

Tue, Part 1

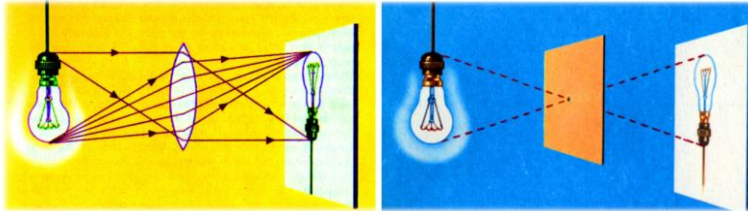
Quantum Physics

Outline

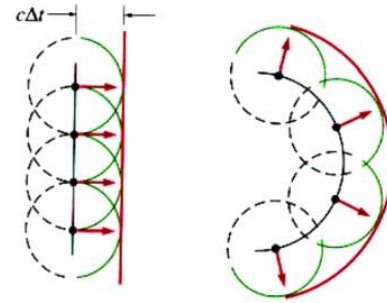
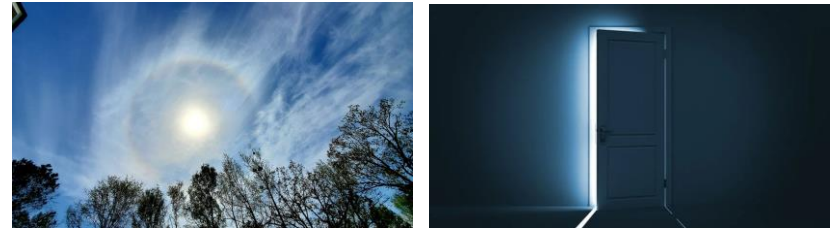
- Light – wave or particles?
- Double slit experiment
- Mach-Zehnder interferometer
- Virtual lab

Light – waves or particles?

Newton's Corpuscular Model of Light

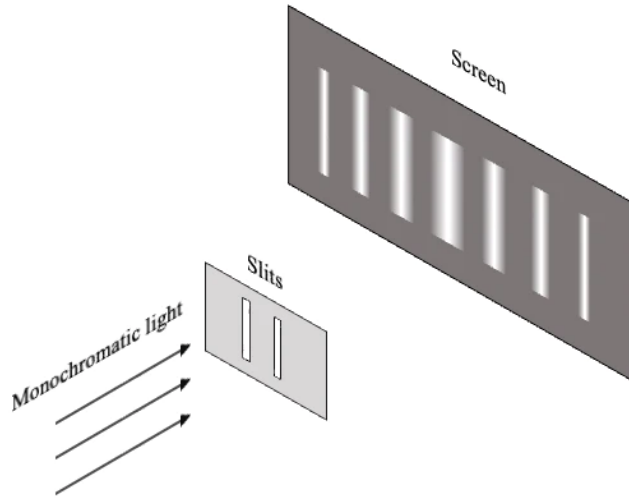


Huygens' Wave Model of Light

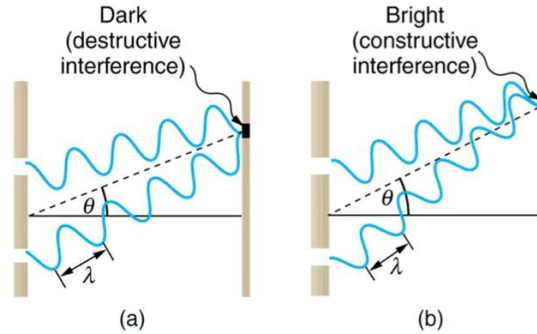
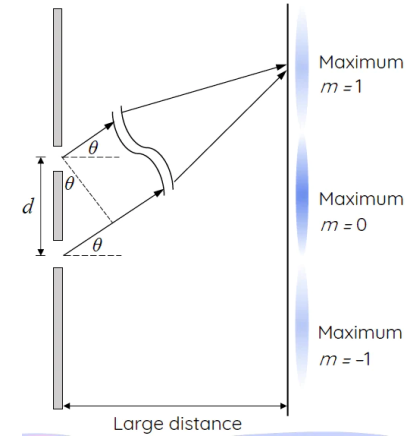


source: <https://scienceready.com.au/> etc.

Light as wave – double slit experiment



source: <https://scienceready.com.au/pages/diffraction>

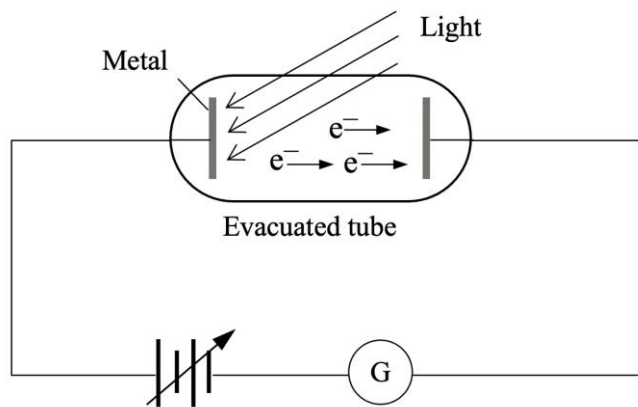


The interference pattern is easy to explain with wave theory: it results from the difference between two paths light takes from the two slits

- bright spots – positive interference, waves arrive *in phase*
$$d \sin \theta = m\lambda$$
- dark spots – negative interference, waves arrive *out of phase*
$$d \sin \theta = (m + \frac{1}{2})\lambda$$

So, light is a wave, right?

