## Standard Forms

literals

product term → $X\bar{Y}Z$

sum term → $X+Y+\bar{Z}$

literals

## Minterms & Maxterms

min term: A product term in which all the variables appear exactly once, either complemented or uncomplemented

three variables X, Y, Z

literals

$\bar{X}\,\bar{Y}Z$

$\bar{X}Y\bar{Z}$

$XY\bar{Z}$

minterms

8 terms for 3 variables

$2^n$ minterms for n variables

minterms for XYZ

$\bar{X}\bar{Y}\bar{Z}$ ← truth table of $m_0$

$\bar{X}\bar{Y}Z$ ← truth table of $m_1$

| | X | Y | Z | min terms | symbol | $m_0$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $m_7$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\bar{X}\bar{Y}\bar{Z}$ | $m_0$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | $\bar{X}\bar{Y}Z$ | $m_1$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | $\bar{X}Y\bar{Z}$ | $m_2$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 1 | $\bar{X}YZ$ | $m_3$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4 | 1 | 0 | 0 | $X\bar{Y}\bar{Z}$ | $m_4$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | $X\bar{Y}Z$ | $m_5$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 6 | 1 | 1 | 0 | $XY\bar{Z}$ | $m_6$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 7 | 1 | 1 | 1 | $XYZ$ | $m_7$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

**max term** { A sum term that contains all the variables complemented or not

3 variables, 8 max terms:

$X + Y + Z$

$\overline{X} + Y + Z$

$\overline{X} + Y + \overline{Z}$

⋮

n variables → $2^n$ max terms

Maxterms for three variables X Y Z

$x+y+z$ ↑ $x+y+\overline{z}$ ↑

| X | Y | Z | Symbol | max term | $M_0$ | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ | $M_6$ | $M_7$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | $M_0$ | $X+Y+Z$ | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | $M_1$ | $X+Y+\overline{Z}$ | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | $M_2$ | $X+\overline{Y}+Z$ | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | $M_3$ | $X+\overline{Y}+\overline{Z}$ | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | $M_4$ | $\overline{X}+Y+Z$ | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | $M_5$ | $\overline{X}+Y+\overline{Z}$ | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | $M_6$ | $\overline{X}+\overline{Y}+Z$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | $M_7$ | $\overline{X}+\overline{Y}+\overline{Z}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

minterm is a function ($\neq 0$) that has the minimum number of 1s in its truth table

maxterm is a function ($\neq 1$) that has the max number of 1s in its truth table

$$\overline{m_3} = \overline{\bar{X}YZ} = X + \bar{Y} + \bar{Z} = M_3$$

$$\overline{M_3} = \overline{X + \bar{Y} + \bar{Z}} = \bar{X}YZ = m_3$$

$$\overline{m_j} = M_j \qquad \overline{M_j} = m_j$$

| | X | Y | Z | F | $\bar{F}$ |
|---|---|---|---|---|---|
| $m_0$ | 0 | 0 | 0 | 1 | 0 |
| $m_1$ | 0 | 0 | 1 | 0 | 1 |
| $m_2$ | 0 | 1 | 0 | 1 | 0 |
| $m_3$ | 0 | 1 | 1 | 0 | 1 |
| $m_4$ | 1 | 0 | 0 | 0 | 1 |
| $m_5$ | 1 | 0 | 1 | 1 | 0 |
| $m_6$ | 1 | 1 | 0 | 0 | 1 |
| $m_7$ | 1 | 1 | 1 | 1 | 0 |

$$F = m_0 + m_2 + m_5 + m_7$$

$$F = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z + XYZ$$

$$F(X,Y,Z) = \sum m(0,2,5,7)$$

$$\bar{F} = m_1 + m_3 + m_4 + m_6$$

$$\bar{F} = \sum m(1,3,4,6)$$

$$F = \bar{\bar{F}} = \overline{\sum m(1,3,4,6)} = \overline{m_1 + m_3 + m_4 + m_6}$$

$$= M_1 M_3 M_4 M_6$$

$$F = \prod M(1,3,4,6)$$

$$= (X + Y + \bar{Z})(X + \bar{Y} + \bar{Z})(\bar{X} + Y + Z)(\bar{X} + \bar{Y} + Z)$$

$$F(X,Y,Z) = \sum m(0,2,5,7) = \prod M(1,3,4,6)$$

sum of minterms — Product of max terms

$\Sigma m(0,1,2,3,4,5,6,7) = 1$

$\Pi M(0,1,2,3,4,5,6,7) = 0$

Examples:

Express $E = \bar{Y} + \bar{X}\bar{Z}$ as sum of min terms

| | X | Y | Z | E |
|---|---|---|---|---|
| $m_0$ | 0 | 0 | 0 | 1 |
| $m_1$ | 0 | 0 | 1 | 1 |
| $m_2$ | 0 | 1 | 0 | 1 |
| $m_3$ | 0 | 1 | 1 | 0 |
| $m_4$ | 1 | 0 | 0 | 1 |
| $m_5$ | 1 | 0 | 1 | 1 |
| $m_6$ | 1 | 1 | 0 | 0 |
| $m_7$ | 1 | 1 | 1 | 0 |

$E(X,Y,Z) = m_0 + m_1 + m_2 + m_4 + m_5$

$= \Sigma m(0,1,2,4,5)$

$= \Pi M(3,6,7)$

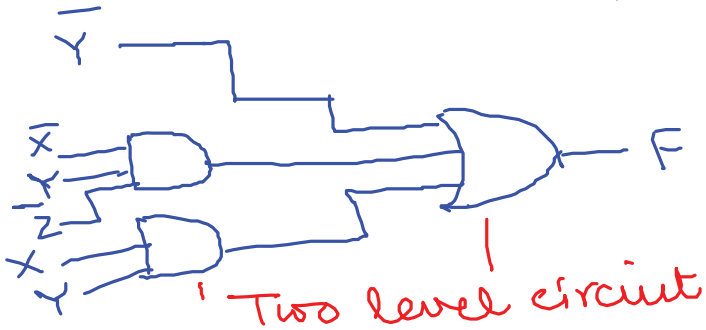
$\bar{E}(X,Y,Z) = \Sigma m(3,6,7)$

$= \Pi M(0,1,2,4,5)$

sum of minterms ⟶ sum of products

$F = \bar{Y} + \bar{X}Y\bar{Z} + XY$

not minterms



E(XYZ) = m0 + m1 + m2 + m4 + m5
= X'Y'Z' + X'Y'Z + X'YZ' + XY'Z' + XY'Z
sum of minterms

Simplify (identities or K-map)

E(XYZ) = Y' + X'Z'
= sum of products (not sum of minterms)

K-maps will be covered in the next lecture