JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS VOL. 40, NO. 2, JUNE 2005

Horses and Rabbits? Trade-Off Theory and Optimal Capital Structure

Nengjiu Ju, Robert Parrino, Allen M. Poteshman, and Michael S. Weisbach*

Abstract

This paper examines the optimal capital structure choice using a dynamic capital structure model that is calibrated to reflect actual firm characteristics. The model uses contingent claim methods to value interest tax shields, allows for reorganization in bankruptcy, and maintains a long-run target debt to total capital ratio by refinancing maturing debt. Using this model, we calculate optimal capital structures in a realistic representation of the traditional trade-off model. In contrast to previous research, the calculated optimal capital structures do not imply that firms tend to use too little leverage in practice. We also estimate the costs borne by a firm whose capital structure deviates from its optimal target debt to total capital ratio. The costs of moderate deviations are relatively small, suggesting that a policy of adjusting leverage infrequently is likely to be reasonable for many firms.

I. Introduction

A central issue in corporate finance research is the question of why firms have fairly low leverage ratios, despite the large tax advantage enjoyed by debt. This question motivated much of the early research on agency theory (Jensen and Meckling (1976), Myers (1977)), important work on information asymmetries (Myers and Majluf (1984)), three American Finance Association presidential addresses (Miller (1977), Myers (1984), and Leland (1998)), and some well-regarded recent research (Graham (2000)). The consensus view underlying this vast literature is that bankruptcy costs alone are too small to offset the value of tax shields and, thus, other factors, such as agency costs, must be introduced into

*Ju, nju@rhsmith.umd.edu, Department of Finance, Smith School of Business, University of Maryland, College Park, MD 20742; Parrino, parrino@mail.utexas.edu, Department of Finance, McCombs School of Business, University of Texas at Austin, Austin, TX 78712; both Poteshman, poteshma@uiuc.edu, and Weisbach, weisbach@uiuc.edu, Department of Finance, College of Business, University of Illinois, Champaign, IL 61820. We thank seminar participants at Arizona State University, Boston College, DePaul University, the University of Illinois, Koc University, the New York Federal Reserve Bank, Princeton University, Tulane University, the University of Nevada at Las Vegas, the University of South Carolina, the University of Texas at Austin, Vanderbilt University, Washington State University, the 13th Annual Conference on Financial Economics and Accounting, and the 2003 Western Finance Association meetings for helpful comments. We also thank Mark Loewenstein, Robert McDonald, Jon Karpoff (the editor), and John Graham (the referee) for helpful suggestions.

the cost-benefit analysis to explain observed capital structures. Miller ((1977), p. 264) memorably characterizes the discrepancy by comparing the trade-off between tax gains and bankruptcy costs as "like the recipe for the fabled horse-and-rabbit stew—one horse and one rabbit."

Researchers have attempted to evaluate the extent to which Miller's intuition captures the essence of the capital structure problem using contingent claims models.[1] Consistent with this intuition, these models typically imply optimal levels of leverage that are substantially higher than those observed in actual firms. The models, however, are generally static and do not consider the rights of bondholders to force firms into bankruptcy.[2] As a result, this literature has not yet provided a compelling answer to the question of whether observed capital structures represent a value-maximizing choice, or whether firms throw away value by substantially underleveraging their assets (Graham (2000), Graham, Lang, and Shackelford (2004)).

This paper estimates the optimal capital structure using a calibrated continuous-time contingent claim model. The model is based on the dynamic framework of Ju (2001), which corresponds to a traditional trade-off approach insofar as the only explicitly modeled factors that affect capital structure are tax shields and bankruptcy costs.[3] In our model, managers make capital structure decisions with the objective of maximizing the total value of the levered firm.[4] Our model predicts that the optimal debt to total capital ratio is 15.29% when we maximize share value for a firm that is calibrated to be similar to the median firm in Standard & Poor's Compustat database. In comparison, the median firm in the Compustat database had a debt to total capital ratio of 22.62% in 2000. The fact that our estimate of the optimal debt to total capital ratio is below the median value of 22.62% implies that, contrary to the dominant view in the literature, the typical firm is not underleveraged.

Several features of our approach are different from those considered previously. First, the model is dynamic in the sense that finite maturity debt is repeatedly issued and refinanced upon maturity to obtain a pre-specified ratio of debt to total capital. This approach is consistent with the firm following a policy of managing the capital structure to maintain a long-run target debt to total capital ratio. Second, we assume that our model's bankruptcy boundary, which determines when firms default on their debt, is an exponential function of time. This type of boundary, originally proposed by Black and Cox (1976), is intended to capture the rights of creditors to force a firm into bankruptcy when its value falls

[1]See, for example, Brennan and Schwartz (1978), Kane, Marcus, and McDonald (1984), (1985), Fischer, Heinkel, and Zechner (1989), Leland (1994), Leland and Toft (1996), Leland (1998), Titman and Tsyplakov (2001), and Morellec (2004).

[2]It should be noted that Morellec (2004) describes a static model that predicts low leverage ratios in the presence of stockholder/manager conflicts. Titman and Tsyplakov (2001) describe a dynamic model that predicts high leverage ratios.

[3]Survey evidence suggests that managers, especially at large firms, consider trade-offs such as these when they make capital structure decisions (see Graham and Harvey (2001)).

