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Theoretical Note

A Note on Mimicking Additive Reaction Time Models

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Ashby (1982a) and Ashby and Townsend (1980) have developed a series of tests of pure insertion based on the reaction time distribution. One of these tests allows one to determine whether an experimental task has inserted into its processing a component that is exponentially distributed. In this note, the test is applied to predictions from the diffusion model of Ratcliff (1978) and to data from two experiments (Ratcliff, 1978, Experiment 2; Hockley, 1984, Experiment 1) that were fitted by the diffusion model. It is found that both the predictions from the diffusion model and the experimental data pass tests for inserted stages at a statistical level (i.e., the analyses provide results that are within the range of the null hypothesis established by Ashby and Townsend (1980)). However, it is shown that the diffusion model does not mimic insertion theoretically. These results demonstrate nonintuitive mimicking that should serve as a warning in the use of such tests for insertion, and the results suggest that the power of tests for pure insertion could be profitably examined using predictions from the diffusion and possibly other sequential sampling models.

It is generally assumed that the processing of information proceeds through a number of stages from encoding through storage and retrieval. One class of models that has been developed to represent this view is the class of discrete stage models, including both parallel and serial models. Much is known about the conditions under which mimicking between serial and parallel models takes place (Townsend & Ashby, 1983) and this work has been advanced recently by the development of tests based on the reaction time distribution that provide evidence for pure insertion of stages.

In this note, I describe a class of nonintuitive mimicking problems that appear at the level of reaction time distributions. The motivation comes from an attempt to evaluate data presented in Ratcliff (1978), data that were used to support the diffusion model for recognition. The data were obtained from a standard Sternberg paradigm (Sternberg, 1966), and accuracy, reaction time, and reaction time

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distribution shape were well described by the diffusion model. Thus, three questions arise: First, do the data pass the tests for pure insertion? Second, do predictions of the diffusion model also pass the tests for pure insertion? (If the fits of diffusion model to the data are very good, then we would expect the same answer to these questions.) Third, if the diffusion model passes the test for pure insertion, is the result analytic (exact) or statistical (within confidence limits derived to test the null hypothesis)? To give these analyses more generality, they were also applied to data and fits of the diffusion model from a study by Hockley (1984). (While it would have been nice to use the Townsend and Roos (1973) data used in the Ashby and Townsend (1980) article, this would have involved typing the raw data into my computer system, so I used the data of Hockley who had histograms for the reaction time distributions already available.)

The tests developed by Ashby (1982a) and Ashby and Townsend (1980) evaluate the assumption that an inserted stage is serial (or parallel of a form that mimics serial processing). Some of the tests concern a stronger assumption and that is that the inserted stage is exponentially distributed. Both of these classes of tests will be examined below. The first test considered (Ashby and Townsend, 1980) is based on a simple relationship between the exponential time constant of the proposed stage and the distribution and density functions of the observed distributions. Suppose that a process has $k-1$ stages with some distribution function, and then another stage with exponential finishing time is added. Then,

$$g_k(t) = g_{k-1}(t) * (1/\tau)\, e^{-t/\tau} \tag{1}$$

for $t > 0$, where τ is the time constant of the exponential stage (equal to $1/V_k$ in Ashby and Townsend's notation) and $*$ represents convolution (i.e., a reaction time from $g_{k-1}(t)$ is added to a reaction time from the exponential). Since τ is the mean of the exponential stage, then τ would be expected to equal $\overline{RT}_k - \overline{RT}_{k-1}$. However, this estimate does not allow one to argue that the distribution of the added stage is exponential. Ashby and Townsend (1980) showed that

$$1/\tau = g_k(t)/(G_{k-1}(t) - G_k(t)) \tag{2}$$

for any $t > 0$, using the method of Laplace transforms (Ashby and Townsend, 1980, Theorem 1). Thus, the expression on the right hand side of Eq. (2) is constant for *all* values of t, so that a plot of the right hand side of Eq. (2) as a function of time should produce a line with slope zero. Such a plot provides a test of the assumption of an inserted exponential stage. If the line has zero slope, then a model with an inserted exponential stage is implied. It is possible to examine insertion of several stages if one has a range of experimental conditions corresponding to a range of values of k.

