Performance Analysis of the Mooney M20R

A final project report submitted for
AE 2200: Introduction to Aerospace Engineering

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SUMMARY

This report provides an analysis of the flight performance of the Mooney M20R aircraft. Given some specifications on the weight, wing span, and propulsion system, as well as a three-view drawing, the aircraft's flight was computationally analyzed at sea level and altitudes of 6,000 ft and 12,000 ft to find key flight information.

The parasitic drag coefficient for the Mooney M20R was estimated to be 0.0298. This value is the minimum drag that the aircraft would experience, when no lift is being produced. The maximum lift-to-drag ratio for this aircraft was determined to be 11.4. Flying at this ratio produces the most lift for the aircraft at a minimal drag.

At the three different altitudes, the power required and power available in relation to the velocity of the aircraft were analyzed. In addition, the stall speed (minimum speed required to maintain flight), speed for minimum power required, speed for maximum lift-to-drag ratio, and maximum speed were calculated for all three altitudes. At sea level, these four speeds were 66.0, 69.6, 91.5, and 158.5 knots respectively.

The climb performance of the aircraft was also analyzed. The maximum rate of climb and corresponding freestream velocities were calculated at the three altitudes. For sea level, the maximum rate of climb was 1294.2 ft/min at 90.0 knots. The time for the Mooney M20R to climb from sea level to 12,000 ft was calculated to be 14.4 min. The best climb angle and its corresponding velocity were also calculated, 9.23 degrees at 71 knots. The maximum rate of climb allows the aircraft to reach its intended altitude in the shortest amount of time, while the best climb angle allows the aircraft to reach its intended altitude in the shortest distance to clear any obstacles in its path.

The maximum range and endurance of the aircraft were calculated to be 1104.9 nautical miles and 11.9 hrs respectively. The range determines how far the aircraft can travel before refueling, while the endurance determines how long the aircraft can stay in the air.

Not only was the climb performance analyzed, but also the glide performance of the aircraft. Assuming a starting altitude of 6,000 ft, the maximum gliding range of the aircraft was 11.2 nautical miles at an indicated airspeed of 99.8 knots. This means that if the aircraft was to experience engine failure at this height, if the pilot maintained this indicated airspeed, the aircraft could land anywhere within an 11.2 nautical mile range.

To evaluate turning flight performance, a V-n diagram was created showing the maximum and minimum load factors, "never exceed" airspeed, and maneuver point. The maneuver point, the maximum speed the aircraft can fly at before structural failure occurs, was found to be 128.6 knots. At this speed, the aircraft has a minimum turn radius of 399.4 ft and a maximum turn rate of 0.544 rad/s.

Finally, the takeoff and touchdown performance of the aircraft was analyzed. The takeoff ground roll distance with maximum weight at standard sea level conditions was 1201.9 ft. The touchdown ground roll distance under the same circumstances was 984.1 ft. These are the minimum runway length requirements under standard conditions for the aircraft.

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NOMENCLATURE

AR	Aspect ratio
b	Wing span, ft
c	Consistent units specific fuel consumption, 1/ft
C_D	Drag polar (drag coefficient)
$C_{D,0}$	Parasitic drag coefficient
C_F	Friction drag coefficient
C_L	Lift coefficient
$C_{L,max}$	Maximum lift coefficient
$C_{L,opt}$	Optimal lift coefficient
c_r	Root chord length, ft
c_t	Tip chord length, ft
D	Drag, lbs
D_{fuel}	Fuel density, lbs/gal
E	Endurance, s
e	Oswald efficiency factor
F_L	Landing force, lbs
F_{TO}	Takeoff force, lbs
g	Acceleration due to gravity, 32.2 ft/s^2
h	Altitude, ft
h_f	Final altitude, ft
h_i	Initial altitude, ft
h_w	Wing height, ft
L	Lift, lbs
n	Load factor
n_{max}	Maximum load factor
P_A	Power available, lb*ft/s
P_E	Excess power, lb*ft/s
P_{max}	Maximum power provided by propulsion system, lb*ft/s
P_R	Power required, lb*ft/s
R	Range, ft
r	Turn radius, ft
r_e	Wing efficiency factor
R_G	Glide range, ft
RC	Rate of climb, ft/s
S	Wing area, ft^2
s_L	Landing rolling distance, ft
s_{TO}	Takeoff rolling distance, ft
S_{wet}	Aircraft wetted area, ft^2
SFC	Specific fuel consumption
t	Climb time, s
T_R	Thrust required, lbs
T_{TO}	Takeoff thrust, lbs
V_∞	Freestream velocity, ft/s
V_{avg}	Average velocity, ft/s
v_{fuel}	Fuel volume, gal
V_G	Indicated glide velocity, ft/s
$V_{G,H}$	Horizontal glide velocity, ft/s
$V_{G,V}$	Vertical glide velocity, ft/s
V_H	Horizontal velocity, ft/s

V_L	Landing velocity, ft/s
V_{stall}	Stall speed, ft/s
V_{TO}	Takeoff velocity, ft/s
V^*	Maneuver speed, ft/s
W	Total weight of aircraft and fuel, lbs
W_0	Initial weight, lbs
W_1	Final weight, lbs
W_f	Total weight after fuel consumption, lbs
W_{fuel}	Fuel weight, lbs
δ	Induced drag factor
η	Propeller efficiency
η_c	Constant propeller efficiency
θ_C	Climb angle, rad
θ_G	Glide angle, rad
λ	Taper ratio
μ_r	Rolling friction coefficient
$\mu_{r,b}$	Rolling friction coefficient (braking)
ρ_∞	Freestream density, slug/ft^3
φ	Ground effect factor
ω	Turn rate, rad/s

INTRODUCTION

The objective of this report is to provide an in-depth analysis of the flight performance of the Mooney M20R. This aircraft is mainly used for high-speed, cross-country flights and can only seat four. The analysis was based only on the geometry of the airplane seen in a three-view drawing, as well as a few given weight and propulsion specifications. The MATLAB coding environment was used to conduct the computations and graphing/plotting required for this analysis. The commented code containing the equations and values used in the analysis can be found in Attachment A of this report. The numerical and graphical results of the analysis and their relevance will be presented in this report by section, along with a detailed explanation of the methods used to determine or calculate the results.

The first section of the report deals with the aircraft’s drag polar. The parasitic drag coefficient and Oswald efficiency factor were estimated using the aerodynamic cleanliness method and historic data. These values were used to calculate the drag coefficient as a function of lift coefficient with the parasitic drag coefficient as a minimum. The lift and drag coefficients were plotted as the drag polar and a separate graph showed the lift-to-drag ratio as a function of the lift coefficient. This was used to determine the maximum lift-to-drag ratio of the aircraft.

The next section calculated the power required for the flight of the Mooney M20R as a function of velocity at sea level, 6,000 ft, and 12,000 ft. The stall speed, minimum power required speed, and maximum lift-to-drag ratio speed were also calculated for each altitude. These were all potted on a graph of power in horsepower versus velocity from 40 to 180 knots.

Using the propulsion system specifications, the power available at all three altitudes was calculated as a function of velocity and plotted on the same graph as the power required. The maximum velocities were also calculated and graphed for each altitude. For minimum speed, maximum lift-to-drag ratio speed, and maximum speed, the velocity, power required, and power available were calculated at each altitude.

The following section of the analysis studied the climb performance of the aircraft. The rate of climb as a function of velocity was calculated for the three altitudes. These were plotted versus their corresponding altitudes. The resulting trendline was then extrapolated to determine the absolute ceiling and service ceiling for the Mooney M20R. By inverting this graph, a trapezoidal integral could be calculated to determine the time to climb from sea level to 12,000 ft. A climb hodograph was then developed for the aircraft at maximum weight. This was used to calculate the best rate of climb, its corresponding climb angle, and velocity, as well as the best climb angle and its rate of climb and velocity.

The maximum range in nautical miles and endurance in hours of the aircraft at 12,000 ft were then calculated. The Breguet Range and Endurance relations were used with a constant specific fuel consumption and propeller efficiency to determine these values. It was also assumed that the aircraft was at maximum weight initially and then used 90% of its fuel. The velocities required to achieve the maximum range and endurance were also determined.

In contrast to the climb performance, a glide hodograph was developed to analyze the aircraft’s glide performance. It was assumed that engine failure occurred at 6,000 ft and that the airplane was still at maximum weight. The maximum glide range and necessary indicated airspeed were calculated based on this hodograph.

In order to analyze the turning flight performance, a V-n diagram for standard sea level conditions was created. This was done by using the given maximum positive and negative load factors as upper and lower bounds for the diagram, as well as using a “never exceed” airspeed as the right boundary. Maximum load factor was plotted as a function of velocity. This graph also showed the load factors produced by stall speed and the maneuver point. Using this information, the minimum turn radius and maximum turn rate were calculated for maximum weight at sea level.

Finally, in the last portion of this analysis, the takeoff and landing performances were investigated. Four conditions were evaluated to estimate their ground roll distances: takeoff with maximum weight at sea level, takeoff with maximum weight at 6,000 ft, takeoff with 2,400 lbs at sea level, and landing with maximum weight at sea level. The se values were calculated by finding the average takeoff and landing velocities and using these, as well as the optimal lift coefficient and ground effect factor, to determine the average thrust and forces experienced by the aircraft during takeoff and landing. These forces were then used in turn to calculate the ground roll distances.

The in-depth analysis on the following pages will further detail the above methodologies and present complete results in numerical and graphical form. The importance of the found results will also be discussed to provide a full analysis of the performance of the Mooney M20R aircraft.

RESULTS

Below in Table 1, the initial specifications of the geometry, weight, and propulsion system for the Mooney M20R aircraft are listed. These are the values that were used in the MATLAB analysis of the aircraft to produce the results provided in this section.

