NEUROECONOMICS

Decision Making and the Brain

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CHAPTER

3

Computational and Process Models of Decision Making in Psychology and Behavioral Economics

Eric J. Johnson and Roger Ratcliff

OUTLINE

INTRODUCTION

History

At first glance, historically, models in decision-making research seem to have very little in common with neuroscience. Most decision-making models have been concerned with predicting outcomes, or more precisely choices, from a set of inputs, the characteristics of the options, and have been mute about the underlying cognitive and neuronal processes underlying choice. This interest in predicting outcomes has been associated with a reliance on algebraic models that specify a transformation of these properties of external options to a rank ordering of the attractiveness of options, an idea developed in some detail in Chapter 1. These models said little about how the brain and cognition might transform these inputs into output.

These days, that first glance would be very misleading. Neuroeconomics today makes great use of both models that are meant to provide accounts of what *should* be chosen (normative models) and models that describe what is *actually* chosen (descriptive models). This has been facilitated by two important trends: the first is that there is increasing evidence of brain processes that correspond, in some areas, to the output generated by the mathematical expressions employed by some of these historical models. For example, for simple choices, the value of an option described by some of these models seems to be encoded by the medial orbitofrontal cortex and ventral striatum (see Chapters 8 and 20; Kable and Glimcher, 2007; Knutson *et al.*, 2007; Plassmann *et al.*, 2007; Rangel *et al.*, 2008). The second is that models of decision making are changing: faced with

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a plethora of possible accounts mapping inputs to options, there is an emerging consensus that models that make predictions about additional data, data emerging from an understanding of cognitive processes, would both winnow this plethora of possible accounts and would help build more robust, useful and reliable models.

This chapter consists of two parts. The first part describes the history of modeling in choice with an emphasis on the psychological, from normative to descriptive and from algebraic to process models. This review of the major areas of the psychology of decision making focuses on three topics that are central to the study of choices: (1) choice under uncertainty (such as deciding whether or not to buy a lottery ticket, a stock, or an insurance policy), where the outcomes are uncertain; (2) choice under certainty where the outcomes are known (such as deciding which car to buy); and finally (3) choice across time (such as deciding whether to study in hopes of doing better in a distant exam or to, instead, party tonight). The second part of this chapter illustrates the newer style of computational process models which describe the psychological and neural processes in addition to predicting choices, and illustrates this class of models in detail using one important subclass that has had a great impact in neuroscience: *diffusion models*.

MODELS OF RISKY CHOICE

Normative Origins

Most theories of choice are either *normative models* that advise people about how they should make choices, or *descriptive models*, portraying how they actually make choices. The origins of normative models of how to make choice under risk occurred in the eighteenth century, in response to questions presented by gambling. Recall that most gambles consist of a set of outcomes and their associated probabilities. Imagine simply flipping a coin to double your money, say an initial stake of \$10, or lose it all. The coin flip gives each outcome a probability of .5, and the two outcomes are either \$0 or \$20. The early normative advice about how to choose between two gambles was simply to choose the one with the highest expected payoff:

$$EV(X) = \sum_{x} p(x) \cdot x \tag{3.1}$$

where x is the payoff for each outcome, $1 \ldots X$, and p is the probability associated with that outcome. Thus, in our coin flip example, $.5 \times \$0 + .5 \times \$20 = \$10$. However appealing the idea of weighting payoffs by their probability might have been in this early *expected value theory*, this approach implied the uncomfortable fact that the value of each increasing dollar to every chooser was the same: it required that the pleasure generated by increasing one's wealth, x, from \$10 to \$20 is, according to Expected Value, exactly the same as the impact of an increase from \$999,980 to \$1,000,000.

In response to this uncomfortable fact, Bernoulli (1738) proposed that the decision maker should instead pick the gamble with the highest expected *utility* where:

$$EU(X) = \sum_{x} p(x)u(x) \tag{3.2}$$

The function that maps actual wealth, x, on the x-axis into utility for wealth, $u(x)$, is in this formulation no longer linear but usually "concave," for example, a power function of the form, $u(x) = x^{\theta}$, where θ is a number less than or equal to 1. The exponent θ is thus a parameter that describes the curvature of this function and serves as an index of an individual's degree of risk aversion. Put another way $\theta < 1$ corresponds to money having diminishing marginal returns, a point developed in Chapter 1. This idea of expected utility has been the dominant normative theory in economics, in part because von Neumann and Morgenstern (1953) provided an intuitively appealing axiomatic foundation for expected utility (EU) maximization, which made it a normatively attractive decision criterion not only for repeated decisions in the long run, but when extended by Savage (1954) also for unique risky decisions even when the true probabilities are not known to the decision maker. Here we gloss over the very foundational conceptual differences between the classical economic approach of Bernoulli and the neoclassical approach of von Neumann and Morgenstern that are the subject of Chapter 1.

