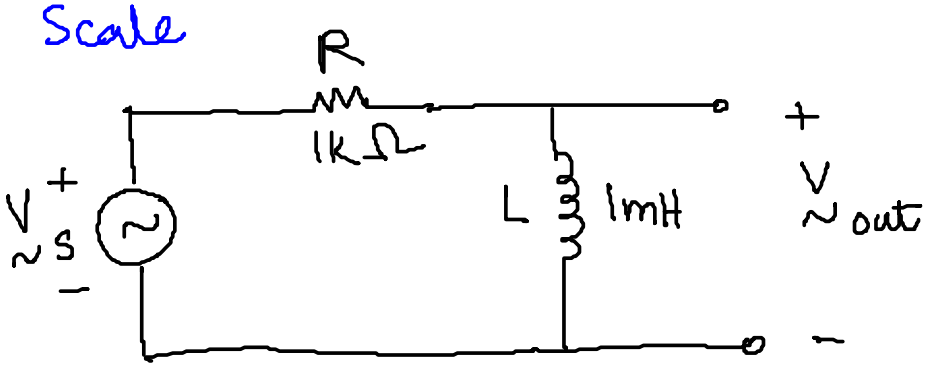
High Pass Filter



$$M_R(\omega_c) = \frac{1}{\sqrt{2}} \qquad \frac{\omega_c CR}{\sqrt{1+\omega_c^2 C^2 R^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{\omega_c^2 C^2 R^2}{1+\omega_c^2 C^2 R^2} = \frac{1}{\sqrt{2}} \qquad \omega_c = \frac{1}{CR} = 1000 \text{ rad/s}$$

dB Scale



$$\tilde{V}_{out} = \frac{j\omega L}{R + j\omega L}\tilde{V}_s$$

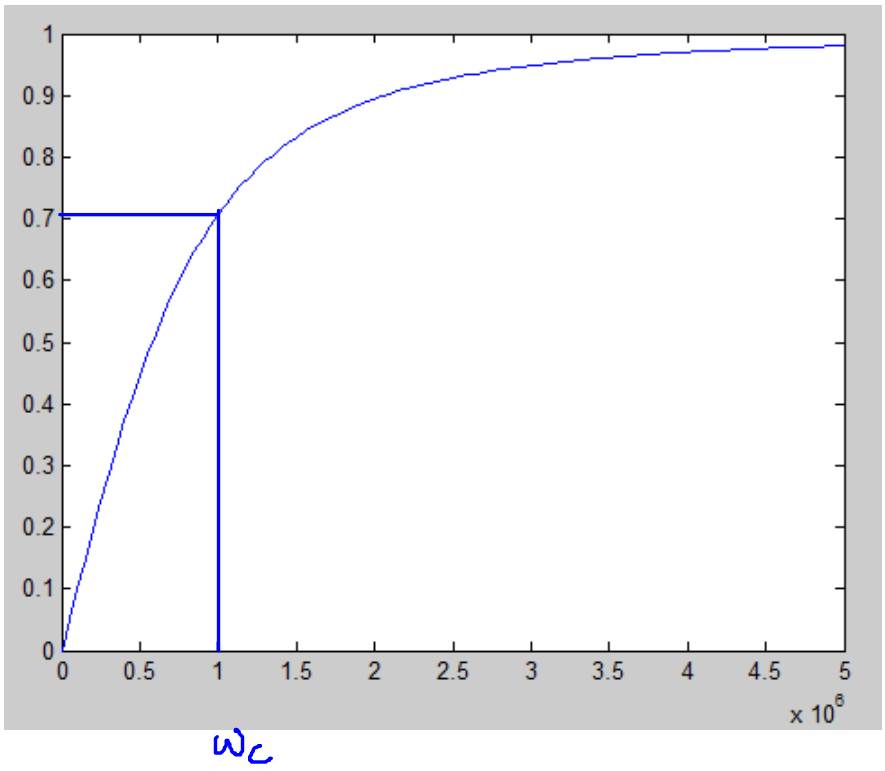
$$H(\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_s} = \frac{j\omega L}{R + j\omega L}$$

$$M(\omega) = |H(\omega)| = \frac{|j\omega L|}{|R + j\omega L|} = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

MATLAB

```
R = 10^3;
L = 10^-3;
w_max = 5*10^6;
w = [0:w_max/100:w_max];
H_mag = (w*L)./sqrt(R^2 + (w*L).^2);
plot(w, H_mag);
```

High Pass Filter



$\frac{1}{\sqrt{2}}$

ω_c

$$M(\omega_c) = \frac{1}{\sqrt{2}} \qquad \frac{\omega_c L}{\sqrt{R^2 + \omega_c^2 L^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{\omega_c^2 L^2}{R^2 + \omega_c^2 L^2} = \frac{1}{2} \qquad \omega_c = \frac{R}{L} = 10^6 \text{ rad/s}$$