Given this result, Ashby and Townsend (1980) proceeded to evaluate data from a standard Sternberg procedure by Townsend and Roos (1973). They used the slope of the regression line from Eq. (2) as an index of goodness-of-fit and found that the slopes for most of the conditions were close to zero and within confidence intervals they derived for the experiment. Thus, the data meet the criteria for insertion of exponentially distributed stages.

In order to examine the possibility of mimicking in this test (Eq. (2)), I performed two sets of studies. First I applied the test to two sets of Sternberg paradigm data from Ratcliff (1978) and Hockley (1984) and second I applied the test to theoretical predictions from the diffusion model (Ratcliff, 1978) for both sets of the data. The diffusion model was found to fit the reaction time distributions for the two experiments, so it was expected that to the extent that the data passed the tests for serial insertion, so would the theoretical predictions for the diffusion model.

Diffusion Model

The diffusion model uses a continuous version of the random walk as the decision component. It is assumed that items are encoded into memory and at test time, the test item is compared with each item in memory in parallel. Each parallel comparison is carried out by a diffusion process: the greater the match between the test item and a memory item, the faster and more accurate are positive decisions. The smaller the match between the test item and a study item, the faster and more accurate are negative decisions. A positive response is generated when one diffusion process terminates with a match and a negative response requires all processes to terminate with nonmatches. For further details see Ratcliff (1978).

The mathematics needed to obtain predictions from the diffusion model are outlined in Ratcliff (1978). Some of the main equations relevant to the analyses carried out in this note are presented here. For a single diffusion process terminating in a nonmatch, the finishing time density function is given by

$$g(t) = \frac{\pi s^2}{a^2} e^{-(zu/s^2)} \sum_{k=1}^{\infty} k \sin\left(\frac{\pi z k}{a}\right) \exp -\frac{1}{2}\left[\frac{u^2}{s^2} + \frac{\pi^2 k^2 s^2}{a^2}\right] t, \tag{3}$$

where zero is the nonmatch boundary of the diffusion process, z is the starting point, a is the match boundary, u is the drift rate, and s^2 is the variance in drift. The model of Ratcliff (1978) makes the additional assumptions that drift is distributed over nominally identical comparisons (i.e., items encoded into memory have variable strength), and comparisons are parallel: self-terminating on matches, exhaustive on nonmatches. To derive the predictions for the Sternberg data, it is necessary to integrate over the distribution of drift rates and to find the maximum of several of these processes for negative responses. For these latter two calculations, numerical solutions are obtained (Eq. (3) represents an infinite sum and numerical methods are necessary to obtain solutions). Further details are presented in Ratcliff (1978). In the experimental data presented below for Experiment 2 in Ratcliff (1978), the negative responses only are considered because positive responses vary as a function of serial position as well as set size and so have fewer observations per condition. In addition, it would be necessary to argue that positive responses were the result of exhaustive processing, and this seems unlikely in view of the strong serial position effects in accuracy and reaction time.

Applications of the Tests: Experiment 2, Ratcliff (1978)

Ratcliff's experiment used two subjects who performed eight experimental sessions. Each session used three set sizes (3, 4, or 5) and there were 480 trials per session. Thus, the number of negative trials per set size was 1280. Thus, sample sizes on which the following tests are based are around 1200 (once errors are removed).