Descriptive Modifications

Starting as early as the 1950s, empirical evidence, however, began to cast doubt on EU as a descriptive choice model (Allais, 1953). While these data did not catch the attention of many economists, by the early 1970s there were a significant number of empirical observations that could not be accounted for by expected utility (Kahneman and Tversky, 1979; see Wu *et al.*, 2004 for a historical overview). While there had been piecemeal attempts to account for each of the failures of expected utility, prospect theory (Kahneman and Tversky, 1984; Kahneman and Tversky, 1979) presented three major changes to expected utility intended to account for many of these known failures as well as several new problems identified by Kahneman and Tversky (see the Appendix for a detailed description of prospect theory). These changes were: (1) introducing a transformation relating objective probabilities to subjective probabilities; (2) defining outcomes (utilities) not on total wealth as in expected utility but rather on gains and losses relative to a dynamic *reference point*; and (3) allowing losses to have a different mapping into value than that of gains, a phenomenon they called *loss aversion*.

Prospect theory is a descriptive theory of choice because it attempts to describe the choices that people make, and not, like a normative theory, how choices *should* be made. In the intervening three decades, prospect theory has flourished as the leading descriptive model of decision under risk, and has been used to account for many empirical phenomena (Kahneman and Tversky, 2000). There have been many successful attempts at implementing prospect theory in realistic settings such as medical decision making (Bleichrodt *et al.*, 2001), consumer reactions to supermarket prices (Hardie *et al.*, 1993) and behavior in labor and real estate markets (Camerer *et al.*, 1997; Genesove and Mayer, 2001). Paralleling recent developments in neuroscience, individual differences in prospect theory parameters are serving as explanations for differences in observed behavior in games (Tanaka *et al.*, 2010) and researchers have developed technologies for measuring these parameters quickly (Toubia *et al.*, 2012). For a comprehensive review of prospect theory see the Appendix; for a review of its applications see Wakker (2010) and Camerer (2004).

Several different descriptive theories have, however, emerged as alternative mappings between options and choices (for example: Birnbaum, 2008; Birnbaum *et al.*, 1999; Loomes, 2010; Loomes and Sugden, 1982). One robust set of findings is that the rank order of the outcomes matters: the extreme outcomes of a gamble have more impact on choices than would be expected. In economics, these so-called *rank-dependent* models (see Quiggin, 1993 for a review) and Tversky and Kahneman's *Cumulative Prospect Theory* (Tversky and Kahneman, 1992) were developed to address these results. The basic intuition that guides these models is that decision makers give more weight to outcomes that are particularly good, or particularly bad in the set of possible outcomes when considering the relative values of the available options.

These models have been very successful in predicting choice, and in establishing insights into phenomena such as *framing* and loss aversion. For all of their success and impact however, prospect theory and these related models share important properties with expected value and expected utility theories: they define a mapping between characteristics of the objects under consideration and their value, but are mute to the cognitive computations that may construct this mapping (Brandstätter *et al.*, 2006; Johnson *et al.*, 2008), as in fact do most of the alternatives to this class of theory.

Two particular properties of this entire class of models that they share are: (1) the assumption that outcomes are weighted by their probabilities; and (2) that all outcomes are examined. Thus, if the decision maker is faced with a complex decision with hundreds of outcomes, all must be examined and combined. As we will see, this is an important feature, perhaps even a constraint, to which we will turn next.

Heuristic Models of Risky Choice

Developed in response to these observations, *heuristic models* (see Payne *et al.*, 1993 for a review) describe shortcuts for making a choice or judgment that do not necessarily include these two properties. For risky choice, heuristic models differ from the preceding classes of *integration models* with regard to both properties we have described: first, these models do not always weight outcomes by their probabilities. Instead they may, for example, calculate the differences in payoffs (González-Vallejo, 2002; González-Vallejo *et al.*, 2003; Tversky, 1969), or make a series of comparisons such as which gamble has the biggest and most likely outcome (Brandstätter *et al.*, 2006). Second, they may intentionally ignore available information; they may for example not even consider outcomes with small payoffs or small probabilities. If a gamble has a small probability (say .01) of a small outcome (say losing $.10) this option might be ignored entirely during the decision process. These approaches present a stark contrast with integration models, such as prospect theory.[1]

Heuristic models have strengths and weaknesses. One important strength is that they often make predictions of not only what is chosen, but also make predictions for other characteristics of the choice process, such as how long it will take to make a choice (reaction time), or what information will and will not be examined while making a choice. This kind of information can be very important in separating models, because often very different models can make very similar predictions. For example, the priority heuristic (Brandstätter *et al.*, 2006) proposed that people made choices through a series of comparisons: first compare the amounts to win in the minimum outcome, then the probabilities of the minimum outcome, then the amounts of the maximum outcome.

[1]The original description of prospect theory described similar ideas as *editing* operations that were applied before gambles were evaluated.