[4]Our model reflects the trade-off in which the only motivation for issuing debt is to obtain the tax benefit. The way that the proceeds of debt are distributed to shareholders does not affect capital structure decisions. For models in which debt affects investment policies, see Mello and Parsons (1992), Parrino and Weisbach (1999), Parrino, Poteshman, and Weisbach (2003), Morellec (2004), and Morellec and Smith (2003).

too low (and presumably when it violates covenants). Finally, like other recent continuous-time contingent claim models, we specify the value of the unlevered assets as an exogenous process. The volatility of changes in this process has often been calibrated to 20% with the justification that this number corresponds to an equity volatility of around 30% (Leland (1994), Leland and Toft (1996), and Goldstein, Ju, and Leland (2001)). In contrast, we calibrate the volatility of changes in the unlevered value of the firm to 38.02%, which produces lower optimal leverage ratios. We calibrate the volatility to this higher value because it results in our model producing credit spreads and bankruptcy recovery rates similar to observed levels.[5]

To examine whether model or calibration differences are more responsible for the lower optimal leverage ratio, we recalibrate three models from the existing literature, namely Leland (1994), Leland and Toft (1996), and a dynamic extension of the Leland (1994) model, using our estimate of volatility. The resulting leverage ratios, while lower than those reported in the original papers, are still substantially higher than those observed for typical firms. This experiment suggests that the important factors affecting ex ante leverage decisions include i) the ability of firms to manage their capital structures to maintain a target leverage ratio, and ii) the rights of bondholders to force a firm into bankruptcy when its value declines too much. However, once these factors are accounted for, relatively straightforward models of capital structure that do not rely on market imperfections, agency costs, or suboptimal behavior of any type can predict leverage ratios close to those observed in practice.

We also examine the relation between firm value and capital structure. Our estimates indicate that the impact on firm value of moderate deviations from optimal capital structure is small. For example, for any debt to total capital ratio between 11.0% and 20.3%, an adjustment to the optimal level of 15.29% would increase firm value by less than 0.5% for the typical company. Insofar as the transactions costs that would be incurred to adjust a firm's ratio of debt to total capital to a targeted level exceed the potential increase in firm value, the optimal policy may be to allow the firm's capital structure to deviate substantially from the target. Our estimates suggest that it can make sense for managers to allow a firm's capital structure to deviate by at least 10 percentage points before recapitalizing the firm. Such a policy is consistent with evidence reported by Welch (2004), who documents that firms do not regularly recapitalize following changes in their equity values. Our analysis thus suggests that similar firms that receive differing shocks to their equity values will not necessarily find it in the best interest of their shareholders to adjust their capital structures back to the target level. The well-documented empirical regularity of otherwise similar firms having very different capital structures is therefore consistent with all firms choosing capital structures optimally along the lines suggested in this paper.

[5]We also directly estimate this volatility for firms with data in the Compustat database and obtain a median value of about 28.5% (0.285). As Section IV.A.2 explains, these estimates are biased downward so that a reasonable number is greater than 30%. It should also be noted that while the 30% equity volatility used to guide the choice of unlevered asset volatility in other studies may be sensible for a typical large industrial firm, it is too low for the equity of a typical firm in a broader sample that includes much smaller firms. For example, as Section IV.A.2 discusses, we estimate the median equity volatility for a typical individual firm to be closer to 70% in late 2000 and early 2001.

We compute numerical comparative statics to evaluate the sensitivity of the estimated optimal capital structure to the major parameters in the model. Not surprisingly, corporate tax rates, bankruptcy costs, and the ability of debtholders to force the firm into bankruptcy all impact the optimal capital structure.

Finally, we calibrate the model to estimate the optimal capital structure for 15 firms. For 10 of these firms, the ratio debt to total capital that is estimated by the model to maximize the firm's stock price is less than the actual ratio of debt to total capital. In general, the model is able to predict within a reasonable degree of error the leverage observed at firms that have small to typical levels of debt, but substantially underestimates the level of debt observed at highly levered firms.

Overall, the results in this paper suggest that the trade-off model performs reasonably well in predicting capital structures for firms with typical levels of debt. Certainly, the "horse and rabbit stew" analogy seems inappropriate as bankruptcy costs are much larger than this analogy implies. Our model indicates that, in addition to the tax shields, important determinants of capital structure include the underlying risk of the firm's assets, the maturity of the debt, the ability of debtholders to force default for a given level of firm value, and the incremental bankruptcy costs, conditional on default.

It is important to recognize that while the trade-off model we describe predicts capital structures that are relatively similar to those observed for a typical firm, it does not consider all determinants of the capital structure choice. For example, our model does not explain cross-sectional variation in capital structures attributable to factors such as strategic considerations or investment opportunities, nor does it account for other factors, such as differences in managerial risk aversion, that would be necessary to explain such variation. Consequently, while this study contributes to our overall understanding of capital structure choice, it certainly does not resolve all issues.

The rest of this paper is organized as follows. Section II describes the model in detail and Section III explains how we calibrate the model to reflect current market data. Section IV then discusses the implications of the calibrated model. Section V concludes. Technical details and an expanded version of the model that incorporates managerial incentives are discussed in three appendices that are available on the JFQA Web site at http://www.jfqa.org.

II. A Dynamic Model of Capital Structure

The model we use is based on Ju (1998), (2001). In this model, the firm issues debt with a maturity of T, which pays a continuous, constant (tax-deductible) coupon. The value process of the firm's assets (that is, the value of the cash flows from operations) follows a geometric Brownian motion.