Before presenting results of the applications of the tests to both the data and the theoretical predictions, it should be noted that all the parameters of the diffusion model used here are those presented in (Ratcliff, 1978). Thus, nothing about the theoretical fits has been changed since then in applying the tests. The probability density function was obtained from (Ratcliff, 1978, Fig. 15). The group distribution method allows group quantiles to be obtained and between each quantile, there is equal probability density. Equal area rectangles are constructed to give the density function and the heights of these rectangles provide the probability densities at the midpoint of the range. I decided to present just the raw data and not perform smoothing such as that performed in Ashby (1982a). Figures 1 and 2 show results of applying the method to both the data from Ratcliff's (1978) Experiment 2 and also to the predictions from the diffusion model with parameter values used in fitting the model to that data. There are two things to note from these figures. First, the plots of V_k are not completely flat both for the data and for the diffusion model. Second, the fits of the diffusion model to the data miss by a significant amount. Both these points show that the method is quite sensitive. Suppose that the diffusion model density function is a reasonable approximation to the observed

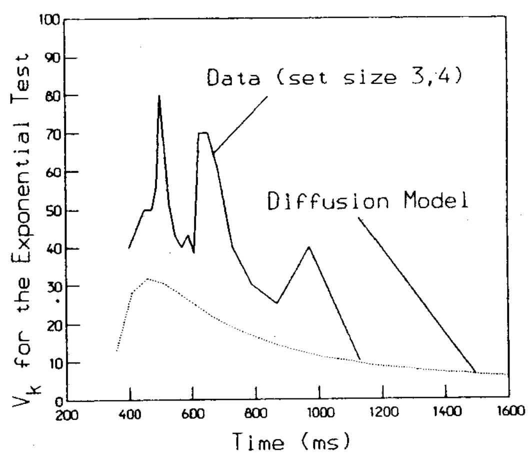

FIG. 1. Application of the test for an exponential inserted process (Eq. (2)) for data from the Sternberg paradigm from Ratcliff (1978, Experiment 2) and for the fit of the diffusion model to that data for set sizes 3 and 4. V_k is in units of $(s)^{-1}$.

density function. Then empirically, the quantiles for the k and $k-1$ distributions differ in this example by as little as 20 *ms*. Thus, if the empirical distribution is shifted by a small amount relative to the theoretical prediction (e.g., 10 *ms*), the deviations between theory and data found in Figs. 1 and 2 will be seen. Thus, the test is extremely sensitive to the relative locations of the reaction time distributions.

The important question concerns the power of the test. Ashby and Townsend (1980) addressed this by performing a series of Monte Carlo simulations in order to find a typical range of values of the slope for the number of observations they used in the simulations. They performed the simulations first with an exponentially distributed stage and then with several kinds of nonexponential distributions inserted. They then chose values of the slope that separated these two sets of simulations (e.g., a value above which the nonexponential distributions produced rejection). For sample sizes of 1000 pseudo-observations and an exponential parameter (and mean) of 25 *ms*, a slope of $\pm 10^{-4}$ in the rate constant (1 divided by the time constant = 25 *ms*) was chosen.

I performed linear regression fits to the data in Figs. 1 and 2 (inverting the ordinate to produce estimates of the rate constant). For the theoretical predictions, the slope and intercept for set sizes 3 and 4 are -0.65×10^{-4} and 0.011 for a range of 300–1100 *ms*. The equivalent slope and intercept for set sizes 4 and 5 are -0.46×10^{-4} and 0.014 for the same range. Thus, the distributions provided by the diffusion model are consistent with a model that assumes an exponential inserted process. Thus with only 1000 observations, it is unlikely that the diffusion model can be discriminated from a model that assumes an exponential inserted process. This can be considered another case of statistical mimicking in the context of this test. The data also slow slopes consistent with exponential pure insertion. The slopes and intercepts were, for set sizes 3 and 4, -0.74×10^{-4} and 0.021. A test for

