

- Schimank U 1996 *Theorien gesellschaftlicher Differenzierung. (Theories of Social Differentiation)*. Leske and Budrich, Opladen, Germany
- Simmel G 1900 *Philosophie des Geldes*. Reprinted (1977) Duncker and Humblot, Berlin. Translation (1978) *Philosophy of Money*. Routledge, London
- Weber M 1919 *Wissenschaft als Beruf. (Science as Vocation)*. Reprinted (1967) Duncker and Humblot, Berlin

U. Schimank

Diffusion and Random Walk Processes

Random walk and diffusion models form one of the major classes of models in the response time domain. They best apply in situations in which subjects make two-choice decisions that are based on a single, 'one shot', cognitive process, decisions for which response times do not average much over one second. The basic assumption of the models is that a stimulus test item provides information that is accumulated over time towards one of two decision criteria, each criterion representing one of the two response alternatives. A response is initiated when one of the decision criteria is reached. The researchers who have developed random walk and diffusion models take the approach that all aspects of experimental data need to be accounted for by a model. This means that models should deal with both correct and error response times, with the shapes of the full distributions of response times, and with the probabilities of correct versus error responses. It should be stressed that dealing with all these aspects of data is much more of a challenge than dealing with only one dependent variable.

Random walk models have been prominent since the 1960s (Laming 1968, Link and Heath, 1975). Diffusion models, continuous versions of the random walk, appeared in the late 1970s (Ratcliff 1978, 1980, 1981). The random walk and diffusion models are close cousins and not competitors of each other as they are with other classes of models (e.g., accumulator models and counter models, e.g., LaBerge 1962, Smith and Vickers 1988).

The earliest random walk models assumed that the accumulation of information occurred at discrete points in time, each piece of information either fixed or variable in size (e.g., Laming 1968, Link and Heath 1975). The models succeeded in accounting for mean correct response time and accuracy, and were sometimes successful with mean error response times, but they rarely addressed the shapes of response time distributions.

In diffusion models, the accumulation of information is continuous over time. Ratcliff's (1978) diffusion model is illustrated in Fig. 1. One parameter of the model is the mean rate of accumulation of

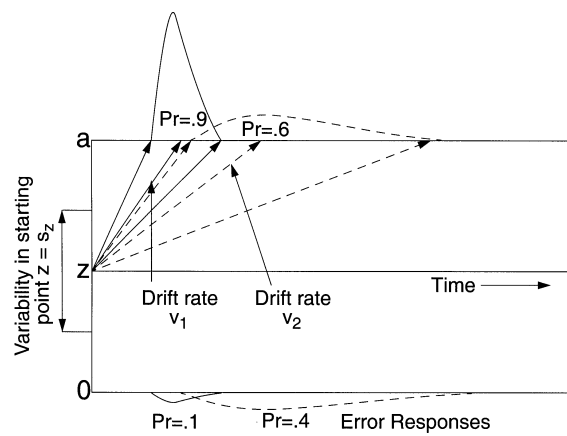


Figure 1
Illustration of Ratcliff's diffusion model

information. In the figure, two processes are shown one with mean drift rate v_1 (solid arrows) and the other with mean drift rate v_2 (dashed arrows). Drift rate varies around the mean as shown by the arrows for the average, the fastest and the slowest processes for the means v_1 and v_2 (these are meant to be illustrative; real processes are highly variable). Besides mean drift rate, the other parameters of the model are the separation of the boundaries (a), the starting point (z), the variability in drift rate across trials (η), and a parameter (T_{er}) that represents all the nondecisional components of response time, such as encoding and response preparation. There is also a parameter representing variability in processing (drift) within a trial (s). This is a scaling parameter; if it were doubled, for example, all the other parameters could be changed to produce the same identical predictions as before the change. s is a fixed parameter (not a free parameter) in fits of the model to data.