Light as particles – photoelectric effect



source: <https://scienceready.com.au/pages/photoelectric-effect>

Assumptions of the wave model

- Light with higher intensity will eject electrons with greater kinetic energy
- Light with higher frequency will result in a greater current
- Light of any intensity and frequency will be able to eject electrons

Observations

- Light with higher intensity results in a greater current
- Light with higher frequency results in electrons with greater kinetic energy
- Light below a certain frequency is unable to eject photoelectrons, no matter what the intensity of light is

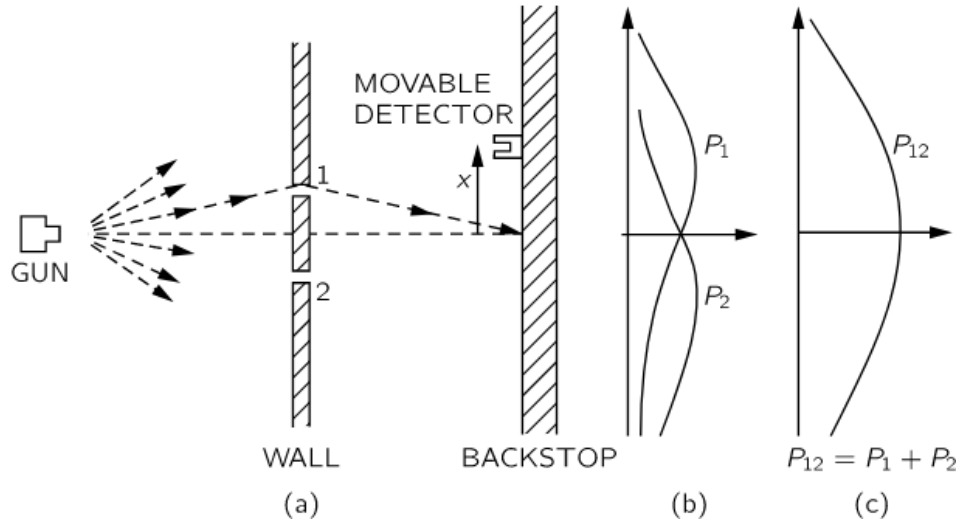
Quantum model of light

- Light consists of photons (Einstein), one photon interact with one electron
- Each photon carries energy proportional to its frequency $E = hf$
- If the energy of the photon is higher than metal's *work function*, photon transfers its energy to an electron – electron is ejected, and the remaining energy is transformed into its kinetic energy.

So, light is a stream of particles, right?