The model is in continuous time. At time zero, the value of the firm's assets is $V(0)$. Without debt in its capital structure, the firm's capital consists of N_{NL} shares of stock with a total market value of $E_{NL}(0) = V(0)$.[6] The value of the firm's assets, $V(t)$, follows a geometric Brownian motion described by

[6]The subscript NL refers to quantities when the firm is not leveraged (that is, when it does not have debt in its capital structure). The subscript L refers to quantities when the firm is leveraged.

$$\frac{dV(t)}{V(t)} = (\mu - \delta)dt + \sigma dZ(t), \tag{1}$$

where μ and $\sigma > 0$ are constants and $Z(t)$ is a standard Wiener process. The firm liquidates assets at a rate of δ of the total value of the firm's assets, so that $\delta V(t)dt$ is equal to a time-varying dividend $div(t)dt$ paid to equity holders over the time interval dt,

$$\delta V(t)dt = div(t)dt. \tag{2}$$

The value of δ is specified exogenously as a model parameter.

At time zero, the manager makes a capital structure decision that consists of choosing a level of debt that maximizes the total value of the levered firm. The debt has a face value of F_L and has a market value when it is issued at time zero of $D_L(0)$. The debt pays a coupon at a constant annualized rate C_L that is set such that the debt is priced at par, that is, $F_L = D_L(0)$. The firm deducts its coupon payments from its taxes at the effective rate τ, and the tax shields of the debt at time zero have a value of $TB_L(0)$. The debt has a protective covenant specifying that if the asset value at any time during the life of the debt $[0, T]$ decreases to an exponential boundary, the firm is forced into bankruptcy.[7] Besides offering tractability, this bankruptcy boundary form contains several default-triggering mechanisms as special cases, for example, the positive net worth-protected debt case in Leland (1994) and the constant default boundary case of Longstaff and Schwartz (1995). When default occurs, the stock becomes worthless and the debtholders recover $1 - \alpha_{BC}$ of the *levered* value of the assets. The fraction of the value of the assets not recovered by the debtholders is assumed to be consumed in the bankruptcy process. The bankruptcy boundary is an exponential curve that increases at rate g and is equal to the face value of debt at time T. Consequently, the bankruptcy boundary is described by $F_L e^{g(t-T)}$. The bankruptcy costs for the firm are the present value of the expected losses in bankruptcy and are denoted by $BC_L(0)$. The levered firm liquidates assets at a rate of δ of the total value of the firm's assets, so that $\delta V(t)dt$ equals the sum of the after-tax coupon paid to debt holders $[(1-\tau)C_L dt]$ and a time-varying dividend $div(t)dt$ paid to equity holders over the time interval dt,

$$\delta V(t)dt = [div(t) + (1-\tau)C_L]dt. \tag{3}$$

[7]As in Black and Cox (1976), our bankruptcy boundary increases over time at an exponential rate until it reaches the face value of the debt at the time that the debt matures. The boundary is intended to act somewhat like covenants in bond indenture agreements that give bondholders the right to seize assets when they are in danger of being lost. Huang and Huang (2003) discuss how firms often continue to operate even when their asset values fall below the face value of outstanding debt. On the other hand, at maturity, the firm's asset value must be at least as high as the face value of the debt to avoid default. Our choice of an increasing default boundary is designed to capture the idea that a firm's ability to operate when firm value is below the face value of the debt declines as debt moves closer to maturity. Our choice of an exponential form is for tractability and is unlikely to affect our results significantly.

Note that although we assume a constant asset payout rate of δ, the dividend payout is time varying and will be less for lower $V(t)$. In particular, (3) may require cash infusions for low asset values.[8]

The value of the debt, the bankruptcy costs, and the debt tax shields are computed from the probability density function for first hitting the exponential bankruptcy boundary. Let $f(t^* ; V(0), A, g, r, \delta, \sigma)$ be the probability density for first hitting a boundary described by Ae^{gt} at a time t^*, where A is a constant, if the variable V initially has a value $V(0) > A$ that follows a geometric Brownian motion with drift $r - \delta$ and volatility σ. In our model, A is the value of the bankruptcy boundary at time zero, so that A is equal to $F_L e^{-gT}$. An explicit expression for $f(t^* ; V(0), A, g, r, \delta, \sigma)$ is provided in Appendix A, which is available on the JFQA Web site at http://www.jfqa.org. We next define

$$(4)\quad G(T, V(0), A, g, r, \delta, \sigma) \equiv \int_0^T f(t^* ; V(0), A, g, r, \delta, \sigma)\, dt^*,$$

$$(5)\quad H(T, V(0), A, g, r, \delta, \sigma) \equiv \int_0^T e^{-rt^*} f(t^* ; V(0), A, g, r, \delta, \sigma)\, dt^*, \quad \text{and}$$

$$(6)\quad I(T, V(0), A, g, r, \delta, \sigma) \equiv \int_0^T e^{-(r-g)t^*} f(t^* ; V(0), A, g, r, \delta, \sigma)\, dt^*.$$

Closed-form solutions for these expressions are derived in Appendix A. The G function is the total probability of default from time zero to T. The H function corresponds to the present value of receiving one dollar upon default, if default occurs between time zero and T. Similarly, the I function represents the present value of receiving e^{gt^*} dollars upon default, if default occurs between time zero and T.

