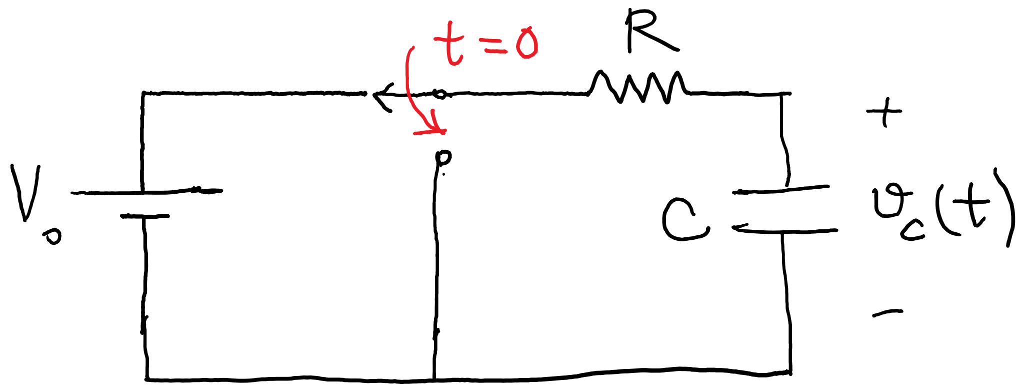
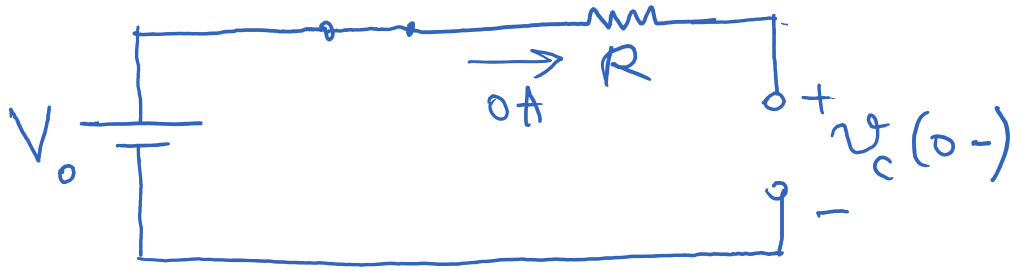
RC & RL First Order Circuits

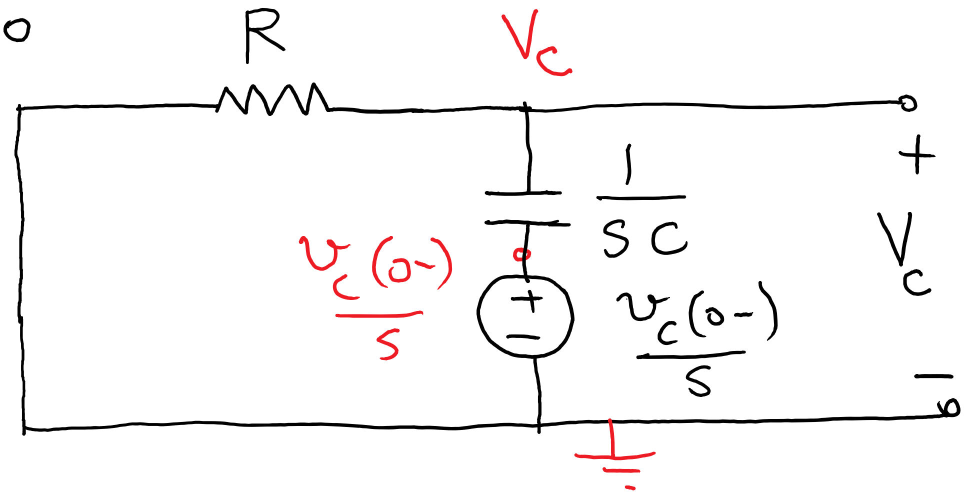


$t = 0-$ dc-conditions



$$v_c(0-) = V_0$$

$t > 0$



Node Analysis:

$$\frac{0 - V_c}{R} + \frac{\frac{v_c(0-)}{s} - V_c}{\frac{1}{sC}} = 0$$

$$V_c\left[\frac{1}{CR}+s\right] = v_c(0-)$$

$$V_c = \frac{v_c(0-)}{\frac{1}{CR}+s}$$

$$v_c(0-) = V_0$$

$$v_c(t) = V_0 e^{-\frac{t}{\boxed{CR}}} u(t)$$

$t > 0$

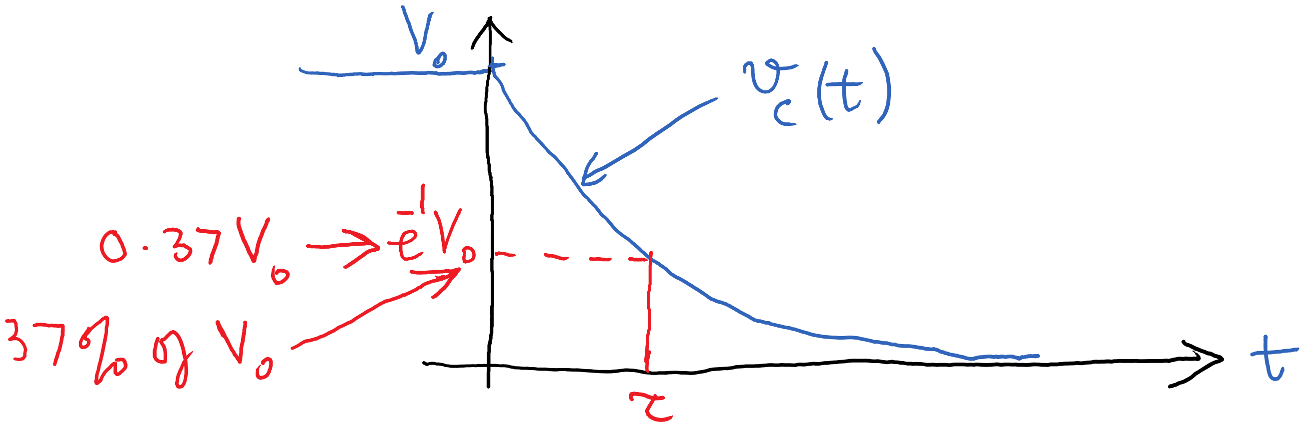
$$\tau = CR$$

(time constant (seconds))

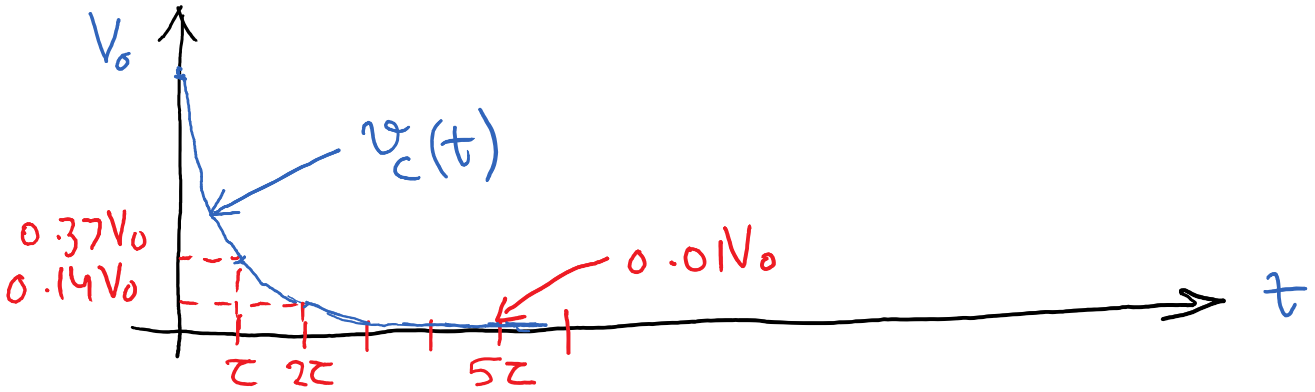
$$v_c(t) = V_0 e^{-\frac{t}{\tau}} u(t)$$