At each stage, if the difference between the options exceeds 10%, the process stops and the gamble better on that attribute is chosen. This process seems very different from the process implied by integration models like prospect theory or expected utility, which both weight outcomes by their probabilities and look at all information. Despite these differences, the priority heuristic makes choices that are very similar to prospect theory and in some cases fit the data better. If it were not for the predictions about what information is examined, the two models would both be strong candidates as choice models. However, when one examines the information that is acquired by the decision maker, the priority heuristic appears to be a poor predictor of information choice (Johnson *et al.*, 2008). That is an important distinction for fields like neuroeconomics where the underlying process is of tremendous importance.

More generally, the emphasis in heuristic models on making predictions for different kinds of heuristic behavior, and not just choices, is a real strength: by making predictions for multiple dependent measures, extra constraints are placed on the model and models that mimic on one measure can often be discriminated when both measures are examined. At the same time, these models have not reached the point where they are easily applied to real-world decision problems: there is not much in the way of off-the-shelf technology that allows these models to be applied to problems of consumer choice and public policy, for example. Thus, models like expected utility, and to a lesser extent, prospect theory, remain the mainstay of applications.

MODELS OF RISKLESS CHOICE

Multi Attribute Utility Theory

Riskless choices involve choosing options where the outcomes are known, like buying a car or smartphone, and these options are usually thought of as consisting of a set of features or attributes. In the case of a car, these attributes might include price, gas mileage, room, appearance, and acceleration.

The history of riskless choice, in many ways, parallels the origins of the choice models described above. The multi-attribute utility model served, in many ways the role of expected utility in more classical models. In such a model the utility of an offer is defined as:

$$U_i = \alpha + \sum\nolimits_{q=1}^{Q} V_{iq} \qquad (3.3)$$

where $V_{iq} \equiv f_q(X_{iq})$ represents a possibly nonlinear *value function* for the qth attribute of the alternative i. This model has lesser normative status because it does not originate from a strong set of appealing axioms, but the similarities between this model and expected utility are striking. In this framework, all the pieces of information about each of the alternatives considered are summed, producing an aggregate utility for each of the alternatives. In our car example, each attribute of each car, like price or gas mileage has a value (called a part-worth in the marketing literature), the V_{iq} in the equation above, and these are summed into an overall utility for each car, the U_i. In some of the most popular versions of this model, the probability of an option being chosen is given by the p(*choosing option* i) $= U_i/\sum U_i$ or the ratio of the utility of the option over the sum of the utilities of all the options. This property is called the Luce Choice Axiom (Luce, 1977).

These models and their close relatives have been the driving force behind much applied work in many social sciences, in particular in marketing and economics and have been useful in thousands of studies. However, such value maximization models (Tversky and Simonson, 1993) make two kinds of strong predictions that suggest that they do not correspond to the underlying psychological properties that are actually producing these choices in people. As detailed in the next section, these concern both predictions for choice probabilities as the number of options changes, and predictions for the choice process itself.

Cognitive Limitations and Context Effects

The multi-attribute utility model, as a descriptive model, falls short on two grounds. The first problem is the assumption that the value of an option should be independent of the choice set. To illustrate briefly, consider Figure 3.1, and imagine that a decision maker is faced with choosing between two choice options t (for target) and c (for competitor). Figure 3.1 plots the options in a space defined by two attributes. We are interested in how the proportion of choices in a set of decision makers might change when we add third options, called decoys (and labeled d in Figure 3.1) to different places in Figure 3.1. If choices obey the Luce choice axiom (as we might hope they would), adding an option can only *reduce* the share of existing options, because the option can only make the denominator in the ratio larger, lowering its probability. This property is called regularity, and says that, if a set of decision makers chose between c and t, then the addition of d_a can only reduce the share of choices of both (Huber *et al.*, 1982; Tversky and Simonson, 1993). Empirically, however, the choice share of the t is usually increased, a result known as the attraction effect because d_a seems to attract choices to t.

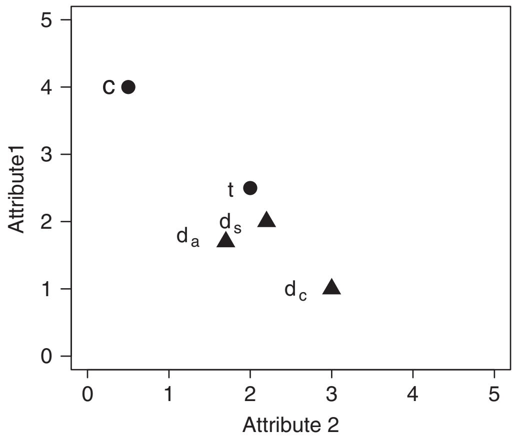

FIGURE 3.1 Context effects in choice.

More generally, these violations are termed *context effects* (discussed in more detail in Chapter 24), because the characteristic of the choice set, the context of the choice, influences what is chosen. Two other context effects also pose problems for most multi-attribute analysis, *compromise*, in which the addition of d_c in Figure 3.1 increases the share of t more than predicted by value maximization and *similarity*, where the addition of d_s reduces the share of t more than predicted. Together these three context effects provide a strong set of constraints for models of riskless choice that have not yet been entirely met by current models.