Following Leland and Toft (1996), the value of the debt at time zero is the sum of a contribution from the coupon, a contribution from the payment to debtholders if bankruptcy occurs, and the repayment of the face value at time T if bankruptcy does not occur, that is,

$$\begin{aligned}(7)\quad D_L(0) = {} & C_L \int_0^T e^{-rt^*} \left(1 - G\left(t^*, V(0), F_L e^{-gT}, g, r, \delta, \sigma\right)\right) dt^* \\ & + \int_0^T e^{-rt^*} (1 - \alpha_{BC}) \frac{TV_L(0)}{V(0)} F_L e^{-g(T-t^*)} \\ & \times f\left(t^*, V(0), F_L e^{-gT}, g, r, \delta, \sigma\right) dt^* \\ & + F_L \left(1 - G\left(T, V(0), F_L e^{-gT}, g, r, \delta, \sigma\right)\right) e^{-rT}, \quad \text{or}\end{aligned}$$

[8]Though our bankruptcy boundary is exogenous, cash infusions are not uncommon in models with an endogenous boundary (e.g., Leland (1994)). Another possible modeling approach is to allow renegotiation when the firm hits the default boundary. See Francois and Morellec (2004) and Fan and Sundaresan (2000) for models with renegotiation.

$$\begin{aligned}(8)\quad D_L(0) &= \frac{C_L}{r}\left(1-\left(1-G\left(T,V(0),F_Le^{-gT},g,r,\delta,\sigma\right)\right)e^{-rT}\right.\\ &\quad \left.-H\left(T,V(0),F_Le^{-gT},g,r,\delta,\sigma\right)\right)\\ &\quad +\left(1-\alpha_{BC}\right)\frac{TV_L(0)}{V(0)}F_Le^{-gT}I\left(T,V(0),F_Le^{-gT},g,r,\delta,\sigma\right)\\ &\quad +F_L\left(1-G\left(T,V(0),F_Le^{-gT},g,r,\delta,\sigma\right)\right)e^{-rT},\end{aligned}$$

where

$$(9)\qquad TV_L(0) = V(0)+TB_L(0)-BC_L(0)$$

is the total levered value of the firm with debt at time zero. If the $TV_L(0)/V(0)$ factor were omitted from equation (7), then the debtholders would receive $(1-\alpha_{BC})$ of the *unlevered* value of the firm's assets upon bankruptcy. The inclusion of this factor implements the modeling decision (reorganization) that upon bankruptcy the debtholders receive $(1-\alpha_{BC})$ of the *levered* value of the remaining assets to a healthy firm. Explicit expressions for $TB_L(0)$ and $BC_L(0)$ are provided below.

Another modeling decision involves the question of whether the firm should refinance the debt when it matures. We consider two alternative models. The first is a "static" model, in which the firm does not replace the maturing debt and therefore is entirely equity financed after time T. The second is a "dynamic" model, in which new debt is reissued when old debt matures. Since the dynamic framework seems more appealing a priori, and given that Ju (1998), (2001) shows the refinancing assumption can affect corporate financing decisions ex ante, we analyze the dynamic model. Nonetheless, it is convenient to present the solution of the dynamic model in terms of that for the static model that we develop below.

A. The Static Model

In the static model, when the firm is forced into bankruptcy at time t^*, the bankruptcy costs are $\alpha_{BC}V(t^*)$. Hence, at time zero the value of the bankruptcy costs is

$$(10)\quad BC_L(0) = \int_0^T \alpha_{BC}F_Le^{g(t^*-T)}e^{-rt^*}f\left(t^*;V(0),F_Le^{-gT},g,r,\delta,\sigma\right)dt^*,$$

or

$$(11)\qquad BC_L(0) = \alpha_{BC}F_Le^{-gT}I\left(T,V(0),F_Le^{-gT},g,r,\delta,\sigma\right).$$

Note that we use the levered value of the remaining assets to price the debt in (7), but we use the unlevered value of the lost assets to compute the bankruptcy costs in (10). We use the unlevered value of the lost assets to compute bankruptcy costs because this corresponds to the cost to the original shareholders before the firm is levered. We also compute bankruptcy costs using the levered value of the lost assets. We omit these calculations, however, because the results using this latter approach are virtually identical to those of the approach that uses the unlevered value.

The interest tax shields accrue to the firm as long as it does not go bankrupt. Consequently, the interest tax shields of debt in the static model can be computed by

$$(12) \qquad TB_L(0) = \int_0^T \tau C_L e^{-rt^*}\left(1 - G\left(t^*, V(0), F_L e^{-gT}, g, r, \delta, \sigma\right)\right) dt^*,$$

or

$$(13) \qquad TB_L(0) = \frac{\tau C_L}{r}\left(1 - \left(1 - G\left(T, V(0), F_L e^{-gT}, g, r, \delta, \sigma\right)\right) e^{-rT} - H\left(T, V(0), F_L e^{-gT}, g, r, \delta, \sigma\right)\right).$$

The value of the equity equals the unlevered value of the assets plus the value of the tax shields from the debt minus the bankruptcy costs minus the value of the debt,

$$(14) \qquad E_L(0) = V(0) + TB_L(0) - BC_L(0) - D_L(0).$$

B. The Dynamic Model

Next we extend the model to a more realistic dynamic setting. If the firm has not gone bankrupt at the end of T years, the firm issues new T-year debt at time T. The new debt has a coupon of $C_L V(T)/V(0)$. Similarly, as Appendix A shows, all other securities are scaled by a factor of $V(T)/V(0)$ because at time T the firm is identical to itself at time zero, except that it is $V(T)/V(0)$ as large. The process of issuing new T-year debt when the old debt matures continues indefinitely unless the firm goes bankrupt. If bankruptcy occurs before the debt matures, at time $t^* < T$, we allow the debtholders to become the new shareholders and they optimally lever the remaining assets, $(1-\alpha_{BC})F_L e^{-g(T-t^*)}$, after the bankruptcy process consumes $\alpha_{BC} F_L e^{-g(T-t^*)}$ of the assets $F_L e^{-g(T-t^*)}$ at the bankruptcy boundary.