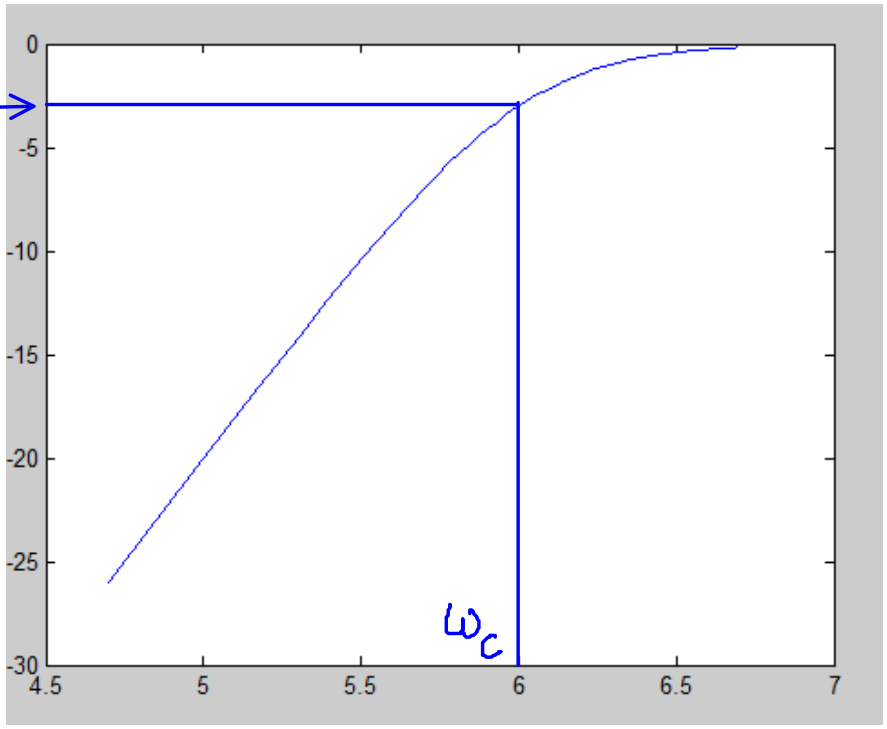
Bode Plot (dB scale)

$20 \log M(\omega)$ vs. $\log(\omega)$

```
w_log = log10(w);
H_mag_dB = 20*log10(H_mag);
plot(w_log, H_mag_dB);
```

MATLAB

Magnitude Bode Plot



-3dB

ω_c

ω 10^5 10^6 10^7 rad/s

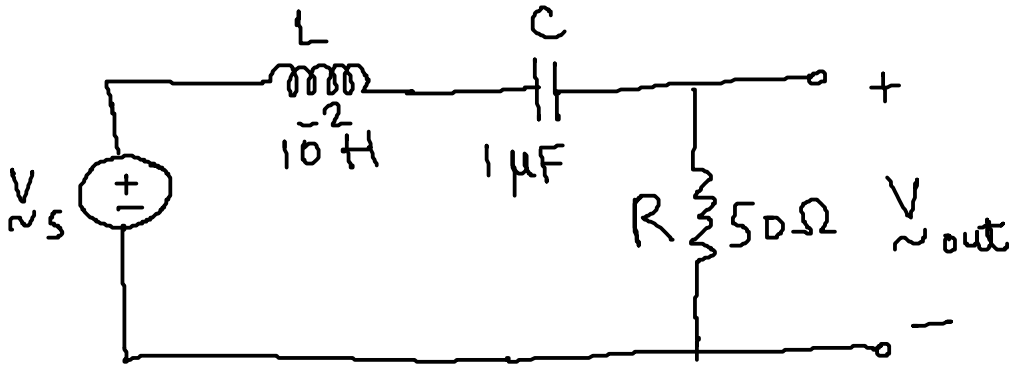
$\log\omega = 5$ $\omega = 10^5$

$\log\omega = 6$ $\omega = 10^6$

$\log\omega = 7$ $\omega = 10^7$

$\omega_c = 10^6$ rad/s

Band Pass Filter



$$V_{out} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} V_{out}$$

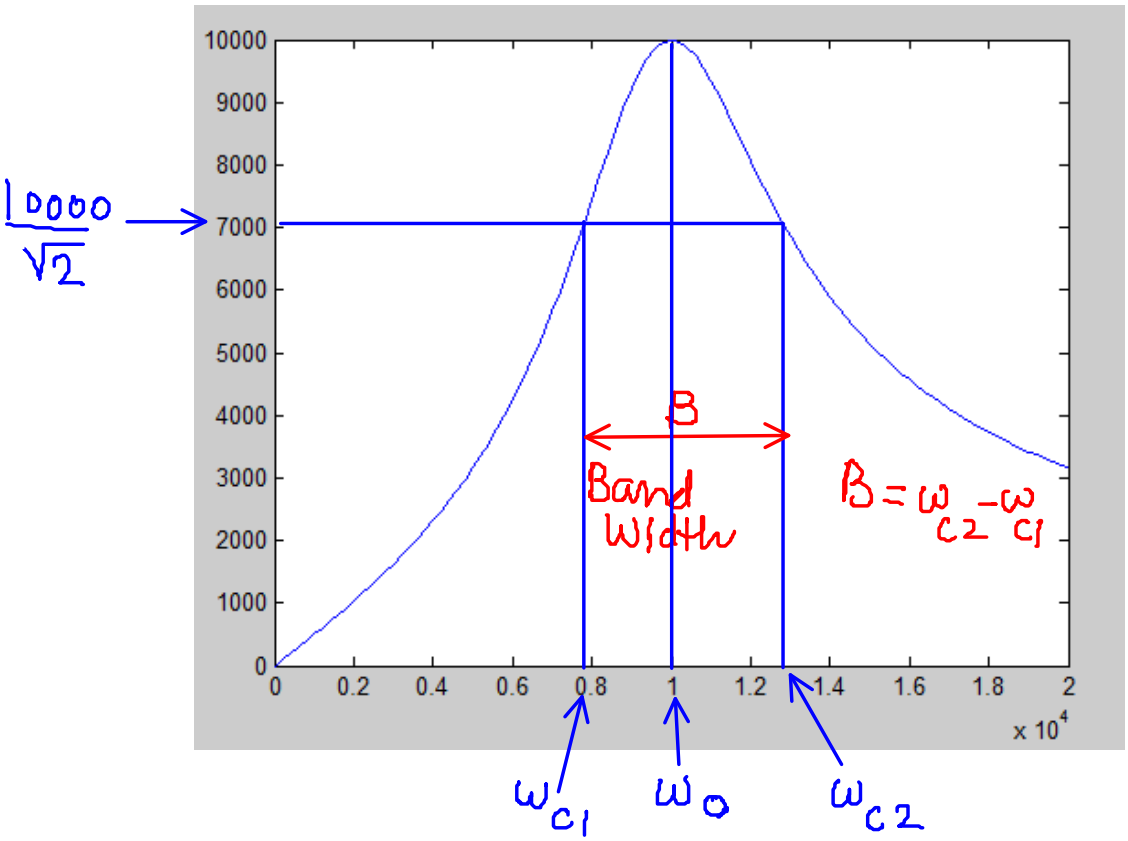
$$H(\omega) = \frac{V_{out}}{V_s} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