Table 1: Given specifications for the Mooney M20R

Maximum Weight	3,368 lbs
Wing Area	174.9 ft^2
Wing Span	36.08 ft
Maximum Lift Coefficient	1.306
Propulsion System Displacement	550 in^3
Maximum Power at Sea Level	280 hp
Maximum Power at 6,000 ft	230 hp
Maximum Power at 12,000 ft	182 hp
Fuel Volume	89 gal
Fuel Density	6 lbs/gal
Specific Fuel Consumption at 12,000 ft	0.465 (lbs/hr)/shp
Wing Height above Ground	2.00 ft

Drag Polar

Using the aerodynamic cleanliness method, the parasitic drag coefficient is expressed by Equation 1.

$$C_{D,0} = \frac{C_F S_{wet}}{S} \quad (1)$$

It represents the base drag coefficient, when there is no lift being produced. To calculate this value, the wetted area of the aircraft was measured by using the three-view drawing of the aircraft to determine the exposed surface area. The total wetted area was determined to be 599.3 ft^2. The friction coefficient was determined by averaging the coefficients for two similar, historical aircraft, the PA-28R and Beech V35. Their friction coefficients were 0.0067 and 0.0049 respectively, so the resulting average was 0.0058. Using the wing span given in Table 1, the parasitic drag coefficient was calculated to be 0.0199. This was significantly less than the collectively agreed upon value of 0.0298. The difference between the two estimated values could stem from the use of the component buildup method for the converged value. It could also be that the PA-28R is closer to the Mooney M20R, so its friction coefficient should have been used. The wetted area of the aircraft could likely have been underestimated as well. All three of these factors could contribute to the smaller estimated parasitic drag coefficient; however, the converged value of 0.0298 was used for this analysis.

Before the Oswald efficiency factor could be determined, several values had to be calculated or estimated. The first of these, the wing efficiency factor, could be between 0.009 and 0.0012. Since little information about the aircraft was known, it was estimated to be 0.0010, in the middle but erring on the side of more efficiency. The wing aspect ratio was calculated using Equation 2.

$$AR = \frac{b^2}{S} \quad (2)$$

The wing span and wing area found in Table 1 above were used in this calculation to produce an aspect ratio of 7.443. The induced drag factor depended on the taper ratio of the wing, calculated using Equation 3.

$$\lambda = \frac{c_t}{c_r} \quad (3)$$

The tip and root cords were measured to be about 32 and 68 in respectively from the three-view drawing provided and resulted in a taper ratio of .4706. Once these values were calculated, a historic graph relating the induced drag factor to the aspect ratio and taper ratio was used to estimate an induced drag factor of

0.013. The wing efficiency factor, aspect ratio, and induced drag factor were used in Equation 4 to calculate the Oswald efficiency factor.

$$e = \frac{1}{r_e \pi AR + 1 + \delta} \tag{4}$$

This represents the efficiency of the wing as a nonelliptical lift distribution, including the effects of induced drag and wing efficiency. The above estimation resulted in a value of 0.80, while the Oswald efficiency factor that was collectively agreed upon was 0.66. The estimated value was higher than the converged value, possibly because of a lower estimated wing efficiency factor. For the analysis in this report, the converged value of 0.66 will be used for the Oswald efficiency factor.

The drag polar could then be calculated using Equation 5.

$$C_D = C_{D,0} + \frac{{C_L}^2}{\pi e AR} \tag{5}$$

The drag polar packages all the aerodynamic properties of the aircraft into one value which can be used as the drag coefficient for the entire airplane. Using a range of lift coefficients, from the negative to positive maximum lift coefficient given above in Table 1, the drag polar was calculated and plotted versus the lift coefficient, as seen in Figure 1 below.

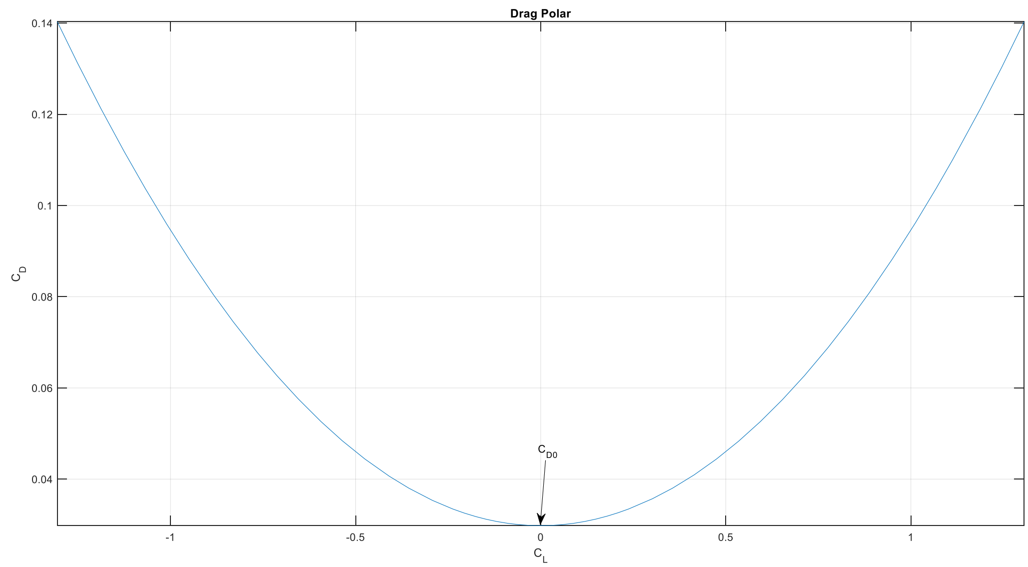

Figure 1: Drag polar for the Mooney M20R with parasitic drag coefficient

To determine the lift-to-drag ratio, the lift coefficients were divided by their corresponding drag coefficients. These resulting ratios were plotted versus their lift coefficients to produce Figure 2 on the next page. This figure also shows the maximum lift-to-drag ratio. Since the thrust required is inversely proportional to the lift-to-drag ratio, the velocity that produces this ratio should also be the velocity required for minimum thrust. By evaluating the lift-to-drag ratios produced, the maximum ratio was determined to be 11.4.

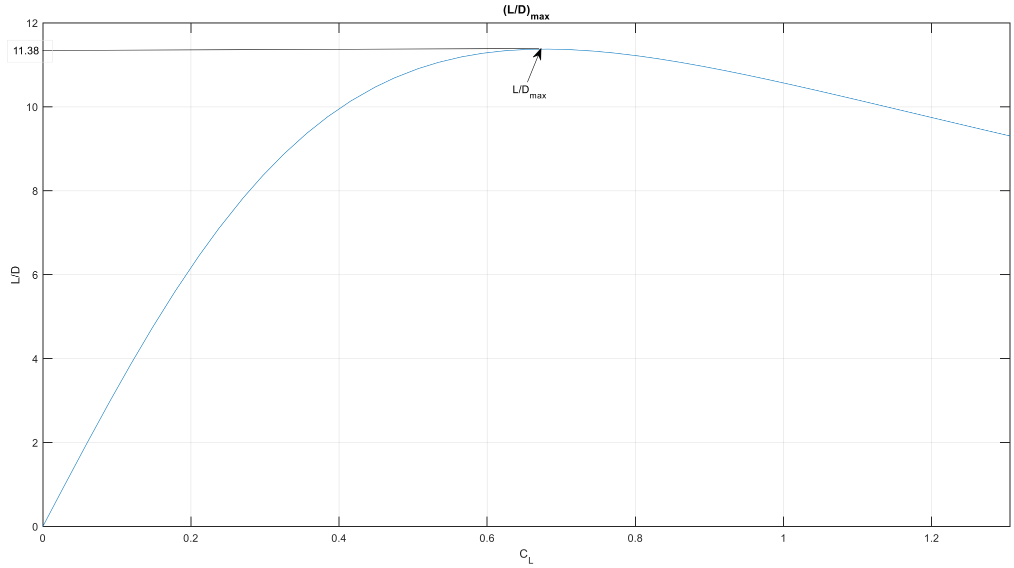

Figure 2: Maximum lift-to-drag ratio for the Mooney M20R

Power Required

During steady, level flight, the lift produced by the motion of the aircraft is equivalent to the weight and can be calculated using Equation 6.

$$L = W = \frac{1}{2}\rho_\infty {V_\infty}^2 S C_L \tag{6}$$

By rearranging this equation, the lift coefficient can be calculated for any freestream velocity at a given freestream density using the maximum weight given in Table 1. This lift coefficient is used to calculate the drag coefficient necessary to calculate the thrust required for steady, level flight, shown in Equation 7.

$$T_R = D = \frac{1}{2}\rho_\infty {V_\infty}^2 S C_D \tag{7}$$

The power required can then be calculated for a given velocity using Equation 8.

$$P_R = V_\infty T_R \tag{8}$$

This is the power required to maintain unaccelerated flight at a constant altitude, given the freestream velocity. For the analysis of the Mooney M20R, the required powers were calculated over a freestream velocity range of 40 to 180 knots and at altitudes of sea level (0 ft), 6,000 ft, and 12,000 ft. The corresponding freestream densities used were 0.0023769, 0.0019869, and 0.0016480 slug/ft^3.

The stall speed at each of the three altitudes was also calculated, using Equation 9.

$$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S C_{L,max}}} \tag{9}$$

These values represent the minimum velocity required for the aircraft to produce enough lift and can be found in Table 2, later in this report, as well as their respective required powers. The speed required to achieve the maximum lift-to-drag ratio (and thus minimum thrust required) and the speed required for minimum power required were determined by analyzing the lift-to-drag ratio and power required curves respectively. The velocities and required powers for maximum lift-to-drag ratio at each altitude can also be found later in Table 2. On the next page, Figure 3 plots the power required in horsepower as a function of velocity for each of the three altitudes. It also shows the stall speeds, speeds for the maximum lift-to-drag ratio, and speeds for minimum power required.