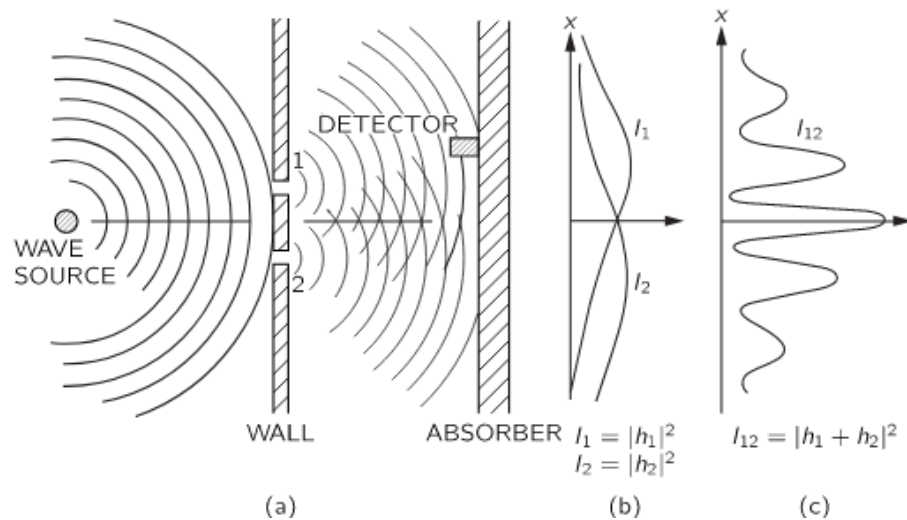
Double slit experiment

Double slit experiment – classical particles

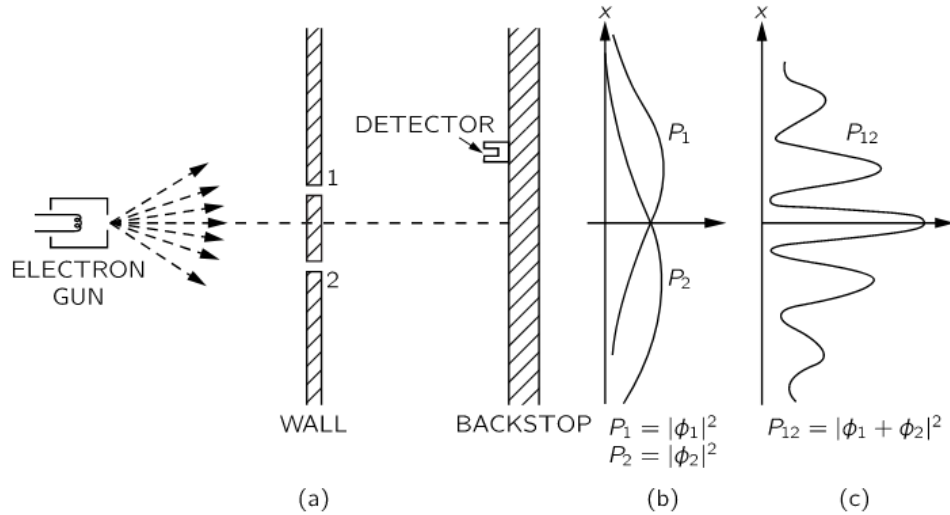


FEYNMAN, Richard P., Robert B. LEIGHTON and Matthew SANDS, no date. *The Feynman Lectures on Physics, Volume III* [online]. B.m.: California Institute of Technology. ISBN 978-0-465-04083-4. Available at: http://www.feynmanlectures.caltech.edu/III_toc.html