In this dynamic setting, the price of the debt is still given by equation (8). The firm value, however, will reflect the costs and benefits of the debt issued in the future. In determining both the total tax shields and the total bankruptcy costs associated with current and future debt issues, the following quantity is useful,

$$(15) \qquad \phi = \int_{F_L}^{\infty} e^{-rT}\frac{V(T)}{V(0)}\rho(V(T)\,;V(0), T, A, g, r, \delta, \sigma)dV(T) + \int_0^T e^{-rt^*}\frac{(1-\alpha_{BC})F_L e^{-g(T-t^*)}}{V(0)} \times f\left(t^*\,; V(0), F_L e^{-gT}, g, r, \delta, \sigma\right) dt^*.$$

In this expression, $\rho(V(T)\,; V(0), T, A, g, \mu, \delta, \sigma)$ is the density function for starting at value $V(0) > A$ and realizing $V(T) > Ae^{gT}$ at time $T > 0$ without ever hitting the boundary Ae^{gt} in the interval $t \in [0,T]$ when the V process follows a geometric Brownian motion with drift $\mu - \delta$ and volatility σ. The first term in equation (15) accounts for the effect of rebalancing at the debt maturity date, T, if there is no default between time zero and T. The second term accounts for the leveraging of the assets that remain after bankruptcy costs if default occurs before T.[9] Appendix A shows that the total tax shields and the total bankruptcy costs of debt are given by

$$(16) \qquad TB_L^{Dynamic}(0) = \frac{TB_L(0)}{1-\phi} \quad \text{and}$$

$$(17) \qquad BC_L^{Dynamic}(0) = \frac{BC_L(0)}{1-\phi}.$$

Similar to equation (14), the value of the equity is equal to the unlevered value of the assets plus the tax shields of debt minus the bankruptcy costs minus the value of the debt,[10]

$$(18) \qquad E_L^{Dynamic}(0) = V(0) + TB_L^{Dynamic}(0) - BC_L^{Dynamic}(0) - D_L(0).$$

III. Calibrating the Model

In choosing the amount of debt that will be issued, a face value, F_L, of 10-year debt (i.e., $T = 10$ years) is chosen to maximize the total value of the levered firm. The total value of the firm's assets before the firm is levered, $V(0)$, is normalized to \$100, which is divided among 100 shares, each worth \$1.

Given these assumptions, calibration of the model requires estimates of the risk-free rate, r, the effective tax rate, τ, the volatility of the total value of the firm, σ, the debtholder bankruptcy recovery rate, $(1 - \alpha_{BC})$, the exponent of the exponential function characterizing the bankruptcy boundary, g, the level of dividends, *DivRate*, paid by the firm, and the drift parameter for the total value of the firm, μ. We estimate these parameters using data from the end of January 2001.

As our estimate of the risk-free rate, we use the rate on 10-year Treasury bonds as of January 30, 2001, as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook*. This rate equals 5.22%.

To estimate the tax rate used to calculate the tax shields from the debt, we use data on estimated marginal tax rates (before interest expense) provided by John Graham, who constructed these estimates using the approach described in Graham (1996). In particular, for the base case, we assume that the tax rate equals the median marginal tax rate of 34% for the 5,519 firms for which 1999 estimates are available.

The volatility of the total value of the firm's assets, σ, the debtholder bankruptcy recovery rate, $(1 - \alpha_{BC})$, and the exponent of the bankruptcy boundary function, g, are selected to yield an expected recovery in bankruptcy equal to 45% of the face value of debt and a spread over the 10-year Treasury bond rate

[9]For more details, see equation (A.9) in Appendix A and the discussion preceding it.

[10]An extended version of the dynamic model, which incorporates managerial incentives, is presented in Appendix C, and is available on the JFQA Web site at http://www.jfqa.org.

for the firm's debt equal to 1.90%, for a firm with the median debt to total capital ratio of 22.62% among firms in the 2000 Compustat database. The 45% recovery target is broadly consistent with recovery rates published by Hamilton, Gupton, and Berhault (2001), who find that for the 1981 to 2000 period, the mean default recovery rates for senior secured bonds, senior unsecured bonds, and subordinated bonds of all ratings equal 53.9%, 47.4%, and 32.3%, respectively. The 1.90% spread over the Treasury bond rate equals the spread for 10-year A-rated corporate debt as of January 30, 2001, as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook.* The volatility of the total value of the firm's assets, σ, is estimated this way to be 38.02% (0.3802). This value implies a volatility of the value of the typical firm's equity of 48.09%. The bankruptcy recovery rate, relative to the levered value to a healthy firm of the remaining assets at the time of default, and the exponent of the bankruptcy boundary function for our base case equal 0.5090 (α_{BC}=0.4910) and 3.69% (g=0.0369), respectively.[11]

In our calibration procedure, we do not, and cannot, distinguish between the effects of the direct and indirect costs of financial distress. The calibrated bankruptcy parameter values, $(1-\alpha_{BC})$ and g, are chosen so that the model output matches the 1.90% spread over the Treasury bond rate and the 45% expected recovery in bankruptcy. Thus, they reflect the total direct and indirect costs of bankruptcy.

We set the dividend rate, *DivRate*, equal to 1.5% in the base case. Because this rate is stated as a percentage of the unlevered value of the firm, we use a number that is on the lower end of the 1.5% to 2.0% dividend yield paid by public firms at the beginning of 2001.