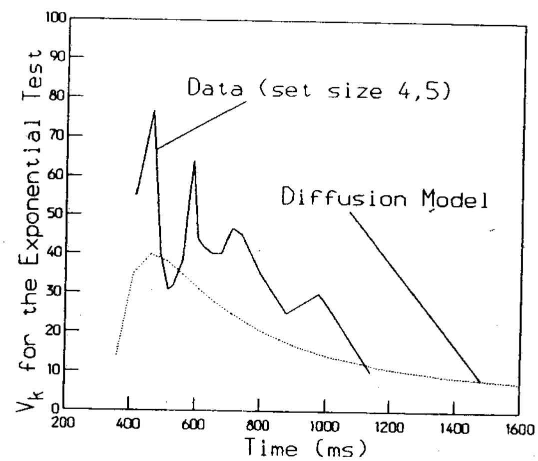

FIG. 2. Application of the test for an exponential inserted process (Eq. (2)) for data from the Sternberg paradigm from Ratcliff, (1978, Experiment 2) and for the fit of the diffusion model to that data for set sizes 4 and 5. V_k is in units of $(s)^{-1}$.

linearity provided $F = 6.4$, with 1, 13 degrees of freedom which was significant $P < 0.05$. With the last point removed, which is based on sparse data at the long reaction times, $F = 2.1$, with $df = 1, 12$, and this is not significant. For set sizes 4 and 5, the slope and intercept were -0.74×10^{-4} and 0.019 and the linear trends were 10.8, $df = 1, 13$, $P < 0.05$, and with the last data point removed, $F = 4.6$, $df = 1, 12$, not significant. Both the slopes and the tests for linearity squeak by the tests for an exponentially inserted process. Thus the data satisfy the assumption of pure exponential insertion. At least for the diffusion model and the data from Ratcliff (1978, Experiment 2), the test produced by Ashby and Townsend is not able to discriminate statistically between the diffusion model and a model with exponentially inserted stages.

Ashby (1982a) developed these methods further and provided two additional tests, one for an exponential inserted stage and one for insertion of a stage that is not necessarily exponential. He showed that if a process has an inserted exponential stage, then the density function of the base process intersects the density function of the combined process at the mode of the latter. This is easily derived from Eq. (2) as follows. Rewriting Eq. (2) gives

$$G_{k-1}(t) - G_k(t) = \tau g_k(t).$$

Differentiating both sides gives

$$g_{k-1}(t) - g_k(t) = \tau\, dg_k(t)/dt.$$

The left hand side is zero when the density functions intersect and the right hand side is zero at the mode of the combined distribution. Figures presented in Ashby (1982a) indicate that when the data of Townsend and Roos (1973) are smoothed, this property is obtained. The data from Ratcliff (1978, Experiment 2) are more noisy and it is hard to tell if this property obtains or not. However, it is possible to

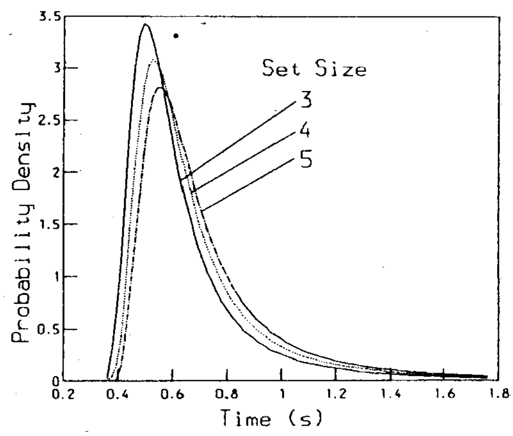

FIG. 3. Theoretical probability density functions for the fits of the diffusion model to data from the Sternberg paradigm (Ratcliff, 1978, Experiment 2).

examine the predictions from the diffusion model using the parameter values from fits to the data, and these are shown in Fig. 3. The theoretical density functions do not appear to satisfy the condition that the curves cross at the mode of the slower condition; there is a 20-*ms* difference for set sizes 3 and 4 and a 30-*ms* difference for set sizes 4 and 5 (approximately). These differences are certainly in the range of differences found by Ashby (1982a, Fig. 3) in his examination of the Townsend and Roos (1973) data. But, it is unlikely that experimental data will be reliable enough to detect such small differences (see Ashby, 1982a, footnote 4, for statistical tests) unless an extremely large experiment is performed. Thus, for the fits of the diffusion model to the data examined here, the diffusion model again produces predictions consistent with pure insertion within the range of statistical mimicking, but not analytic mimicking.