$$H(\omega) = \frac{j\omega RC}{(1 - \omega^2 LC) + j\omega RC}$$

$$M(\omega) = |H(\omega)| = \frac{|j\omega RC|}{|(1 - \omega^2 LC) + j\omega RC|}$$

$$M(\omega) = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}}$$

```
R = 50;
L = 10^-2;
C = 10^-6;
w_max = 2*10^4;
w = [0:w_max/100:w_max];
H_mag = (w*R*L)./sqrt(  (1 - w.^2*L*C).^2 + (w*R*C).^2);
plot(w, H_mag);
```



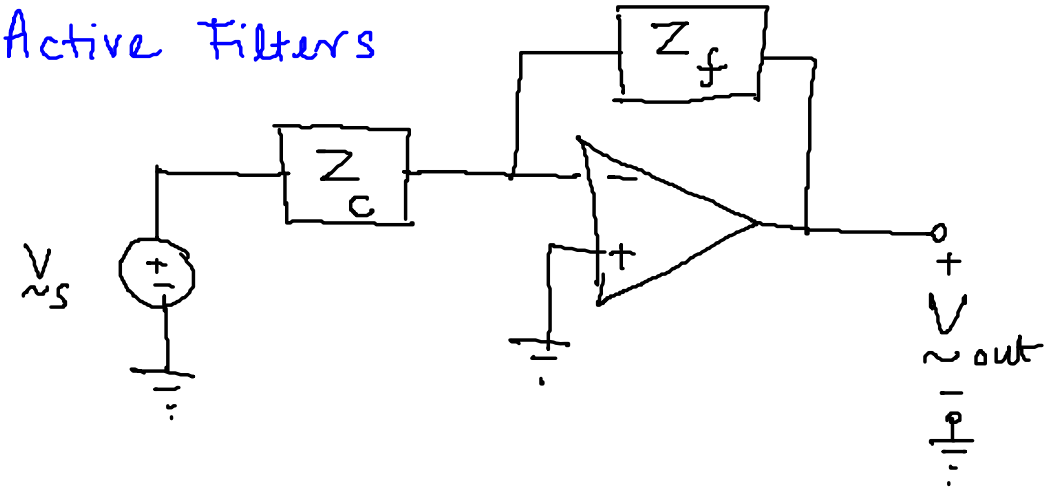
$$\omega_{c_1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}},$$

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

$$\omega_{c_2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}.$$

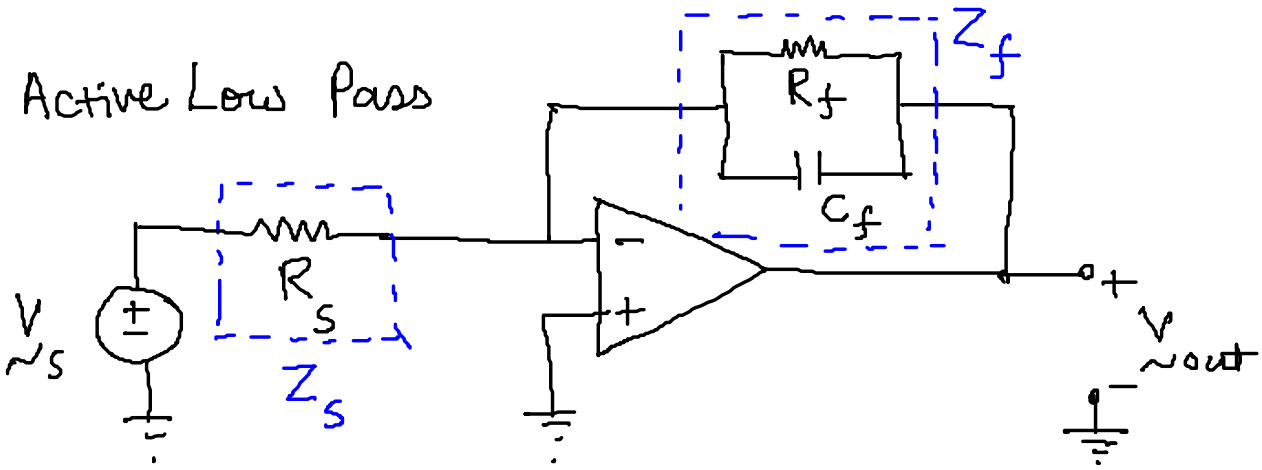
$$B = \omega_{c_2} - \omega_{c_1} = \frac{R}{L}.$$