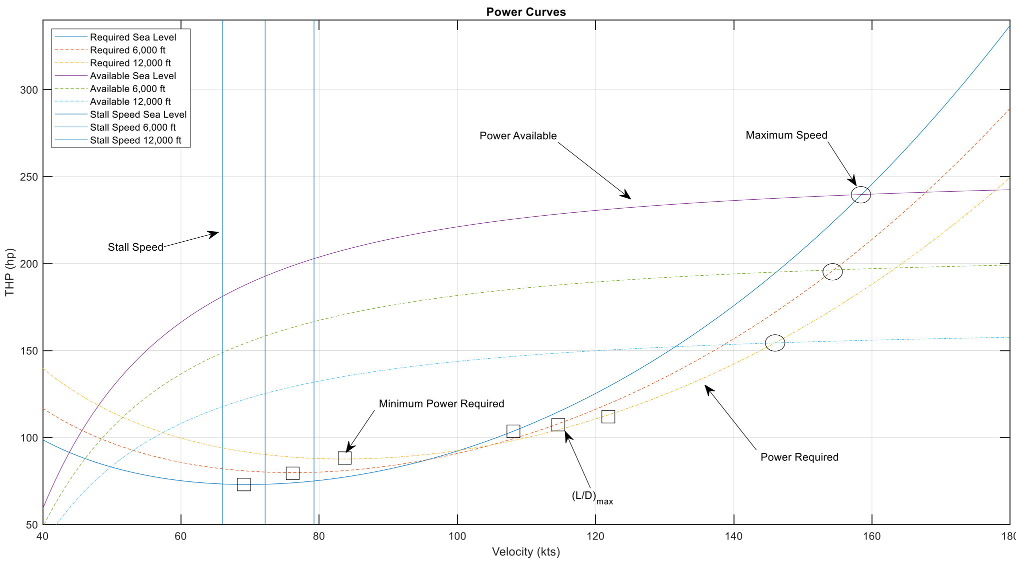

Figure 3: Power curves for the Mooney M20R with stall, minimum power required, maximum lift-to-drag, and maximum speeds

Power Available

The propeller efficiency for the Mooney M20R was dependent on the freestream velocity and is given below in Equation 10.

$$\eta = 0.90\left(1 - \left(\frac{35}{V_\infty}\right)^2\right) \tag{10}$$

For this equation, the freestream velocity must have units of knots instead of the usual ft/s. This value represents the efficiency of the airplane propeller as velocity changes. Using the propeller efficiency and the maximum powers provided by the propulsion system, the available powers can be calculated using Equation 11.

$$P_A = P_{max}\eta \tag{11}$$

This is the power available at each altitude to provide thrust for the aircraft. The power required cannot exceed the power available. Thus, the maximum speed can be located at the point where the power required and the power available are equivalent. The power available in horsepower at each altitude as a function of velocity is plotted above in Figure 3 as well. The maximum speeds are also marked on this graph and are included in Table 2 below, along with their corresponding required powers. In addition, the power available was calculated for each set of values in Table 2.

Table 2: Velocities, required powers, and available powers at given altitudes for the Mooney M20R

	Minimum Speed			Speed for $(L/D)_{max}$			Maximum Speed		
Altitude	V_∞	P_R	P_A	V_∞	P_R	P_A	V_∞	P_R	P_A
Sea Level	66.0 kts	73.3 hp	181.1 hp	91.5 kts	83.1 hp	215.1 hp	158.5 kts	239.7 hp	239.7 hp
6,000 ft	72.2 kts	80.2 hp	158.3 hp	100.0 kts	90.9 hp	181.7 hp	154.5 kts	196.4 hp	196.4 hp
12,000 ft	79.3 kts	88.0 hp	131.9 hp	109.8 kts	99.8 hp	147.2 hp	146.0 kts	154.5 hp	154.4 hp

Climb Performance

The difference between the power available and power required curves is known as the excess power and can be calculated using Equation 12.

$$P_E = P_A - P_R \tag{12}$$

The excess power can be used in conjunction with the weight of the aircraft to determine the rate of climb, as seen in Equation 13.

$$RC = \frac{P_E}{W} \tag{13}$$

The rate of climb represents how fast the aircraft can ascend at a given freestream velocity. The rate of climb in ft/min at each altitude is plotted against velocity in knots in Figure 4 below. The maximum rates were determined from the curve as well as the velocities at which they occur. They are marked on the graph as well and presented in Table 3 underneath it.

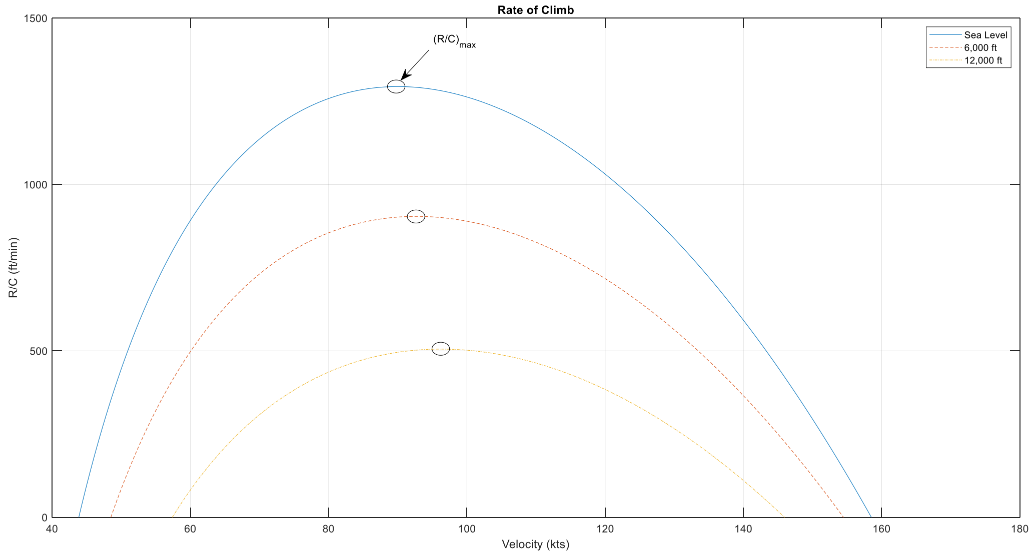

Figure 4: Rate of climb for the Mooney M20R with maximum rates

Table 3: Maximum rate of climb for altitude for the Mooney M20R

Altitude	Maximum Rate of Climb	Velocity
Sea Level	1294.2 ft/min	90.0 knots
6,000 ft	904.1 ft/min	92.7 knots
12,000 ft	505.4 ft/min	96.2 knots

The altitudes were also plotted versus their corresponding maximum rates as shown in Figure 5 on the next page. A trendline was fitted to the three points in order to extrapolate the maximum rate of climb to 0 ft/min to find the absolute ceiling of the Mooney M20R. This represents the maximum altitude of the aircraft and was found to be 19,413 ft using the equation of the trendline. The service ceiling, located at a maximum rate of climb of 100 ft/min, represents the practical and safe upper altitude limit at 17,963 ft.

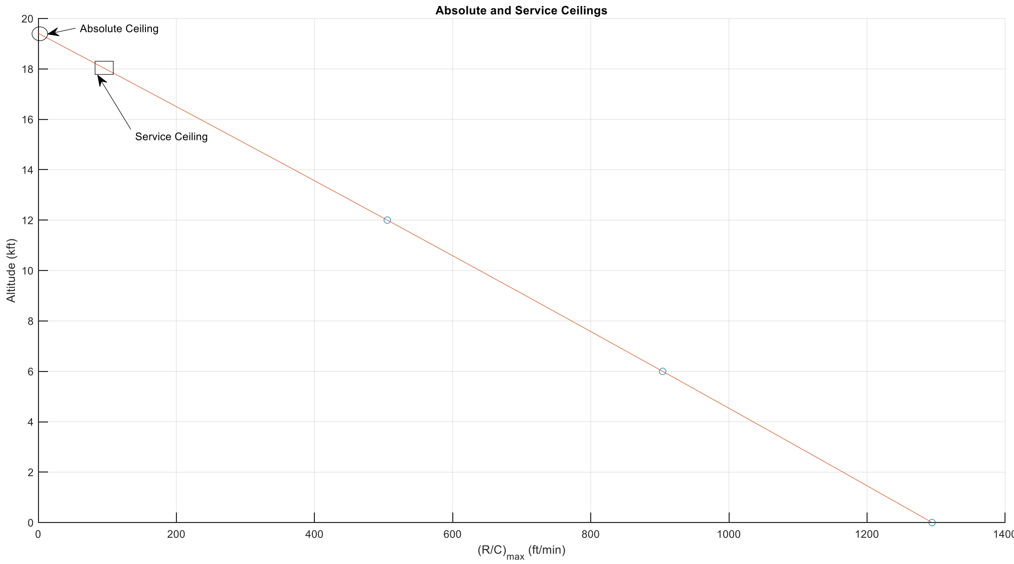

Figure 5: Absolute and service ceilings for the Mooney M20R

To calculate the time it would take to climb to 12,000 ft from sea level, Equation 14 was used below.

$$t = \int_{h_i}^{h_f} \frac{1}{RC} dh \tag{14}$$

To calculate the integral, the relation shown in Figure 5 was inverted so that maximum rate of climb was dependent on altitude. A trapezoidal integral was calculated in MATLAB from 0 to 12,000 ft to determine a climb time of 14.4 min.

A climb hodograph was created showing the relationship between the vertical and horizontal flight velocities at sea level. The axes were made to scale so that the angle measured between the curve and the horizontal axis is equivalent to the actual climb angle at that velocity. The distance from the origin to the curve also represents the true airspeed for those specific vertical and horizontal velocities. This graph is shown below in Figure 6.