The next property concerns ordering of the distribution and hazard functions. Dropping the requirement that the inserted stage is exponential, Townsend and Ashby (1978) showed that the distribution with the inserted stage must have a cumulative distribution function that is always greater than that for the distribution without the inserted stage. Thus, $G_{k-1}(t) \geqslant G_k(t)$ for all $t > 0$. This property is satisfied by the predictions from the diffusion model. A stronger condition is that the hazard function obeys this inequality: $h_{k-1}(t) \geqslant h_k(t)$ for all $t \geqslant 0$ (the hazard function condition implies the cumulative distribution condition but not vice versa). However, this latter ordering only holds when $h_{k-1}(t)$ is *nondecreasing* in k. Figure 4 presents data from Ratcliff's (1978) Experiment 2 showing noisy hazard functions and Figs. 5 and 6 show two predictions from the diffusion model. Because the hazard functions are not strictly increasing, the ordering property cannot be tested.

Despite the hazard functions' being noisy, they are all nonmonotonic as a function of time (see also examples in Luce, 1986, Chap. 4). For the diffusion model, I considered two cases, one in which the drift rate changed (with parameter values typical of fits to perceptual matching data (Ratcliff, 1981)) and one in which

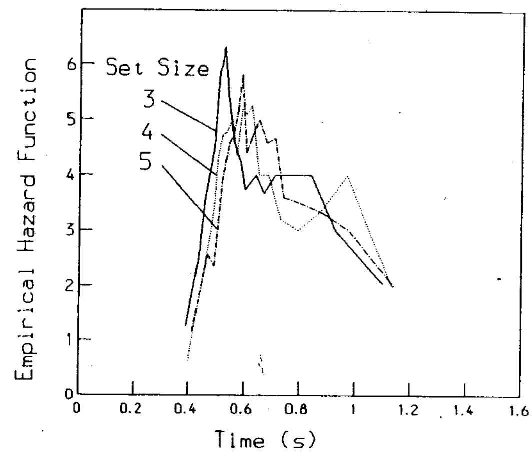

FIG. 4. Empirical hazard functions for the Sternberg paradigm data (Ratcliff, 1978, Experiment 2).

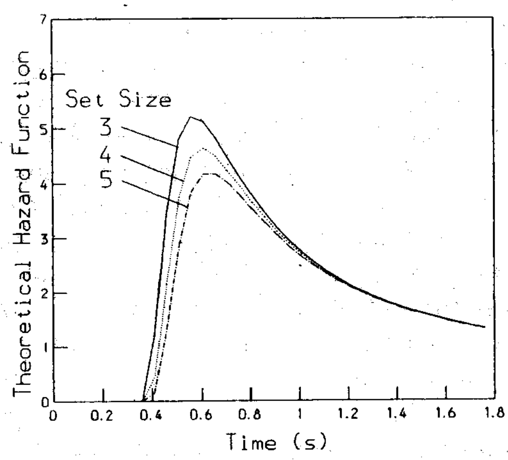

FIG. 5. Theoretical hazard functions from the fits of the diffusion model to the Sternberg paradigm data (Ratcliff, 1978, Experiment 2).

the number of parallel diffusion processes changed as in the fits to the Sternberg data noted above (with parameters from those fits to the Sternberg data). All these functions (both theory and data) show nonmonotonic hazard functions so that the conditions for Ashby's test are not met. However, the theoretical hazard function mimics gross properties of the empirical hazard function for Ratcliff's data. For example, for the three theoretical hazard functions for different set sizes, each rises to the same height of the empirical function at the mode and falls to about the same height in the tail as the empirical hazard function.