The second challenge is plausibility: as was the case for integration models of risky choice such as expected utility and prospect theory, a multi-attribute utility model of riskless choice must by design consider all relevant information, for all the alternatives. This seems computationally implausible for larger sets of options and attributes. This is, essentially, the same assumption that proved problematic for risky choice. Patterns of information search have been examined using eye movement recording, verbal reports, and manual information acquisition, all of which show that for large choice sets some information is ignored. In fact, a common pattern of search suggests that multiple processes are employed with large sets of options: the first compares the options on a limited number of attributes, followed by a closer and more complete examination of a small subset of attributes. The observation that decision makers can be quite selective in information acquisition, particularly with larger sets of options and attributes, supports the idea that heuristic shortcuts are involved in riskless choice as well.

Heuristic Models of Riskless Choice

Together, these considerations led to the development of a set of simplified choice procedures for making choices. Paralleling heuristics for risky choice, these heuristic models both ignore information and combine information using shortcuts such as direct comparison or by computing differences. A large list of potential alternative choice procedures arose from these changes in assumptions. One example would be *elimination by aspects* (Tversky, 1972), a model that suggests that a decision maker has a cutoff for each attribute (for price: "I won't buy any car costing more than \$30,000") and compares each to that standard, selecting the option that first passes all of these cutoffs. Other heuristic choice procedures include *comparison of alternatives on each attribute*, the *additive difference rule* (Russo and Dosher, 1983; Tversky, 1969), and procedures that chose the alternative that is best on the most important attribute, a *lexicographic procedure* (Johnson and Payne, 1985; Gigerenzer and Goldstein, 1996). These heuristics stand in stark contrast to the idea that the value of each option is calculated exhaustively, an idea that motivates much contemporary neuroeconomic thinking about choice. But that may be changing. It has recently been argued that relative evaluation, how options compare to each other, is a very important, if not the most important, component of choice even in a neuroeconomic domain (Vlaev *et al.*, 2011).

While there are clearly a profusion of models, the richness of description seems necessary to understand choice in complex settings with many options and/or attributes. But this richness may also be a weakness: many different procedures appear to be used, even in the course of a simple decision (Payne *et al.*, 1991). This complexity makes the application of these procedures for understanding choice challenging, and these applications have been quite limited.

MODELS OF CHOICE OVER TIME

A third major stream of theory concerns choice that involves time, a subject dealt with in detail in Chapter 10. This is, prototypically, a choice between a smaller reward that is received sooner, and a larger reward that is received afterwards. Again an algebraic input–output model, discounted utility, has served as the basis for much theoretical and empirical work and has normative basis in a set of axioms (Rubinstein and Fishburn, 1986). The basic idea of the model is that one assumes two things: first that the longer one has to wait for a reward the less it is worth and second that this rate of decline in value with delay is exponential. That is to say that the value of a reward is assumed to decline by a constant fraction at each period of time. The standard form for this model describes the utility of consuming a reward at each time period as simply the product of the reward's intrinsic utility and the fractional decline in value imposed by a delay of duration d:

$$U(x,d) = U(x)\alpha^{d} \tag{3.4}$$

where the utility of x is reduced by a proportion α, each time period or α to the d for each of the d time periods. The proportion, α is often called the discount rate.

While the idea was not originally thought of as a normative model (Frederick *et al.*, 2002), this equation has been appealing, in part, because it has a marked similarity to the standard formulas for discounting cash flows and compounding interest in the financial world.

While this form has been the dominant model and basis for extensive work in economics, empirically, there have been many departures from its predictions. In their classic review Frederick *et al.* (2002) describe several findings that are inconsistent with Discounted Utility. The first, *hyperbolic discounting*, refers to the fact that the empirically observed discount rate is not constant but decreases with time; the longer the delay the lower the apparent discount rate. The second, the *magnitude effect*, refers to the fact that the observed discount rate depends upon the amounts involved, with larger amounts being discounted at lower rates. Finally while the model would say that discounting should be constant independent of whether one is accelerating a reward closer to now, moving consumption forward, or delaying a reward to later in time, the *direction effect* shows that the observed rate of discounting depends on whether one is accelerating or delaying a reward. Roughly speaking, people are about twice as impatient when a reward is delayed than when it is pushed forward.

The first of these anomalies is usually modeled by changing the way that rewards are valued, from assuming a constant discount rate to one which discounts rewards more when they will be received sooner, these models are called hyperbolic (Kirby and Herrnstein, 1995) or quasi-hyperbolic (Laibson, 1997) models and have been the subject of intense interest and debate in neuroscience, much of which is discussed in Chapter 10 (Glimcher *et al.*, 2007; Kable and Glimcher, 2007; McClure *et al.*, 2007, 2004). In contrast, modeling the other two anomalies has not drawn as much attention until recently. Scholten and Read (2010) have recently proposed a model that provides accounts for all three anomalies. It, like many of the heuristic models of risky and riskless choice suggests that decision makers compare the rewards that will be received and the times at which they will be received.