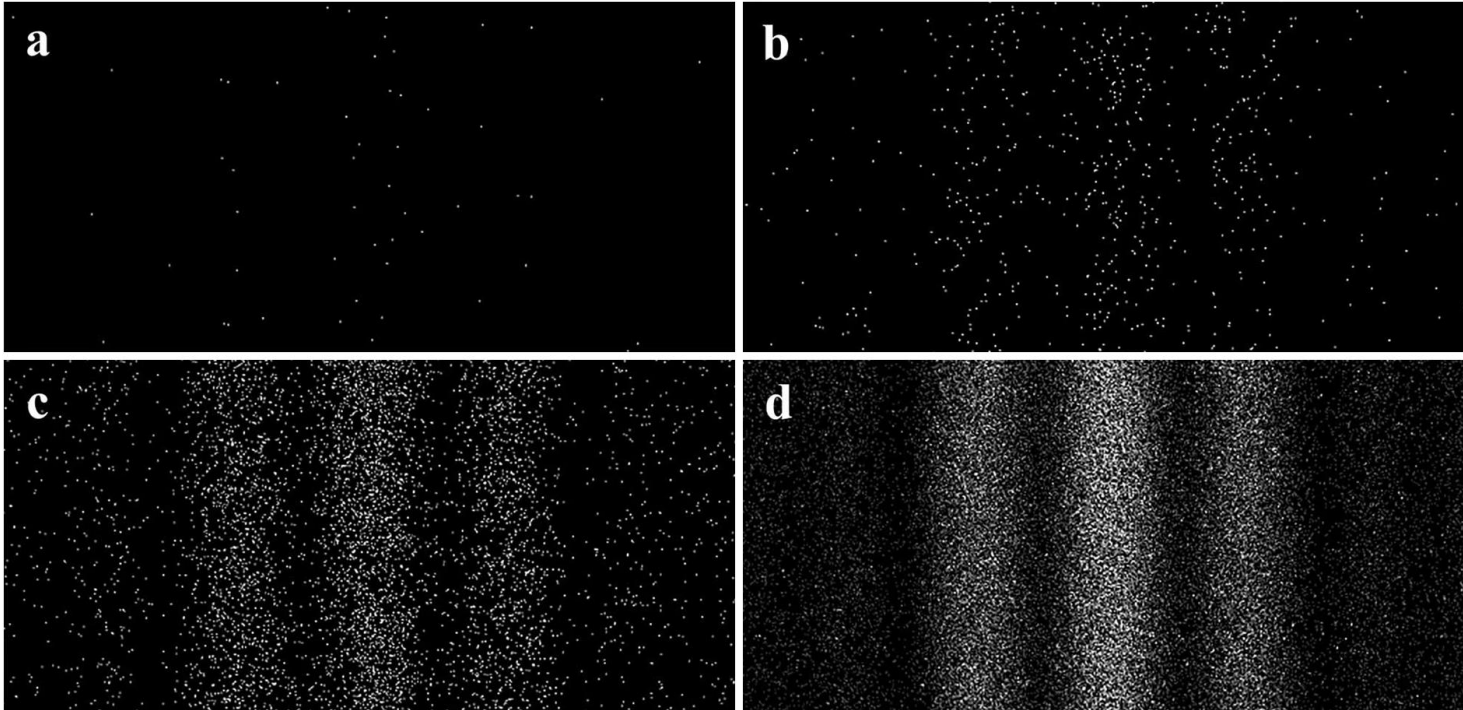
Double slit experiment – waves



Double slit experiment – microparticles

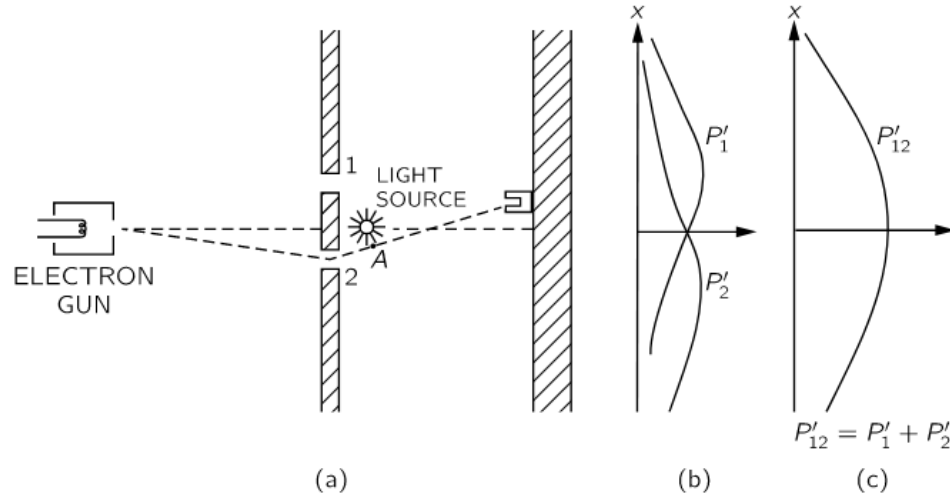


Double slit experiment – microparticles



Harada, Ken, Tetsuya Akashi, Kodai Niitsu, Keiko Shimada, Yoshimasa A. Ono, Daisuke Shindo, Hiroyuki Shinada, and Shigeo Mori. 2018. 'Interference Experiment with Asymmetric Double Slit by Using 1.2-MV Field Emission Transmission Electron Microscope'. *Scientific Reports* 8 (1): 1008. <https://doi.org/10.1038/s41598-018-19380-4>.

Double slit experiment – observation



Three principles

1. In an ideal experiment, the **probability** P of an event is equal to the **absolute square** of the complex number φ which is called the **probability amplitude**:

$$P = |\varphi|^2$$

2. If an event occurs in several ways which are **indistinguishable**, the probability amplitude is the sum of amplitudes for each of them. In this case, the interference effect occurs.

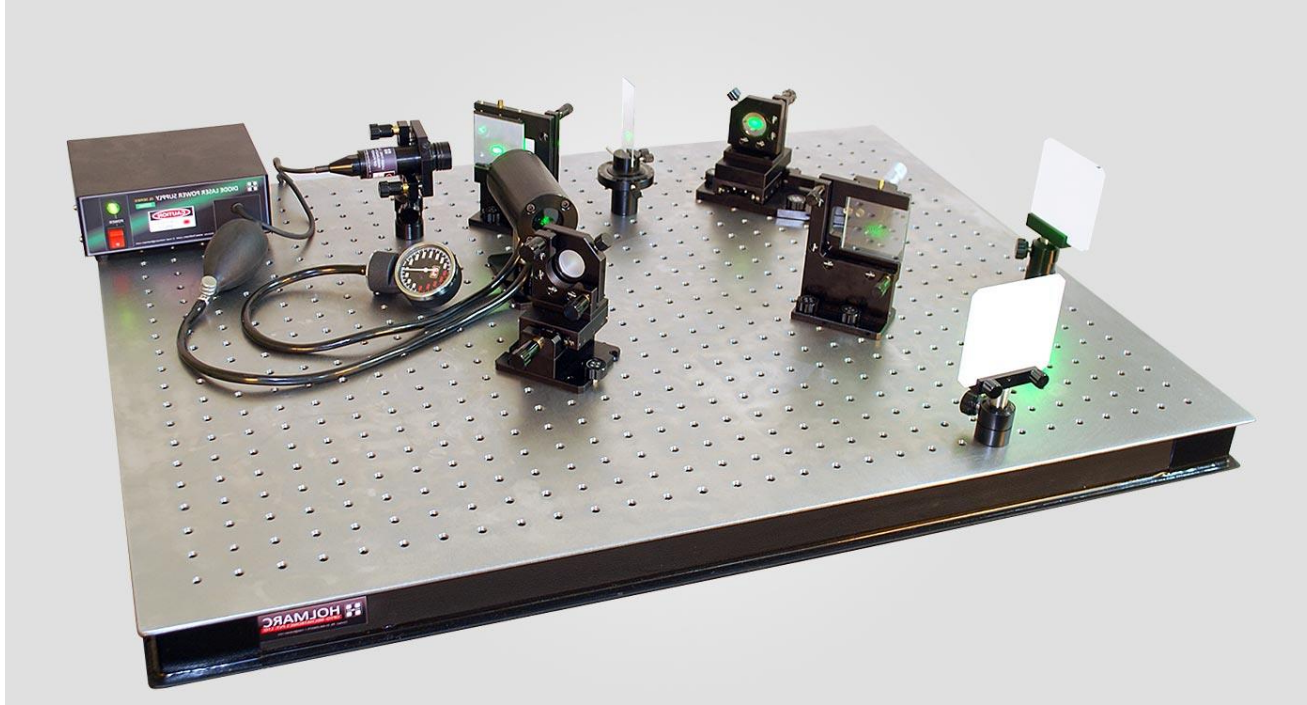
$$\begin{aligned}\varphi &= \varphi_1 + \varphi_2 \\ P &= |\varphi_1 + \varphi_2|^2\end{aligned}$$