The basic diffusion process is used in many areas of research, including heat transfer, Brownian motion, neural processes, financial calculus, and fluid mechanics. As a result, the process has been examined in great detail and many explicit solutions for the distributions of the 'first passage' times ($G(t, \xi)$) and probabilities of reaching a boundary ($P(\xi)$) (where ξ is drift rate) are available.

$$P(\xi) = (e^{-(2\xi a/s^2)} - e^{-(2\xi z/s^2)}) / (e^{-(2\xi a/s^2)} - 1)$$

$$G(t, \xi) = P(\xi) - \frac{\pi s^2}{a^2} e^{-(\xi^2/s^2)}$$

$$\times \sum_{k=1}^{\infty} \frac{2k \sin(k\pi z/a) e^{-\frac{1}{2}(\xi^2/s^2 + \pi^2 k^2 s^2/a^2)t}}{(\xi^2/s^2 + \pi^2 k^2 s^2/a^2)}$$

But stochastic differential equations are difficult, and numerical solutions are required for examination of

more complex characteristics of the diffusion process, such as the effects of changes in drift rate and boundary positions during the course of information accumulation. Smith (2000) has developed alternative numerical methods to solve for the first passage times above; these are based on recursive expressions for the passage of a process to a response boundary. The idea is that the probability that a process will hit a response boundary at time t is the integral over time t_1 of the product of the probability that it first hit the boundary at time t_1 and the probability that it reaches the boundary starting at time t_1 and ending at time t . This solution allows the calculation of first passage times even when drift rate and boundary positions are not fixed. Another solution is to model the diffusion process with a discrete random walk process with very small step sizes, using Monte Carlo methods to obtain response probabilities and first passage time distributions for correct and error responses.

Ornstein-Uhlenbeck (OU) models are variants of diffusion models in which drift rate is offset by a decay term that tends to push the accumulation of information back towards the starting point. The size of the decay term is a linear function of the distance moved away from the starting point. OU models have been used to describe the behaviors of single neurons, they have been applied to simple reaction time tasks (Smith 1995), and they have been applied to decision-making and choice behavior (Busemeyer and Townsend 1993). Ratcliff and Smith (1999) have found that the OU process mimics the diffusion process, and vice versa, in many situations, but further work is needed to decide whether and how the two kinds of models can be discriminated.

1. Diffusion Models and Empirical Data

1.1 Scaling and Distribution Shape

The two dependent variables, response time and accuracy, have different scale properties. Response time has a minimum value and its variance increases as response time increases. Accuracy is bounded at probabilities 0.5 and 1.0, and as probability correct approaches 1.0, variance decreases. Diffusion models account for these scale properties automatically as a result of the geometry of the diffusion process. When mean drift rate is high, the probability of a correct response is near 1.0 and decision processes approach the correct boundary quickly with little spread in arrival times (e.g., the processes with mean v_1 in Fig. 1). When mean drift rate is nearer zero (processes with mean v_2 in Fig. 1), variability leads some processes to hit one boundary, other processes to hit the other boundary, and accuracy nears 0.5; the arrival times at boundaries are long and highly variable. In addition to these scale properties, the geometry of the diffusion process also gives response time distributions skewed

to the right; as mean response time increases (e.g., from drift rate v_1 to v_2), the fastest responses slow a little and the slowest responses slow a lot.

1.2 Speed vs. Accuracy

Human subjects can manipulate their speed performance relative to their accuracy performance. For example, if in some blocks of an experiment they are instructed to respond as quickly as possible, and in other blocks they are instructed to respond as accurately as possible, mean response times can vary between the two kinds of blocks by as much as 500ms and mean accuracy can vary by 10 percent (Ratcliff and Rouder 1998). Ratcliff's diffusion model can account for these differences with only the boundary separation parameter; moving the boundaries close together produces fast responses for which variability in drift can cause large numbers of errors; moving the boundaries apart produces slower responses that are more likely to be accurate.

1.3 Error Response Times

A major problem that restricted the development of random walk and diffusion models was their inability to accurately predict error response times. According to the models, errors and correct responses should have the same mean response times when the correct and error boundaries are equidistant from the starting point. However, experimental data showed that in some situations, errors are faster than correct responses and in some situations they are slower, and both patterns can even occur in different conditions in the same experiment (Luce 1986, Ratcliff et al. 1999, Ratcliff and Rouder 2000, Smith and Vickers 1988). In early attempts to deal with this problem, Laming (1968) added variability to the starting point to produce fast errors, and Link and Heath (1975) used variability in the sizes of the steps toward correct and error boundaries to account for fast errors in some experiments and slow errors in others.