We select a value for the drift parameter of the firm, μ, by implementing an argument similar to the one provided in Merton (1974). We begin by formally writing the dynamics of the equity's value as

$$dE = (\mu_E - \delta_E)E\,dt + \sigma_E E\,dZ_E. \tag{19}$$

By Ito's lemma and the dynamics of the firm under the physical measure given in equation (1), we can also write the dynamics for E as

$$dE = \left[\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 E}{\partial V^2} + (\mu - \delta)V\frac{\partial E}{\partial V} + \frac{\partial E}{\partial t}\right] dt + \sigma V \frac{\partial E}{\partial V}\,dZ. \tag{20}$$

Matching the coefficients on the drift components of equations (19) and (20) yields

$$\mu = \frac{(\mu_E - \delta_E)\,E - \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 E}{\partial V^2} - \frac{\partial E}{\partial t}}{V\frac{\partial E}{\partial V}} + \delta. \tag{21}$$

We set μ_E equal to 11.22% by assuming an equity risk premium of 6% over our risk free rate of 5.22%. When the rest of the quantities on the right-hand side of

[11]The asset volatility, σ, in our model is the *annual* volatility of the returns of the firm's asset value. While the model is in continuous time and the parameter values, like the interest rate, r, and σ, are annualized, the spread of any *finite* maturity bond over a similar Treasury bond is consistent with the use of continuous time and annualized parameter values.

equation (21) are computed from the calibrated values for our standardized firm with a ratio of debt to total capital of 22.62%, the equation yields our base case value for μ of 10.63%.

Panel A of Table 1 summarizes our parameter choices. These choices are used to derive the values of the variables that are presented in Panel B of Table 1.

TABLE 1

Model Parameters

Panel A. Chosen Parameters

Variable	Calibrated Value	Variable Description
T	10	Time at which debt matures
$V(0)$	\$100	Value of assets without debt
N_{NL}	100	Total shares outstanding without debt
r	5.22%	Annualized risk-free rate
τ	34%	Effective tax rate for debt tax shields
σ	38.02%	Volatility of value of firm assets
α_{BC}	0.4910	Bankruptcy cost parameter (1 − debtholder bankruptcy recovery rate)
g	3.69%	Exponent of bankruptcy boundary function
DivRate	1.5%	Dividend payout rate to equity holders as a percentage of the unlevered value of the firm.
μ	10.63%	Drift of value of firm assets

Panel B. Derived Variables

Variable	Variable Description
F_L	Face value of debt
C_L	Constant annualized coupon rate paid on debt, set to price the debt at par
$D_L(0)$	Initial total value of debt
N_L	Total shares outstanding with debt
$E_L(0)$	Initial total value of equity with debt
$BC_L(0)$	Initial total value of bankruptcy costs
$TB_L(0)$	Initial total value of tax shields of debt
ϕ	Scaling factor that accounts for future rebalancing for total tax shields and default costs
$E_L^{Dynamic}(0)$	Initial total value of equity with debt
$BC_L^{Dynamic}(0)$	Initial total value of bankruptcy costs
$TB_L^{Dynamic}(0)$	Initial total value of tax shields of debt
δ	After tax cash payout rate to both debtholders and equity holders as a percentage of the unlevered value of the firm

IV. Optimal Capital Structure

In our model, each potential capital structure implies different values for the debt tax shields and bankruptcy costs, and ultimately different price distributions for the firm's securities. We define an optimal capital structure as the debt value that maximizes the total levered value of the firm and calculate optimal capital structures for various firm characteristics.

A. Optimal Capital Structures for a Representative Firm

1. The Shareholders' Perspective

Table 2 presents estimates of optimal capital structures. Each column represents a different level of asset volatility. Except for the debt maturity, which varies across the panels, the values of all other parameters are as described in Section III. The optimal capital structure for 10-year debt is shown in Row 1. Optimal leverage levels are clearly very sensitive to asset volatility, equaling 40.71% when asset volatility is 13.00% and 8.34% when asset volatility is 53.00%. The negative relation between leverage and volatility is consistent with casual empiricism, as well as with studies suggesting that riskier firms do in fact use less leverage (see, e.g., Titman and Wessels (1988), or Rajan and Zingales (1995)).

TABLE 2

Model Output for Firms with Different Firm Asset Volatilities Where Objective is to Maximize Share Value

	Volatility of Firm Asset Value								
Row Variable	13.00%	18.00%	23.00%	28.00%	33.00%	38.02%	43.00%	48.00%	53.00%
Panel A. $T = 10$									
1. Debt/total capital	40.71%	33.10%	27.14%	22.39%	18.52%	15.29%	12.60%	10.30%	8.34%
Equity:									
2. Value of equity w/o debt	$100	$100	$100	$100	$100	$100	$100	$100	$100
3. No. of shares w/o debt	100	100	100	100	100	100	100	100	100
4. Value of equity w debt	$74.39	$80.68	$84.90	$87.83	$89.96	$91.59	$92.90	$94.02	$94.98
5. No. of shares w debt	59.29	66.90	72.86	77.61	81.48	84.71	87.40	89.70	91.66
6. Change in share price	$0.255	$0.206	$0.165	$0.132	$0.104	$0.081	$0.063	$0.048	$0.036
Debt:									
7. Face value of debt	$51.08	$39.91	$31.63	$25.34	$20.44	$16.54	$13.40	$10.80	$8.64
8. Value of debt	$51.08	$39.91	$31.63	$25.34	$20.44	$16.54	$13.40	$10.80	$8.64
9. Coupon	$2.737	$2.181	$1.771	$1.460	$1.216	$1.018	$0.854	$0.712	$0.589
10. Bankruptcy costs	$2.366	$3.230	$3.963	$4.531	$4.915	$5.110	$5.119	$4.957	$4.644
11. Tax benefits	$27.835	$23.817	$20.492	$17.698	$15.315	$13.238	$11.416	$9.773	$8.271
12. Cumulative default rate	0.00%	0.01%	0.28%	2.02%	6.51%	9.98%	13.40%	16.98%	23.65%
Panel B. $T = 5$									
13. Debt/total capital	40.47%	33.10%	27.21%	22.45%	18.53%	15.27%	12.56%	10.28%	8.35%
Panel C. $T = 20$									
14. Debt/total capital	49.07%	42.51%	37.58%	33.87%	31.08%	29.01%	27.53%	26.53%	25.96%
Panel D. $T = 10$, Liquidation at Bankruptcy									
15. Debt/total capital	39.67%	32.02%	26.13%	21.48%	17.73%	14.64%	12.07%	9.89%	8.03%