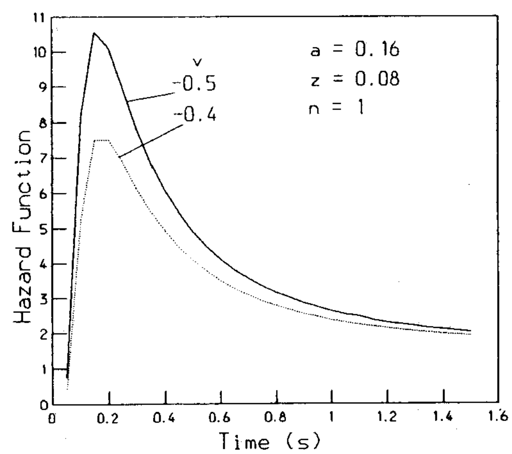

FIG. 6. Theoretical hazard functions for one diffusion process as a function of drift rate (parameter values are set to be typical of those from the perceptual matching paradigm (Ratcliff, 1981)).

Application of the Tests: Experiment 1, Hockley (1984)

To demonstrate the generality of the ability of the diffusion model to mimic exponential insertion, I fitted the diffusion model to data presented by Hockley (1984, Experiment 1, Sternberg paradigm data) and used the obtained parameter values to generate tests as in Eq. (2). The method of fitting involves several steps in which summaries of reaction time distribution shape are used to eventually fit both distributions and accuracy. The first step involves fitting a summary distribution to the empirical reaction time distribution. The distribution used was the convolution of normal and exponential distributions (see Ratcliff, 1978; 1979; Ratcliff and Murdock, 1976). This distribution has parameters μ and σ for the mean and standard deviation of the normal, and τ for the mean and parameter of the exponential. The next step involves generating predictions from the diffusion model for accuracy and reaction time distributions. To the predictions for the reaction time distribution, the convolution model is fitted. The diffusion model is then fitted by minimizing the squared differences (weighted by standard errors) between the theoretical and the empirical values of accuracy and the convolution model parameters μ and τ. This is done using the SIMPLEX minimization routine (Nelder and Mead, 1965). It

TABLE I

Fits of the Diffusion Model to the Data of Hockley (1984)

Parameter theory	Set size 3	4	5	6
a	0.186	0.186	0.174	0.184
z	0.087	0.084	0.070	0.070
u	0.405	0.331	0.325	0.294
v	−0.453	−0.447	−0.403	−0.387
T_{er}	0.340	0.336	0.349	0.346
Predictions				
CR accuracy	0.976	0.967	0.930	0.906
CR μ	0.496	0.509	0.517	0.536
CR τ	0.182	0.204	0.242	0.293
H accuracy	0.982	0.956	0.947	0.927
H μ	0.449	0.458	0.475	0.493
$H\tau$	0.153	0.203	0.210	0.259
Data				
CR accuracy	0.982	0.975	0.948	0.933
CR μ	0.495	0.510	0.520	0.535
CR τ	0.185	0.220	0.255	0.295
H accuracy	0.974	0.970	0.965	0.919
H μ	0.450	0.445	0.460	0.470
H τ	0.150	0.210	0.225	0.280

Note. H, hits; CR, correct rejections.

should be noted that the diffusion model approximates the convolution model very well (see Ratcliff, 1978, Fig. 7).

Hockley's analysis of his reaction time data involved an analysis of the reaction time distributions using fits of the convolution model to the empirical data. He found that 2 out of 48 fits were significant by the χ^2 test at the 0.05 level; thus the empirical distribution provided an excellent summary of shape. I fitted the diffusion model to the data for correct negative responses for each set size. Table I contains the values of the parameters of the diffusion model and the theoretical and empirical values of accuracy, μ, and τ. In the fits I chose to let the parameters shown vary freely. The major trend is a fall in u and v as a function of set size. It should be noted that adequate fits could have been obtained by fixing parameters such as T_{er}, a, and z at constant values across set sizes.

