Number Systems

Binary numbers
base, radix = 2

$A_{n-1} \quad A_1 A_0 \quad A_{-1} A_{-2} \quad A_{-m}$

$A_i \in \{0, 1\}$ $\quad A_i \rightarrow$ binary digit (bit)

Binary number
$\hookrightarrow 1011.01\ 1101 = (101101\ 1101)_2$

bit bit binary point

Converting from binary to decimal

$$(110101.11)_2 = 1\times2^5 + 1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$$
$$+ 1\times2^{-1} + 1\times2^{-2} = (53.75)_{10}$$

5 4 3 2 1 0 −1 −2

Notation: Code Composer Studio

$(110101)_2 \longrightarrow$ 110101b

binary

msd (most significant digit)

110101

lsd (least significant digit)

decimal integer to binary

$$(41)_{10} = (?)_2$$

$$\frac{41}{2} = 20 + \frac{1}{2}$$

$$\frac{20}{2} = 10 + \frac{0}{2}$$

$$\frac{10}{2} = 5 + \frac{0}{2}$$

$$\frac{5}{2} = 2 + \frac{1}{2} \qquad (1\ 0\ 1\ 0\ 0\ 1)_2$$

$$\frac{2}{2} = 1 + \frac{0}{2}$$

$$\frac{1}{2} = 0 + \frac{1}{2}$$

Hexadecimal numbers

Hexadecimal radix = 16 $\quad 16 = 2^4$ $\quad$ radix is a power of 2

useful to represent binary numbers easily in compact form (easier on human beings than binary representation)

$$\underbrace{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \overset{10}{A}, \overset{11}{B}, \overset{12}{C}, \overset{13}{D}, \overset{14}{E}, \overset{15}{F}}_{\text{16 hexadecimal digits}}$$

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^0 + 15 \times 16^0$$
$$= (46687)_{10}$$

In Code Composer Studio:

zero

$(B65F)_{16} \longrightarrow$ 0XB65F (C style)

$\longrightarrow$ B65Fh (TI style)

Important table for converting bases among binary, octal and hexadecimal number representation

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Converting from binary to hexadecimal

$$(1011000110101011)_2 = (?)_{16}$$

(use Table)

$$\underbrace{0010}_{2}\ \underbrace{1100}_{C}\ \underbrace{0110}_{6}\ \underbrace{1011}_{B}$$

pad zeros if required

start from here

$$= (2C6B)_{16}$$

Converting from hexadecimal to binary

$$(3A6)_{16} = (?)_2$$

(use Table)

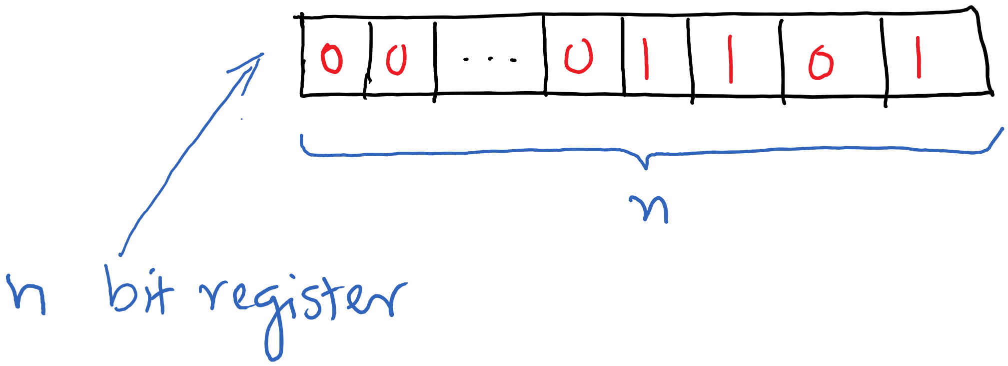
discard

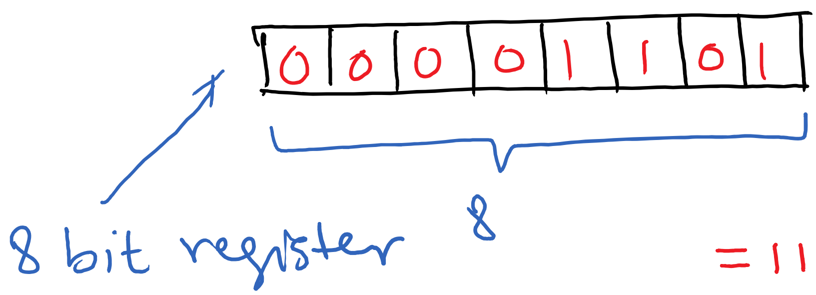
$$(3A6)_{16} = \underbrace{\cancel{00}11}_{3}\ \underbrace{1010}_{A}\ \underbrace{0110}_{6}$$

$$= (1110100110)_2$$

Compliments of Binary numbers.

