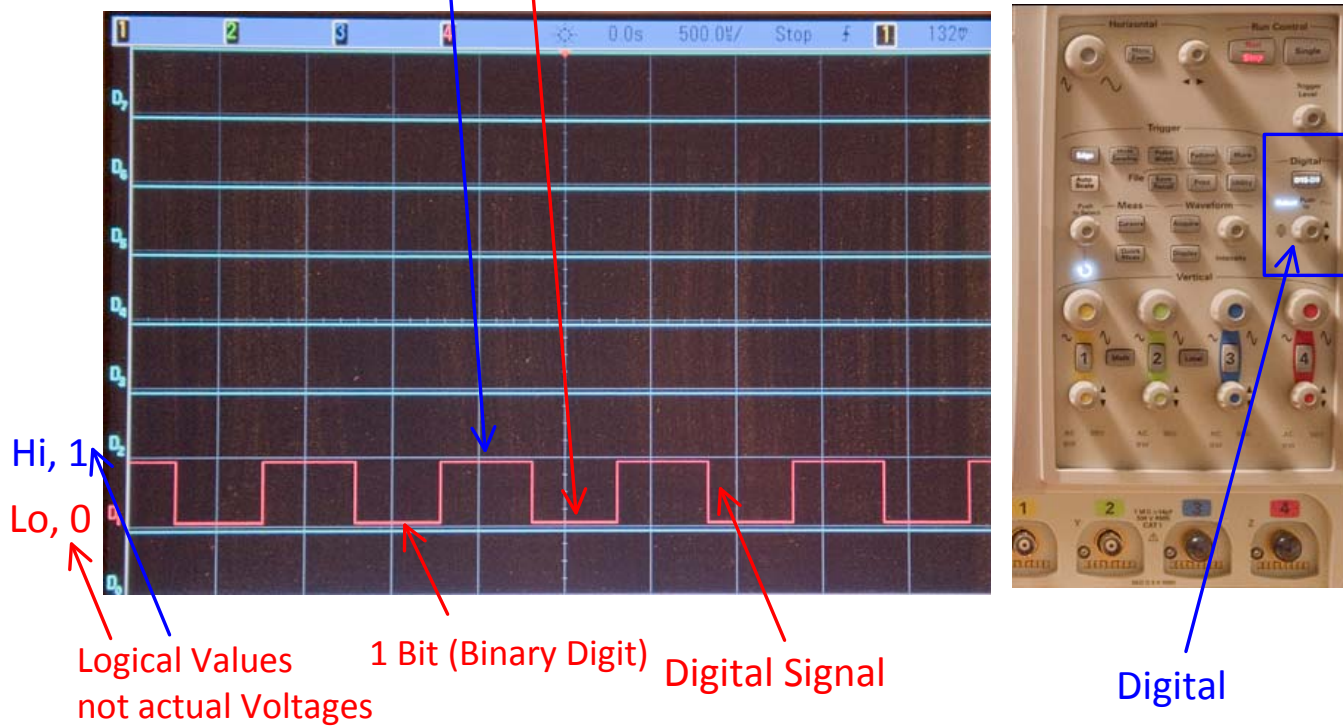
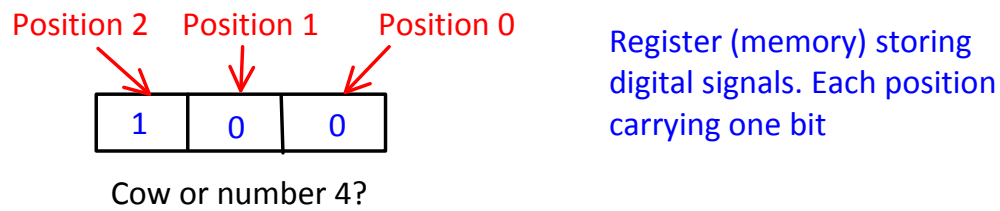
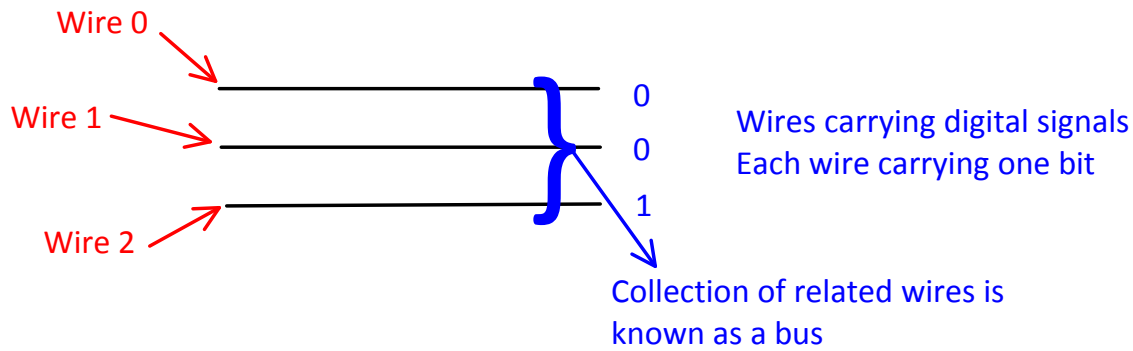