Recently, a more complete solution has been provided by Ratcliff et al. (1999; see also Ratcliff 1981), showing that the complete patterns of error versus correct response times can be explained by combining variability in starting point and variability in drift rate across trials. This solution to the problem was so late in coming because earlier applications of diffusion models did not have enough computer power to automatically search the complete parameter space (e.g. Ratcliff 1978).

1.4 Time Course of Accumulation of Information

Diffusion and random walk models are explicitly based on a process that accumulates information over

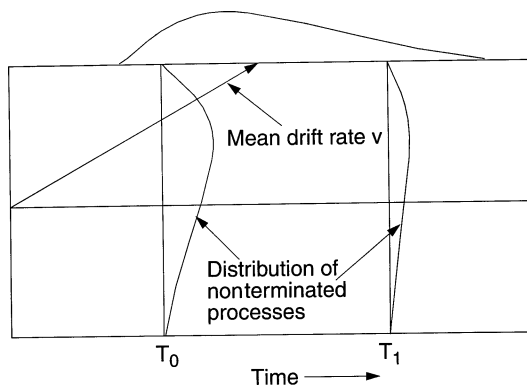


Figure 2
Illustration of the distribution of nonterminated processes within a diffusion process at two times T_0 and T_1 . The top curve represents the distribution of finishing times

time. Empirical examination of the time course of processing is possible with deadline and response signal procedures in which subjects are asked to respond at experimenter-determined times (a signal is presented after each test item to indicate exactly when a response to the item is to be made). The dependent variable is accuracy. The first diffusion model approach to this kind of data was to assume that there are no boundaries on the diffusion process; when the signal to respond is given, the position of the process is evaluated and, if it is above the starting point, one response is given; if it is below the starting point, the other response is given. This assumption leads to a simple formula for d' (see *Signal Detection Theory*) as a function of time:

$$d'(t) = d'_a / \sqrt{1 + s^2 / (\eta^2 t)}$$

An alternative approach (Ratcliff 1988), more in line with subjects' intuitions, is to assume that responses come from a mixture of two kinds of processes: processes that have not yet reached a boundary, like those just described, and processes that have reached a boundary. Figure 2 shows what happens with a test item for which the mean drift rate is v . The figure shows the distribution of the processes not terminated with this mean drift rate at time T_0 . Most of these processes are above the starting point z , and so, if a signal to respond is given at time T_0 , most responses from processes not yet terminated at a response boundary will produce the response represented by the top boundary. At time, T_1 , responses will be a mixture of processes with mean drift rate v , which have terminated, and other processes which have not.

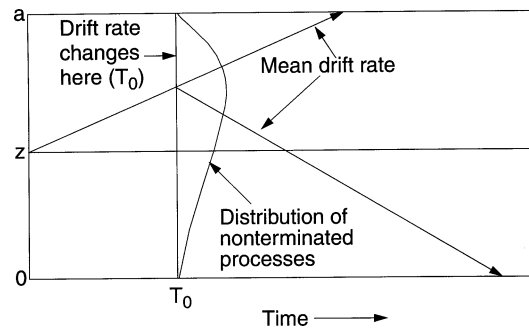


Figure 3
An illustration of the way drift rate might change in the diffusion process. At some point T_0 the drift rate changes and the distribution of nonterminated processes becomes a distribution of starting points for the second phase of the diffusion process

1.5 Nonmonotonic Functions

With some kinds of test stimuli, it can be shown empirically that the accumulation of information does not proceed at a constant rate. One source of information drives the process early in the time course and a different source of information drives the process later (e.g., Ratcliff and McKoon 1982). If the two sources of information contradict each other, then drift rate changes substantially during the time course of processing. Typically, it is the latter source of information that yields a correct response, and so accuracy first decreases from chance, and then later increases above chance, showing a nonmonotonic function across time. Figure 3 shows how this is handled in Ratcliff's diffusion model (Ratcliff 1980). When the second source of information becomes available, at time T_0 , mean drift rate for nonterminated processes changes from a positive to a negative value and the distribution of nonterminated processes at T_0 serves as the starting point for the second process.

1.6 Constraints on Diffusion Models

In many paradigms, accuracy is high and differences in accuracy values across experimental conditions are too small to be statistically significant. The consequence is that the effects of experimental manipulations can be observed only in response time. Data like this do not provide strong tests of diffusion models because they can be fit with any of a number of different parameter values.