$t > 0$

time constant: Measure of how fast or how slow a circuit responds to a change



t	$v_c(t)$
τ	$.37V_0$
2τ	$.14V_0$
$\vdots$	
5τ	$0.01V_0 \leftarrow$ 1% of V_0



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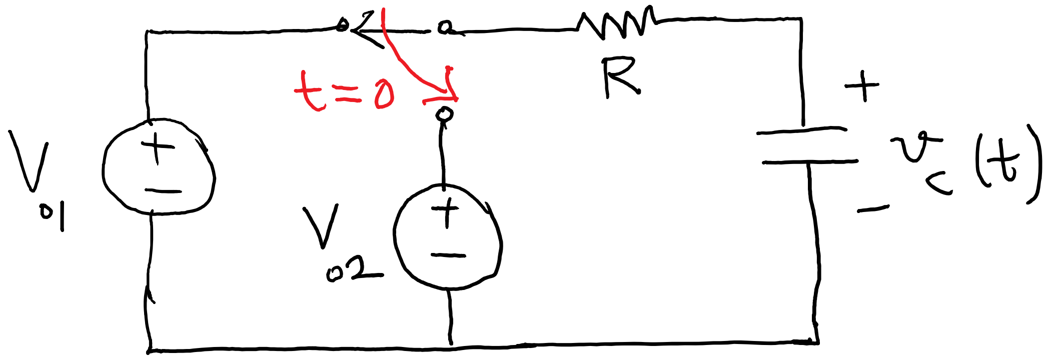
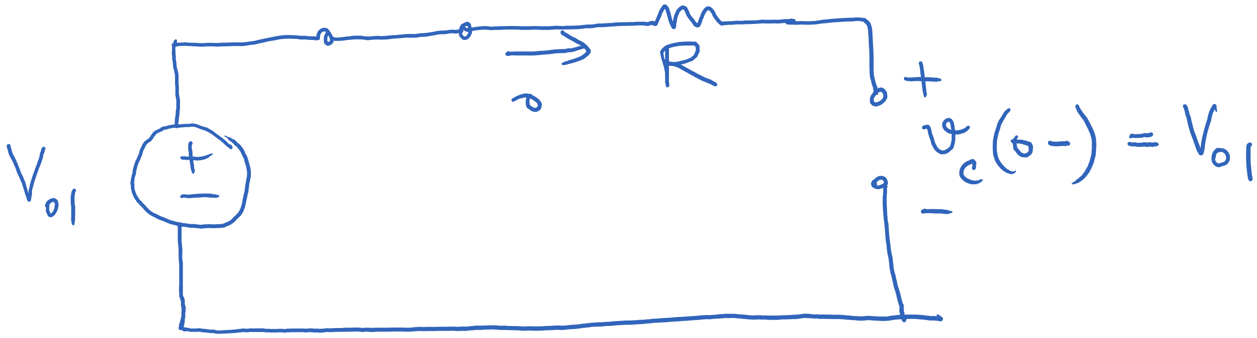
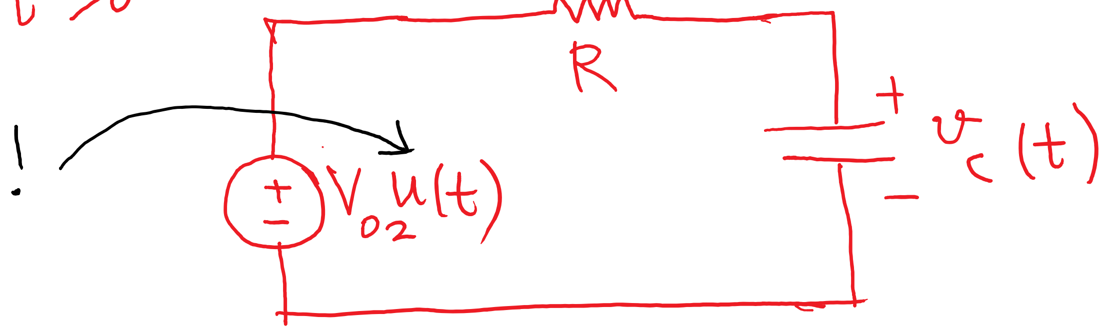


