Unit 4

Two-Level circuit optimization :

Karnaugh Map K-map

Two variable map X, Y

$X \backslash Y$	0	1
0	$\overline{X}\,\overline{Y}$ m_0	$\overline{X}Y$ m_1
1	$X\overline{Y}$ m_2	XY m_3

$$F = X\overline{Y} + XY = \sum m(2,3)$$

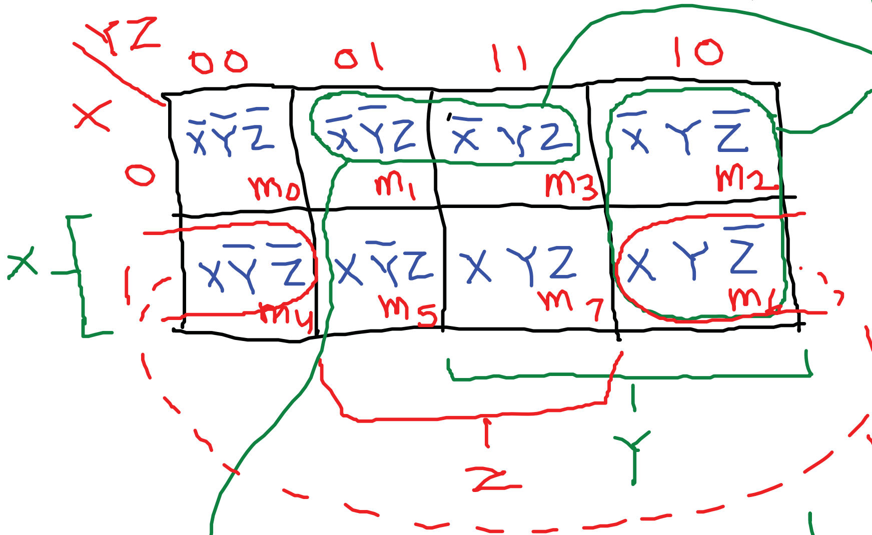
	$X \backslash Y$	0 ($\overline{Y}$)	1 (Y)
$\overline{X}$	0		
X	1	1	1

