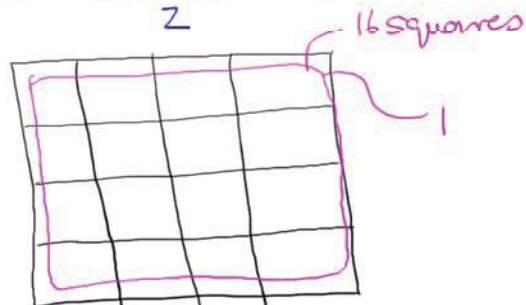
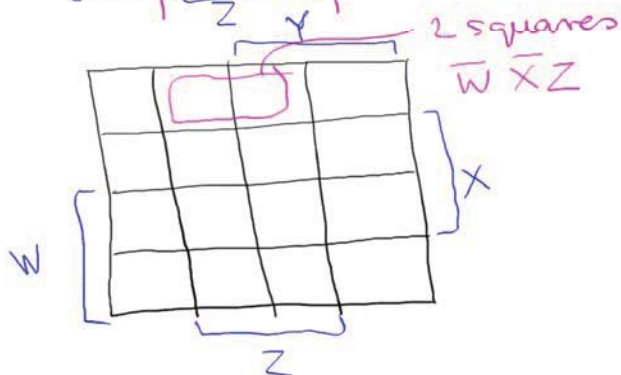
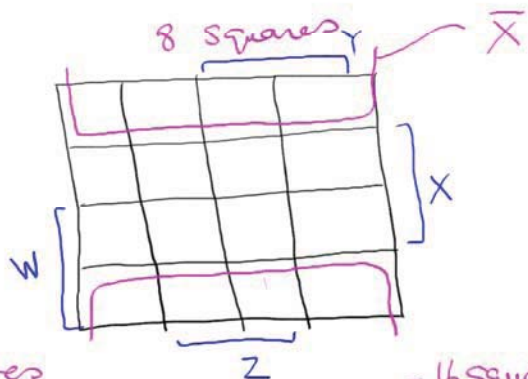
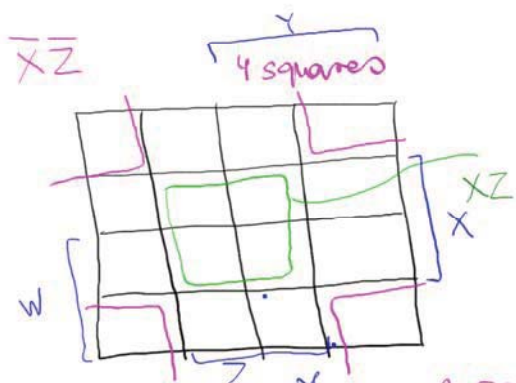
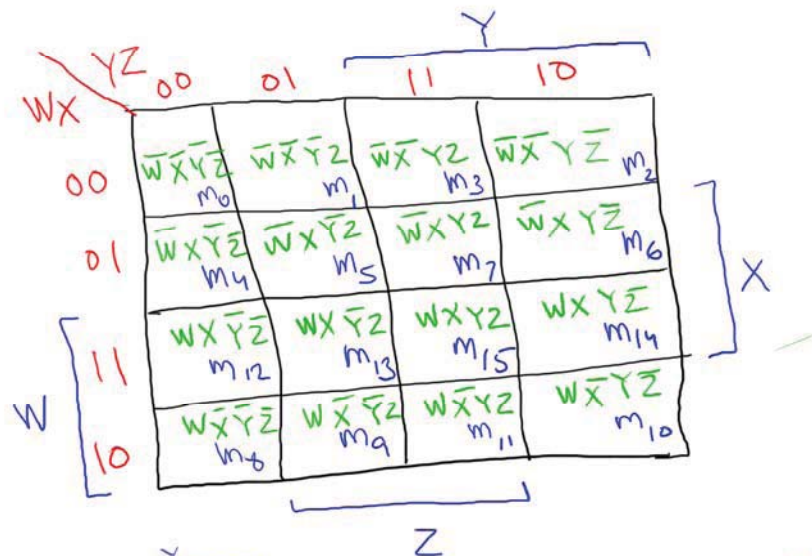


