The GZK Effect

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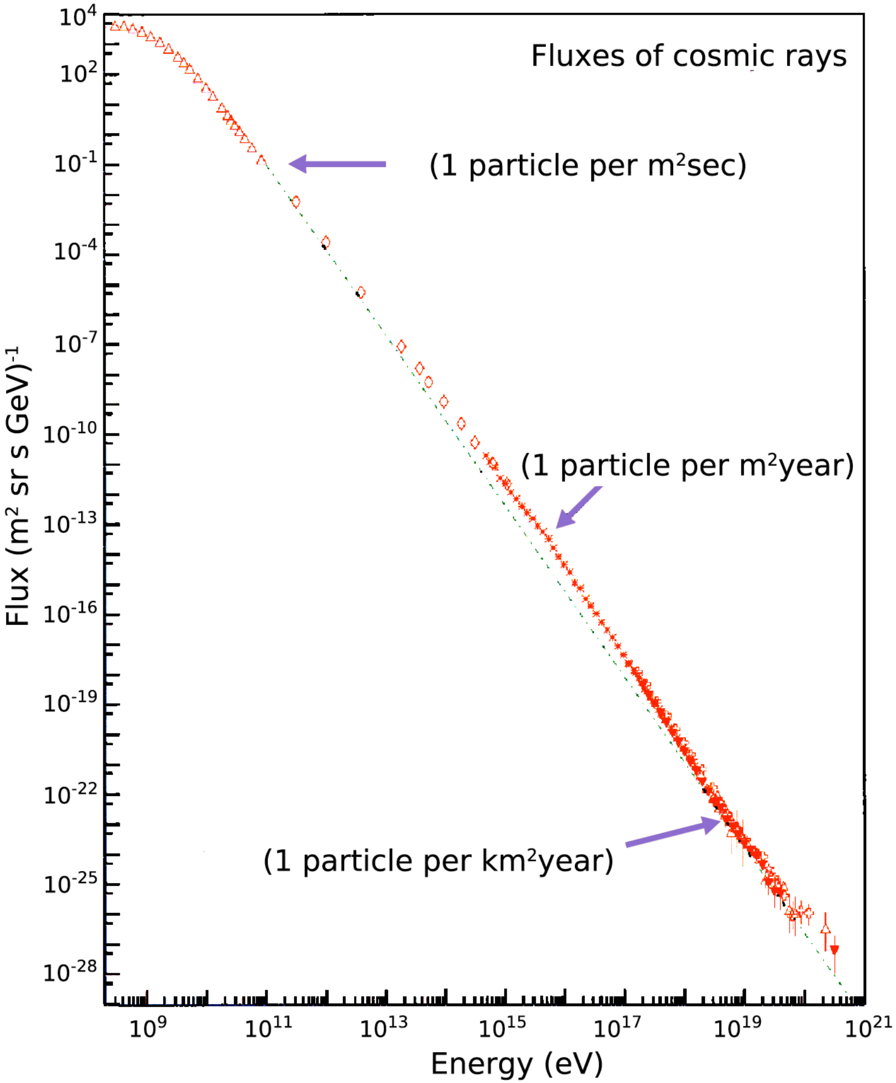
Missing Cosmic Rays

- Cosmic ray spectrum has steep drop-off above ~$10^{19.5}$ eV
- Where did all the CR go?

Historical Sidenote: The cutoff was experimentally observed only in the last twenty years. It serves as nice motivation for the calculation, but only so in hindsight.



GZK Effect

- In 1966, Greisen[1] and Zatsepin & Kuz'min [2] gave a theoretical explanation
- What if the protons were interacting with the CMB?

$$g_{CMB} + p \rightarrow D^{+} \rightarrow p + p^{0}$$

$$g_{CMB} + p \rightarrow D^{+} \rightarrow n + p^{+}$$

- Calculation went as follows:
  - Assume the CR spectrum is primarily protons
  - Calculate at what energy the protons would interact with the CMB
  - Does this "cutoff energy" match the losses observed?