Active Filters



$$H(\omega) = \frac{V_{\sim out}}{V_{\sim s}} = -\frac{Z_f}{Z_c}$$

Active Low Pass



$$Z_f = R_f \,||\, \frac{1}{j\omega C_f}$$

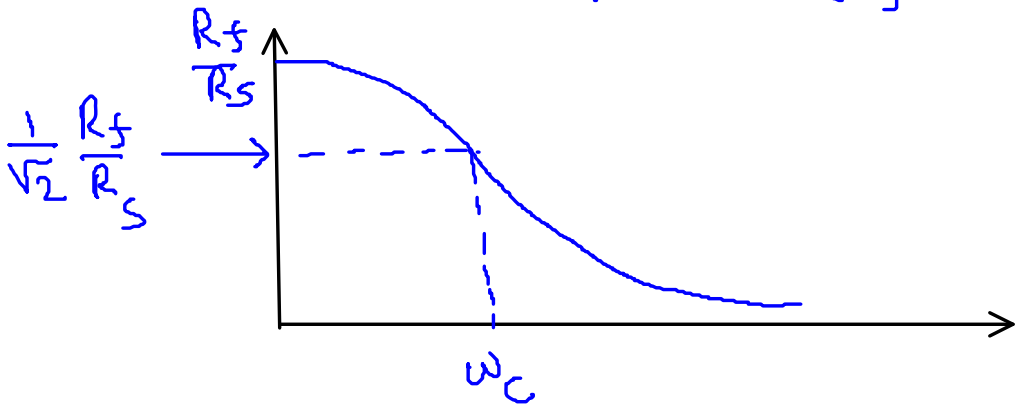
$$\frac{1}{Z_f} = \frac{1}{R_f} + \frac{1}{\frac{1}{j\omega C_f}}$$

$$Z_f = \frac{R_f}{1 + j\omega R_f C_f}$$

$$H(\omega) = \frac{V_{out}}{V_s} = -\frac{Z_f}{Z_s} = -\frac{\frac{R_f}{1 + j\omega R_f C_f}}{R_s}$$

$$H(\omega) = -\frac{R_f}{R_s}\,\frac{1}{1 + j\omega R_f C_f}$$

$$M(\omega) = |H(\omega)| = \frac{R_f}{R_s}\,\frac{1}{\sqrt{1 + \omega^2 R_f^2 C_f^2}} = \frac{1}{\sqrt{2}}\frac{R_f}{R_s}$$

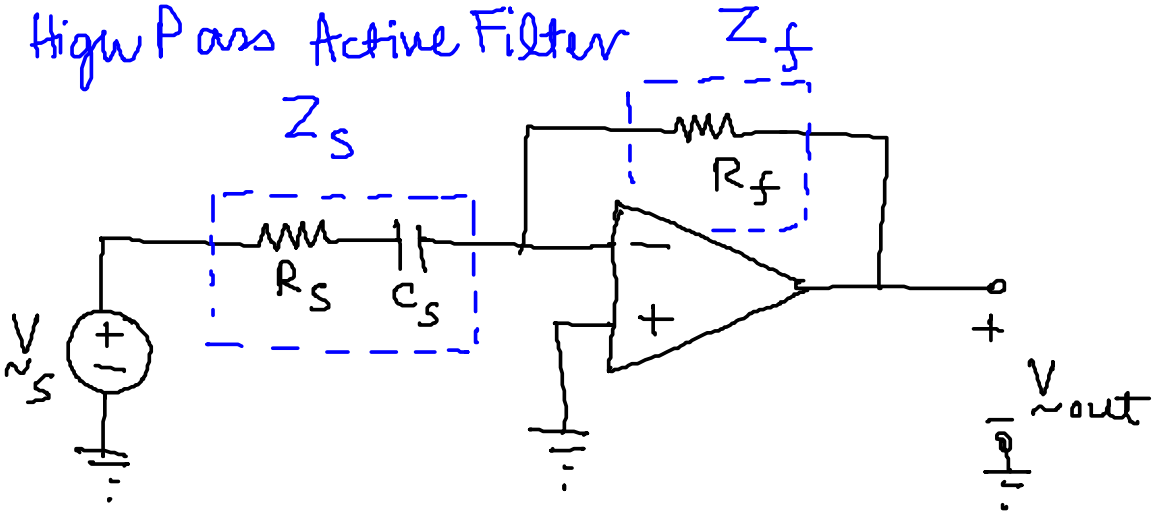


$$\omega_c = \frac{1}{R_f C_f}$$

$$H(\omega) = -\frac{R_f}{R_s}\,\frac{1}{1 + j\left(\frac{\omega}{\omega_c}\right)}$$

LP Filter

High Pass Active Filter



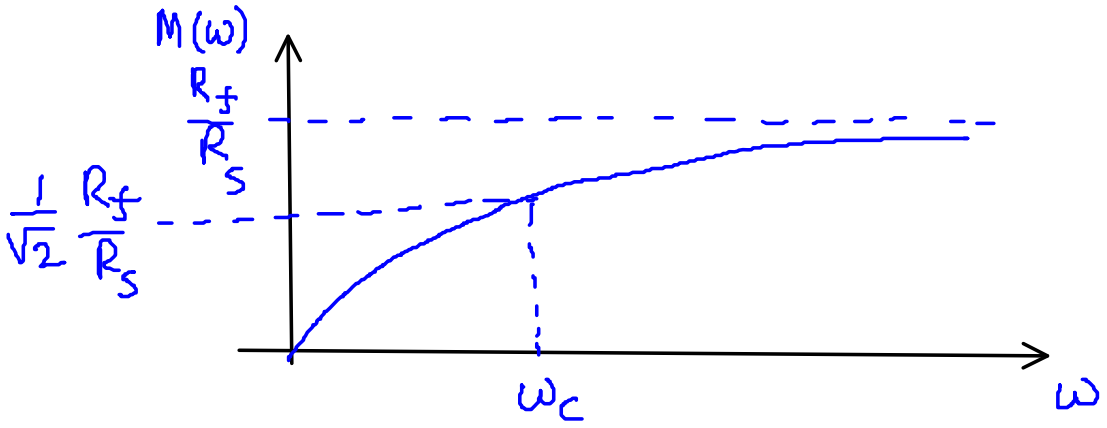
$$Z_s = R_s + \frac{1}{j\omega C_s}$$

$$Z_f = R_f$$

$$H(\omega) = \frac{V_{out}}{V_s} = -\frac{Z_f}{Z_s} = -\frac{R_f}{R_s + \frac{1}{j\omega C_s}}$$