3. If the experiment allow to **distinguish between alternatives**, then the probability of an event is the sum of contributions of probability for each of them. Interference effect is not present.

$$\begin{aligned}P &= |\varphi_1|^2 + |\varphi_2|^2 \\ P &= P_1 + P_2\end{aligned}$$

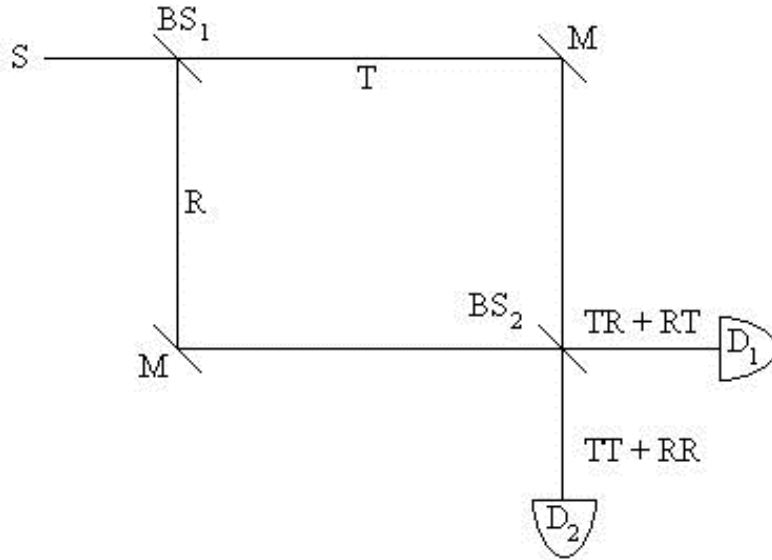
Mach-Zehnder interferometer

Mach-Zehnder interferometer



source: https://www.holmarc.com/mach_zehnder_interferometer.php

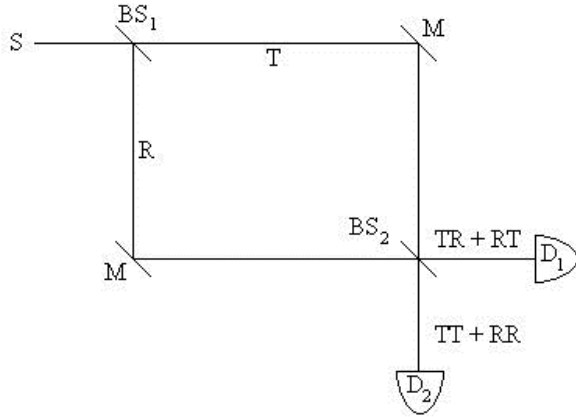
Mach-Zehnder interferometer



- how does it work?
 - S – source of light
 - BS – beam splitters
 - T – light transmitted
 - R – light reflected
 - M – mirrors
 - D – light detectors
- two detectors – where does the light go?

source: <https://faculty.csbsju.edu/frioux/super/MM-super.htm>

Mach-Zehnder interferometer



- initial state: ψ_S
- probability amplitude for transmission: $\langle \psi_T | \psi_S \rangle = \frac{1}{\sqrt{2}}$
- probability amplitude for reflection: $\langle \psi_R | \psi_S \rangle = \frac{i}{\sqrt{2}}$

wave function (at a fixed point) $\psi_S = e^{-i\omega t}$

$$\psi_T = \frac{1}{\sqrt{2}} e^{-i\omega t}$$

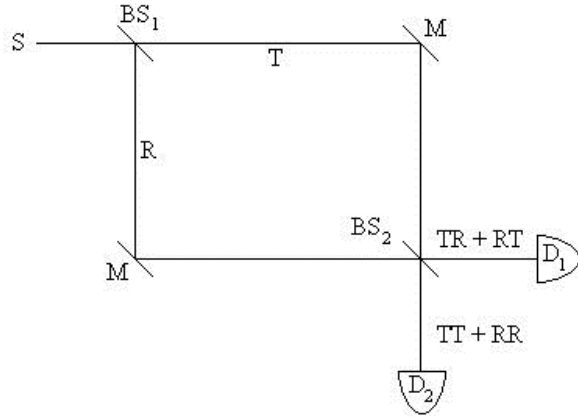
$$\langle \psi_T | \psi_S \rangle = \frac{1}{\sqrt{2}} e^{i\omega t} \cdot e^{-i\omega t} = \frac{1}{\sqrt{2}} e^{i\omega t - i\omega t} = \frac{1}{\sqrt{2}} e^0 = \frac{1}{\sqrt{2}}$$

$$\psi_R = \frac{i}{\sqrt{2}} e^{-i\omega t}$$

$$\langle \psi_R | \psi_S \rangle = \frac{i}{\sqrt{2}} e^{i\omega t} \cdot e^{-i\omega t} = \frac{i}{\sqrt{2}} e^{i\omega t - i\omega t} = \frac{i}{\sqrt{2}} e^0 = \frac{i}{\sqrt{2}}$$