An additional anomaly concerns discount rates themselves. In real economic settings there should be a relationship between how much it costs to borrow money, the market interest rate, and a person's discount rate. The reason is simple: if one is tempted by an immediate reward, but if waiting increases the reward, one could always borrow the larger amount now, and repay the loan when the larger amount arrives. This is profitable if the discount implied by the choice is greater than the cost of borrowing. Thus, people should never choose a smaller sooner option if the rewards to waiting are larger than the cost of borrowing. However, not only are stated personal discount rates higher than the cost of borrowing in surveys and laboratory experiments, but this is also true in classic field studies with real and financially costly choices. For example, members of the military frequently chose lump sum payments (Warner and Pleeter, 2001) rather than a series of payments that were much larger, implying a personal discount rate of over 20%. There is to date no formal model that accounts for this, but the idea that people overestimate future money and time resources (Zauberman and Lynch, 2005) is one appealing explanation. There is both behavioral (Read *et al.*, 2005) and neural evidence (Peters and Büchel, 2010) that the predicted accessibility mediates discounting: concrete dates for future events increases patience.

COMPUTATIONAL PROCESS MODELS

A separate effort to model very simple choices originated in psychology over the last 30 years that differs in several ways from the models described in the previous section. The main domain of study for these popular models has been the analysis of tasks that require two alternative decisions that are made reasonably quickly in what is assumed to be a one-step process. Decisions with mean reaction times (RTs) less than 1.0−2.0 s are the typical subject of these models. Very importantly, these models make predictions not only about the choices of subjects but also about other features of the decision process like the RTs or the reported confidence that subjects have in their decisions. This is a point developed in detail in Chapter 13.

Such models have recently been quite influential in neuroscience, in part because the tasks they describe are close to the simple tasks typically used in both human brain imaging experiments and in the study of nonhuman primate decision making. They have also been influential because they make specific predictions about the structure of the underlying neural processes. The remainder of the chapter focuses on one class of these models that has proven to be very useful, *diffusion models*. Two closely related accounts, important because they have been able to account for some context effects of the kind described above are: Decision Field Theory (described in Chapter 4; Roe *et al.*, 2001) and Leaky Competing Accumulator Models (Usher and McClelland, 2004).

DIFFUSION MODELS OF RAPID DECISIONS

Diffusion models assume that noisy information is accumulated to one of two or more decision criteria, and that the time taken by the model to reach a criterion accounts for the response time (RT) distributions observed for both correct and error responses made by subjects. In the classic diffusion model of Ratcliff and colleagues (Ratcliff, 1978; Ratcliff and McKoon, 2008), we begin by assuming a process which drifts to the right at a constant rate from an initial position as shown in Figure 3.2. Typically time is treated as a discrete variable that starts at 0. The horizontal position of the particle at any time, t, is thus simply t. The particle is hypothesized to diffuse upward or downward a small amount at each increment of time with the upward and downward directions reflecting two possible alternatives in a two alternative choice task. The magnitude of the particle's diffusion is controlled by the amount of evidence provided to the model in that increment supporting either the "upward" or "downward" decision. The vertical position of the particle at any given point of time thus reflects the sum of the evidence for the upwards and downwards decisions available at that time. Once the particle is observed to cross a fixed upper or lower criterion line, the model records a decision, as shown in Figure 3.2. If, as is usually assumed to be the case, the evidence is noisy or stochastic then the path traced by the process enroute to the boundaries will vary from repetition to repetition – yielding an estimate of the reaction time distribution for each alternative under any given set of conditions.

To take a concrete example, consider a brightness discrimination task in which subjects report whether or not a stimulus is brighter or darker than some remembered reference intensity. In a diffusion model of that decision-making process, a bright stimulus will induce a positive drift rate (towards the top boundary) and if the accumulated evidence reaches the top boundary, a "bright" response is executed – a "dark" response would then correspond to the bottom boundary. In Figure 3.2A, the arrows show the drift rate from the starting point to the "bright" boundary for a bright stimulus (red arrow), a slightly bright stimulus (green arrow) and a dark stimulus (blue arrow).

The three paths in Figure 3.2A show three different decisions all from the same drift rate. Because the decision process is noisy, particles can of course occasionally hit the wrong boundary, produce error responses like those observed in behavior. Interestingly, both correct responses and errors show right-skewed distributions of response times consistently with the same shape just like those observed in real behavior (see Ratcliff and McKoon, 2008, Figure 8). This is one of the very powerful things about the model. Empirical RT distributions are right skewed with an approx. exponential tail and hundreds of subjects have been fit that keep showing this shape. The diffusion model keeps producing exactly this shape.