$t = 0-$ dc-conditions

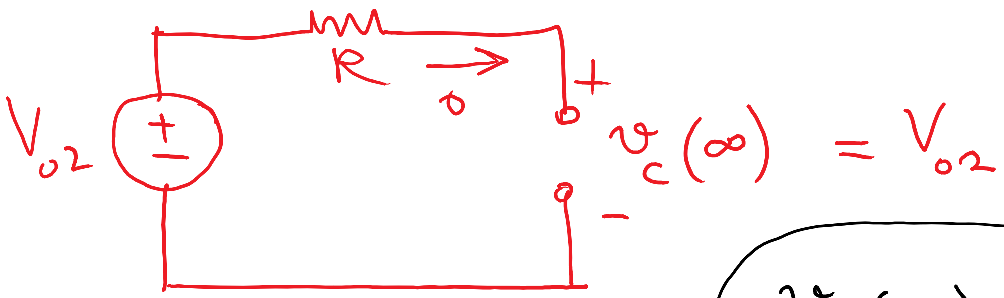


$v_c(0-) = V_{01}$

$t > 0$



$t \rightarrow \infty$ dc-conditions



$v_c(\infty) = V_{02}$

$$\boxed{v_c(\infty) = V_{02}}$$

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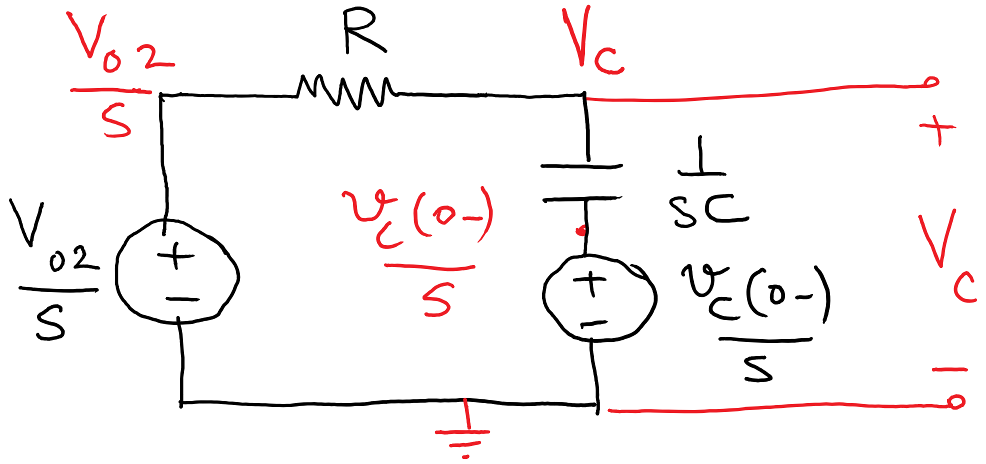


$v_c(0-) = V_{01}$

Node Analysis :

$$\frac{\frac{V_{02}}{s} - V_c}{R} + \frac{\frac{v_c(0-)}{s} - V_c}{\frac{1}{sC}} = 0$$

$$V_c\left(\frac{1}{RC} + s\right) = \frac{1}{RC}\frac{V_{02}}{s} + V_{01}$$

$$V_c = \frac{1}{RC}\frac{V_{02}}{s\left(s+\frac{1}{RC}\right)} + \frac{V_{01}}{s+\frac{1}{RC}}$$

$$\frac{1}{s\left(s+\frac{1}{RC}\right)} = \frac{A}{s} + \frac{B}{s+\frac{1}{RC}}$$