source: <https://faculty.csbsju.edu/frioux/super/MM-super.htm>

Mach-Zehnder interferometer



- initial state: ψ_S
- probability amplitude for transmission: $\langle \psi_T | \psi_S \rangle = \frac{1}{\sqrt{2}}$
- probability amplitude for reflection: $\langle \psi_R | \psi_S \rangle = \frac{i}{\sqrt{2}}$

$$\psi_R = i \cdot \frac{1}{\sqrt{2}} e^{-i\omega t}$$

$$i \cdot e^{-i\omega t} = ?$$

$$i \cdot e^{-i\omega t} = i \cdot \cos(\omega t) + \sin(\omega t)$$

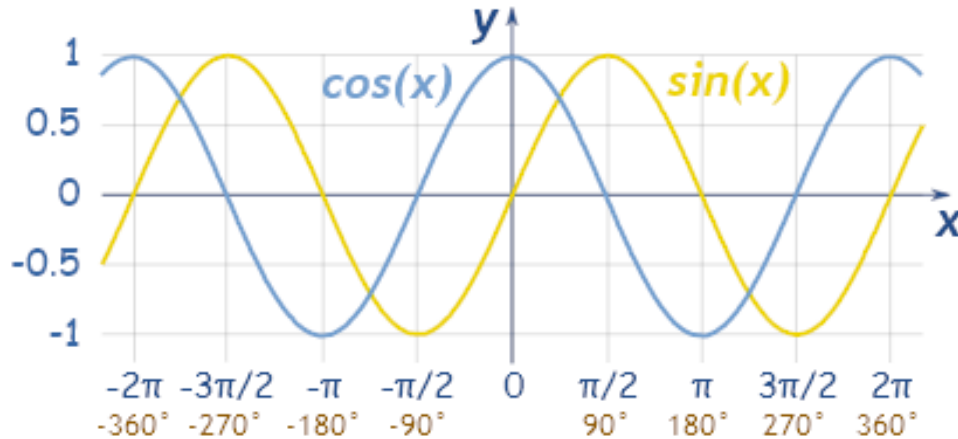
$$i \cdot e^{-i\omega t} = \sin(\omega t) + i \cdot \cos(\omega t)$$

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

$$e^{-i\omega t} = \cos(\omega t) - i \sin(\omega t)$$

Mach-Zehnder interferometer

- initial state: ψ_S
- probability amplitude for transmission: $\langle \psi_T | \psi_S \rangle = \frac{1}{\sqrt{2}}$
- probability amplitude for reflection: $\langle \psi_R | \psi_S \rangle = \frac{i}{\sqrt{2}}$



$$i \cdot e^{-i\omega t} = ?$$

$$i \cdot e^{-i\omega t} = i \cdot \cos(\omega t) + \sin(\omega t)$$

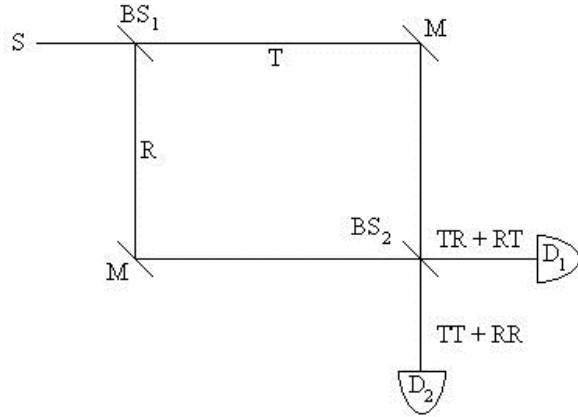
$$i \cdot e^{-i\omega t} = \sin(\omega t) + i \cdot \cos(\omega t)$$

$$i \cdot e^{-i\omega t} = \cos\left(\omega - \frac{\pi}{2}\right)t - i \cdot \sin\left(\omega - \frac{\pi}{2}\right)t$$

$$i \cdot e^{-i\omega t} = e^{-i\left(\omega - \frac{\pi}{2}\right)t}$$

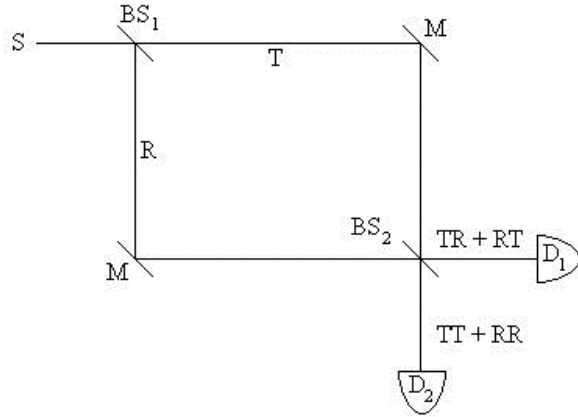
- reflection on beam splitter shifts the phase of the wave by $\frac{\pi}{2}$