Figure 7 shows the value of the estimate of V_k as a function of time for the three differences (set sizes 3–4, 4–5, and 5–6) for the predicted distributions for the diffusion model. These replicate the functions shown above for the theoretical predictions of the diffusion model for the experiment of Ratcliff (1978). Straight lines were fitted to the functions and the slopes again lay inside the confidence intervals established by Ashby and Townsend (1980). The slopes and intercepts were -3.60×10^{-5}, 0.0599; -3.04×10^{-5}, 0.0506; and -2.38×10^{-5}, 0.0400 for set sizes 3–4, 4–5, and 5–6, respectively. These slopes all have absolute values less than 10^{-4}, the confidence interval established by Ashby and Townsend for somple size 1000. The sample size in the Hockley experiment is about 200 observations per subject per condition, so with six subjects, the theory is fitted to data of equivalent sample size as in the Ratcliff (1978) example.

To obtain the values of V_k for the data, the reaction time distributions must be combined across subjects. The method used for this involves computing quantiles

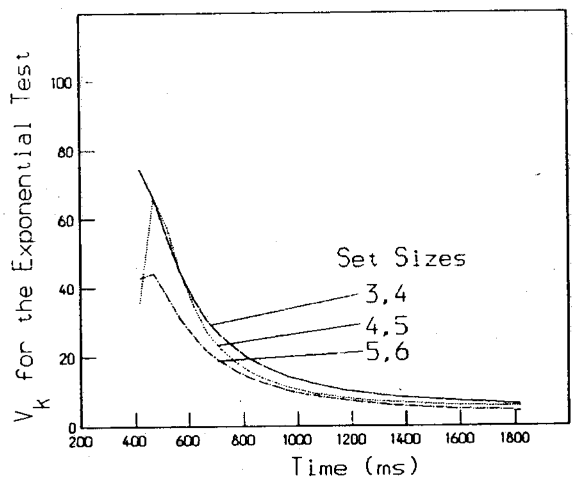

FIG. 7. Application of the test for an exponential inserted process for predictions of the diffusion model for fits to the data from the Sternberg paradigm from Hockley (1984, Experiment 1). V_k is in units of $(s)^{-1}$.

for each set size for each subject. These quantiles are then averaged across the six subjects leading to group quantiles that can be used to calculate values of V_k (see Ratcliff, 1978, 1979). This is the method that was used above for Experiment 2 (Ratcliff, 1978). Figure 8 shows the values of V_k as a function of time and the slopes and intercepts are as follows: -3.9×10^{-5}, 0.055; -8.9×10^{-5}, 0.106; and -6.0×10^{-5}, 0.083 for set sizes 3–4, 4–5, and 5–6, respectively. As above all these have absolute values of slopes less than 10^{-4}. The test for linearity gave F values as follows: $F=3.5$, $df=1, 6$, not significant; $F=4.8$, $df=1, 6$, not significant; and $F=3.3$, $df=1, 6$, not significant for set sizes 3–4, 4–5, and 5–6, respectively.

These two analyses show that both theory and data pass statistical tests for pure insertion of exponentially distributed processes. The figures show that both the data and theory provide V_k versus time functions that are initially high and then fall. These are consistent with the functions shown in Figs. 1 and 2. While both sets of data pass the tests for serial insertion of exponential processes, the data are well fitted by the diffusion model.

One more comparison between statistical and analytic mimicking can be made using the analyses of Ashby (1982b) as applied to the cascade model of McClelland (1979). Ashby derived predictions for the right hand side of Eq. (2) and showed that the functions were not flat so the cascade model did not mimic exponential insertion. I fitted the results shown in Fig. 3 of Ashby (1982b) with a straight line and obtained a slope of 7.4×10^{-6} and intercept of 0.0144 for a range of 200–1000 *ms* (scaling the processing rate to be $(50 \text{ ms})^{-1}$). The slope lies within the range that would not allow rejection of the null hypothesis for the number of observations that Ashby and Townsend used to set their confidence interval. This again illustrates the difference between analytic and statistical mimicking.