$$A = \left.\frac{1}{s+\frac{1}{RC}}\right|_{s=0} = RC$$

$$B = \left.\frac{1}{s}\right|_{s=-\frac{1}{RC}} = -RC$$

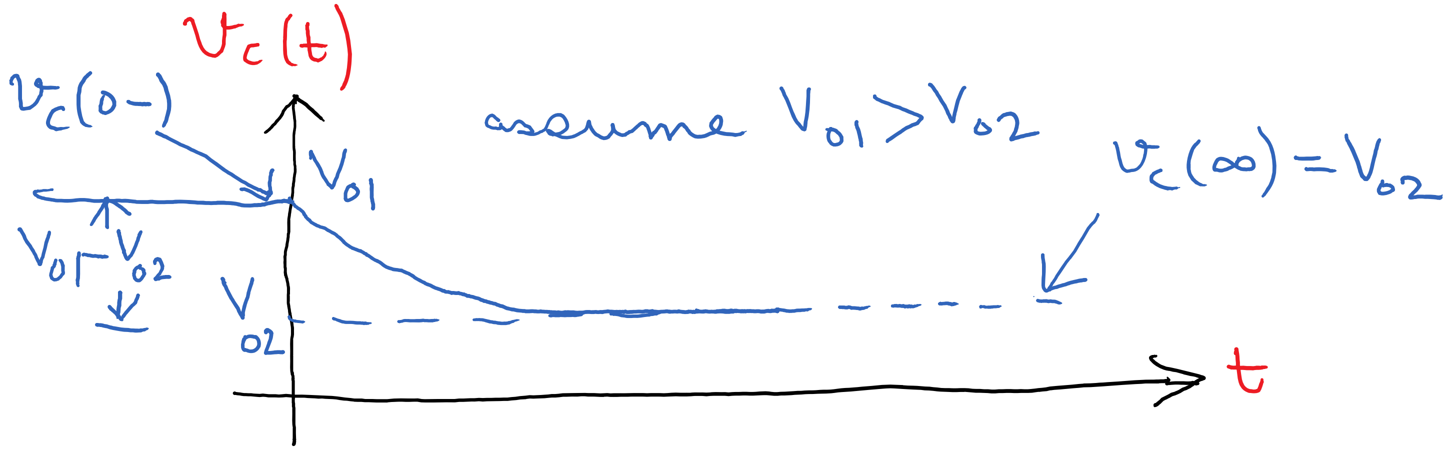
$$\frac{1}{s\left(s+\frac{1}{RC}\right)} = \frac{RC}{s} - \frac{RC}{s+\frac{1}{RC}}$$

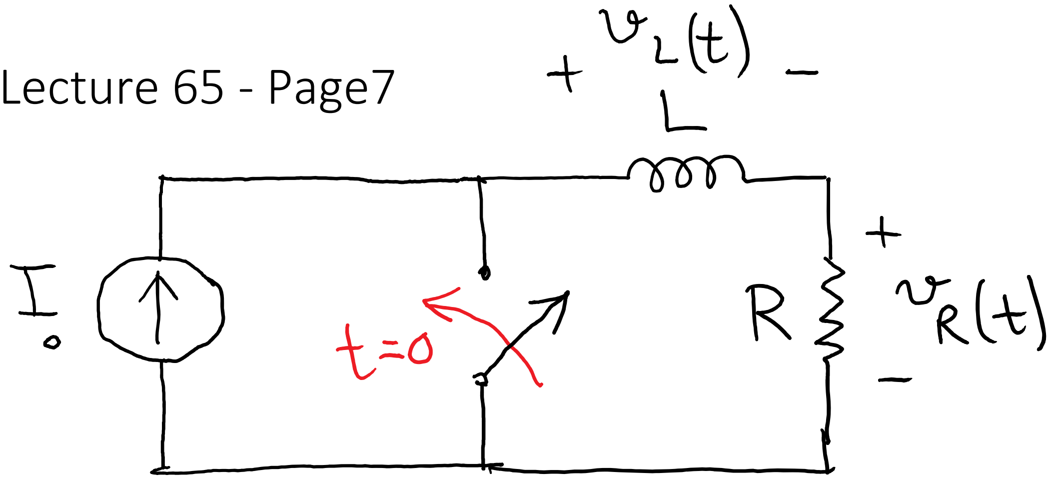
$$V_c = \frac{V_{02}}{s} - \frac{V_{02}}{s+\frac{1}{RC}} + \frac{V_{01}}{s+\frac{1}{RC}}$$

$$v_c(t) = V_{02}u(t) - V_{02}e^{-\frac{t}{RC}}u(t) + V_{01}e^{-\frac{t}{RC}}u(t)$$

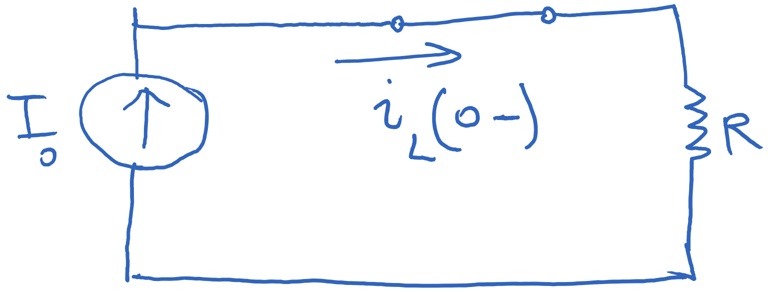
$$v_c(t) = \left[(V_{01}-V_{02})e^{-\frac{t}{\tau}} + V_{02}\right]u(t)$$

$\tau = RC$ time constant (sec.)