Interpretation

Digital signal is Hi or 1 if analog signal voltage > blue voltage

Digital signal is Lo or 0 if analog signal voltage < red voltage





A Computer Code

- 0 0 0 -> Pig
- 0 0 1 -> Dog
- 0 1 0 -> Cat
- 0 1 1 -> Mouse
- 1 0 0 -> Cow**
- 1 0 1 -> Sheep
- 1 1 0 -> Skunk
- 1 1 1 -> Horse

Binary Numbers

$$1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = \text{decimal number } 4$$

Number Systems

decimal number system
 ↑ every day arithmetic

$$724.5 = 7 \times 10^2 + 2 \times 10^1 + 4 \times 10^0 + 5 \times 10^{-1}$$

Base 10
Radix 10

decimal point
↓

$$A_{n-1} A_{n-2} \dots A_2 A_1 A_0 \cdot A_{-1} A_{-2} \dots A_{-m+1} A_{-m}$$

$$A_i \in \{0, 1, 2, \dots, 9\}$$

10 digits

$$A_{n-1} \times 10^{n-1} + A_{n-2} \times 10^{n-2} + \dots + A_1 \times 10^1 + A_0 \times 10^0 + A_{-1} \times 10^{-1} + \dots + A_{-m} \times 10^{-m}$$

In general base (radix r) number

r digits 0, 1, 2, ..., r-1

$$A_{n-1} \dots A_1 A_0 \cdot A_{-1} A_{-2} \dots A_{-m}$$

radix point

$$A_{n-1} \times r^{n-1} + \dots + A_1 \times r^1 + A_0 \times r^0 + A_{-1} \times r^{-1} + \dots + A_{-m} \times r^{-m}$$

$$(A_{n-1} A_{n-2} \dots A_1 A_0 \cdot A_{-1} A_{-2} \dots A_{-m})_r \leftarrow \text{optional}$$