$$H(\omega) = -\frac{R_f}{R_s}\,\frac{j\omega C_s R_s}{1 + j\omega C_s R_s}$$

$$|H(\omega)| = \frac{R_f}{R_s}\,\frac{\omega C_s R_s}{\sqrt{1 + \omega^2 C_s^2 R_s^2}}$$



$$M(\omega_c) = \frac{1}{\sqrt{2}}$$

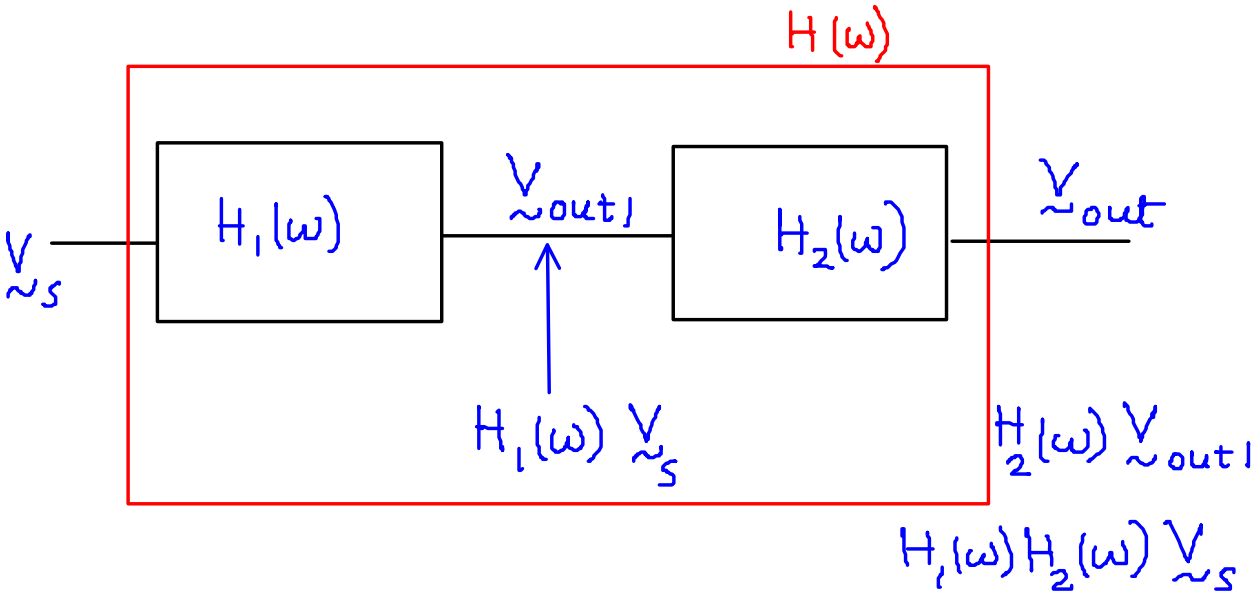
$$\frac{R_f}{R_s}\frac{\omega C_s R_s}{\sqrt{1+\omega^2 C_s^2 R_s^2}} = \frac{1}{\sqrt{2}}\frac{R_f}{R_s} \qquad \omega_c = \frac{1}{C_s R_s}$$

$$H(\omega) = -\frac{R_f}{R_s}\frac{j\omega C_s R_s}{1+j\omega C_s R_s}$$

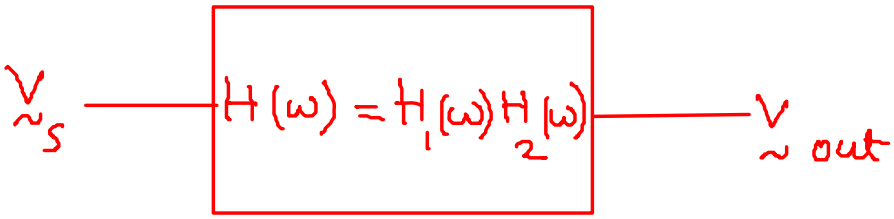
$$\boxed{H(\omega) = -\frac{R_f}{R_s}\frac{j\frac{\omega}{\omega_c}}{1+j\frac{\omega}{\omega_c}}}$$

