

3.4. AAAA.

Recall the AA table from section 3.3.

AA

1. $A \cup. fA \subseteq A \cup. gA. \neg INF. \neg AL. \neg ALF. \neg FIN. \neg NON.$
2. $A \cup. fA \subseteq A \cup. gB. \neg INF. \neg AL. \neg ALF. \neg FIN. \neg NON.$
3. $A \cup. fA \subseteq A \cup. gC. \neg INF. \neg AL. \neg ALF. \neg FIN. \neg NON.$
4. $B \cup. fA \subseteq A \cup. gA. \neg INF. AL. \neg ALF. \neg FIN. NON.$
5. $B \cup. fA \subseteq A \cup. gB. \neg INF. AL. \neg ALF. \neg FIN. NON.$
6. $B \cup. fA \subseteq A \cup. gC. \neg INF. AL. \neg ALF. \neg FIN. NON.$
7. $C \cup. fA \subseteq A \cup. gA. \neg INF. AL. \neg ALF. \neg FIN. NON.$
8. $C \cup. fA \subseteq A \cup. gB. \neg INF. AL. \neg ALF. \neg FIN. NON.$
9. $C \cup. fA \subseteq A \cup. gC. \neg INF. AL. \neg ALF. \neg FIN. NON.$

It is clear that there is no reason to further consider clauses 1-3 from the AA table, as all of our five proposition attributes already come out false. So we instead work with the following reduced AA table. Note that we have renumbered the clauses.

REDUCED AA

1. $B \cup. fA \subseteq A \cup. gA. \neg INF. AL. \neg ALF. \neg FIN. NON.$
2. $B \cup. fA \subseteq A \cup. gB. \neg INF. AL. \neg ALF. \neg FIN. NON.$
3. $B \cup. fA \subseteq A \cup. gC. \neg INF. AL. \neg ALF. \neg FIN. NON.$
4. $C \cup. fA \subseteq A \cup. gA. \neg INF. AL. \neg ALF. \neg FIN. NON.$
5. $C \cup. fA \subseteq A \cup. gB. \neg INF. AL. \neg ALF. \neg FIN. NON.$
6. $C \cup. fA \subseteq A \cup. gC. \neg INF. AL. \neg ALF. \neg FIN. NON.$

We need only consider ordered pairs of these clauses i, j , where $i < j$.

- 1,2. $B \cup. fA \subseteq A \cup. gA, B \cup. fA \subseteq A \cup. gB.$
- 1,3. $B \cup. fA \subseteq A \cup. gA, B \cup. fA \subseteq A \cup. gC.$
- 1,4. $B \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gA.$
- 1,5. $B \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gB.$
- 1,6. $B \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gC.$
- 2,3. $B \cup. fA \subseteq A \cup. gB, B \cup. fA \subseteq A \cup. gC.$
- 2,4. $B \cup. fA \subseteq A \cup. gB, C \cup. fA \subseteq A \cup. gA.$ Equivalent to 1,6.
- 2,5. $B \cup. fA \subseteq A \cup. gB, C \cup. fA \subseteq A \cup. gB.$
- 2,6. $B \cup. fA \subseteq A \cup. gB, C \cup. fA \subseteq A \cup. gC.$
- 3,4. $B \cup. fA \subseteq A \cup. gC, C \cup. fA \subseteq A \cup. gA.$ Equivalent to 1,5.
- 3,5. $B \cup. fA \subseteq A \cup. gC, C \cup. fA \subseteq A \cup. gB.$

- 3,6. $B \cup. fA \subseteq A \cup. gC, C \cup. fA \subseteq A \cup. gC$. Equivalent to 2,5.
 4,5. $C \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gB$. Equivalent to 1,3.
 4,6. $C \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gC$. Equivalent to 1,2.
 5,6. $C \cup. fA \subseteq A \cup. gB, C \cup. fA \subseteq A \cup. gC$. Equivalent to 2,3.