$$X\overline{Y} + XY = X(\overbrace{Y + \overline{Y}}^{=1}) = X$$

rectangle $= X$

$$\boxed{F = X}$$

Three variable map



rectangles

minterms differ by only a single variable

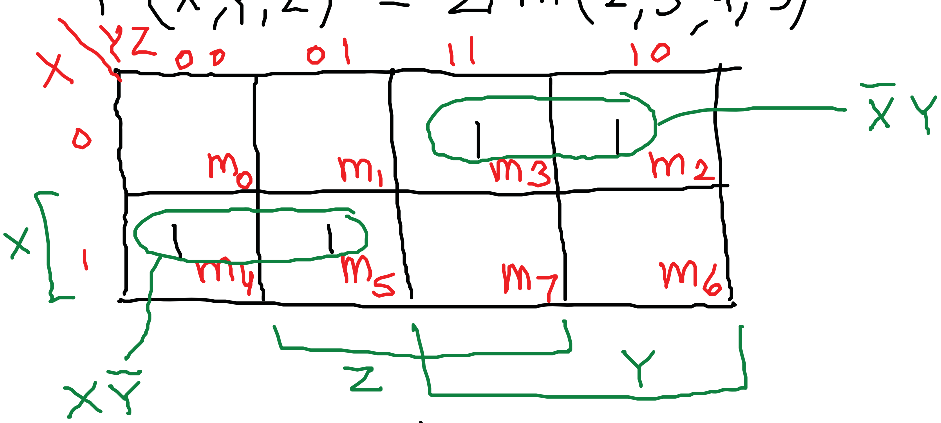
rectangle

$XY\bar{Z} + X\bar{Y}\bar{Z}$

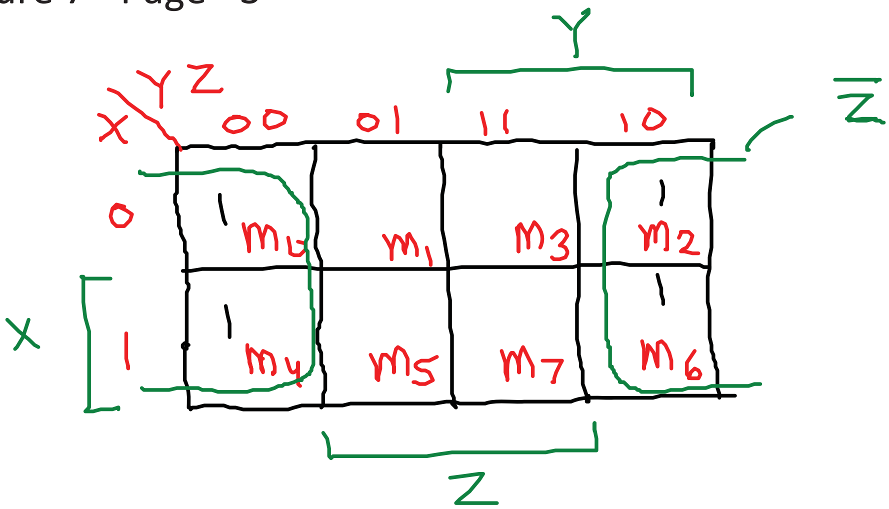
$X\bar{Z}(Y+\bar{Y}) = X\bar{Z}$

$$\bar{X}\bar{Y}Z + \bar{X}YZ = \bar{X}Z\underbrace{(\bar{Y}+Y)}_{=1} = \bar{X}Z$$

$$F(X,Y,Z) = \sum m(2,3,4,5)$$



$$F(X,Y,Z) = X\bar{Y} + \bar{X}Y$$



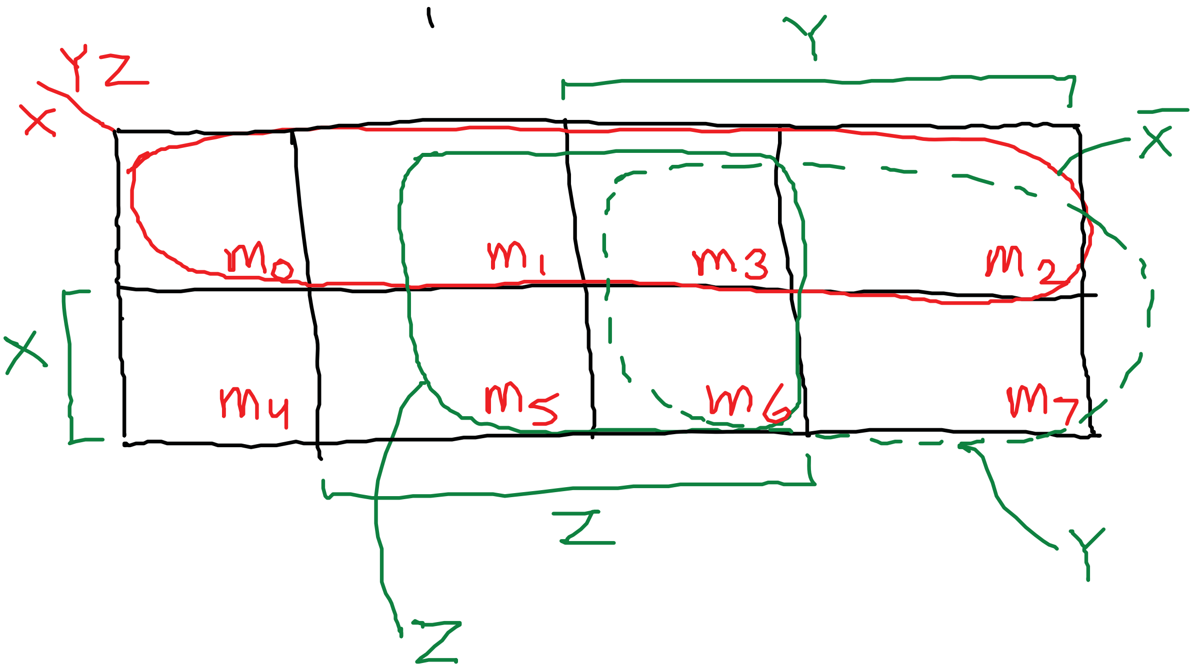
$$m_0 + m_2 + m_4 + m_6$$

$$\bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z}$$

$$= \bar{Z}(\bar{X}\bar{Y} + \bar{X}Y + X\bar{Y} + XY)$$

$$= \bar{Z}[\bar{X}(\underbrace{\bar{Y}+Y}_{=1}) + X(\underbrace{\bar{Y}+Y}_{=1})]$$