HP Filter

Cascading Active Filters



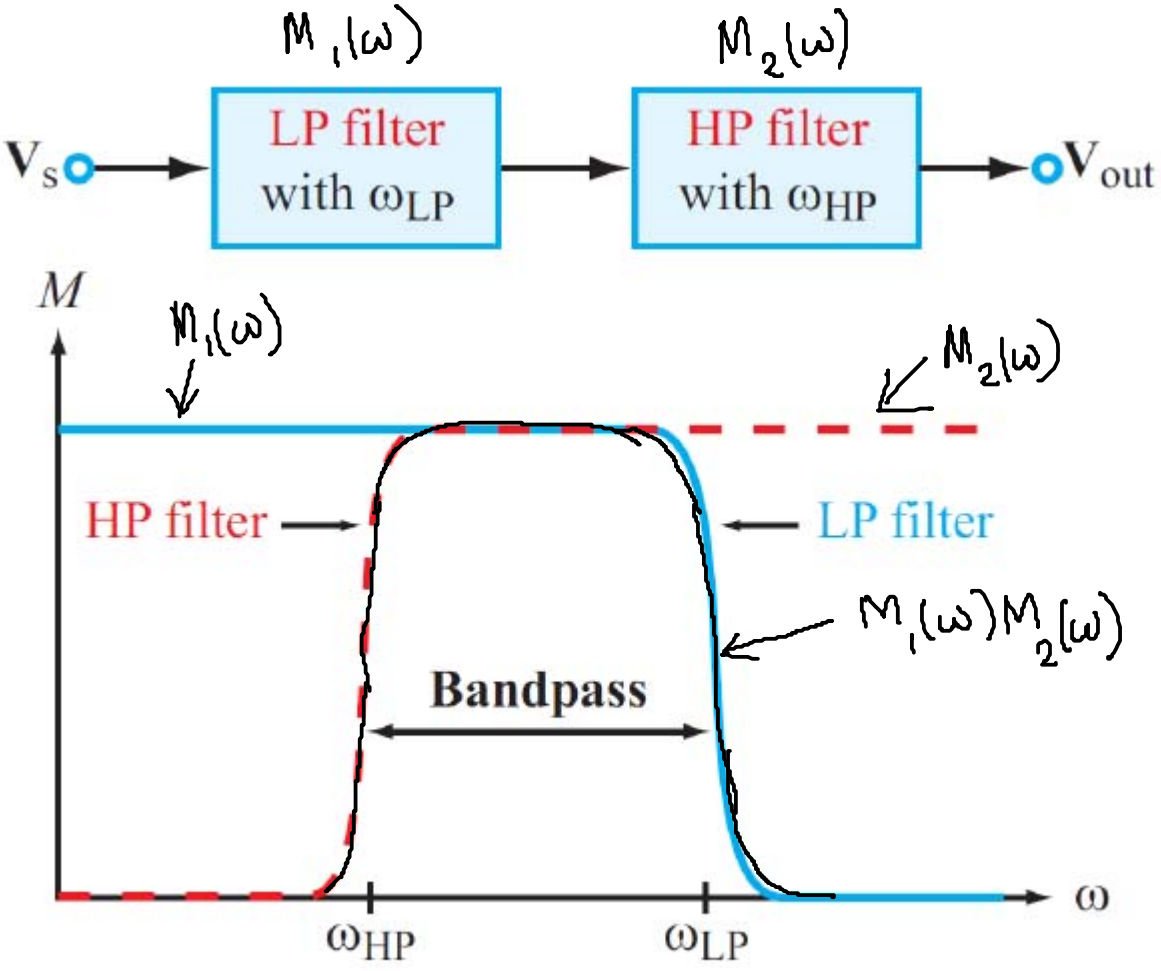
$$\underset{\sim}{V}_{out} = H_1(\omega)\, H_2(\omega)\, \underset{\sim}{V}_s$$



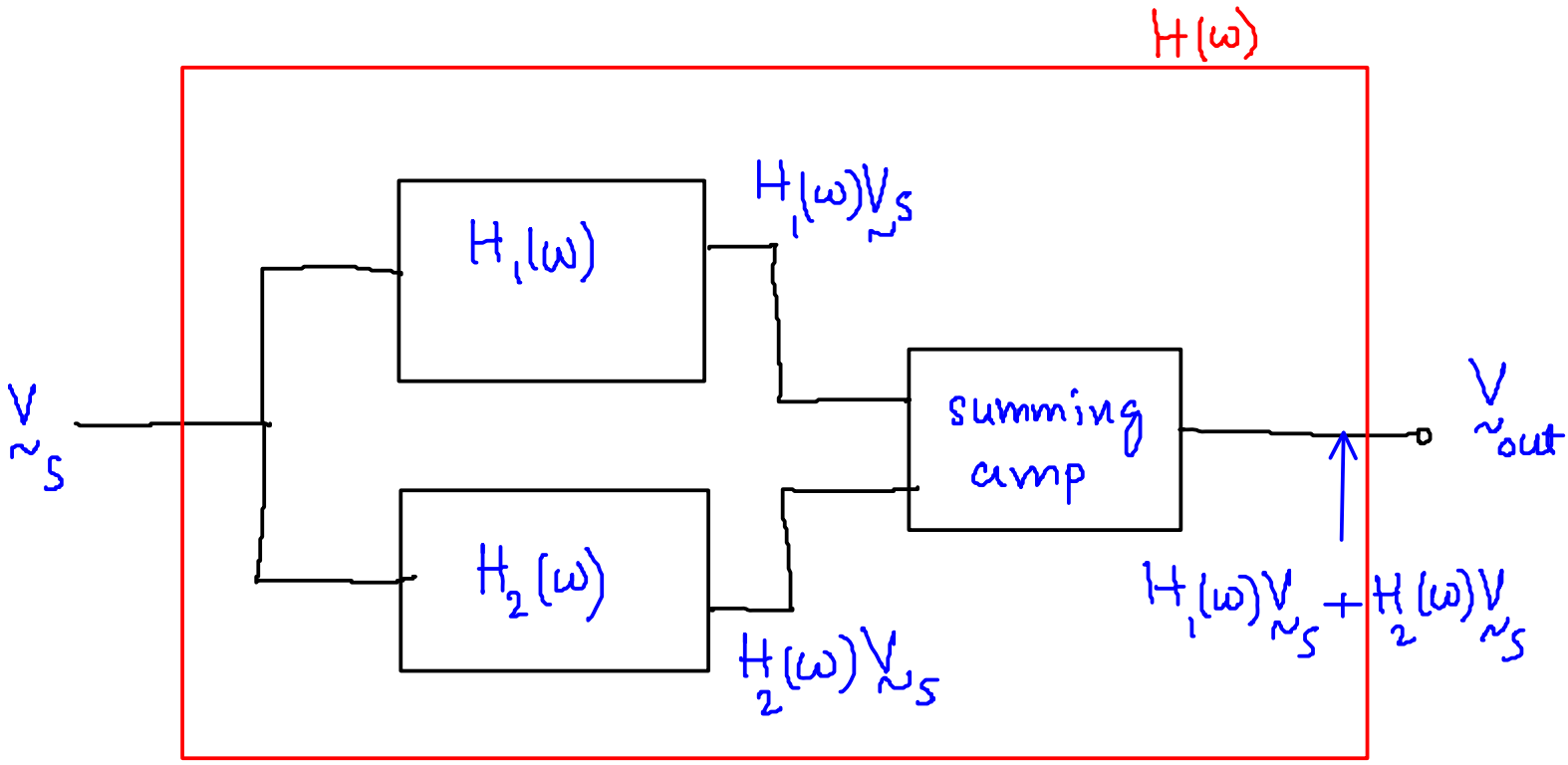
$$|H(\omega)| = |H_1(\omega)|\,|H_2(\omega)|$$