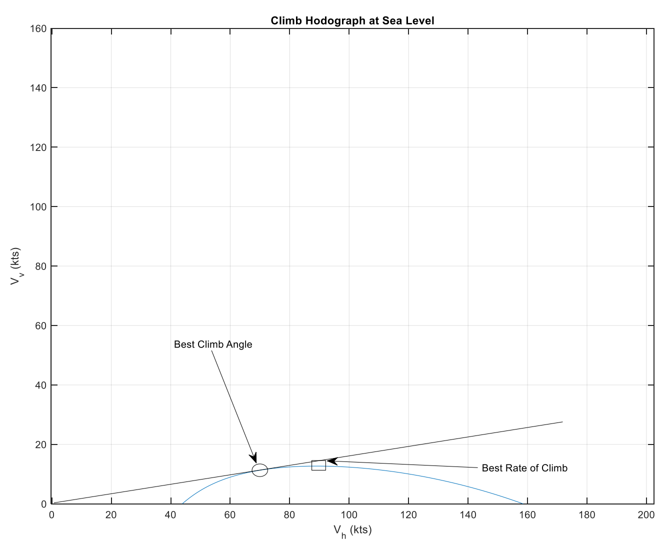

Figure 6: Climb hodograph at sea level for the Mooney M20R with best climb angle and rate of climb

Equation 15 shows how to calculate the freestream velocity during climb.

$$V_\infty = \sqrt{RC^2 + {V_H}^2} \tag{15}$$

The climb angle at any point can be calculated using Equation 16.

$$\tan\theta_C = \frac{RC}{V_H} \tag{16}$$

The rate of climb and climb angle curves were used to determine the respective maximum values at sea level. The maximum rate of climb is the velocity which will reach the desired altitude the fastest. The best climb angle is the velocity which will reach the desired altitude over the shortest distance. For each maximum value, a set was created including rate of climb, climb angle, and freestream velocity. These results are shown in Table 4 below.

Table 4: Maximum rate of climb and climb angle at sea level for the Mooney M20R

	Rate of Climb	Climb Angle	Velocity
Best Rate of Climb	1294.2 ft/min	8.16°	90.0 knots
Best Climb Angle	1153.5 ft/min	9.23°	71.0 knots

Range and Endurance

For the analysis of the range and endurance of the aircraft, the aircraft was assumed to be flying at an altitude of 12,000 ft at initial maximum weight. The specific fuel consumption and propeller efficiency remained constant at 0.465 (lb/hr)/shp and 0.90 respectively and it was assumed that only 90% of the fuel was used. The range is the maximum distance the aircraft can travel before having to refuel while the endurance is the maximum time the aircraft can stay in the air without having to refuel. The Breguet equations were used to complete this analysis and in order to use them a couple new values had to be determined.

The given specific fuel consumption was not in consistent units, so it could not be used in the equation. Equation 17 shows the conversion to consistent units.

$$c = \frac{SFC}{3600*550} \tag{17}$$

The final weight of the aircraft also needed to be calculated. Equation 18 shows how to determine the weight of the fuel tank of fuel. After determining the weight of 90% of the fuel, Equation 19 shows how to calculate the final total weight of the aircraft.

$$W_{fuel} = v_{fuel}D_{fuel} \tag{18}$$

$$W_f = W - W_{fuel} \tag{19}$$

The Breguet Range equation is shown in Equation 20 and the Breguet Endurance equation is shown in Equation 21.

$$R = \frac{\eta_c}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W_1} \tag{20}$$

$$E = \frac{\eta_c}{c}\frac{{C_L}^{\frac{3}{2}}}{C_D}(2\rho_\infty S)^{\frac{1}{2}}\left({W_1}^{-\frac{1}{2}} - {W_0}^{-\frac{1}{2}}\right) \tag{21}$$

The maximum range was 1104.9 nautical miles and the maximum endurance was 11.9 hours when only consuming 90% of the fuel.

The velocity required to achieve the maximum range of the aircraft occurs when the lift-to-drag ratio is maximized. This value was determined to be 109.8 knots at 12,000 ft based on the lift-to-drag ratio curve. The velocity required to achieve the maximum endurance of the aircraft occurs when the lift$^{3/2}$-to-drag ratio is maximized. This value was determined to be 80.5 knots at 12,000 ft by calculating the curve for $L^{3/2}/D$ and determining its maximum.

Glide Performance

Much like with the climb performance, a hodograph was made to describe the glide performance of the Mooney M20R at 6,000 ft. To do so, the glide angle as a function of velocity was calculated using Equation 22.

$$\tan\theta_G = \frac{1}{\left(\frac{L}{D}\right)} \tag{22}$$

Once the glide angle is known, the glide freestream velocity (indicated airspeed) can be calculated with Equation 23.

$$V_G = \sqrt{\frac{2\cos\theta_G}{\rho_\infty C_L}\frac{W}{S}} \tag{23}$$

The vertical and horizontal glide velocities can be calculated using Equation 24 and Equation 25 respectively.

$$V_{G,V} = -V_G \sin\theta_G \tag{24}$$

$$V_{G,H} = V_G \cos\theta_G \tag{25}$$

The hodograph displays the relationship between the vertical and horizontal glide velocities. Since the axes of the graph are scaled equally, the actual between the curve and horizontal axis of the graph is the actual glide angle. The distance from the origin to the curve is once again the freestream velocity or indicated airspeed. The glide hodograph is below in Figure 7.

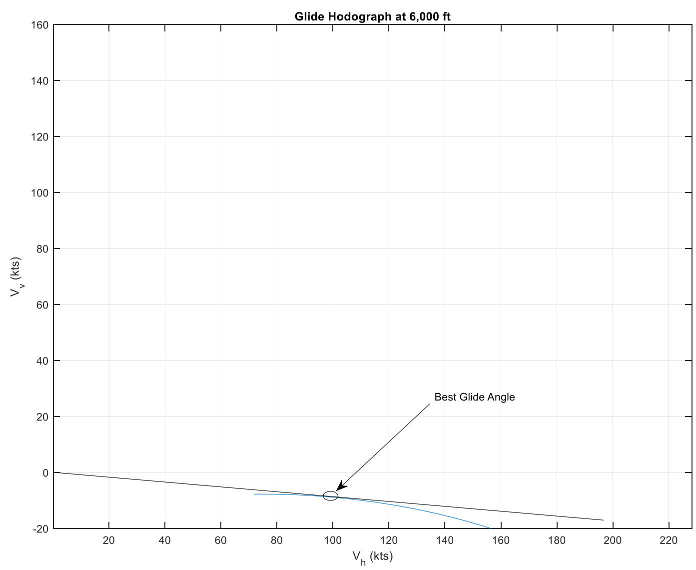

Figure 7: Glide hodograph at 6,000 ft for the Mooney M20R with best glide angle

The best glide angle was determined from the glide hodograph. This is the angle that will achieve maximum glide range and time. In case of engine failure, the pilot should maintain this glide angle to increase his/her chances of finding a safe place to land. The best glide angle for the Mooney M20R at 6,000 ft, assuming complete engine failure, was 5.02° below the horizon. This glide angle is achieved by maintaining an indicated airspeed of 99.8. The glide range is calculated using Equation 26.

$$R_G = \frac{h}{\tan\theta_G} \tag{26}$$

Thus, the maximum glide range for the best glide angle was 11.2 nautical miles.

Turning Flight

To best analyze the turning flight, a V-n diagram was created for standard sea level conditions. The maximum positive and negative load factors were given as 3.8 and -1.5 respectively, along with the "never exceed" velocity of 195 knots. The load factor for a given velocity is described in Equation 27 below.

$$n = \frac{1}{2}\rho_\infty {V_\infty}^2 \frac{C_{L,max}}{\frac{W}{S}} \tag{27}$$

The main point of significance on a V-n diagram is the maneuver point. This is the point at which the load factor reaches its maximum positive value. At this velocity, the aircraft has its best turning performance. It is also the maximum safe velocity; any speeds greater than this will result in structural damage to the aircraft. To calculate the maneuver point velocity, Equation 28 below is used.

$$V^* = \sqrt{\frac{2n_{max}}{\rho_\infty C_{L,max}}\frac{W}{S}} \tag{28}$$

At standard sea level conditions, the maneuver point velocity for the Mooney M20R was calculated to be 128.6 knots.

On the V-n diagram, the maximum positive and negative load factors form upper and lower bounds for the right side of the diagram. Any values higher or lower than these respectively result in structural failure. The right bound of the diagram is formed by the "never exceed" velocity. The left boundaries are formed by the graph of load factor as a function of velocity, both positive and negative. Figure 8 below shows the V-n diagram for the aircraft at sea level and points out the positive and negative load factors at stall speed. At the stall speed, the load factor was 1.0.

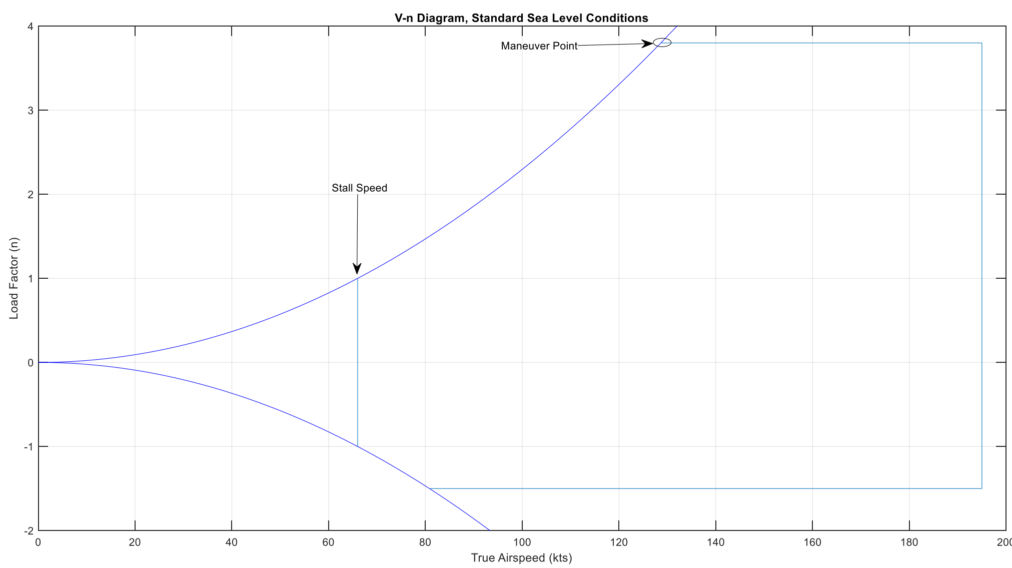

Figure 8: V-n diagram, standard sea level conditions for the Mooney M20R with maneuver point

The calculated maneuver point velocity was used to determine the minimum turn radius and the maximum turn rate of the aircraft. The minimum turn radius gives the smallest area in which the aircraft can turn around, while the maximum turn rate tells how fast the airplane is able to complete a turn. The turn radius is calculated using Equation 29. The turn rate is calculated using Equation 30.

$$r = \frac{{V_\infty}^2}{g\sqrt{n^2 - 1}} \tag{29}$$

$$\omega = \frac{g\sqrt{n^2 - 1}}{V_\infty} \tag{30}$$

With a maximum positive load factor of 3.8, the minimum turn radius was 399.4 ft, while the maximum turn rate was 0.544 rad/s at the maneuver point velocity of 128.6 knots.