Given our estimate of asset volatility of 38.02%, the model does reasonably well at predicting capital structures. For the median firm, the model predicts a debt to total capital ratio of 15.29% (equal to book debt of $16.54 divided by book debt of $16.54 plus market value of equity of $91.59). In comparison, for a sample of 2,609 firms for which sufficient data are available on Compustat for 2000 and for which three months of returns leading up to January 2001 are available on CRSP, the median ratio of debt to total capital (computed as book debt/book debt plus market equity) equals 22.62%.[12]

The likelihood of default during the life of the debt is also reasonably consistent with the average cumulative default rates reported by Hamilton, Gupton, and Berhault (2001). Row 12 in Table 2 reports the probability that the firm defaults, at any time before maturity, on 10-year debt for each volatility level. The default probability is clearly sensitive to the volatility of firm asset value. For example, when the volatility is at the low level of 13% and the optimal leverage ratio is at 40.71%, the probability of default during the life of the 10-year debt is practically zero. On the other hand, when the volatility is at the high level of 53% and the leverage ratio is only 8.34%, the default probability is 23.65%. For the base case, where the volatility is 38.02%, the probability of default over the 10-year life of the bond is 9.98% or roughly 1% per year. This value is virtually the same as the average cumulative 10-year default rate during the 1983 to 2000 period of 9.61% reported by Hamilton, Gupton, and Berhault (2001) in their Exhibit 42 for all corporate issues and is only modestly higher than the corresponding 8.96% rate they report for the 1970 to 2000 period in their Exhibit 41.

To investigate the sensitivity of the results to the choice of a debt maturity of 10 years, we examine estimated optimal capital structures for alternative debt maturities. Rows 13 and 14 in Table 2 report the optimal debt to capital ratios corresponding to debt maturities of five and 20 years, respectively. The results show that while reducing the maturity from 10 to five years has little effect, lengthening it from 10 to 20 years significantly increases the optimal debt to total capital ratios. For example, for the base case volatility of 38.02%, the ratio of debt to total capital rises from 15.29% to 29.01%. Debt maturity affects leverage ratios by changing the frequency of capital structure rebalancing. Since the next opportunity to rebalance the firm's capital structure is far in the future with long maturity debt, the model with long maturity debt is much like a static one in which the firm does not have the opportunity to rebalance at all.

We do not attempt to address the issue of optimal debt maturity here. To do so would require incorporating transactions costs for issuing debt into our model. The optimal maturity would be very sensitive to these costs.[13] Without transactions costs, a firm would choose to rebalance its capital structure continuously, issuing essentially riskless debt and rolling it over instantly. Our goal is to take the observed maturity as given and to examine leverage ratios that are implied when firms issue debt with maturities similar to those observed in practice.

When default occurs, part of the asset value is lost in the bankruptcy process. An attractive feature of our model is that when the firm defaults, it optimally leverages its remaining assets. This assumption is in contrast to much of the literature, which assumes that when the firm defaults, it liquidates and sells off the remaining assets. To gauge the importance of this alternative assumption, the last row in Table 2 presents model output for the case in which the firm liquidates its assets when it hits the bankruptcy boundary. These results indicate that liquidation

[12]We impose the CRSP return availability requirement so that we can later compute equity return volatility from the same set of firms.

[13]See Ju (2001) for a related model that does include endogenous debt maturity.

at bankruptcy leads to less leverage ex ante, although the differences are not as large as those discussed above.

2. Why Are Our Predicted Leverage Ratios so Low?

The estimates of optimal ratios of debt to total capital presented in Table 2 are substantially lower than those presented elsewhere in the literature, and are generally consistent with observed leverage ratios. There are several reasons why these estimates are so much lower than those of other studies.

Asset Volatility Estimates. First, the model output is sensitive to our choice of parameters, especially the asset volatility estimate. Table 2 highlights the importance of accurately measuring asset volatility when estimating optimal capital structure in a model such as ours. The procedure we use, which identifies the model parameters that match both the yield spread on the firm's bonds and the expected recovery rate, conditional on reaching bankruptcy, to levels typically observed in practice, yields a value of 38.02% for asset volatility. The credibility of the estimates from our model, as well as those from other capital structure models, depends greatly on this parameter choice.

As an independent check on the plausibility of our asset volatility estimate, we compute the standard deviation of the annual change in firm value (estimated as the market value of equity plus the book value of debt) for each of the 1,043 firms for which sufficient data are available on Compustat between 1980 and 1999. Calculated in this way, the median estimate of the standard deviation of the change in firm value is 28.5%. However, this value is likely to understate the standard deviation of the change in asset value at a typical firm for two reasons. First, the procedure used to obtain this estimate is subject to survivorship bias. Since more volatile firms are more likely to leave the sample than less volatile firms, estimating volatility on the basis of firms that survived throughout the sample period will lead to a lower value than if that data were available for all firms. Second, this calculation implicitly assumes that the market value of debt equals its book value. Since there is a positive relation between a firm's equity and debt values, assuming that the market value of debt is equal to its book value will tend to lower estimated volatilities as well. It is not clear how to quantify precisely the extent to which these two factors lead the Compustat-based estimate of 28.5% to be understated. However, they do suggest that if the estimate of 38.02% produced by our calibration procedure is too high, it is not too high by much; surely a number greater than 30% is appropriate.