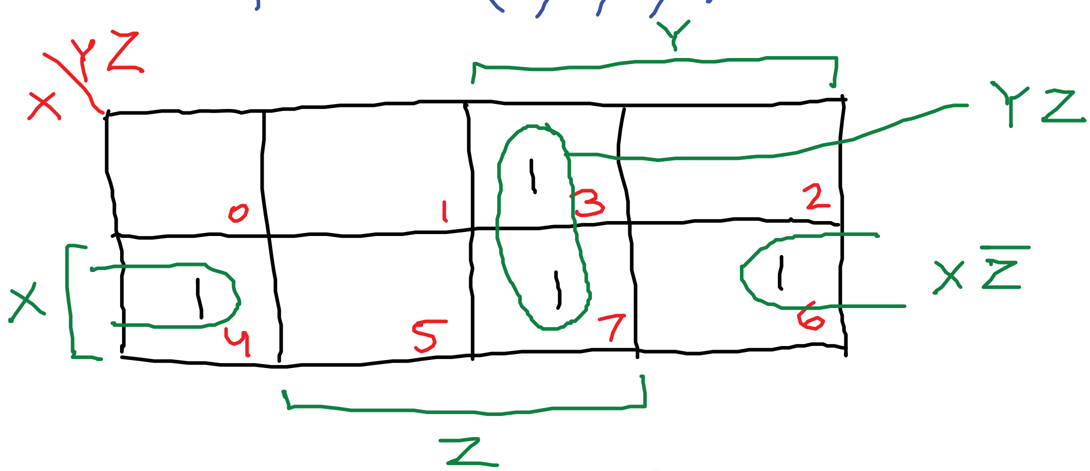
$$= \bar{Z}[\underbrace{\bar{X}+X}_{=1}] = \bar{Z}$$



- A square represents a minterm of three literals
- A rectangle of two squares represents a product term of two literals
- A rectangle of four squares represents a product term of one literal
- A rectangle of eight squares equals to logic 1