$$M(\omega) = M_1(\omega)\, M_2(\omega)$$

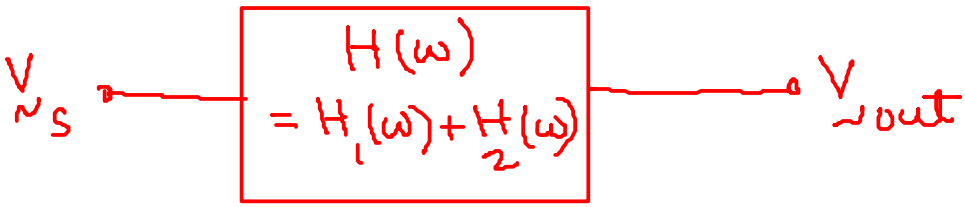
$$20 \log M(\omega) = 20 \log M_1(\omega) + 20 \log M_2(\omega)$$



(a) Bandpass filter

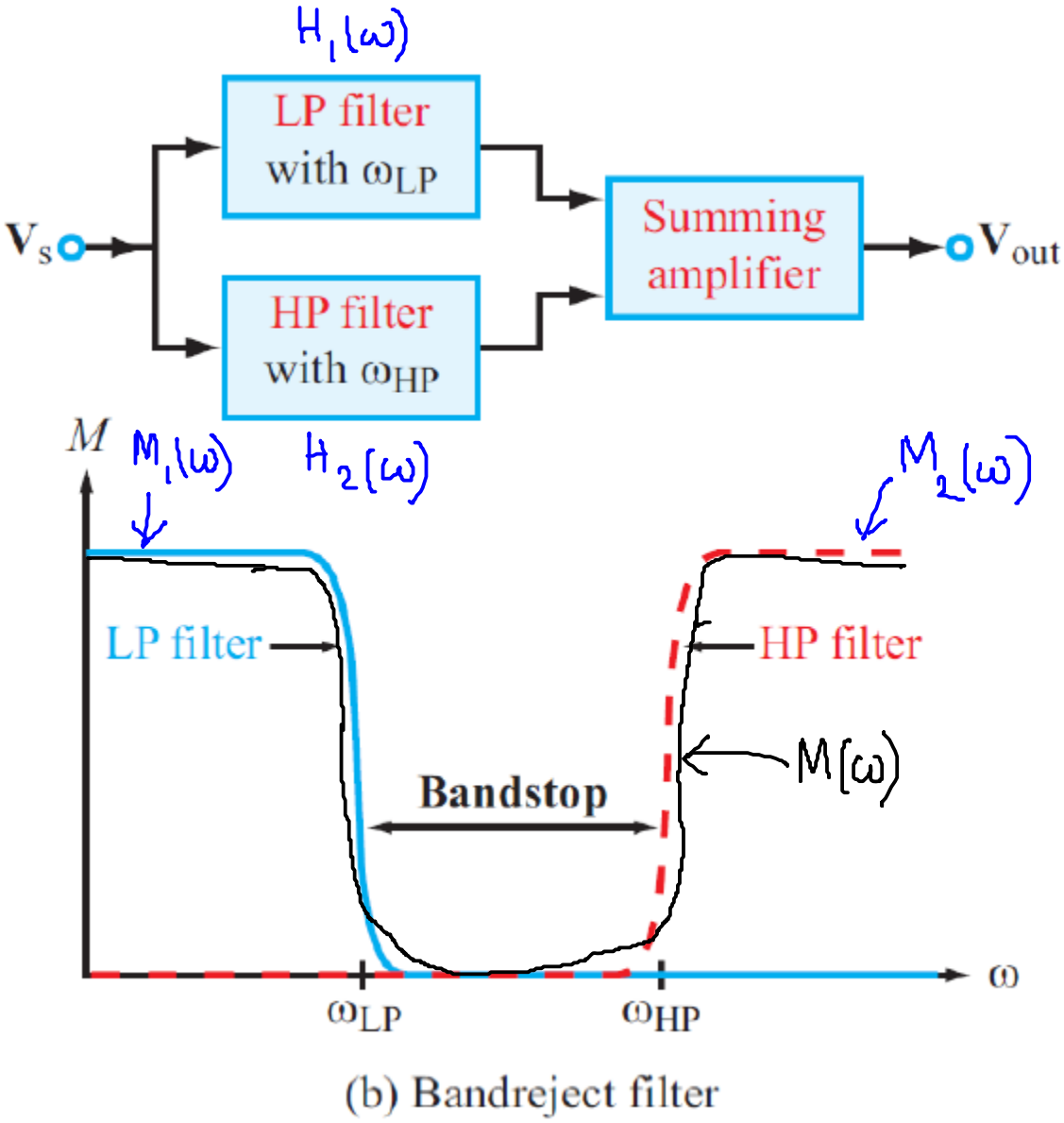


$$V_{out} = [H_1(\omega) + H_2(\omega)]\, V_s$$



$$H(\omega) = \frac{V_{out}}{V_s} = H_1(\omega) + H_2(\omega)$$

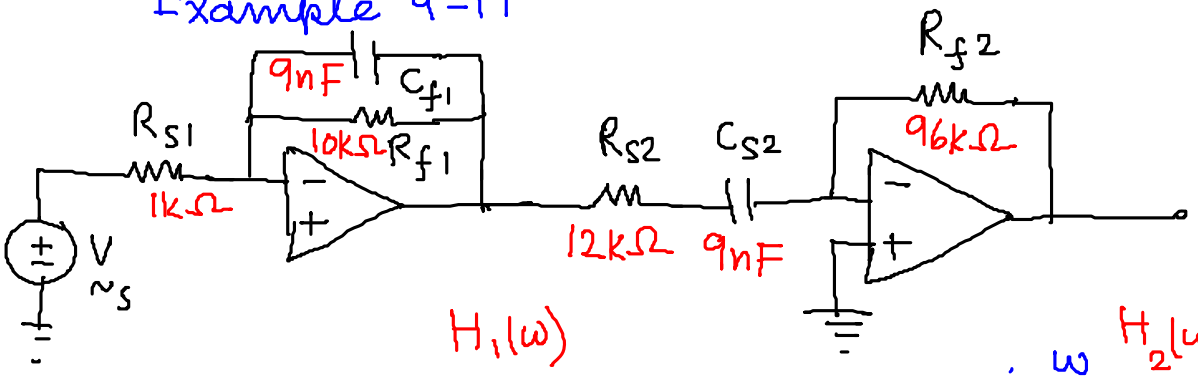
$$|H(\omega)| = |H_1(\omega) + H_2(\omega)|$$



(b) Bandreject filter

Screen clipping taken: 5/15/2012, 11:58 PM

Example 9-11



$H_1(\omega)$ $H_2(\omega)$

$$-\left(\frac{R_{f1}}{R_{S1}}\right)\frac{1}{1+j\frac{\omega}{\omega_{CLP}}} \qquad -\left(\frac{R_{f2}}{R_{S2}}\right)\frac{j\frac{\omega}{\omega_{CHP}}}{1+j\frac{\omega}{\omega_{CHP}}}$$