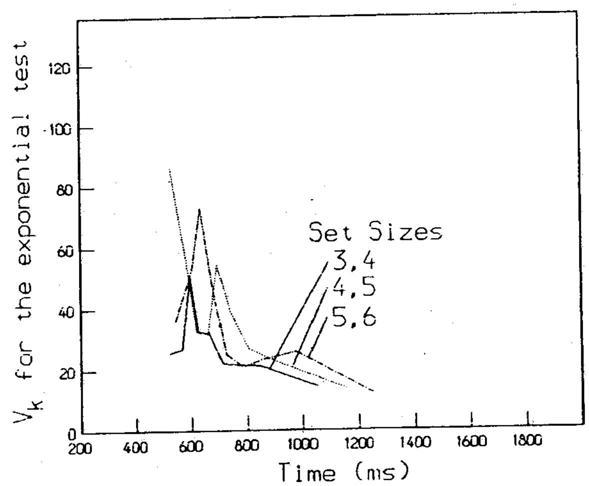

FIG. 8. Application of the test for an exponential inserted process for data from the Sternberg paradigm from Hockley (1984, Experiment 1). V_k is in units of $(s)^{-1}$.

DISCUSSION

The explorations presented in this article demonstrate the difficulty that can be encountered in testing assumptions about pure insertion in information processing models. The tests proposed by Ashby (1982a) and Ashby and Townsend (1980) are satisfied by predictions from the diffusion model within statistical limits for large numbers of observations. The basis for the predictions of the diffusion model involve no pure insertion. One way to view these results is that the data are relatively consistent with pure exponential insertion. Thus, any model consistent with the data will produce predictions that are consistent with pure exponential insertion at least at the level of statistical mimicking. In the case of the diffusion model, these predictions are surprisingly consistent with the tests of Ashby, and Ashby and Townsend.

These mimicking results should also serve as a warning in the application of methods to extract component processes of proposed serial models. While the results presented here are specific to a particular set of tests and particular paradigm (and just two published experiments), the results do suggest that the wider problem of mimicking should be considered. Specifically, the methods examined here provide an interpretable decomposition into inserted exponential processes for the experiments examined; however, they provide no guarantee that there is an inserted exponential process. More generally, several impressive methods have been developed that allow the distribution of a component serial process to be extracted given the empirical distribution for all the component processes and the empirical distribution of the processes without that serial component. For example, Green and Luce (1971) have used Fourier transform methods, Bloxom (1979) has used spline methods, and Kohfeld, Santee, and Wallace (1981) have used a linear systems approach. All these methods extract the component serial process, but none provides further validation that the serial process model is in fact correct. The work presented in this article warns that other competing explanations should be considered because application of the techniques to theoretical predictions from other nonserial models may provide incorrect, but interpretable, serial process decompositions.

It is my belief that the best way to address such mimicking problems within the class of these reaction time models is to apply the models to a wider range of data and experimental paradigms such as accuracy (after all, focusing on the dependent variable reaction time and not accuracy is simply selecting one aspect of a whole when the ultimate aim is to model both variables) and data from deadline or response signal procedures. It is likely that the models will cease to mimic each other when the full range of dependent variables and tasks is examined (even within one experimental paradigm). When the models are then compared across experimental paradigms, the models will be able to be compared on other grounds such as the scope of application and parameter invariance across experiments.

These results also raise the issue of strategy in evaluating mathematical models. Within the domain of reaction time, I argued for more comprehensive evaluations of models using all dependent variables and other tasks within the same experimental paradigm. The alternate view is that we should be both developing these tests for insertion further and perhaps performing experiments with more power that will allow accurate evaluations of the notion of inserted stages. As a general strategy, both approaches need to be followed. The contribution of this paper is to raise questions about the power of current tests of insertion and to show that another well developed model that does not assume insertion passes those tests within statistical limits. These results cannot be generalized to other kinds of tests of this nature, but the results should serve as a serious warning that power cannot be ignored.

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