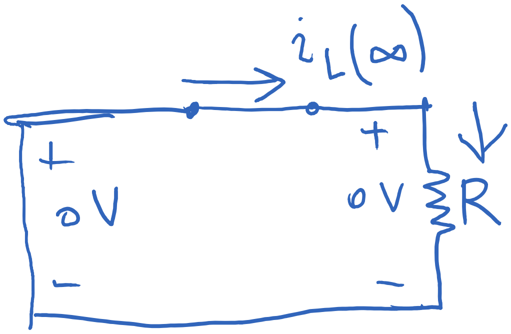
$t = 0-$ dc-conditions



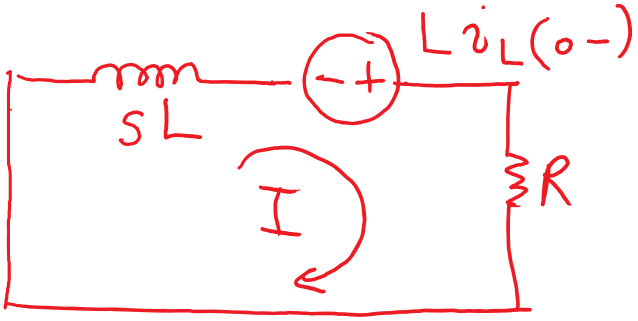
$i_L(0-) = I_0$

$t > 0$

$t \to \infty$ dc-conditions



$i_L(\infty) = 0$



Mesh Analysis: $sLI - Li_L(0-) + RI = 0$

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$$I(sL+R) = L i_L(0-) \qquad i_L(0-) = I_0$$

$$I = \frac{LI_0}{sL+R} = \frac{I_0}{s+\frac{R}{L}}$$

$$\boxed{V_L = sLI - Li_L(0-)}$$

$$V_L = \frac{sL}{s+\frac{R}{L}} I_0 - LI_0$$

$$= LI_0\left(\frac{s}{s+\frac{R}{L}} - 1\right)$$

$$\begin{array}{r|l} & 1 \\ \hline s+\frac{R}{L} & \cancel{s} \\ & -\cancel{s} \pm \frac{R}{L} \\ \hline & -\frac{R}{L} \end{array} \qquad \frac{s}{s+\frac{R}{L}} = 1 - \frac{\frac{R}{L}}{s+\frac{R}{L}}$$

$$V_L = LI_0\left(\cancel{1} - \frac{\frac{R}{L}}{s+\frac{R}{L}} - \cancel{1}\right)$$