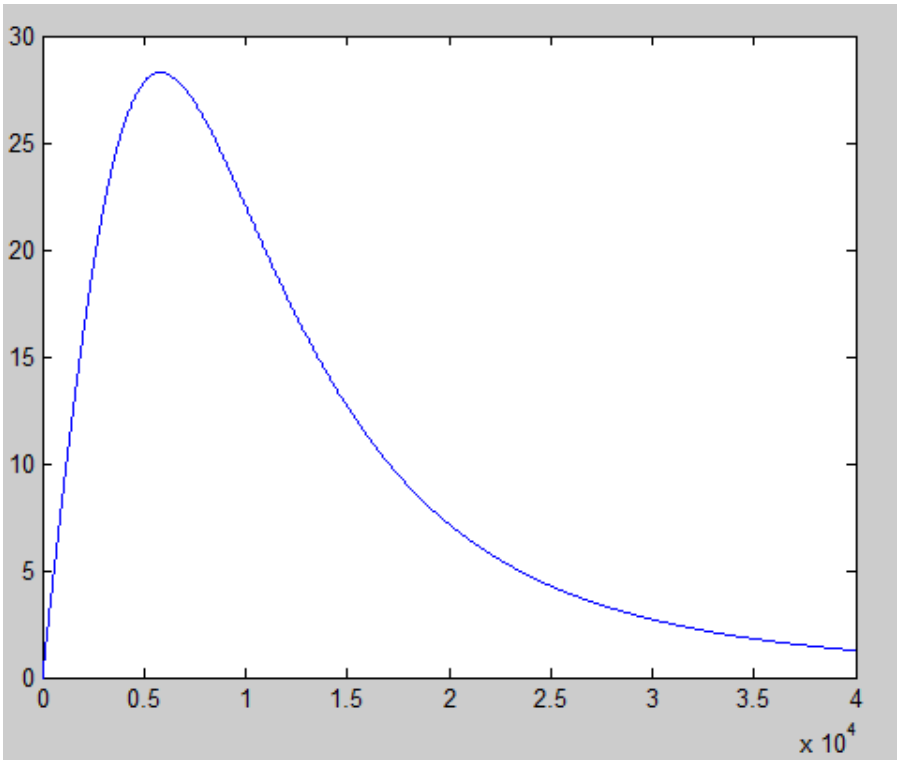
$$\frac{1}{R_{f1}C_{f1}} = 11.11 \text{ krad/s} \qquad \frac{1}{R_{S2}C_{S2}} = 9.26 \text{ krad/s}$$

$$H(\omega) = \frac{R_{f1}R_{f2}}{R_{S1}R_{S2}} \cdot \frac{1}{1+j\frac{\omega}{\omega_{CLP}}} \cdot \frac{j\frac{\omega}{\omega_{CHP}}}{1+j\frac{\omega}{\omega_{CHP}}}$$

```
Rs1 = 10^3;
Rf1 = 10*10^3;
Rs2 = 12*10^3;
Rf2 = 96*10^3;
Cf1 = 9*10^-9;
Cs2 = 9*10^-9;
wcLP = 1/(Rf1*Cf1);
wcHP = 1/(Rs2*Cs2);

w_max = 4*10^4;
w = [0:w_max/5000:w_max];

H_mag1 = (Rf1/Rs1)*1./(1 + (w/wcLP).^2);
H_mag2 = (Rf2/Rs2)*(w/wcHP)./(1 + (w/wcHP).^2);

H_mag = H_mag1 .* H_mag2;
plot(w, H_mag);
```

MATLAB

M(w)



Magnitude plot