Strong tests are provided when the models must account jointly for a broad range of response times (and the shapes of the response time distributions) and accuracy values from experimental conditions that are

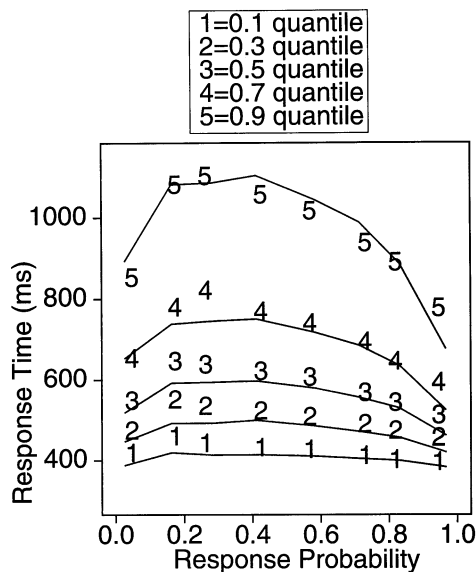


Figure 4

A quantile-probability function. The points represent the quantiles from the data and the solid lines the fit of a model to the data. Errors are to the left of the 0.5 probability point and correct responses corresponding to the errors to the right (mirror images). Each vertical line of quantiles represents a different condition in the experiment

manipulated in such a way that subjects cannot alter response criteria as a function of test item type (e.g., number of repetitions of words in a list where some studied items have few repetitions, some many). The manipulations should yield accuracy values varying from near floor (50 percent correct) to near ceiling (100 percent correct).

The issue arises of how to display this amount of data. Latency-probability functions have been used to simultaneously display mean response times and probabilities, with mean response time plotted on the y-axis and response probability on the x-axis. Different conditions are different points on the function, each condition represented by a pair of points, one for correct responses at probability p and a mirror image for errors at probability $1-p$. A latency-probability function gives a good picture of how accuracy and response time covary, showing, for example, whether response time changes quickly in conditions for which accuracy is high and changes little, or whether errors are faster than correct responses.

However, latency-probability functions do not show information about the shapes of response time distributions. One way to show this information is to plot the quantile response times for each of several quantiles (e.g., fifths) of the distributions as a quantile-probability function. Figure 4 shows an example using

five quantiles. The numbers represent the data and the solid lines show a theoretical fit of the diffusion model. The figure shows that the speed of the fastest responses (the 0.1 quantile) does not change much across conditions (i.e., across response probability values). Responses do slow from the conditions with high accuracy to the conditions with lower accuracy (where probability of a correct response is about 0.6), but they slow because the distributions spread out with the slowest responses (in the fifth quantile) increasing the most in response time.

2. Applications

Diffusion and random walk models have been applied to a wide variety of tasks. Here, we summarize the most frequent and salient applications.

2.1 Recognitions Memory

Diffusion models were first applied in the recognition memory domain (Ratcliff 1978). They provide a good account of correct response times, response probabilities, and the shapes of response time distributions. With the assumptions of variability in starting point and across-trial drift rates, they can also explain the patterns of correct vs. error response times.

2.2 Signal Detection

In one class of signal detection experiments, subjects are presented with stimuli and probabilistic feedback to those stimuli (e.g., Ratcliff et al. 1999). The diffusion model provides an extension to signal detection theory to explain the standard accuracy-based results as well as reaction time. Experimental manipulations allow accuracy to be varied from ceiling to floor and the experimental paradigm has served as a useful procedure for testing the diffusion model (Ratcliff and Rouder 1998, Ratcliff et al. 1999). This procedure also serves as one of the main procedures to test and evaluate models of categorization (e.g., Ashby 2000, Nosofsky and Palmeri 1997).

2.3 Perception

Ratcliff's diffusion model has been used to study the time course of processing when a stimulus is presented briefly, then masked (Ratcliff and Rouder 2000). In this situation, the amount of information coming into the decision process could either increase during stimulus presentation and then begin to fall during mask presentation, or the information coming into the decision process could be an integration of the

total information available from the stimulus, from its onset until the mask is presented. Only with the latter assumption could the data be well fit by the diffusion model.