Takeoff and Landing Performance

The takeoff and landing performance of an aircraft are analyzed in very similar ways. For the takeoff of an aircraft, the takeoff velocity can be calculated using Equation 31.

$$V_{TO} = 1.2V_{stall} \tag{31}$$

This is the velocity required for takeoff from the ground. The average takeoff velocity can be determined by substituting the takeoff velocity for velocity in Equation 32.

$$V_{avg} = 0.707V \tag{32}$$

Using the average takeoff velocity, the average thrust produced for takeoff can be calculated with Equation 33.

$$T_{TO} = \frac{P_A}{V_{TO}} \tag{33}$$

To calculate the total force acting on the aircraft during takeoff, a few new values needed to be calculated. The ground effect is given by Equation 34, using the height of the wing above the ground given in Table 1 at the beginning of the results section of this report.

$$\varphi = \frac{(\frac{16h_w}{b})^2}{1 + (\frac{16h_w}{b})^2} \tag{34}$$

This value accounts for the reduced induced drag caused by the downwash from wingtip vortices when an aircraft flies close to the ground. It was calculated to be 0.440. It is also used along with the rolling friction coefficient of 0.02 to calculate the optimal lift coefficient as shown in Equation 35.

$$C_{L,opt} = \frac{\mu_r \pi ARe}{2\varphi} \tag{35}$$

The result was an optimal lift coefficient of 0.35. Once these values have been calculated, the average resultant force acting on the aircraft during takeoff can be found using Equation 36 and the rolling coefficient value, as well as substituting the average thrust and velocity values.

$$F_{TO} = T_{TO} - \frac{1}{2}\rho_\infty {V_{TO}}^2 \left(SC_{D,0} + \varphi \frac{{C_{L,opt}}^2}{\pi Are}\right) - \mu_r(W - \frac{1}{2}\rho_\infty {V_{TO}}^2 SC_{L,opt}) \tag{36}$$

The average force is then used to calculate the takeoff rolling distance using Equation 37.

$$s_{TO} = \frac{1.44W^2}{g\rho_\infty SC_{L,max}F_{TO}} \tag{37}$$

This analysis was completed for three different sets of conditions. These can be viewed in Table 5 on the next page.

For landing performance, similar equations were used. Equation 38 shows how to calculate the landing velocity.

$$V_L = 1.3V_{stall} \tag{38}$$

The average landing velocity is calculated in the same way as before, and there is no thrust being provided. The ground effect remains the same, as does the optimal lift coefficient. Therefore, the average force experienced during landing can be calculated by Equation 29, using a rolling friction coefficient (braking) of 0.4.

$$F_L = -\frac{1}{2}\rho_\infty {V_L}^2 S\left(C_{D,0} + \varphi \frac{{C_{L,opt}}^2}{\pi ARe}\right) - \mu_{r,b}\left(W - \frac{1}{2}\rho_\infty {V_L}^2 SC_{L,opt}\right) \tag{39}$$

Finally, this equation can be used in Equation 40 on the next page to produce the landing rolling distance.

$$s_L = \frac{1.69W^2}{g\rho_\infty S C_{L,max} F_L} \tag{40}$$

The takeoff and landing rolling distance correspond to the minimum runway length requirements under each set of conditions. These limit which airports the Mooney M20R can utilize for takeoff and for landing. The results of the takeoff and landing performance of the aircraft are displayed below in Table 5.

Table 5: Rolling distances for the Mooney M20R

Conditions	Takeoff/Landing Velocity	Average Takeoff/Landing Velocity	Average Thrust	Average Force	Rolling Distance
Takeoff at Maximum Weight at Sea Level	133.7 knots	94.5 knots	893.5 lbs	777.4 lbs	1201.9 ft
Takeoff at Maximum Weight at 6,000 ft	146.2 knots	103.4 knots	741.7 lbs	625.5 lbs	1786.9 ft
Takeoff at 2,400 lbs at Sea Level	133.7 knots	94.5 knots	893.5 lbs	796.7 lbs	595.5 ft
Landing at Maximum Weight at Sea Level	144.8 knots	102.4 knots	-	-1114.3 lbs	984.1 ft

CONCLUSION

This report presented a detailed flight analysis of the Mooney M20R at altitudes of sea level, 6,000 ft, and 12,000 ft given a three-view drawing of the aircraft and basic specifications for the weight, geometry, and propulsion system. MATLAB was used to perform the calculations, graphing, and analysis. The numerical and graphical results were provided along with an in-depth discussion of their implications.

Based on the collectively agreed upon values for parasitic drag coefficient and Oswald efficiency factor, the maximum lift-to-drag ratio was calculated to be 11.4. At this ratio, the minimum amount of thrust is required to maintain steady, level flight. The required and available powers were calculated at all three altitudes examined in this report. The stall, minimum power required, maximum lift-to-drag ratio, and maximum speeds were calculated for the tree altitudes, as well as their corresponding required and available powers. The time it would take the aircraft to climb from sea level to 12,000 ft was calculated to be 14.4 min. The service ceiling for the aircraft was calculated to be 17,963 ft. This is the safest maximum altitude for the aircraft. The best climb rate was 1294.2 ft/min while the best climb angle was 9.23°. These represent the fastest time to climb and the shortest distance to climb respectively.

The maximum range of the Mooney M20R using 90% of its fuel at 12,000 ft was calculated to be 1104.9 nautical miles and the maximum endurance was 11.9 hours. These determine how far the aircraft can travel or how long it can stay in the air before it must refuel. They cannot both be achieved at the same time. In case of engine failure at 6,000 ft, the maximum range of the aircraft was calculated to be 11.2 nautical miles at a glide angle of 5.02° below the horizon. It was determined that the velocity at the maneuver point was 128.6 knots, which resulted in a minimum turn radius of 399.4 ft and a maximum turn rate of 0.544 rad/s. The maneuver point is the fastest velocity the aircraft can achieve before it starts experiencing structural failure from the load factor. It is also the best velocity for maximizing turn performance. Finally, the takeoff rolling distance with maximum weight at sea level was calculated to be 1201.9 ft, while the landing rolling distance under the same conditions was 984.1 ft. These determine the minimum runway lengths required for each situation.

Beyond just analyzing the flight of the Mooney M20R, this project explored the usefulness of MATLAB for numerical analysis. It provided a way to practically apply the theories that were learned in the course while providing a new way to engage with the material. This project also revealed how the flight of an aircraft is only dependent on a few specifications, showing how the many components of flight are all interconnected.