Mach-Zehnder interferometer – undisturbed



- interferometer undisturbed
 - D_1 : TR and RT
 - phase difference is $(\frac{\pi}{2} - \frac{\pi}{2}) = 0$
 - positive interference
 - D_2 : TT and RR
 - phase difference is $(0 - \pi) = -\pi$
 - negative interference
- all light hit detector D_1 , no light reaches D_2

Mach-Zehnder interferometer – detector no.1



- initial state: ψ_S
- probability amplitude for transmission: $\langle \psi_T | \psi_S \rangle = \frac{1}{\sqrt{2}}$
- probability amplitude for reflection: $\langle \psi_R | \psi_S \rangle = \frac{i}{\sqrt{2}}$

$$\begin{aligned} \langle \psi_{D1} | \psi_S \rangle_{TR} &= \frac{i}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \\ \langle \psi_{D1} | \psi_S \rangle_{TR} &= \frac{i}{2} \end{aligned}$$

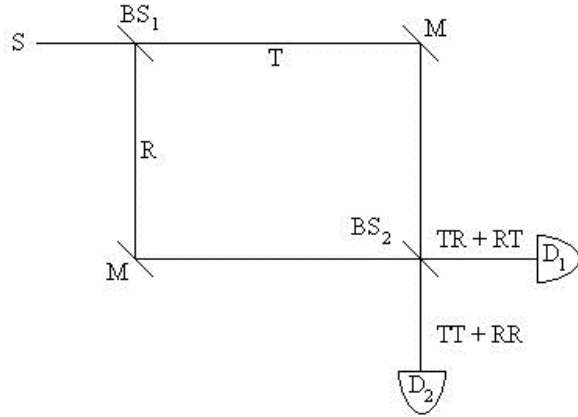
$$\begin{aligned} \langle \psi_{D1} | \psi_S \rangle_{RT} &= \frac{1}{\sqrt{2}} \cdot \frac{i}{\sqrt{2}} \\ \langle \psi_{D1} | \psi_S \rangle_{RT} &= \frac{i}{2} \end{aligned}$$

$$\begin{aligned} \langle \psi_{D1} | \psi_S \rangle &= \langle \psi_{D1} | \psi_S \rangle_{TR} + \langle \psi_{D1} | \psi_S \rangle_{RT} \\ \langle \psi_{D1} | \psi_S \rangle &= \frac{i}{2} + \frac{i}{2} = i \end{aligned}$$

$$\begin{aligned} P(D_1) &= |\langle \psi_{D1} | \psi_S \rangle|^2 = |i|^2 \\ P(D_1) &= 1 \end{aligned}$$

source: <https://faculty.csbsju.edu/frioux/super/MM-super.htm>

Mach-Zehnder interferometer – detector no.2



- initial state: ψ_S
- probability amplitude for transmission: $\langle \psi_T | \psi_S \rangle = \frac{1}{\sqrt{2}}$
- probability amplitude for reflection: $\langle \psi_R | \psi_S \rangle = \frac{i}{\sqrt{2}}$

$$\langle \psi_{D2} | \psi_S \rangle_{TT} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$$

$$\langle \psi_{D2} | \psi_S \rangle_{TT} = \frac{1}{2}$$

$$\langle \psi_{D2} | \psi_S \rangle_{RR} = \frac{i}{\sqrt{2}} \cdot \frac{i}{\sqrt{2}}$$

$$\langle \psi_{D2} | \psi_S \rangle_{RR} = -\frac{1}{2}$$

$$\langle \psi_{D2} | \psi_S \rangle = \langle \psi_{D2} | \psi_S \rangle_{TT} + \langle \psi_{D2} | \psi_S \rangle_{RR}$$

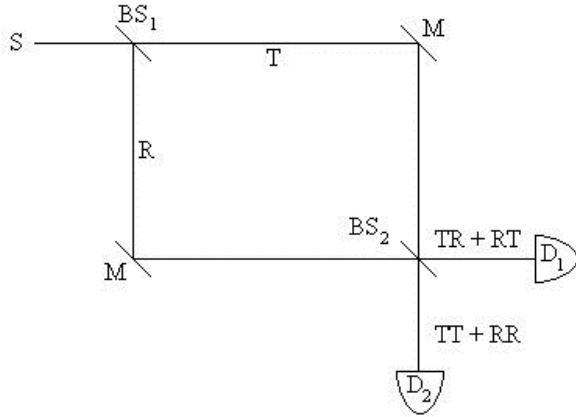
$$\langle \psi_{D2} | \psi_S \rangle = \frac{1}{2} - \frac{1}{2} = 0$$

source: <https://faculty.csbsju.edu/frioux/super/MM-super.htm>

$$P(D_1) = |\langle \psi_{D1} | \psi_S \rangle|^2 = |0|^2$$

$$P(D_1) = 0$$

Mach-Zehnder interferometer – observed



- what happens if we observe which way has the photon ‘chosen’
 - third principle – when alternative paths are distinguished by the experiment, resulting probability is a sum of probabilities alongside the alternative paths

$$p(D_1)_{ob} \neq |\langle \psi_{D1} | \psi_S \rangle_{TR} + \langle \psi_{D1} | \psi_S \rangle_{RT}|^2$$

$$p(D_1)_{ob} = |\langle \psi_{D1} | \psi_S \rangle_{TR}|^2 + |\langle \psi_{D1} | \psi_S \rangle_{RT}|^2$$

$$p(D_1)_{ob} = \left| \frac{i}{2} \right|^2 + \left| \frac{i}{2} \right|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