Thus we need only examine

REDUCED AAAA

- 1,2. $B \cup. fA \subseteq A \cup. gA, B \cup. fA \subseteq A \cup. gB$. $\neg INF. \neg AL. \neg ALF. \neg FIN. \neg NON$.
 1,3. $B \cup. fA \subseteq A \cup. gA, B \cup. fA \subseteq A \cup. gC$. $\neg INF. AL. \neg ALF. \neg FIN. NON$.
 1,4. $B \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gA$. $\neg INF. AL. \neg ALF. \neg FIN. NON$.
 1,5. $B \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gB$. $\neg INF. \neg AL. \neg ALF. \neg FIN. \neg NON$.
 1,6. $B \cup. fA \subseteq A \cup. gA, C \cup. fA \subseteq A \cup. gC$. $\neg INF. \neg AL. \neg ALF. \neg FIN. \neg NON$.
 2,3. $B \cup. fA \subseteq A \cup. gB, B \cup. fA \subseteq A \cup. gC$. $\neg INF. AL. \neg ALF. \neg FIN. NON$.
 2,5. $B \cup. fA \subseteq A \cup. gB, C \cup. fA \subseteq A \cup. gB$. $\neg INF. AL. \neg ALF. \neg FIN. NON$.
 2,6. $B \cup. fA \subseteq A \cup. gB, C \cup. fA \subseteq A \cup. gC$. $\neg INF. AL. \neg ALF. \neg FIN. NON$.
 3,5. $B \cup. fA \subseteq A \cup. gC, C \cup. fA \subseteq A \cup. gB$. $\neg INF. AL. \neg ALF. \neg FIN. NON$.

Note that we have used an entirely different method for compiling the ordered pairs to be analyzed than the purely syntactic method used in section 3.1 to compile the master list for AAAA that is used in the Annotated Table, section 3.14. Here we take full advantage of the fact that $\neg NON$ implies $\neg INF, \neg AL, \neg ALF, \neg FIN$. The result is that on the master list for AAAA, there are 20 entries, whereas on the above Reduced AAAA list, there are only 9 entries.

The same considerations apply in sections 3.5 - 3.13, where the number of ordered pairs actually requiring analysis is considerably smaller than the number of ordered pairs in the relevant part of the Annotated Table.

By the reduced AA table, we see that all of these pairs must have \neg INF, \neg ALF, \neg FIN. It remains to determine the status of AL and NON.

In the next Lemma, we use this method of substitution: Suppose α, β are pairs of clauses, where β is the result of substituting one letter by another letter in α . Then any of our five attributes that holds of β also holds of α . As a consequence, if the negation of any of our five attributes holds of α then that negation also holds of β .

LEMMA 3.4.1. 1,3, 1,4 have AL.

Proof: From the reduced AA table, $B \cup. fA \subseteq A \cup. gA$ has AL. In the cited ordered pairs, replace C by A, and C by B, respectively. QED

The following pertains to 1,2, 1,5, 1,6.

LEMMA 3.4.2. $fX \subseteq A \cup. gA, fA \subseteq A \cup. gY, Y \cap fA = \emptyset$ has \neg NON.

Proof: Let f be as given by Lemma 3.2.1. Let $g \in \text{ELG}$ be defined by $g(n) = 2n+1$. Let $fX \subseteq A \cup. gA, fA \subseteq A \cup. gY, Y \cap fA = \emptyset$, where A, B, C are nonempty.

Let $n \in fA \cap 2N$. Then $n \in A$. Hence $fA \cap 2N \subseteq A$. By Lemma 3.2.1, fA is cofinite. Since $Y \cap fA = \emptyset$, Y is finite. Hence A is cofinite. This contradicts $A \cap gA = \emptyset$. QED

The following pertains to 2,3, 2,5, 2,6.

LEMMA 3.4.3. $B \cup. fA \subseteq A \cup. gB, X \cup. fA \subseteq A \cup. gY$ has AL, provided $X, Y \in \{B, C\}$.

Proof: From the reduced AA table, $B \cup. fA \subseteq A \cup. gB$ has AL. In the cited ordered pairs, replace C by B. QED

The following pertains to 3,5.

LEMMA 3.4.4. $B \cup. fA \subseteq A \cup. gC, C \cup. fA \subseteq A \cup. gX$ has AL, provided $X \in \{B, C\}$.

Proof: From the reduced AA table, $B \cup. fA \subseteq A \cup. gB$ has AL. In the cited ordered pair, replace C by B. QED