decimal $32.65 = (32.65)_{10}$

radix 8
(octal) $32.65 = (32.65)_8$

not the same number

$$(312.4)_5 = 3 \times 5^2 + 1 \times 5^1 + 2 \times 5^0 + 4 \times 5^{-1}$$

$$= (75)_{10} + (5)_{10} + (2)_{10} + (0.8)_{10} = (82.8)_{10}$$

Binary numbers
base, radix = 2

$$A_{n-1} \dots A_1 A_0 . A_{-1} A_{-2} \dots A_{-m}$$

$$A_i \in \{0, 1\} \quad A_i \rightarrow \text{binary digit (bit)}$$

Binary number

$$\rightarrow 101101.1101 = (101101.1101)_2$$

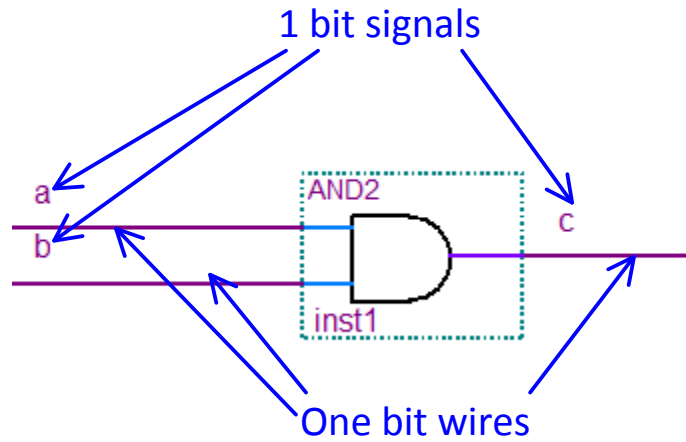
↑ bit ↑ bit ↑ binary point

Converting from binary to decimal

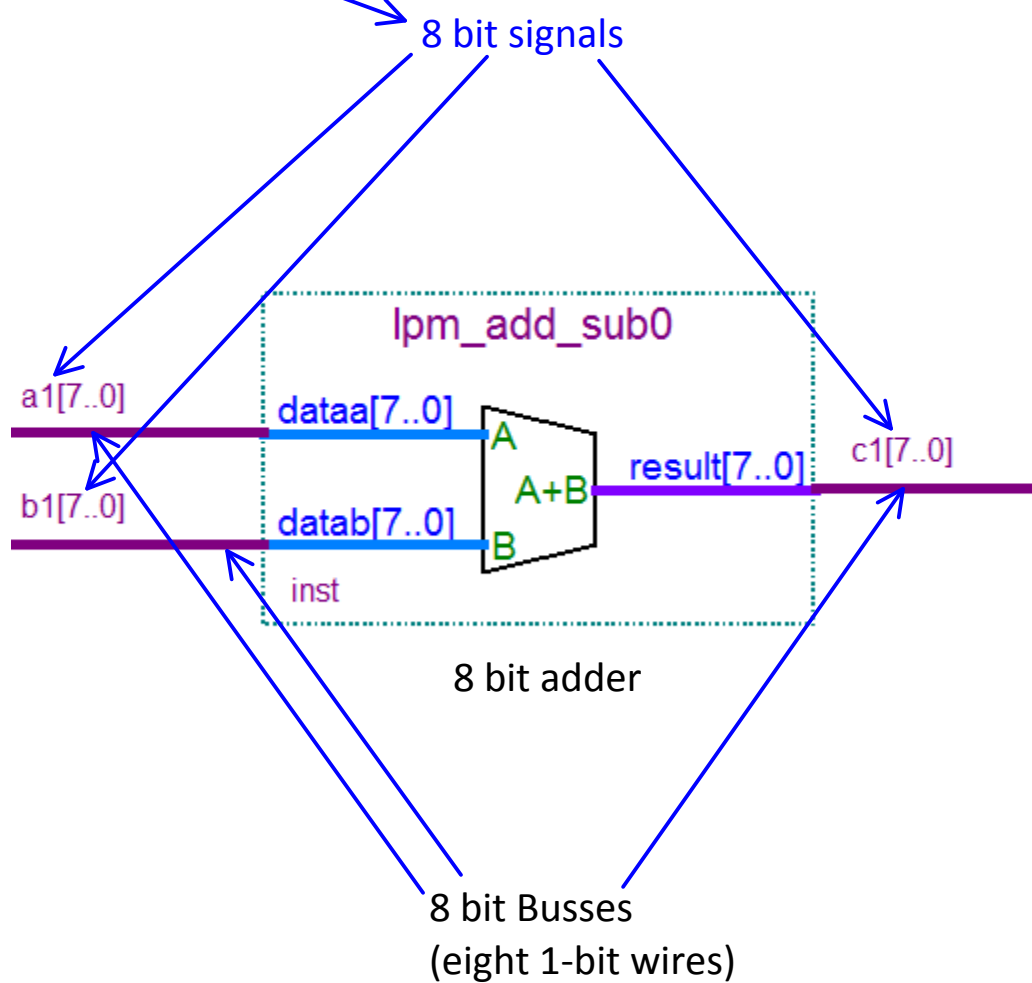
$$(110101.11) = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$+ 1 \times 2^{-1} + 1 \times 2^{-2} = (53.75)_{10}$$

5 4 3 2 1 0 -1 -2



could for example represent an 8 bit unsigned integer



Octal & Hexadecimal numbers

Octal radix = 8 $8 = 2^3$ \rightarrow radix is a power of 2

Hexadecimal radix = 16 $16 = 2^4$

useful to represent binary numbers easily in compact form (easier on human beings than binary representation)

Octal numbers: $\underbrace{\hspace{10em}}_{\text{8 octal digits}}$
0, 1, 2, 3, 4, 5, 6, 7

$$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1}$$

$$= (87.5)_{10}$$

Hexadecimal numbers

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

$\begin{matrix} 10 & 11 & 12 & 13 & 14 & 15 \\ \nearrow & \nearrow & \nearrow & \nearrow & \nearrow & \nearrow \end{matrix}$

16 hexadecimal digits

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0$$

$$= (46687)_{10}$$

Important table for converting bases among binary, octal and hexadecimal number representation

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Converting from binary to octal (use table)

groups of three

pad zeros if required

add zeros here if needed to complete group of 3

start from here

$(26153.7406)_8$

Converting from binary to hexadecimal

pad zeros if required

start from here

pad zeros if required

$(2C6B.F\phi6)_{16}$

Converting from octal to binary

this zero is not needed

$(673.12)_8 = 110111011.001010$

Converting from hexadecimal to binary

discard

(use Table)

$(3A6.C)_{16} = 001110100110.110$

$(1110100110.11)_2$