Step 1: Energy of CMB Photon

- Universe is blackbody with energy density $\epsilon(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu}{e^{h\nu/kT} - 1}$

- Get total energy by integrating over all frequencies, with variable substitution $y = h\nu / kT$ for simplicity of the integral

$$\varepsilon_{rad} = \frac{8\pi k^4}{h^3 c^3} T^4 \int_0^\infty \frac{y^3 dy}{e^y - 1} = \frac{\pi^2 k^4}{15\hbar^3 c^3} T^4 = \left(7.5657 \times 10^{-16} \frac{J}{m^3 K^4}\right) T^4$$

- Plug in numbers (temperature of CMB is ~3K), find:

$$\varepsilon_{rad} \equiv E_{\gamma_{CMB}} = 7.06 \times 10^{-4} eV$$

Step 2: COM Collision

- Threshold calculation: COM collision between proton and photon, outgoing proton/pion have no kinetic energy

- In COM: proton & pion are at rest
- In LAB: proton & pion have equal velocities

Step 2: COM Collision (cont)

- Write down energy and momentum conservation in COM frame (energy and three momenta of a photon are the same)

$$E_P + E_{\gamma_{CMB}} = E'_P + E_\pi$$

$$|\vec{p}_P| + (-E_{\gamma_{CMB}}) = |\vec{p}'_P| + |\vec{p}_\pi|$$

- Write down the corresponding on-shell conditions

$$E_P = \sqrt{\vec{p}_P^2 + m_P^2} \qquad E'_P = \sqrt{\vec{p}'^2_P + m_P^2} \qquad E_\pi = \sqrt{\vec{p}'^2_\pi + m_\pi^2}$$

- Write down equal velocity condition of outgoing particles

$$\frac{|\vec{p}'_P|}{E'_P} = \frac{|\vec{p}_\pi|}{E_\pi}$$

Step 3: Ultra-Relativistic Assumption

- This is three equations for three unknowns: E_P, E'_p, E_π
- BUT, a priori hard to solve (the radicals are very messy to untangle)
- Utilize that the particles are ultra-relativistic, and Taylor expand their momenta

$$\vec{p}^2 = E^2 - m^2 \rightarrow |\vec{p}| = \sqrt{E^2 - m^2} = E\sqrt{1 - \frac{m^2}{E^2}} \approx E\left(1 - \frac{m^2}{2E^2} + \dots\right)$$

$$|\vec{p}_P| \approx E_p - \frac{m^2{}_p}{2E_p} \quad |\vec{p}'_P| \approx E'_P - \frac{m^2{}_p}{2E'_p} \quad |\vec{p}_\pi| \approx E_\pi - \frac{m^2{}_\pi}{2E_\pi}$$

Step 4: Solve

- With these simplified momenta, we can solve for the particle energies

$$E_P + E_{\gamma_{CMB}} = E'_P + E_\pi$$
$$|\vec{p}_P| + (-E_{\gamma_{CMB}}) = |\vec{p}'_P| + |\vec{p}_\pi|$$
$$\frac{|\vec{p}'_P|}{E'_P} = \frac{|\vec{p}_\pi|}{E_\pi}$$

$\longrightarrow$

$$E_P + E_{\gamma_{CMB}} = E'_P + E_\pi$$
$$\frac{m_P{}^2}{2E_P} + 2E_{\gamma_{CMB}} = \frac{m_P{}^2}{2E'_P} + \frac{m_\pi{}^2}{2E_\pi}$$
$$\frac{m_P{}^2}{E'_P{}^2} = \frac{m_\pi{}^2}{E_\pi{}^2}$$

- Eliminate E_π, solve for E_P, E'_p

Conclusion

- Compute the final numbers:

$$E_P = 1.07 \times 10^{20} eV$$

$$E'_P = 9.45 \times 10^{19} eV$$

- Which is right where we saw the drop off in the CR flux (but GZK just wanted to show you wouldn't measure them)
- Sidenote: those pions and neutrons live on to provide the neutrino flux we hope to measure

$$\pi^+ \to \mu^+ + \nu_\mu, \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

$$n \to e^- + p + \bar{\nu}_e$$

Thanks! Questions?

References

1. Greisen, Kenneth (1966). "End to the Cosmic-Ray Spectrum?". *Physical Review Letters* **16** (17). 748-750.
2. Zatsepin, G.T.; Kuz'min, V.A. (1966). "Upper Limit of the Spectrum of Cosmic Rays". *Journal of Theoretical Physics Letters* **4**: 78-80.
3. QFT 8808.01 at OSU