A DWBA Treatment of the d + d Reaction at Low Energies

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The nucleon polarization produced in the d + d reaction at low energies (< 500 keV) has been a subject of considerable theoretical interest. The first extensive analysis [1] formulated in terms of deuteron penetrability factors was unsatisfactory in that a strong $^3P \rightarrow {}^3F$ transition was needed [2] to explain the observed nucleon polarization. More recently [3, 4] attention has been focused on the transitions $^1D \rightarrow {}^3D$ and $^3P \rightarrow {}^1P$ to explain the polarization data. These transitions are of particular interest [2] since in first order $^3P \rightarrow {}^1P$ is not allowed and $^1D \rightarrow {}^3D$ occurs only if a two-nucleon $\vec{L}\cdot\vec{S}$ force operates. It is possible to build second order effects into a distorted wave treatment [5, 6] of the reaction in which case polarization effects can arise from interference between the different J values in $^3P \rightarrow {}^3P$ transitions (J = 0, 1, 2).

The analysis reported here is based on the formulation of ref. [5]. The T-matrix element is written as

$$T_{fi} = \langle \psi_f^- | V_i | \chi_i^+ \rangle$$

where χ_i^+ describes the initial two deuteron state distorted only by the Coulomb field and $V_i = V_{13} + V_{24} + V_{14} + V_{23}$ is the sum of the two nucleon potentials appropriate to the initial state interaction (1 and 3 refer to protons and 2 and 4 to neutrons). The above form for T_{fi} is valid even after antisymmetrization is taken into account. In our approximation ψ_f^- is constructed from Coulomb functions and p-^{3}H (or n-^{3}He) elastic scattering phase shifts. The integration over the coordinate $\vec{r}_p = \vec{r}_1 - 1/3(\vec{r}_2 + \vec{r}_3 + \vec{r}_4)$ of the outgoing proton is cut off for $r_p < R$, the p-^{3}H contact radius (3 ~ 4 fm). This approximation is not as drastic as it might first appear, since there is a large contribution to T_{fi} from the restricted range of integration due to the large width (~ 7 fm) of the deuteron. An immediate consequence of this approximation is that V_{13} and V_{14} have negligible contribution in comparison with V_{24} and V_{23}. Our treatment differs from that of Boersma [6], in this respect, and in the way the relative motion wave functions are constructed. Physically, we have a cut-off stripping approximation, but with all the exchange terms included and a formulation that avoids nuclear distortion of the deuteron wave.

The results obtained for some of the T-matrix elements at E_d = 200 keV (lab) are shown in table 1. Well-known Gaussian potentials and internal wave functions have been used

$$\text{central:}\quad V_{ij}^{c} = -\frac{1}{8}(3 - \vec{\tau}_i\cdot\vec{\tau}_j - \vec{\sigma}_i\cdot\vec{\sigma}_j)V_0^c \exp(-r_{ij}^2/r_0^2)$$

$$\text{tensor:}\quad V_{ij}^{T} = -\frac{1}{4}(1 - \vec{\tau}_i\cdot\vec{\tau}_j)\, S_{ij} V_0^T (r_{ij}^2/r_0^2) \exp(-r_{ij}^2/r_0^2)$$

$$\Phi_d(12) = (2\alpha^2/\pi)^{3/4} \exp(-\alpha^2 r_{12}^2);$$

$$\Phi_T(234) = (12/\pi^2)^{3/4} \beta^3 \exp\{-\beta^2(r_{23}^2 + r_{24}^2 + r_{34}^2)\}$$

$$r_0 = 1.58\ \text{fm},\ \alpha = 0.167\ \text{fm}^{-1},\ \beta = 0.255\ \text{fm}^{-1},$$

$$V_0^c = 59\ \text{MeV},\ V_0^T = 107\ \text{MeV}.$$

Parameters of the reaction which have been considered are the nucleon polarization P, the ratio C_2/C_0 for the Legendre polynomial coefficients of the unpolarized distribution, and B_3/C_0, where B_3 is the coefficient of $P_1^1(\cos\theta)$ in the angular distribution produced with vector-polarized incident deuterons.

Table 1. d+d reaction matrix elements at E_d = 200 keV (lab)

	Matrix element J	Real	Imag.	Potentials which will contribute*	$p-{}^3H$ phase shift δ_ℓ^J
$^1S \to {}^1S$	$0(\alpha_0)$	+1.517	−2.910	*central*	−63°
$^5S \to {}^3D$	$2(\gamma_1)$	+0.605	≈ 0	*tensor*, $\vec{L}\cdot\vec{S}$	− 5°
$^3P \to {}^3P$	$0(\alpha_{10})$	−0.399	−0.121	central	+15°
$^3P \to {}^3P$	$1(\alpha_{11})$	+0.138	+0.091	*tensor*	+30°
$^3P \to {}^3P$	$2(\alpha_{12})$	−0.275	−0.182	$\vec{L}\cdot\vec{S}$	+30°

*Potential in italics is the one for which the matrix element has been calculated.

The quintet-singlet transition γ_1 is particularly interesting and in the past has often been neglected. Apart from the small transition ($^1D \to {}^3D$), γ_1 is needed to obtain a non-zero B_3 in first order [4].

$$B_3 = -\frac{9\sqrt{10}}{8}\ \text{Im}\,(\gamma_1\alpha_{11}^* - \gamma_1\alpha_{12}^*).$$

With only tensor forces included the a_{1J} are not large enough to produce the observed polarizations for reasonable phase shifts δ_1^J. Computation of the central force contribution to a_{1J} has not yet been made. However, the behavior of the integrations suggests that the central force will be about twice as effective as the tensor force and generate matrix element components of the same sign. Both V_{24} and V_{23} contribute for the central force case, whereas for the tensor force, V_{24} has no effect, since its spin matrix element vanishes identically. To gain some idea of the effectiveness of the $^3P \rightarrow {}^3P$ transitions, parameters have been computed for values of a_{1J} a factor of 3 greater than those given in table 1, to take into account the central force. The results are $P(54°) = -0.05$, $C_2/C_0 = +0.19$, $B_3 = +0.09$. The values of all of these coefficients are sensitive to the splitting of the p-wave phase shifts δ_1^J. In particular $P = B_3 = 0$ if there is no splitting. The values chosen for the phase shifts are based on the values obtained [7] for p-^{3}He elastic scattering. It is encouraging to see that the polarizations have the correct signs. While the magnitudes are low by a factor of 2 it is conceivable that including the $\vec{L}\cdot\vec{S}$ force will lead to an improvement.

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