Four variable maps:

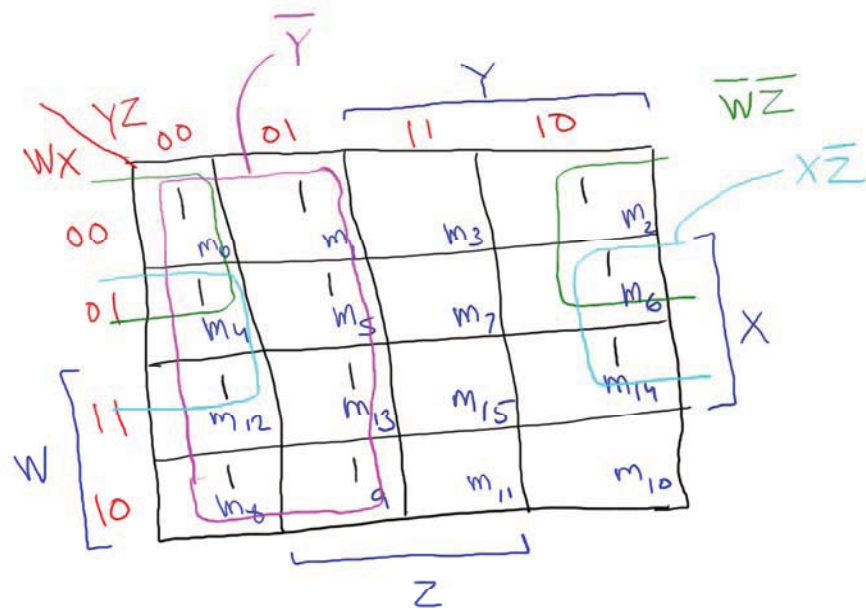
four variables $W, X, Y, Z \rightarrow 16$ min terms $\rightarrow 16$ squares



1 Square $\rightarrow$ 4 literals
 rectangle of 2 squares $\rightarrow$ 3 "
 " " 4 " $\rightarrow$ 2 literals
 " " 8 " $\rightarrow$ 1 "

Example: Simplify

$$F(W, X, Y, Z) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

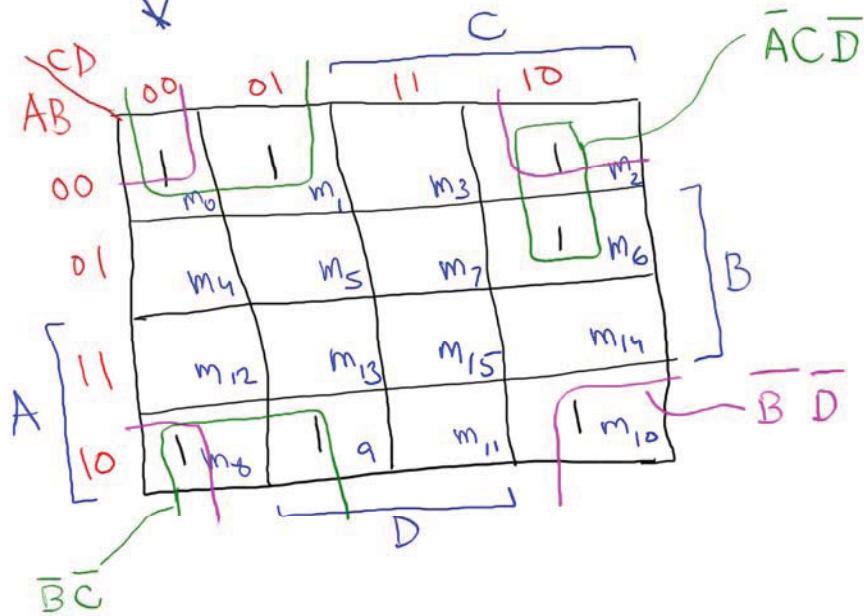


$$F = \bar{Y} + \bar{W}\bar{Z} + X\bar{Z}$$

Example: Simplify $F = \overline{A}\overline{B}\overline{C} + \overline{B}C\overline{D} + A\overline{B}\overline{C} + \overline{A}BC\overline{D}$

truth table

not min terms



$$F(A,B,C,D) = \overline{B}\overline{D} + \overline{B}\overline{C} + \overline{A}C\overline{D}$$