ATTACHMENT A: MATLAB CODE

```
%AERO 2200 Final Project: Performance Analysis of the Mooney M20R
%Jonathan Richmond on 5 December 2018
clear;
clc;

%Given/Determined values
Weight_lbs = 3368; %Weight (lbs)
WingArea_ft = 174.9; %Wing Area (square ft)
WingSpan_ft = 36.08; %Wing Span (ft)
ParasiticDrag = 0.0298; %Parasitic Drag Coefficient
OswaldEfficiency = 0.66; %Oswald Efficiency Factor
MaximumLift = 1.306; %Maximum Lift Coefficient
MaximumPowerSL_hp = 280; %Maximum Power at Sea Level (hp)
Displacement_in = 550; %Displacement (cubic in)
MaximumPower6000_hp = 230; %Maximum Power at 6000 ft (hp)
MaximumPower12000_hp = 182; %Maximum Power at 12000 ft (hp)
FuelVolume_gal = 89; %Volume of Fuel (gal)
FuelDensity_lbs = 6; %Density of Fuel (lbs/gal)
SpecificFuelConsumption_shp = 0.465; %Specific Fuel Consumption at 12000 ft (lbs/hr/shp)
Density_SL_slug = .0023769; %Density at Sea Level (slug/cubic ft)
Density_6000_slug = .0019869; %Density at 6000 ft (slug/cubic ft)
Density_12000_slug = .0016480; %Density at 12000 ft (slug/cubic ft)
Velocity_knot = 40:0.1:180; %Velocity Vector (knot)
ConstantPropellerEfficiency = 0.90; %Constant Propeller Efficiency
NeverExceedVelocity_knot = 195; %"Never Exceed" Velocity (knot)
MaximumPositiveLoadFactor = 3.8; %Maximum Positive Load Factor
MaximumNegativeLoadFactor = -1.5; %Maximum Negative Load Factor
Height_ft = 2; %Wing Height above Ground (ft)
RollingFriction = 0.02; %Coefficient of Rolling Friction
NewWeight_lb = 2400; %Weight (lb)
BrakingFriction = 0.4; %Coefficient of Rolling Friction (Braking)

%Task I: Drag Polar
%Plot the drag polar as C_D vs. C_L and clearly mark C_D_0 on the polar
AspectRatio = (WingSpan_ft ^ 2) / WingArea_ft; %Aspect Ratio

figure
fplot(@(x) ParasiticDrag + ((x ^ 2) / (pi * OswaldEfficiency * AspectRatio)), [-MaximumLift,
MaximumLift])
grid on
title('Drag Polar')
xlabel('C_{L}')
ylabel('C_{D}')

%%
%%Create a separate plot of L/D vs. C_L and determine (L/D)_max
figure
fplot(@(x) x / (ParasiticDrag + ((x ^ 2) / (pi * OswaldEfficiency * AspectRatio))), [0, MaximumLift])
grid on
```

```
axis([0 1.306 0 12])
title('(L/D)_{max}')
xlabel('C_{L}')
ylabel('L/D')


%%
%Task II: Power Required
%Determine the Power required as a function of velocity for flight at three altitudes: sea level, 6,000 ft, and 12,000 ft
%Present the data on a single graph
%Locate the stall speed, speed for minimum PR, and speed for maximum lift-to-drag ratio on this graph for each altitude
%The graph should cover the speed range from 40 to 180 knots
%Units of velocity must be knots, and units of power must be horsepower
PowerRequired_SL = (Velocity_knot .* 1.68781) .* (.5 .* Density_SL_slug .* ((Velocity_knot .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_SL_slug .* ((Velocity_knot .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required at Sea Level Vector (hp)
PowerRequired_6000 = (Velocity_knot .* 1.68781) .* (.5 .* Density_6000_slug .* ((Velocity_knot .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_6000_slug .* ((Velocity_knot .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required at 6000 ft Vector (hp)
PowerRequired_12000 = (Velocity_knot .* 1.68781) .* (.5 .* Density_12000_slug .* ((Velocity_knot .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_12000_slug .* ((Velocity_knot .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required at 12000 ft Vector (hp)

MinimumPowerRequired_SL = min(PowerRequired_SL); %Minimum Power Required at Sea Level (hp)
MinimumPowerRequired_6000 = min(PowerRequired_6000); %Minimum Power Required at 6000 ft (hp)
MinimumPowerRequired_12000 = min(PowerRequired_12000); %Minimum Power Required at 12000 ft (hp)

PowerRequired_SLIndex = find(PowerRequired_SL==MinimumPowerRequired_SL);
PowerRequired_6000Index = find(PowerRequired_6000==MinimumPowerRequired_6000);
PowerRequired_12000Index = find(PowerRequired_12000==MinimumPowerRequired_12000);

Velocity_MinimumPowerRequired_SL = Velocity_knot(PowerRequired_SLIndex); %Velocity for Minimum Power Required at Sea Level (knot)
Velocity_MinimumPowerRequired_6000 = Velocity_knot(PowerRequired_6000Index); %Velocity for Minimum Power Required at 6000 ft (knot)
Velocity_MinimumPowerRequired_12000 = Velocity_knot(PowerRequired_12000Index); %Velocity for Minimum Power Required at 12000 ft (knot)

StallSpeed_SL = (((2 * Weight_lbs) / (Density_SL_slug * WingArea_ft * MaximumLift)) ^ .5) / 1.68781; %Stall Speed at Sea Level (knot)
StallSpeed_6000 = (((2 * Weight_lbs) / (Density_6000_slug * WingArea_ft * MaximumLift)) ^ .5) / 1.68781; %Stall Speed at 6000 ft (knot)
StallSpeed_12000 = (((2 * Weight_lbs) / (Density_12000_slug * WingArea_ft * MaximumLift)) ^ .5) / 1.68781; %Stall Speed at 12000 ft (knot)
```

PowerRequired_StallSpeed_SL = (StallSpeed_SL .* 1.68781) .* (.5 .* Density_SL_slug .* ((StallSpeed_SL .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_SL_slug .* ((StallSpeed_SL .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required for Stall Speed at Sea Level (hp)
PowerRequired_StallSpeed_6000 = (StallSpeed_6000 .* 1.68781) .* (.5 .* Density_6000_slug .* ((StallSpeed_6000 .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_6000_slug .* ((StallSpeed_6000 .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required for Stall Speed at 6000 ft (hp)
PowerRequired_StallSpeed_12000 = (StallSpeed_12000 .* 1.68781) .* (.5 .* Density_12000_slug .* ((StallSpeed_12000 .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_12000_slug .* ((StallSpeed_12000 .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required for Stall Speed at 12000 ft (hp)

Lift = 0:0.01:MaximumLift; %Lift Coefficient Vector
Lift3_2 = Lift .^ (1.5); %Lift Coefficient^3/2
Drag = ParasiticDrag + ((Lift .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)); %Drag Coefficient Vector
Lift3_2PerDrag = Lift3_2 ./ Drag; %Lift Coefficient^3/2 / Drag Coefficient
MaximumLift3_2PerDrag = max(Lift3_2PerDrag); %Maximum Lift Coefficient^3/2 / Drag Coefficient
Lift3_2PerDragIndex = find(Lift3_2PerDrag==MaximumLift3_2PerDrag);
Lift_MaximumLift3_2PerDrag = Lift3_2(Lift3_2PerDragIndex); %Lift Coefficient at Maximum Lift Coefficient^3/2 / Drag Coefficient
Velocity_MaximumLift3_2PerDrag = (((2 * Weight_lbs) / (Density_12000_slug * WingArea_ft * Lift_MaximumLift3_2PerDrag)) ^ .5) / 1.68781; %Velocity at Maximum Lift Coefficient^3/2 / Drag Coefficient (knot)

LiftPerDrag = Lift ./ Drag; %L/D Vector
MaximumLiftPerDrag = max(LiftPerDrag); %Maximum L/D
LiftPerDragIndex = find(LiftPerDrag==MaximumLiftPerDrag);
Lift_MaximumLiftPerDrag = Lift(LiftPerDragIndex); %Lift Coefficient at Maximum L/D

Velocity_MaximumLiftPerDrag_SL = (((2 * Weight_lbs) / (Density_SL_slug * WingArea_ft * Lift_MaximumLiftPerDrag)) ^ .5) / 1.68781; %Velocity at Maximum L/D at Sea Level (knot)
Velocity_MaximumLiftPerDrag_6000 = (((2 * Weight_lbs) / (Density_6000_slug * WingArea_ft * Lift_MaximumLiftPerDrag)) ^ .5) / 1.68781; %Velocity at Maximum L/D at 6000 ft (knot)
Velocity_MaximumLiftPerDrag_12000 = (((2 * Weight_lbs) / (Density_12000_slug * WingArea_ft * Lift_MaximumLiftPerDrag)) ^ .5) / 1.68781; %Velocity at Maximum L/D at 12000 ft (knot)