$$V_L = -I_0R\frac{1}{s+\frac{R}{L}}$$

$$v_L(t) = -I_0Re^{-\frac{R}{L}t}u(t) = -I_0Re^{-\frac{t}{\tau}}u(t)$$

$\tau = \frac{L}{R}$ time constant

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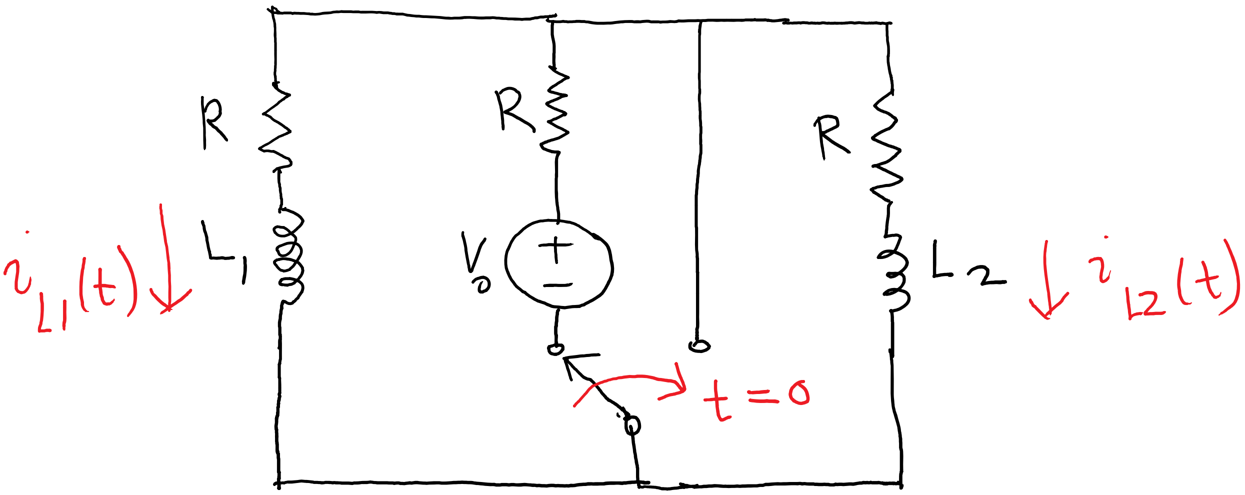
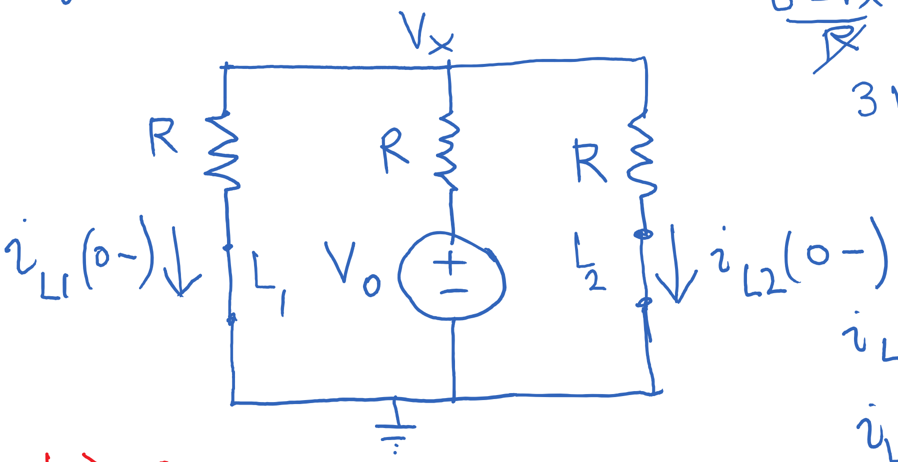


t = 0- dc conditions



$$\frac{0-V_x}{\cancel{R}}+\frac{V_0-V_x}{\cancel{R}}+\frac{0-V_x}{\cancel{R}}=0$$