2.4 Simple Reaction Time

Simple reaction time tasks require the subject to respond when a stimulus is presented. Smith (1995) used an OU model with assumptions about sustained and transient processing channels to model the effects of stimulus intensity in these tasks. The model was able to simultaneously account for the shapes of response time distributions as a function of stimulus intensity and the time course of the availability of stimulus information (see *Stochastic Dynamic Models (Choice, Response, and Time)*).

2.5 Decision Making

Bussemeyer and Townsend (1993) developed a dynamic model of decision making using an OU process to represent the time course of processing for decision making under uncertainty. This model has been further developed to provide an integrative framework for understanding several seemingly diverse empirical phenomena in the area of multialternative preferential choice and also this kind of model has been extended to represent the time course of processing for multi-attribute decision making (see *Dynamic Decision Making; Sequential Decision Making*).

2.6 Categorization

Random walk and diffusion have been married to models of categorization to account for response time data in categorization. In exemplar-based models (Nosofsky and Palmeri 1997), each exemplar provides an increment in a random walk. In decision-bound models (Ashby 2000), the drift rate is determined by the distance of the test stimulus from a boundary that separates the categories on the relevant dimensions. These models generalize the earlier categorization models to allow them to account for response times as well as accuracy and similarity measures.

2.7 Neural Processes

Recent data suggest that neural processes can be represented by random walk or diffusion processes (Hanes and Schall 1996). Neurons in the eye movement system in monkeys behave as though they increase their firing rates until a fixed response

criterion is reached and this is the signal for the response (in the form of an eye movement) to be made. Diffusion processes are continuous processes and so are not exactly consistent with neural processes that are discrete. But if the diffusion process is replaced with a random walk with one step corresponding to 1 ms (about the same time scale as neural counts), then the approximation between the diffusion process and a discrete random walk (with time steps of 1 ms) is very good. Also, if a diffusion decision process were to be represented neurally, it would be represented by a population of neurons and this would lead to quite a good approximation between a population firing rate and a continuous process.

3. Conclusion

Random walk and diffusion models are under continuous testing. As members of the class of sequential sampling models, they appear to account for experimental data more successfully than any other class of models. Perhaps the most encouraging sign is that fits of the models to data are beginning to show invariances in parameter values across subjects and across tasks, which means that some experimental variables are represented in the model by single parameters. This kind of success is something we do not always see in modeling in psychology.

See also: Categorization and Similarity Models; Categorization and Similarity Models: Neuroscience Applications; Decision and Choice: Random Utility Models of Choice and Response Time; Decision Theory: Classical; Dynamic Decision Making; Letter and Character Recognition, Cognitive Psychology of; Luce's Choice Axiom; Sequential Decision Making; Stochastic Dynamic Models (Choice, Response, and Time)

Bibliography

- Ashby F G 2000 A stochastic version of general recognition theory. *Journal of Mathematical Psychology* **44**: 310–29
- Bussemeyer J R, Townsend J T 1993 Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review* **100**: 432–59
- Feller W 1967 *An Introduction to Probability Theory and its Applications*, 3rd edn. Wiley, New York
- Hanes D P, Schall J D 1996 Neural control of voluntary movement initiation. *Science* **274**: 427–30
- LaBerge D A 1962 A recruitment theory of simple behavior. *Psychometrika* **27**: 375–96
- Laming D R J 1968 *Information Theory of Choice Reaction Time*. Wiley, New York
- Link S W, Heath R A 1975 A sequential theory of psychological discrimination. *Psychometrika* **40**: 77–105