Product - of - Sums Optimization

$\overline{F} \rightarrow$ sum of min terms

$\downarrow$ simplify (K-map)

$\overline{F} \rightarrow$ sum of products

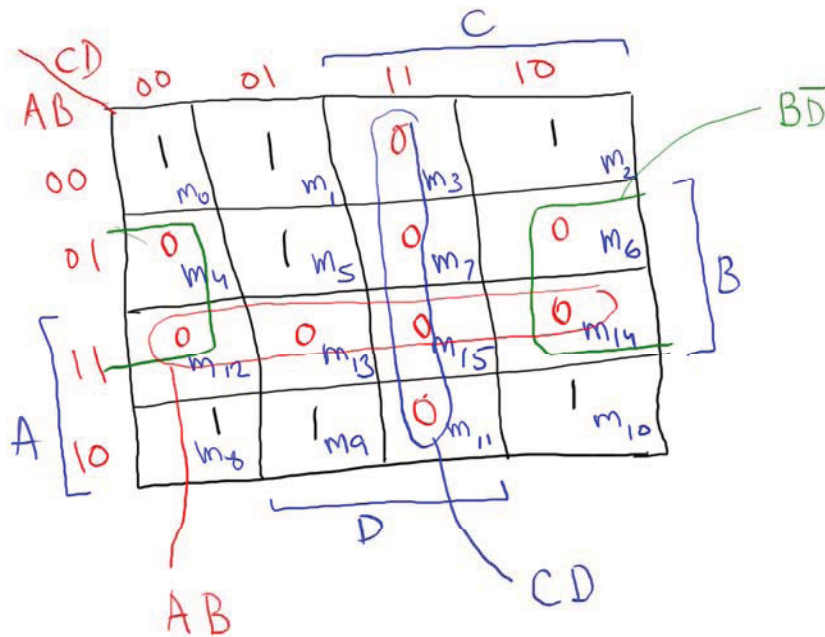
$F = \overline{\overline{F}} \rightarrow$ sum of products

$\downarrow$
product of sums

Example:

Simplify to product of sums

$$F(A, B, C, D) = \sum m(0, 1, 2, 5, 8, 9, 10)$$



$$\overline{F} = AB + CD + \overline{B}D$$

$$F = (\overline{A} + \overline{B})(\overline{C} + \overline{D})(\overline{B} + D)$$

Example $F = (\overline{A} + \overline{B} + C)(B + D)$

$$\overline{F} = ABC + \overline{B}\overline{D}$$

↓
sum of minterms

↓ simplify

sum of products

$$F = \overline{\overline{F}} = \overline{\text{sum of products}}$$

= product of sums

Don't care conditions:

Incompletely
specified
function

$$\begin{cases} F(A, B, C, D) = \sum m(1, 3, 7, 11, 15) \\ d(A, B, C, D) = \sum m(0, 2, 5) \end{cases}$$