PowerRequired_MaximumLiftPerDrag_SL = (Velocity_MaximumLiftPerDrag_SL .* 1.68781) .* (.5 .* Density_SL_slug .* ((Velocity_MaximumLiftPerDrag_SL .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_SL_slug .* ((Velocity_MaximumLiftPerDrag_SL .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required for Maximum L/D at Sea Level (hp)
PowerRequired_MaximumLiftPerDrag_6000 = (Velocity_MaximumLiftPerDrag_6000 .* 1.68781) .* (.5 .* Density_6000_slug .* ((Velocity_MaximumLiftPerDrag_6000 .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_6000_slug .* ((Velocity_MaximumLiftPerDrag_6000 .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .* OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required for Maximum L/D at 6000 ft (hp)
PowerRequired_MaximumLiftPerDrag_12000 = (Velocity_MaximumLiftPerDrag_12000 .* 1.68781) .* (.5 .* Density_12000_slug .* ((Velocity_MaximumLiftPerDrag_12000 .* 1.68781) .^ 2) .* WingArea_ft .* (ParasiticDrag + ((((Weight_lbs) ./ (.5 .* Density_12000_slug .*

```
((Velocity_MaximumLiftPerDrag_12000 .* 1.68781) .^ 2) .* WingArea_ft)) .^ 2) ./ (pi .*
OswaldEfficiency .* AspectRatio)))) ./ 550; %Power Required for Maximum L/D at 12000 ft (hp)

figure
plot(Velocity_knot, PowerRequired_SL, '-')
hold on
plot(Velocity_knot, PowerRequired_6000, '--')
plot(Velocity_knot, PowerRequired_12000, '-.')
grid on
axis([40 180 50 340])
title('Power Curves')
xlabel('Velocity (kts)')
ylabel('THP (hp)')


%Task III: Power Available
%Using the engine data provided, along with the propeller efficiency data, determine the power available
at the three altitudes
%On the same graph as the one developed in Task II, plot PA as a function of velocity
PropellerEfficiency = 0.90 .* (1 - ((35 ./ Velocity_knot) .^ 2)); %Propeller Efficiency

PropellerEfficiency_StallSpeed_SL = 0.90 * (1 - ((35 / StallSpeed_SL) ^ 2)); %Propeller Efficiency for
Stall Speed at Sea Level
PropellerEfficiency_StallSpeed_6000 = 0.90 * (1 - ((35 / StallSpeed_6000) ^ 2)); %Propeller Efficiency
for Stall Speed at 6000 ft
PropellerEfficiency_StallSpeed_12000 = 0.90 * (1 - ((35 / StallSpeed_12000) ^ 2)); %Propeller
Efficiency for Stall Speed at 12000 ft

PropellerEfficiency_MaximumLiftPerDrag_SL = 0.90 * (1 - ((35 / Velocity_MaximumLiftPerDrag_SL)
^ 2)); %Propeller Efficiency for Maximum L/D at Sea Level
PropellerEfficiency_MaximumLiftPerDrag_6000 = 0.90 * (1 - ((35 /
Velocity_MaximumLiftPerDrag_6000) ^ 2)); %Propeller Efficiency for Maximum L/D at 6000 ft
PropellerEfficiency_MaximumLiftPerDrag_12000 = 0.90 * (1 - ((35 /
Velocity_MaximumLiftPerDrag_12000) ^ 2)); %Propeller Efficiency for Maximum L/D at 12000 ft

PowerAvailable_SL = MaximumPowerSL_hp * PropellerEfficiency; %Power Available at Sea Level
(hp)
PowerAvailable_6000 = MaximumPower6000_hp * PropellerEfficiency; %Power Available at Sea Level
(hp)
PowerAvailable_12000 = MaximumPower12000_hp * PropellerEfficiency; %Power Available at Sea
Level (hp)

PowerAvailable_StallSpeed_SL = MaximumPowerSL_hp * PropellerEfficiency_StallSpeed_SL; %Power
Available for Stall Speed at Sea Level (hp)
PowerAvailable_StallSpeed_6000 = MaximumPower6000_hp * PropellerEfficiency_StallSpeed_6000;
%Power Available for Stall Speed at 6000 ft (hp)
PowerAvailable_StallSpeed_12000 = MaximumPower12000_hp *
PropellerEfficiency_StallSpeed_12000; %Power Available for Stall Speed at 12000 ft (hp)

PowerAvailable_MaximumLiftPerDrag_SL = MaximumPowerSL_hp *
PropellerEfficiency_MaximumLiftPerDrag_SL; %Power Available for Maximum L/D at Sea Level (hp)
```

```
PowerAvailable_MaximumLiftPerDrag_6000 = MaximumPower6000_hp *
PropellerEfficiency_MaximumLiftPerDrag_6000; %Power Available for Maximum L/D at 6000 ft (hp)
PowerAvailable_MaximumLiftPerDrag_12000 = MaximumPower12000_hp *
PropellerEfficiency_MaximumLiftPerDrag_12000; %Power Available for Maximum L/D at 12000 ft
(hp)

plot(Velocity_knot, PowerAvailable_SL, '-')
plot(Velocity_knot, PowerAvailable_6000, '--')
plot(Velocity_knot, PowerAvailable_12000, '-.')
legend('Required Sea Level', 'Required 6,000 ft', 'Required 12,000 ft', 'Available Sea Level', 'Available
6,000 ft', 'Available 12,000 ft', 'Location', 'northwest')
line([StallSpeed_SL StallSpeed_SL], [50 340])
line([StallSpeed_6000 StallSpeed_6000], [50 340])
line([StallSpeed_12000 StallSpeed_12000], [50 340])
hold off


%%
%Task IV: Climb Performance
%Prepare a graph of rate of climb (ft/min) vs. V (knots) for the three altitudes
%Note the maximum rate of climb and the true airspeed (V) that corresponds to the maximum rate of
climb
RateOfClimb_SL = ((PowerAvailable_SL - PowerRequired_SL) .* (550 .* 60)) ./ Weight_lbs; %Rate of
Climb at Sea Level (ft/min)
RateOfClimb_6000 = ((PowerAvailable_6000 - PowerRequired_6000) .* (550 .* 60)) ./ Weight_lbs;
%Rate of Climb at 6000 ft (ft/min)
RateOfClimb_12000 = ((PowerAvailable_12000 - PowerRequired_12000) .* (550 .* 60)) ./ Weight_lbs;
%Rate of Climb at 12000 ft (ft/min)

figure
plot(Velocity_knot, RateOfClimb_SL, '-')
hold on
plot(Velocity_knot, RateOfClimb_6000, '--')
plot(Velocity_knot, RateOfClimb_12000, '-.')
grid on
axis([40 180 0 1500])
title('Rate of Climb')
xlabel('Velocity (kts)')
ylabel('R/C (ft/min)')
legend('Sea Level', '6,000 ft', '12,000 ft', 'Location', 'northeast')
hold off

%%
%Plot the results on an altitude vs. R/C graph and extrapolate the results to estimate the absolute and
service ceilings
MaximumRateOfClimb_SL = max(RateOfClimb_SL); %Maximum Rate of Climb at Sea Level (ft/min)
RateOfClimb_SLIndex = find(RateOfClimb_SL==MaximumRateOfClimb_SL);
Velocity_MaximumRateOfClimb_SL = Velocity_knot(RateOfClimb_SLIndex); %Velocity for
Maximum Rate of Climb at Sea Level (knot)

MaximumRateOfClimb_6000 = max(RateOfClimb_6000); %Maximum Rate of Climb at 6000 ft (ft/min)
```

```
RateOfClimb_6000Index = find(RateOfClimb_6000==MaximumRateOfClimb_6000);
Velocity_MaximumRateOfClimb_6000 = Velocity_knot(RateOfClimb_6000Index); %Velocity for
Maximum Rate of Climb at 6000 ft (knot)

MaximumRateOfClimb_12000 = max(RateOfClimb_12000); %Maximum Rate of Climb at 12000 ft
(ft/min)
RateOfClimb_12000Index = find(RateOfClimb_12000==MaximumRateOfClimb_12000);
Velocity_MaximumRateOfClimb_12000 = Velocity_knot(RateOfClimb_12000Index); %Velocity for
Maximum Rate of Climb at 12000 ft (knot)

p = polyfit([MaximumRateOfClimb_SL, MaximumRateOfClimb_6000, MaximumRateOfClimb_12000],
[0, 6, 12], 2); %Trendline Coefficients
x = 0:MaximumRateOfClimb_SL;
f = polyval(p, x); %Trendline

AbsoluteCeiling = 1000 * polyval(p, 0); %Absolute Ceiling (ft)
ServiceCeiling = 1000 * polyval(p, 100); %Service Ceiling (ft)

figure
scatter([MaximumRateOfClimb_SL, MaximumRateOfClimb_6000, MaximumRateOfClimb_12000], [0,
6, 12])
hold on
plot(x, f)
grid on
axis([0 1400 0 20])
title('Absolute and Service Ceilings')
xlabel('(R/C)_{max} (ft/min)')
ylabel('Altitude (kft)')
hold off

%Use the results to determine the time to climb from sea level to an altitude of 12,000 ft
p1 = polyfit ([0, 6, 12], [1 / MaximumRateOfClimb_SL, 1 / MaximumRateOfClimb_6000, 1 /
MaximumRateOfClimb_12000], 2);
x1 = 0:12;
f1 = polyval(p1, x1);
TimeToClimb = 1000 * trapz(x1, f1); % Time to Climb to 12000 ft (min)

%%
%Develop a climb hodograph for sea level climb at maximum takeoff weight to determine the true
airspeeds (V) for best angle of climb and best rate of climb
%On the plot for the climb hodograph, the units for airspeed must be knots on both axes
VerticalVelocityClimb = RateOfClimb_SL ./ 60 ./ 1.68781; %Vertical Velocity during Climb at Sea
Level (knot)
HorizontalVelocityClimb = ((Velocity_knot .^ 2) - (VerticalVelocityClimb .^ 2)) .^ .5; %Horizontal
Velocity during Climb at Sea Level (knot)
ClimbAngle = atan(VerticalVelocityClimb ./ HorizontalVelocityClimb) .* (180 ./ pi); %Climb Angle at
Sea Level (deg)

BestRateOfClimb = max(RateOfClimb_SL); %Best Rate of Climb at Sea Level (ft/min)
RateOfClimbIndex = find(RateOfClimb_SL==BestRateOfClimb);
```

```
ClimbAngle_BestRateOfClimb = ClimbAngle(RateOfClimbIndex); %Climb Angle for Best Rate of Climb (deg)
Velocity_BestRateOfClimb = Velocity_knot(RateOfClimbIndex); %Velocity for Best Rate of Climb (knot)

BestClimbAngle = max(ClimbAngle); %Best Climb Angle at Sea Level (deg)
ClimbAngleIndex = find(ClimbAngle==BestClimbAngle);
RateOfClimb_BestClimbAngle = RateOfClimb_SL(ClimbAngleIndex); %Rate of CLimb for Best Climb Angle (ft/min)
Velocity_BestClimbAngle = Velocity_knot(ClimbAngleIndex); %Velocity for Best Climb Angle (knot)

figure
plot(HorizontalVelocityClimb, VerticalVelocityClimb)
grid on
axis([0 160 0 160])
axis equal
title('Climb Hodograph at Sea Level')
xlabel('V_{h} (kts)')
ylabel('V_{v} (kts)')


%%
%Task V: Range and Endurance
%Use the Breguet Range and Endurance relations to determine the maximum range and endurance while cruising at 12,000 ft and using 90% of the fuel on board
%For this cruise flight, consider the specific fuel consumption and propeller efficiency to be constant
%Note the airspeed(s) that the pilot must maintain during these flights in order to maintain the optimum conditions assumed in the Breguet equations
%Range values should be reported in nautical miles and endurance values should be reported in decimal hours
ConsistentSpecificFuelConsumption = SpecificFuelConsumption_shp / 3600 / 550; %Constant Consistent Specific Fuel Consumption (1/ft)
FuelWeight = FuelVolume_gal * FuelDensity_lbs; %Weight of Fuel (lbs)
FuelWeight90 = .9 * FuelWeight; %Weight of 90% of Fuel (lbs)
FinalWeight = Weight_lbs - FuelWeight90; %Total Weight with 10% Fuel (lbs)

Range = (ConstantPropellerEfficiency / ConsistentSpecificFuelConsumption) * MaximumLiftPerDrag * log(Weight_lbs / FinalWeight) * 0.000164579; %Maximum Range at 12000 ft (nmi)
Endurance = (ConstantPropellerEfficiency / ConsistentSpecificFuelConsumption) * MaximumLift3_2PerDrag * ((2 * Density_12000_slug * WingArea_ft) ^ .5) * ((FinalWeight ^ (-.5)) - (Weight_lbs ^ (-.5))) / 3600; % Maximum Endurance at 12000 ft (hrs)


%Task VI: Glide Performance
%Develop a glide hodograph, with VH and VV in knots (on both axes) for maximum takeoff weight at an altitude of 6,000 ft
GlideAngle = atand(1 ./ LiftPerDrag); %Glide Angle (deg)
BestGlideAngle = min(GlideAngle); %Best Glide Angle (deg)
GlideVelocity = (((2 .* (Weight_lbs ./ WingArea_ft) .* cosd(GlideAngle)) ./ (Density_6000_slug .* Lift)) .^ .5) .* 0.592484; %Glide Velocity (knot)
GlideVerticalVelocity = -1 .* GlideVelocity .* sind(GlideAngle); %Glide Vertical Velocity (knot)
```

```
GlideHorizontalVelocity = GlideVelocity .* cosd(GlideAngle); %Glide Horizontal Velocity (knot)

figure
plot(GlideHorizontalVelocity, GlideVerticalVelocity)
grid on
axis ([0 180 -20 160])
axis equal
title('Glide Hodograph at 6,000 ft')
xlabel('V_{h} (kts)')
ylabel('V_{v} (kts)')

%%
%Assuming engine failure at a pressure altitude of 6,000 ft at maximum takeoff weight, determine the maximum distance (in nmi) that the aircraft can glide to reach an airfield situated at sea level
%What indicated airspeed (in knots) should the pilot fly in order to maximize glide distance
BestGlideRange = (6000 / tand(BestGlideAngle)) * 0.000164579; %Best Glide Range (nmi)
GlideRangeIndex = find(GlideAngle==BestGlideAngle);
BestGlideLift = Lift(GlideRangeIndex); %Best Lift Coefficient for Glide
BestGlideVelocity = (((2 * (Weight_lbs / WingArea_ft) * cosd(BestGlideAngle)) / (Density_6000_slug * BestGlideLift)) ^ .5) * 0.592484; %Best Glide Velocity (knot)


%Task VII: Turning Flight
%Create a V-n diagram for standard sea level conditions if the “never exceed” airspeed is 195 knots, the maximum positive load factor is 3.8, and the maximum negative load factor is -1.5
%Clearly define the maneuver point on the diagram
NewVelocities_knot = 0:0.1:NeverExceedVelocity_knot; %New Velocity Vector (knot)
LoadFactor = .5 .* Density_SL_slug .* ((NewVelocities_knot .* 1.68781) .^ 2) .* (MaximumLift ./ (Weight_lbs ./ WingArea_ft)); %Load Factor
LoadFactor_StallSpeed = .5 * Density_SL_slug * ((StallSpeed_SL * 1.68781) ^ 2) * (MaximumLift / (Weight_lbs / WingArea_ft)); %Load Factor at Stall Speed

ManeuverPoint = (((2 * MaximumPositiveLoadFactor * (Weight_lbs / WingArea_ft)) / (Density_SL_slug * MaximumLift)) ^ .5) * 0.592484; %Velocity for Maximum Positive Load Factor (Maneuver Point) (knot)
Velocity_MaximumNegativeLoadFactor = (((2 * (-1 * MaximumNegativeLoadFactor) * (Weight_lbs / WingArea_ft)) / (Density_SL_slug * MaximumLift)) ^ .5) * 0.592484; %Velocity for Maximum Negative Load Factor (knot)

figure
plot(NewVelocities_knot, LoadFactor, 'b')
hold on
plot(NewVelocities_knot, -LoadFactor, 'b')
line([ManeuverPoint NeverExceedVelocity_knot], [MaximumPositiveLoadFactor MaximumPositiveLoadFactor])
line([Velocity_MaximumNegativeLoadFactor NeverExceedVelocity_knot], [MaximumNegativeLoadFactor MaximumNegativeLoadFactor])
line([StallSpeed_SL StallSpeed_SL], [-LoadFactor_StallSpeed LoadFactor_StallSpeed])
line([NeverExceedVelocity_knot NeverExceedVelocity_knot], [MaximumNegativeLoadFactor MaximumPositiveLoadFactor])
grid on
```

```
axis([0 200 -2 4])
title("V-n Diagram, Standard Sea Level Conditions")
xlabel("True Airspeed (kts)")
ylabel("Load Factor (n)")
hold off

%%
%%What is the minimum turn radius and the maximum turn rate at maximum takeoff weight at standard sea level conditions
TurnRadius = ((ManeuverPoint * 1.68781) ^ 2) / (32.2 * (((MaximumPositiveLoadFactor ^ 2) - 1) ^ .5)); %Turn Radius (ft)
MinimumTurnRadius = min(TurnRadius); %Minimum Turn Radius (ft)
TurnRate = (32.2 * (((MaximumPositiveLoadFactor ^ 2) - 1) ^ .5)) / (ManeuverPoint * 1.68781); %Turn Rate (rad/s)
MaximumTurnRate = max(TurnRate); %Maximum Turn Rate (rad/s)


%Task VIII: Takeoff and Landing Performance
%Takeoff ground roll distance at maximum takeoff weight at standard sea level conditions
%Takeoff ground roll distance at maximum takeoff weight at a high altitude airport (6000 ft)
TakeoffVelocity_SL = 1.2 * StallSpeed_SL * 1.68781; %Takeoff Velocity Sea Level (ft/s)
TakeoffVelocity_6000 = 1.2 * StallSpeed_6000 * 1.68781; %Takeoff Velocity 6000 ft (ft/s)

AverageTakeoffVelocity_SL = 0.707 * TakeoffVelocity_SL; %Average Takeoff Velocity Sea Level (ft/s)
AverageTakeoffVelocity_6000 = 0.707 * TakeoffVelocity_6000; %Average Takeoff Velocity 6000 ft (ft/s)

AveragePropellerEfficiency_SL = 0.90 * (1 - ((35 / (AverageTakeoffVelocity_SL / 1.68781)) ^ 2)); %Average Propeller Efficiency Sea Level
AveragePropellerEfficiency_6000 = 0.90 * (1 - ((35 / (AverageTakeoffVelocity_6000 / 1.68781)) ^ 2)); %Average Propeller Efficiency 6000 ft

AveragePowerAvailable_SL = AveragePropellerEfficiency_SL * MaximumPowerSL_hp * 550; %Average Power Available Sea Level (ft*lb/s)
AveragePowerAvailable_6000 = AveragePropellerEfficiency_6000 * MaximumPower6000_hp * 550; %Average Power Available 6000 ft (ft*lb/s)

AverageThrust_SL = AveragePowerAvailable_SL / AverageTakeoffVelocity_SL; %Average Thrust Sea Level (lbs)
AverageThrust_6000 = AveragePowerAvailable_6000 / AverageTakeoffVelocity_6000; %Average Thrust 6000 ft (lbs)

GroundEffect = ((16 * Height_ft / WingSpan_ft) ^ 2) / (1 + ((16 * Height_ft / WingSpan_ft) ^ 2)); %Ground Effect
OptimalLift = (RollingFriction * pi * AspectRatio * OswaldEfficiency) / (2 * GroundEffect); %Optimal Lift Coefficient

AverageForce_SL = AverageThrust_SL - (.5 * Density_SL_slug * (AverageTakeoffVelocity_SL ^ 2) * WingArea_ft * (ParasiticDrag + (GroundEffect * ((OptimalLift ^ 2) / (pi * AspectRatio * OswaldEfficiency))))) - (RollingFriction * (Weight_lbs - (.5 * Density_SL_slug * (AverageTakeoffVelocity_SL ^ 2) * WingArea_ft * OptimalLift))); %Average Thrust Sea Level (lb)
```