Non-Decision Component

Outside of the accumulation of evidence captured by the drifting particle, there are other processes such as stimulus encoding, memory access, and movement processes that produce the behavioral response of the subject. There is also the process of transforming the stimulus representation into a decision-related variable that drives the decision process. These components of processing take time that is not captured directly by

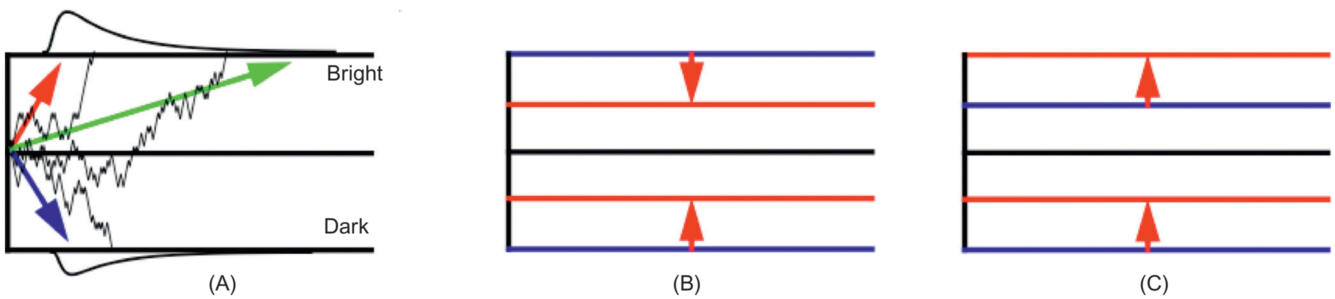

FIGURE 3.2 (A) Quality of evidence from perception or memory. (B) Speed/accuracy tradeoff reflected in boundary separation changes. (C) Bias towards top boundary (blue) changes to bias towards bottom boundary (red).

the model and are combined into one "non-decision" component which has a mean duration of T_{er}. The total processing time for a decision is thus taken to be the sum of the time taken by the decision process and the time taken by the non-decision component.

Boundaries, Speed-Accuracy, and Bias Effects

Experiments have examined biasing subjects using instructions such as "be fast" or "be accurate," by rewarding the two responses differentially or by making one response more likely to be correct than the other. Perhaps surprisingly, nearly all of these variables can be captured in the model by adjusting the locations of the upper and lower criteria, or boundaries. For example, as shown by the red arrows in Figure 3.2B, responses can be speeded at the expense of a higher error rate by moving the decision criteria closer to each other, from the blue to the red settings. The model can be biased toward one versus another response by moving the decision criteria from the blue to red settings as in Figure 3.2C. And perhaps most interesting is the observation that changes in accuracy and RT with manipulations that change either the speed–accuracy tradeoff or that induce bias in responding are well accounted for by changes in the simple boundaries that represent the two decision criteria.

Across-Trial Variability

There was a significant problem with early models of these kinds, which were originally implemented as random walk models (the discrete version of a diffusion process) (Stone, 1960), as well as with the simplest diffusion model. If the starting point is midway between the boundaries, correct and error RT distributions are identical – a feature that is not observed in real data. One way to handle this problem is to allow model parameters to vary from trial-to-trial. If parameters are drawn from a distribution, observed patterns of correct versus error RTs are easily produced. This is presumed to reflect the notion that subjects *cannot* hold the parameter values exactly constant from one trial to the next (Ratcliff, 2013). There is direct evidence for variability in drift rates from trial to trial in perceptual judgments using single trial EEG regressors to divide data based on the quality of the stimulus as judged by the electrical signal (regressor). Drift rate estimates differ as a function of the regressor (Ratcliff *et al.*, 2009).

Figure 3.3 illustrates how this mixing of parameters works with just two values of the parameter instead of with a full distribution as would normally be done. Figure 3.3A shows two drift rates, the red one produces high accuracy and fast responses, the blue one produces lower accuracy and slow responses. The mixture of these produces slow errors because 5% of the 400-ms process averaged with 20% of the 600-ms process gives a weighted mean of 560 ms which is slower than the weighted mean for correct responses (491 ms). Figure 3.3B shows the effect of different starting points: the red distributions are for processes that start further away from the correct boundary. Processes that start near to the correct boundary have few errors and the errors are slow (because there is a greater distance to travel), while processes that start further away have more errors and the errors are fast. The combination leads to errors faster than correct responses.

Model Constraints

The separation of drift rate from the decision criteria and non-decision processes is one key contribution of the model. In the model, stimulus difficulty affects drift rate but not criteria, and speed–accuracy shifts are represented in the criteria, not the drift rate.