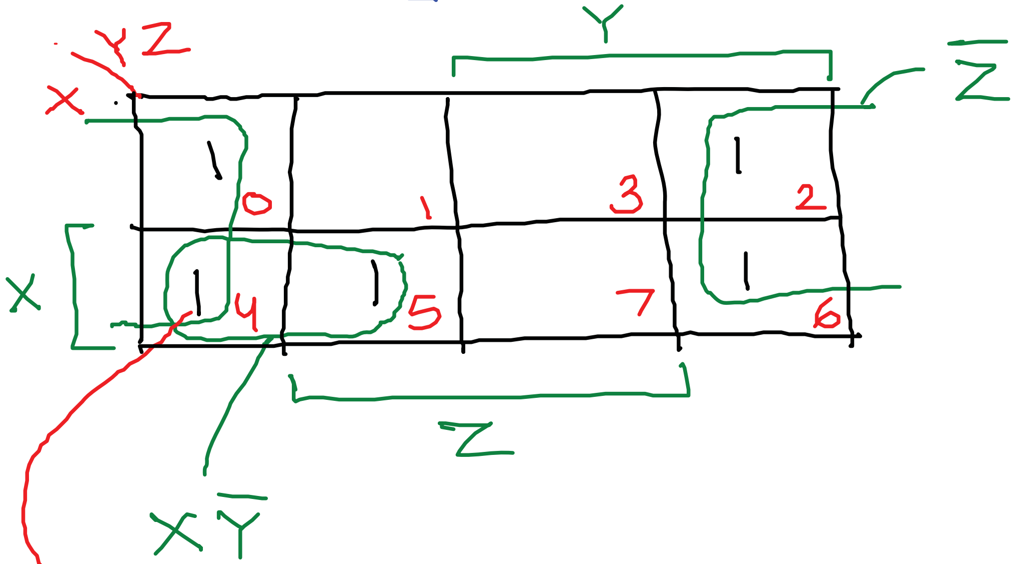
Example:
Simplify with maps

$$F_1 = \sum m(3,4,6,7)$$



$$F_1 = YZ + X\bar{Z}$$

Simplify $F_2 = \sum m(0,2,4,5,6)$

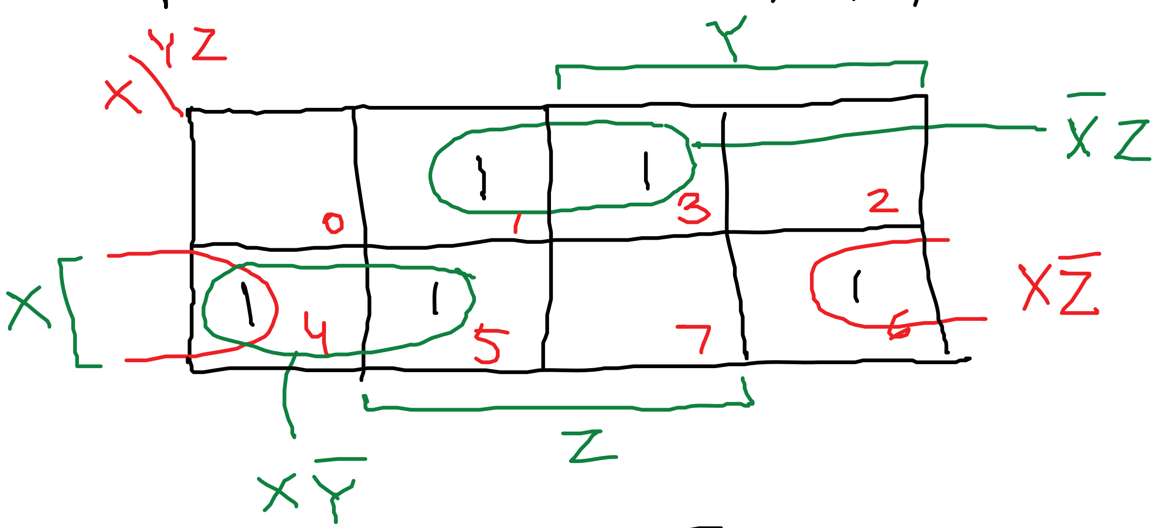


this is permitted! $X+X=X$

$m_0 + m_2 + m_6 + m_4 + m_5$

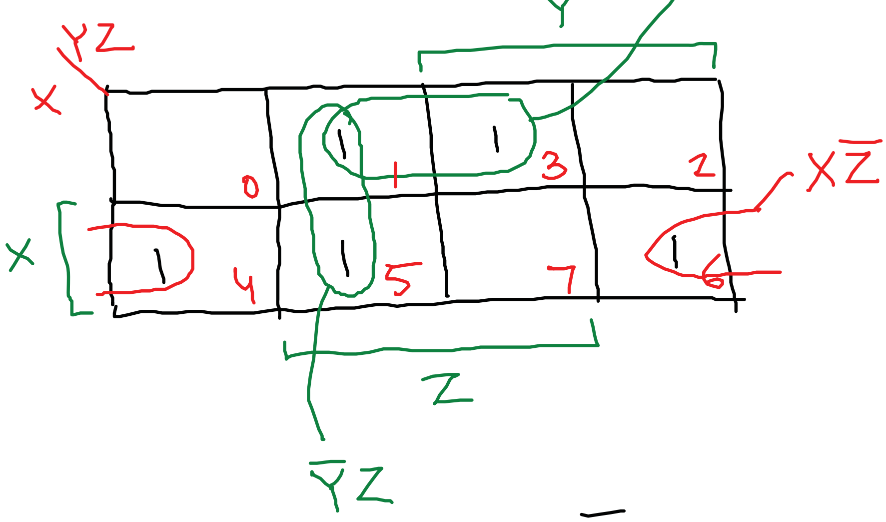
$\underbrace{m_0 + m_2 + m_6 + m_4}_{\bar{Z}} + \underbrace{m_4 + m_5}_{X\bar{Y}}$

Example $F = \sum m(1,3,4,5,6)$



$F = \bar{X}Z + X\bar{Y} + X\bar{Z}$

Alternate way



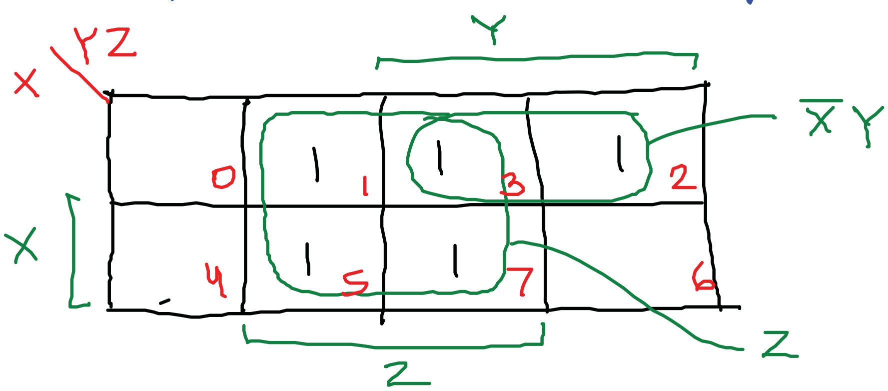
$$F = \bar{X}Z + \bar{Y}Z + X\bar{Z}$$

Example

$$F = \bar{X}Z + \bar{X}Y + X\bar{Y}Z + YZ$$

not a sum of minterms

first convert to sum of min terms → truth table



$$F = Z + \bar{X}Y$$