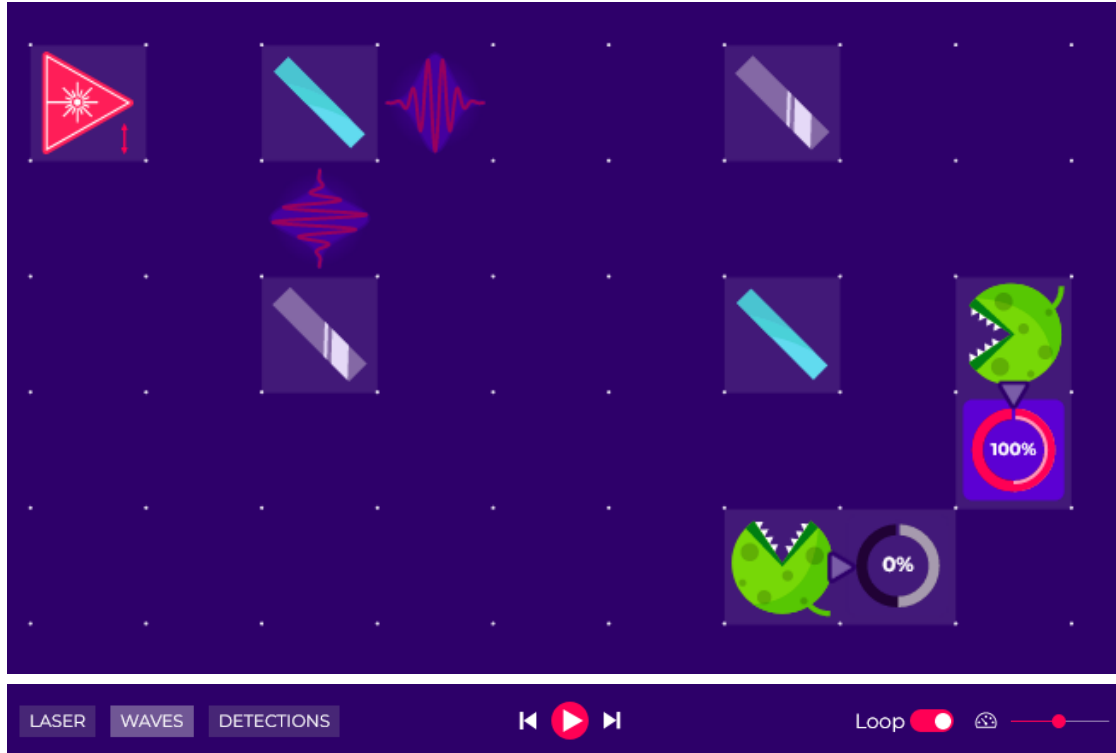
$$p(D_2)_{ob} \neq |\langle \psi_{D2} | \psi_S \rangle_{TT} + \langle \psi_{D2} | \psi_S \rangle_{RR}|^2$$

$$p(D_2)_{ob} = |\langle \psi_{D2} | \psi_S \rangle_{TT}|^2 + |\langle \psi_{D2} | \psi_S \rangle_{RR}|^2$$

$$p(D_2)_{ob} = \left| \frac{1}{2} \right|^2 + \left| -\frac{1}{2} \right|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

source: <https://faculty.csbsju.edu/frioux/super/MM-super.htm>

Virtual Lab by Quantum Flytrap



Virtual Lab by Quantum Flytrap

- Lv.10: <https://lab.quantumflytrap.com/game/level/interference>
- Lv.11: <https://lab.quantumflytrap.com/game/level/smash-it>
- Lv.12: <https://lab.quantumflytrap.com/game/level/redirecting-light>
- Lv.13: <https://lab.quantumflytrap.com/game/level/split-so-you-can-join>
- Lv.14: <https://lab.quantumflytrap.com/game/level/peeking-at-a-photon>

References

- Feynman, Richard P., Robert B. Leighton and Matthew Sands, no date. *The Feynman Lectures on Physics*, Volume III [online]. B.m.: California Institute of Technology. ISBN 978-0-465-04083-4. Available at: http://www.feynmanlectures.caltech.edu/III_toc.html
- Harada, Ken, Tetsuya Akashi, Kodai Niitsu, Keiko Shimada, Yoshimasa A. Ono, Daisuke Shindo, Hiroyuki Shinada, and Shigeo Mori. 2018. 'Interference Experiment with Asymmetric Double Slit by Using 1.2-MV Field Emission Transmission Electron Microscope'. *Scientific Reports* 8 (1): 1008. <https://doi.org/10.1038/s41598-018-19380-4>.
- quantum experiments:
 - Virtual Lab by Quantum Flytrap <https://lab.quantumflytrap.com/game/>

Q&A