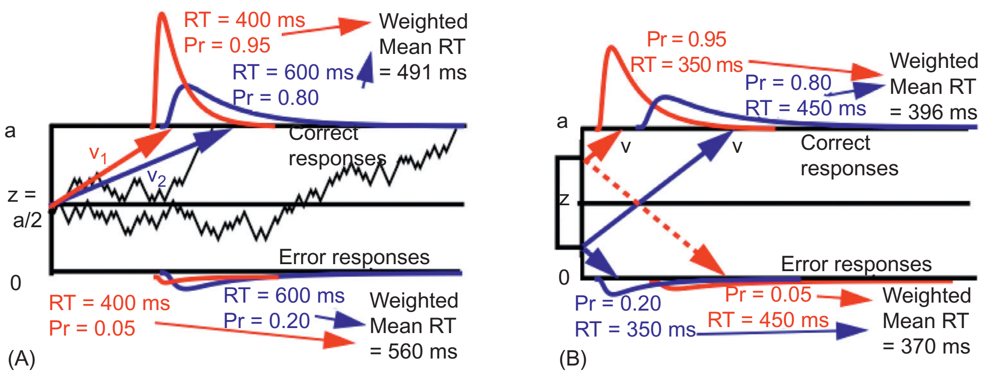

FIGURE 3.3 (A) Effect of different drift rates. (B) Effect of different starting points.

Thus if difficulty varies, changes in drift rate alone must accommodate all the changes in performance-accuracy and the changes in the spreads and locations of the correct and error RT distributions. Likewise, changes in the criteria affect all the aspects of performance. In these ways, the model is tightly constrained by data.

Model Fitting

It is important to note that to fit this model to data, accuracy and RT distributions for correct and error responses have to be simultaneously fitted as described in the next chapter. Also, it is worth noting that in any data set there is a potential problem imposed by so-called outlier RTs (Ratcliff, 1993). To fit RT distributions, a good compromise that reduces the influence of outliers is to use quantiles of the RT distribution and fit the model to the proportion of responses between the quantiles. Because proportions are used, accuracy is automatically included in this computation. For details of how to fit the model to data, see Ratcliff and Tuerlinckx (2002) and fitting packages by Vandekerckhove and Tuerlinckx (2007) and Voss and Voss (2007).

Mapping from Accuracy and RT to Drift Rate

In simple tasks, as task difficulty increases, accuracy goes from near 100% correct to chance (50% correct) and RT changes from fast to slow. An example of this is shown in Figure 3.4 in a simple numerosity discrimination task in which an array of asterisks is presented on the screen and subjects have to decide whether the number is larger or smaller than 50. The diffusion model was fitted to these data and the right panel shows drift rate, which is approximately linear. This shows that the diffusion model (which fits the accuracy values as well as RT distributions for correct and error responses) extracts a simple (in this case) linear function from the nonlinear accuracy and RT functions.

Applications

One aspect of research on diffusion models in psychology is in their applications to answer questions about the effects of differences in groups of subjects and individual differences among subjects. These applications are usually embedded in the literatures of, for example, aging, clinical applications, development, sleep deprivation, ADHD, dyslexia, numeracy, and so on. In aging research, for example, in many but not all tasks, age does not affect drift rates but does result in a larger non-decision time and wider boundary settings (though boundary separation can be altered in older adults by convincing them it is ok to go faster at the expense of making a few more errors). In the same studies, differences in IQ across individuals within age groups affect drift rates but not non-decision times or boundary separations. The values of these three model parameters for each subject are highly correlated across tasks which show that the model uncovers stable individual differences in quite different tasks (e.g., numeracy, word/nonword discrimination, and memory, see Ratcliff *et al.*, 2010, 2011). This set of dissociations produces quite a different view of the effect of age on speed of processing. Also, the size of these effects is large and individual differences three- to five-times larger than estimation error are achieved in just one 40-min session of data collection per task. From a practical point of view, because of the stable individual differences obtained from fitting the model, this approach offers the possibility of practical applications in clinical and neuropsychological testing domains.

Competing Models

The diffusion model described to this point is one of a class of models that have related features. Others include the leaky competing accumulator model (Usher and McClelland, 2001) that assumes two racing diffusion processes, and the linear ballistic accumulator (Brown and Heathcote, 2008) that assumes two racing deterministic accumulators. Both these models also

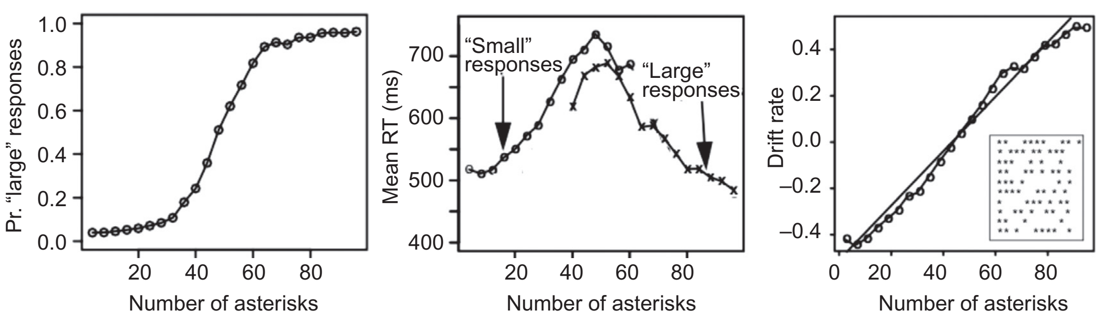

FIGURE 3.4 Numerosity discrimination.

assume variability in model components across trials. In studies that have been carried out, interpretations of the major effects of different independent variables are the same across the models (Donkin *et al*., 2011; Ratcliff *et al*., 2005). This means, that within reason, conclusions from one model will produce about the same conclusions for the other model.