AverageForce_6000 = AverageThrust_6000 - (.5 * Density_6000_slug * (AverageTakeoffVelocity_6000 ^ 2) * WingArea_ft * (ParasiticDrag + (GroundEffect * ((OptimalLift ^ 2) / (pi * AspectRatio * OswaldEfficiency))))) - (RollingFriction * (Weight_lbs - (.5 * Density_6000_slug * (AverageTakeoffVelocity_6000 ^ 2) * WingArea_ft * OptimalLift))); %Average Thrust 6000 ft (lb)

TakeoffRollingDistance_SL = (1.44 * (Weight_lbs ^ 2)) / (32.2 * Density_SL_slug * WingArea_ft * MaximumLift * AverageForce_SL); %Takeoff Ground Roll Distance Sea Level (ft)
TakeoffRollingDistance_6000 = (1.44 * (Weight_lbs ^ 2)) / (32.2 * Density_6000_slug * WingArea_ft * MaximumLift * AverageForce_6000); %Takeoff Ground Roll Distance 6000 ft (ft)

%Takeoff ground roll distance at a takeoff weight of 2400 lbs at standard sea level conditions
AverageForce_NewWeight = AverageThrust_SL - (.5 * Density_SL_slug * (AverageTakeoffVelocity_SL ^ 2) * WingArea_ft * (ParasiticDrag + (GroundEffect * ((OptimalLift ^ 2) / (pi * AspectRatio * OswaldEfficiency))))) - (RollingFriction * (NewWeight_lb - (.5 * Density_SL_slug * (AverageTakeoffVelocity_SL ^ 2) * WingArea_ft * OptimalLift))); %Average Thrust Sea Level (lb)
TakeoffRollingDistance_NewWeight = (1.44 * (NewWeight_lb ^ 2)) / (32.2 * Density_SL_slug * WingArea_ft * MaximumLift * AverageForce_NewWeight); %Takeoff Ground Roll Distance Sea Level (ft)

%Landing ground roll distance at maximum weight and standard sea level conditions
TouchdownVelocity = 1.3 * StallSpeed_SL * 1.68781; %Touchdown Velocity Sea Level (ft/s)
AverageTouchdownVelocity = 0.707 * TouchdownVelocity; %Average Touchdown Velocity Sea Level (ft/s)

AverageTouchdownForce = -(.5 * Density_SL_slug * (AverageTouchdownVelocity ^ 2) * WingArea_ft * (ParasiticDrag + (GroundEffect * ((OptimalLift ^ 2) / (pi * AspectRatio * OswaldEfficiency))))) - (BrakingFriction * (Weight_lbs - (.5 * Density_SL_slug * (AverageTouchdownVelocity ^ 2) * WingArea_ft * OptimalLift))); %Average Thrust Sea Level (lb)
TouchdownRollingDistance = (1.69 * (Weight_lbs ^ 2)) / (32.2 * Density_SL_slug * WingArea_ft * MaximumLift * -AverageTouchdownForce); %Touchdown Ground Roll Distance Sea Level (ft)