↘ don't care

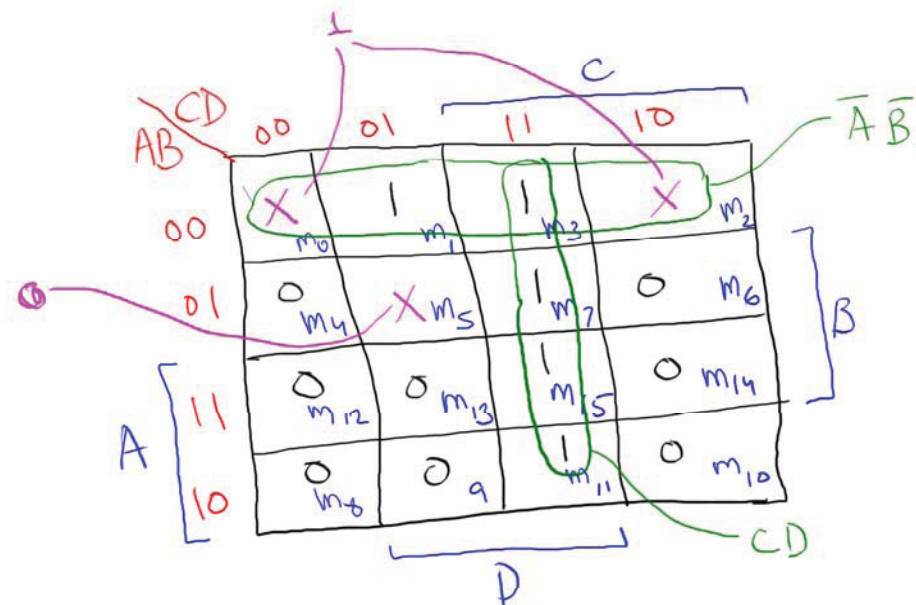
AB \ CD		C			
		00	01	11	10
A	00	X _{m₀}	1 _{m₁}	1 _{m₃}	X _{m₂}
	01	0 _{m₄}	X _{m₅}	1 _{m₇}	0 _{m₆}
	11	0 _{m₁₂}	0 _{m₁₃}	1 _{m₁₅}	0 _{m₁₄}
	10	0 _{m₈}	0 _{m₉}	1 _{m₁₁}	0 _{m₁₀}

D

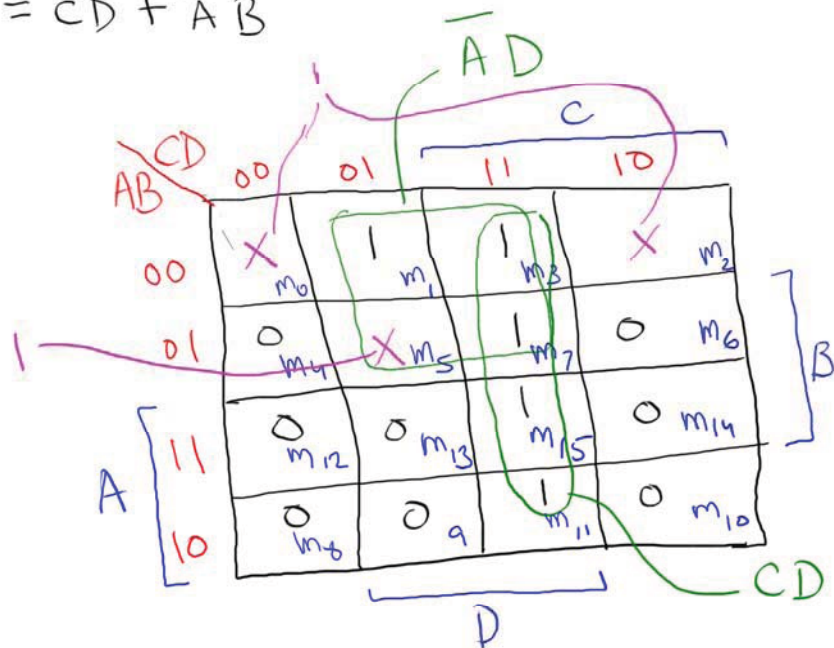
B

X → can be 1
or 0,
we don't
care!

choose x to be 1 or 0 $\rightarrow$ whatever gives the best simplification



$$F = CD + \overline{A}\overline{B}$$



$$F = CD + \overline{A}D$$

Note: $CD + \bar{A}\bar{B} \neq CD + \bar{A}D$
 they have different truth tables!

optimized product-of-sums expression

	C			
	00	01	11	10
AB	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	0 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	0 m_{10}

$$\bar{F} = \bar{D} + \bar{A}\bar{C}$$

$$F = \overline{\bar{F}} = D(\bar{A} + C)$$