- Luce R D 1986 *Response Times*. Oxford University Press, New York
- Nosofsky R M, Palmeri T J 1997 An exemplar based random walk model of speeded classification. *Psychological Review* **104**: 266–300
- Ratcliff R 1978 A theory of memory retrieval. *Psychological Review* **85**: 59–108
- Ratcliff R 1980 A note on modelling accumulation of information when the rate of accumulation changes over time. *Journal of Mathematical Psychology* **21**: 178–84
- Ratcliff R 1981 A theory of order relations in perceptual matching. *Psychological Review* **88**: 552–72
- Ratcliff R 1988 Continuous versus discrete information processing: Modeling the accumulation of partial information. *Psychological Review* **95**: 238–55
- Ratcliff R, McKoon G 1982 Speed and accuracy in the processing of false statements about semantic information. *Journal of Experimental Psychology: Learning, Memory and Cognition*. **8**: 16–36
- Ratcliff R, Rouder J N 1998 Modeling response times for two-choice decisions. *Psychological Science* **9**: 347–56
- Ratcliff R, Rouder J N 2000 A diffusion model account of masking in letter identification. *Journal of Experimental Psychology: Human Perception and Performance* **26**:127–40
- Ratcliff R, Smith P L 1999 Comparing stochastic models for reaction time. Paper presented at the 32nd Annual Meeting of the Society for Mathematical Psychology, Santa Cruz, CA
- Ratcliff R, Van Zandt T, McKoon G 1999 Connectionist and diffusion models of reaction time. *Psychological Review* **106**: 261–300
- Smith P L 1995 Psychologically principled models of visual simple reaction time. *Psychological Review* **102**: 567–91
- Smith P L 2000 Stochastic dynamic models of response time and accuracy: A foundational primer. *Journal of Mathematical Psychology* **44**: 408–63
- Smith P L, Vickers D 1988 The accumulator model of two-choice discrimination. *Journal of Mathematical Psychology* **32**: 135–68

R. Ratcliff

Diffusion: Anthropological Aspects

In anthropology, diffusion has been taken to be the process by which material and immaterial cultural and social forms spread in space. A number of specialized fields of inquiry in cultural diffusion developed in anthropology throughout the twentieth century. At the beginning, and going back to the late nineteenth century, stood cultural history. After World War I came acculturation and culture contact studies, which continued for several decades. The 1970s marked a growing anthropological interest in world-system studies and questions of cultural imperialism. In the late 1970s and early 1980s, transnational and globalization studies became a subfield of anthropology. This is a rough chronology without claim to compre-

hensiveness—in particular if one considers that so much of anthropology has been about cultural interaction and change and thus, in some sense, about the distribution of cultural and social forms across space.

The anthropological interest in diffusion began as an argument against evolutionary interpretations of history. Such interpretations had become part of popular culture in late-nineteenth-century Europe and Euro-America. The general idea was that cultural evolution and cumulative reason had taken humankind from a stage of savagery to one of civilization. The title of one of Lewis Henry Morgan's (1818–81) works put it succinctly: *Ancient Society, or Researches in the Lines of Human Progress from Savagery through Barbarism to Civilization* (1877). Evolutionary theoreticians like Morgan, Edward B. Tylor (1832–1917), and Adolf Bastian (1826–1905), held that, while cultural borrowing between social groups certainly had occurred throughout human history, similar cultural and social forms emerged in different places and times, mostly due to the psychic similarity of humans the world over, and to a general uniformity in evolutionary stages.

Friedrich Ratzel (1844–1904) was the first to explicitly reject this explanation for cultural and social similarities. He argued that no culture had evolved in isolation, and that contact and borrowing between groups should be considered to be far more common in human history than instances of social and cultural innovation. Since Ratzel believed that the principal mechanism of diffusion was migration, he arrived at the conclusion that 'migration theory is the fundamental theory of world history' (Ratzel 1882, p. 464). The object of study for Ratzel was the reconstruction of how cultural and social forms had spread from centers of innovation. This reconstructive effort went under the name of cultural history (*Kulturgeschichte* in German). Among the most notable cultural historians were: Bernhard Ankermann (1859–1943), who, unlike the majority of cultural historians who wrote universal histories of humankind, believed that insufficient data only allowed for the reconstruction of diffusion in particular regions; Leo Frobenius (1873–1938), a student of Ratzel who elaborated the latter's migration theory into a theory of cultural areas (*Kulturkreise* in German); and Fritz Graebner (1877–1934), the author of *Methode der Ethnologie* (1911), arguably the most influential theoretical and methodological work of cultural history. At the University of Vienna, Fathers Wilhelm Schmidt and Wilhelm Koppers formulated a theory of primary cultural units, which were said to be at the root of all existing cultures and to have evolved through processes of diffusion from the primeval culture of hunters and gatherers. At University College, London, Grafton Elliot Smith and his disciple William Perry tried to prove that from a single center of innovation, namely ancient Egypt, culture had diffused to the rest of the world:

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