$$3V_x = V_0$$

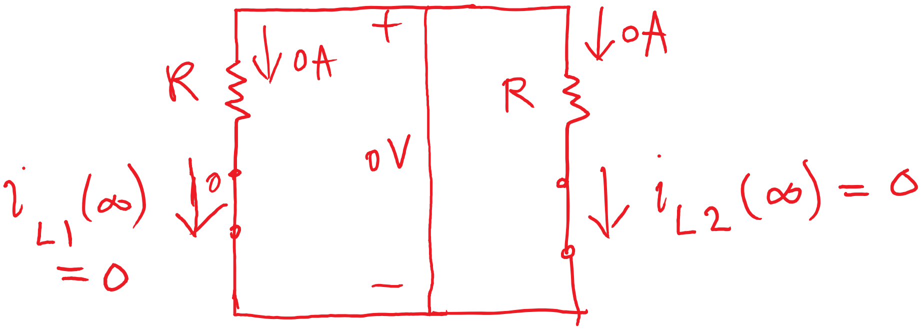
$$V_x = \frac{V_0}{3}$$

$$i_{L1}(0-)=\frac{V_x}{R}=\frac{1}{3}\frac{V_0}{R}$$

$$i_{L2}(0-)=\frac{V_x}{R}=\frac{1}{3}\frac{V_0}{R}$$

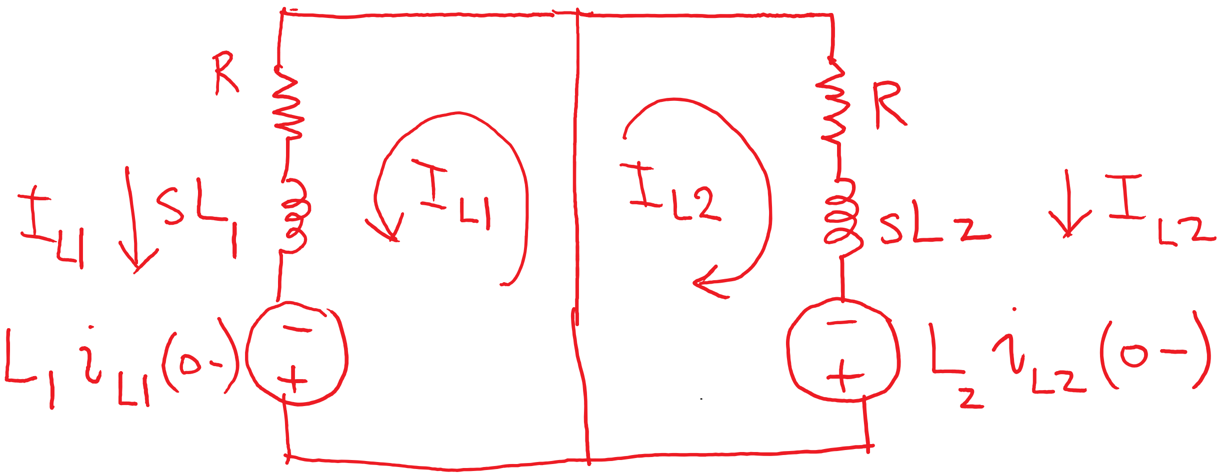
t > 0

t → ∞ dc-conditions



$$i_{L1}(\infty) = 0$$

$$i_{L2}(\infty) = 0$$



$$RI_{L2} + sL_2 I_{L2} - L_2 i_{L2}(0-) = 0$$

$$i_{L2}(0-) = \frac{1}{3}\frac{V_0}{R}$$

$$(R + sL_2) I_{L2} = \frac{1}{3} L_2 \frac{V_0}{R}$$

$$I_{L2} = \frac{1}{3}\frac{V_0}{R} \frac{1}{s + \frac{R}{L_2}}$$

$$i_{L2}(t) = \frac{1}{3}\frac{V_0}{R} e^{-\frac{R}{L_2}t} u(t) = \frac{1}{3}\frac{V_0}{R} e^{-\frac{t}{\tau_2}} u(t)$$

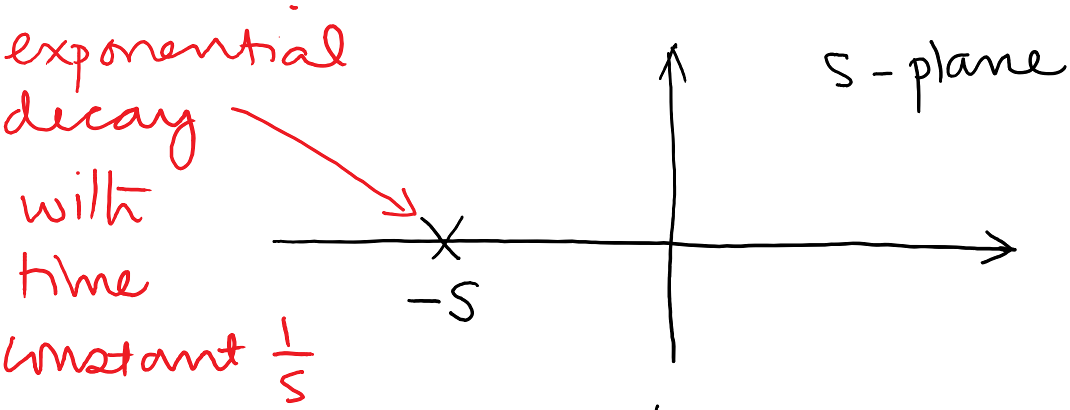
$$\tau_2 = \frac{L_2}{R}$$

similarly

$$i_{L1}(t) = \frac{1}{3}\frac{V_0}{R} e^{-\frac{t}{\tau_1}} u(t) \quad \tau_1 = \frac{L_1}{R}$$

In general for a stable first order LTI system

single real pole $-s$



$\downarrow$

exponential decay

$e^{-st}u(t)$

with time constant

$\tau = \frac{1}{s}$