Partial Fraction Expansion by Hand

$$Y(s) = \frac{N(s)}{D(s)}$$ ← proper fraction

Degree of $D(s)$ is greater than the degree of $N(s)$

$N(s)$, $D(s)$: polynomials

$$Y(s) = \frac{N(s)}{(s+P_1)(s+P_2)\cdots(s+P_N)}$$

poles $-P_1, -P_2, \cdots -P_N$

Distinct Poles:
(non repeated poles)

$$Y(s) = \cdots + \frac{A_n}{s+P_n} + \cdots$$

partial fraction exp.

Double Pole

$$Y(s) = \cdots + \frac{A_{n1}}{s+P_n} + \frac{A_{n2}}{(s+P_n)^2} + \cdots$$

partial fraction exp.

Summary:

Single pole P_n:

$$Y(s) = \cdots + \frac{A_n}{s + P_n} + \cdots$$

$$A_n = (s + P_n)\, Y(s)\Big|_{s=-P_n}$$

Double pole P_n:

$$Y(s) = \cdots + \frac{A_{n1}}{(s + P_n)} + \frac{A_{n2}}{(s + P_n)^2} + \cdots$$

$$A_{n1} = \frac{d}{ds}\left\{Y(s)(s + P_n)^2\right\}\Big|_{s=-P_n}$$

$$A_{n2} = Y(s)(s + P_n)^2\Big|_{s=-P_n}$$