Binary Number N = 1101 Stored in an n bit register.





= 1101 stored in an 8 bit register

N = 1101
n = 8 (word length)

1's Compliment of Binary Numbers

$$\text{1's compliment of } N \rightarrow (\underset{\text{register size}}{2^n} - 1) - \underset{\text{number}}{N}$$

Change all the 1's to 0, and all the 0's to 1 in N to get 1's compliment of N

$10110010 \xrightarrow{1\text{'s comp.}} 01001101$

$111 \xrightarrow{1\text{'s comp}} 11111000$

2's compliment of N $\rightarrow \begin{cases} 2^n - N & \text{for } N \neq 0 \\ 0 & \text{for } N = 0 \end{cases}$

2's comp. of N = 1's comp. of N + 1

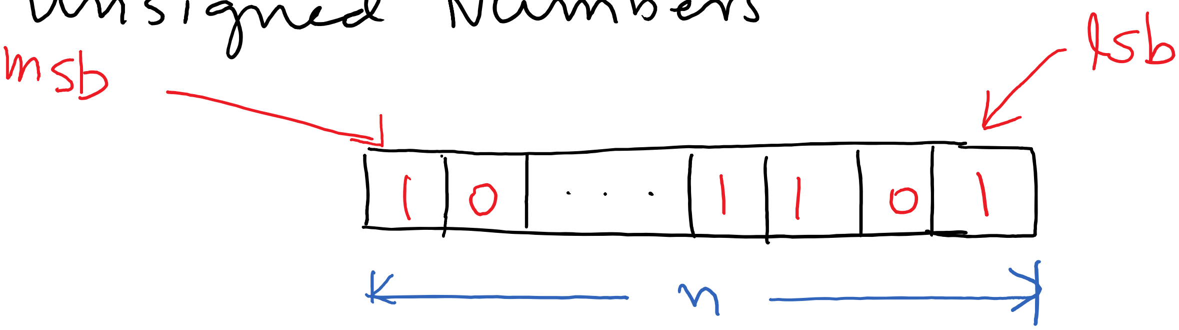
$$101100 \xrightarrow{2\text{'s comp}} \begin{array}{r} 010011 \leftarrow 1\text{'s comp} \\ +\ \ \ 1 \\ \hline 010100 \leftarrow 2\text{'s comp.} \end{array}$$

"Trick" to find 2's compliment of N:

Leave all the least significant zeros and the first 1 unchanged - replace all the remaining 0's by 1's, and 1's by 0's.

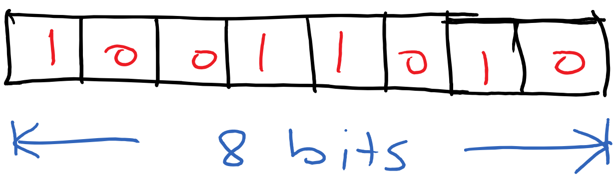
2's comp. of $\underbrace{1011}_{\text{invert these}}\underbrace{00}_{\text{leave these unchanged}} \longrightarrow 010100$

Unsigned Numbers



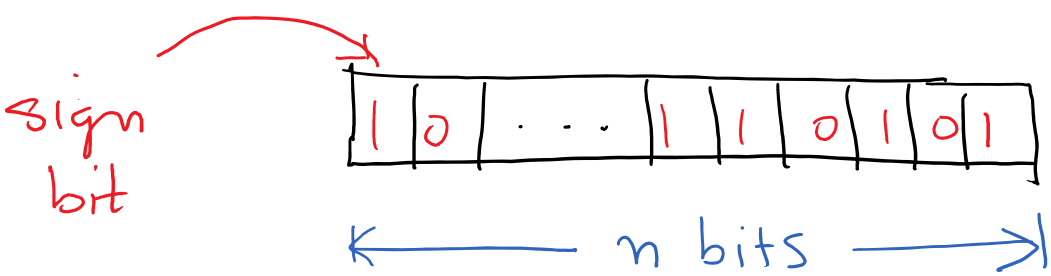
n bit register

All the bits in the register represent the magnitude of the number.



Number $N = +(10011010)_2$

Signed Numbers
(2's compliment representation)



example 8 bit registers

$N = (9)_{10} \longrightarrow 0\underbrace{0001001}_{\text{magnitude}}$

Sign bit

0 = +

magnitude = 0001001 = $(9)_{10}$

$N = (-9)_{10} \longrightarrow$ 2's comp of 00001001

= 11110111

Sign bit

1 = −

Example

11101010

Signed bit is 1 ∴ number is −ve

2's comp. of 11101010 is 00010110 = $(22)_{10}$

∴ 11101010 is $(-22)_{10}$

00011011

Signed bit 0 ∴ number is +ve

00011011 is $(27)_{10}$