Multichoice Decision Making, Confidence, and Simple RT

The two-choice diffusion model is quite well established, but there is considerably less research on (but a growing interest in) how these models can be extended to choice problems with more than two options being considered. It is more difficult to conduct well designed experiments that vary the number of alternatives alone and there is more model freedom because different parameters are needed for the different choices. Models have been advanced to examine visual search (Basso and Wurtz, 1998; Purcell *et al*., 2010) to motion discrimination (Niwa and Ditterich, 2008), and other more cognitive approaches (Leite and Ratcliff, 2010). Also, confidence judgments in decisions and memory are multichoice decision and these are being seriously attacked (Pleskac and Busemeyer, 2010; Ratcliff and Starns, (in press); Van Zandt, 2002). In many of these approaches, a variety of competing models are compared, but as yet, conclusions about which architectures are more promising are only just starting to develop.

In contrast, relatively little work has been done recently on simple RT or "one-choice" decisions. In these kinds of tasks, there is only one key to hit when a stimulus is detected. Ratcliff and Van Dongen (2011) presented a model that used a single diffusion process to represent the process accumulating evidence. The main application was to the psychomotor vigilance task, a task in which the subject is seated in front of a millisecond timer which starts some variable time after the prior response. The data are a single RT distribution and the model is designed to fit that (there is no accuracy measure). Results showed good fits, and that drift rate tracked an independent measure of alertness. There were correlations in drift rates for a simple brightness detection task and a two-choice brightness discrimination task. These results provided validation beyond goodness of fit of the model to data.

Use in Neuroscience

One of the major advances in understanding decision making is in neuroscience applications using single cell recording in monkeys (and rats), and human brain activity including fMRI, EEG, and MEG. All these domains have had interactions between diffusion model theory and neuroscience measures. Hanes and Schall (1996) suggested a connection between diffusion models and single cell recording data, and this was taken up in work by Shadlen and colleagues (e.g., Gold and Shadlen, 2001). Ratcliff and colleagues (2003) showed that the buildup in single neuron activity in the monkey superior colliculus neurons was mirrored by simulated paths in a diffusion model, so that the average of a number of paths moving to a decision bound in the model matched the average firing rate for neurons involved in producing a decision. Pouget and colleagues (2011) have used single cell data (from the frontal eye fields) and behavioral data in a visual search paradigm with monkeys to discriminate among classes of diffusion models. Their research program is aimed at directly linking neural and behavioral levels of analysis by using the input from one class of visual neurons in the FEF to drive decision-related neurons. There have also been significant modeling efforts to related models based on spiking neurons to diffusion models (e.g., Deco *et al*., 2012; Roxin and Ledberg, 2008; Wong and Wang, 2006). Diffusion models are also being used in human neuroscience using fMRI and EEG techniques, although often in non-reaction time tasks. One effort is to look for stimulus independent areas that implement decision making (e.g., vmPFC, Heekeren *et al*., 2004). Other approaches have mapped diffusion model parameters onto EEG signals (Philiastides *et al*., 2006). Reviewing this research would require a chapter by itself, and indeed, Chapters 8 and 20 provide just such reviews.

JUDGMENT

We have emphasized choice in our short review of concepts from Judgment and Decision-making research. There are many other topics, normally grouped under the label of "judgment" that should be of interest to neuroeconomics. Concepts in areas such as probabilistic inference have strong parallels to the issues that we have examined in choice: there are normative models in many cases, and heuristic models that have more descriptive accuracy, but known departures from normative predictions. One general theme that might be both useful for, and informed by neuroeconomics is the idea of attribute substitution (Kahneman, 2003; Morewedge and Kahneman, 2010; Shah and Oppenheimer, 2008). The basic idea pursued in this line of research is that if one is trying to estimate one quantity, such as the soundness of an argument, one might, without awareness, substitute another quantity that is easier to compute or more available, such as the ease with which one can read the font in which the argument is presented. These, and many other related issues would benefit from an understanding of the neural substrate of these meta-cognitive processes.

CONCLUSION

In this brief overview, we have shown two different approaches used in psychology to model choice. The first, with a long historical tradition covers both simple and complex choices in three different domains: decisions under risk, riskless choice, and over time. While broadly they can predict choices in many domains, the development of concerns with underlying cognitive and neuronal processes is relatively recent. This contrasts with more recent computational process models that are concerned with simpler choices but make predictions for choices and errors, and have a strong conceptual connection to neural firing rates. As seen in Chapter 8, we are starting to see models at the intersection of the two, for example explaining context effects using a type of diffusion model. We think this melding of approaches is a very promising area for future explorations.

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