As a final approach to estimating asset volatility, we use daily data on equity prices to compute the standard deviation of equity returns for the 2,609 firms for which there are both sufficient data on the 2000 Compustat to compute the book debt/book debt plus market equity ratio, and no missing CRSP daily returns for the three months ending January 31, 2001, the point in time at which we calibrate our model. The median standard deviation of the three months of equity returns for the firms in this sample is 73%. This number is considerably higher than the 30% equity volatility used to justify the 20% asset volatility used in previous studies. Although equity returns may have been more volatile than usual during the three months ending January 31, 2001, the equity volatility of 73% suggests that the 38.02% value we use for asset volatility is not too high.

Calibration vs. Model Specification. We next consider the extent to which our relatively low estimate for the optimal debt to total capital ratio is driven by our parameter choices, rather than by our model specification. To do so, we compute the optimal debt to capital ratios predicted by three models from the literature, calibrating those models with our parameter values, including our 38.02% estimate for asset volatility. We consider the Leland (1994) model, the Leland and Toft (1996) model using two alternative maturities, and a dynamic version of the Leland (1994) model using both 1% and 2% transactions costs.[14]

Table 3 presents the output from these models. The ratios of debt to total capital estimated using the Leland (1994) and Leland and Toft (1996) models with our asset volatility estimate of 38.02% are lower than those in the original papers, but are still above 50% (Panels A, B, and C). The leverage ratios in the dynamic versions are even lower, but are still around 40%, which is higher than those observed at typical firms (Panels D and E). It appears that while parameter choices partially explain our relatively low leverage ratios, modeling differences are at least as important.

TABLE 3

Output from Models in Other Studies for Firms with Different Firm Asset Volatilities Where Objective is to Maximize Share Value

	Volatility of Firm Asset Value								
Row Variable	13.00%	18.00%	23.00%	28.00%	33.00%	38.02%	43.00%	48.00%	53.00%
Panel A. Leland (1994)									
1. Debt/total capital	78.26%	71.88%	67.08%	63.41%	60.56%	58.30%	56.51%	55.04%	53.84%
2. Coupon	5.783	5.422	5.295	5.332	5.493	5.754	6.098	6.518	7.007
3. Bankrupt level	59.218	48.123	40.353	34.817	30.778	27.753	25.465	23.681	22.273
Panel B. Leland and Toft (1996): T = 20									
4. Debt/total capital	73.97%	67.17%	62.45%	59.09%	56.62%	54.75%	53.32%	52.20%	51.31%
5. Coupon	5.646	5.229	5.083	5.123	5.296	5.572	5.932	6.369	6.876
6. Bankrupt level	63.961	53.273	45.49	39.747	35.416	32.06	29.445	27.346	25.643
Panel C. Leland and Toft (1996): T = 10									
7. Debt/total capital	72.96%	65.54%	60.69%	57.35%	54.96%	53.16%	51.80%	50.74%	49.90%
8. Coupon	5.890	5.270	5.057	5.075	5.241	5.518	5.877	6.314	6.818
9. Bankrupt level	68.439	57.245	49.171	43.202	38.652	35.079	32.245	29.934	28.032
Panel D. Dynamic Version of Leland (1994), Transaction Costs = 1%									
10. Debt/total capital	64.93%	56.02%	49.65%	44.90%	41.22%	38.28%	35.89%	33.88%	32.17%
11. Coupon	7.043	5.911	5.388	5.181	5.165	5.283	5.502	5.806	6.186
12. Bankrupt level	49.942	37.966	30.075	24.682	20.853	18.025	15.895	14.225	12.889
13. Restructuring level	146.416	157.067	166.53	174.882	182.231	188.72	194.396	199.44	203.925
Panel E. Dynamic Version of Leland (1994), Transaction Costs = 2%									
14. Debt/total capital	67.21%	58.23%	51.74%	46.89%	43.12%	40.10%	37.65%	35.59%	33.84%
15. Coupon	6.868	5.833	5.340	5.140	5.122	5.232	5.438	5.726	6.086
16. Bankrupt level	51.033	38.937	30.906	25.395	21.472	18.574	16.391	14.681	13.315
17. Restructuring level	164.808	179.894	193.694	206.118	217.215	227.129	235.877	243.703	250.697

[14]The dynamic Leland (1994) model is developed fully in Appendix B, which is available on the JFQA Web site at http://www.jfqa.org. The original Leland (1994) model is static because the debt is perpetual and is never refinanced. In the dynamic extension, the perpetual debt is callable and is optimally called by shareholders when firm value becomes sufficiently high. In contrast, the refinancing in our model always occurs at the pre-specified maturity.

Model Specification. The dynamic nature of our model is partially responsible for the lower optimal debt to total capital ratio estimates. The fact that future debt levels increase with subsequent increases in firm value limits the aggressiveness with which debt is initially issued. In contrast, firms in the static models described in the literature issue debt more aggressively because they cannot change debt levels in the future if firm value changes (see, e.g., Leland (1994) and Leland and Toft (1996)). Intuitively, static models yield misleadingly high optimal debt levels because the initial debt level is selected knowing that the debt level will remain constant even though firm value is likely to increase over time. The longer a firm is locked into a particular level of debt, the higher it will want that level to be because its total value will grow on average over time and presumably the firm will balance the costs and benefits of incremental debt over the entire time period for which the debt is outstanding. This effect is evident in the context of our model from the fact that longer debt maturities produce higher debt levels.

Another aspect of our model that leads to lower ratios of debt to total capital is the mechanism that triggers default. In the Leland-style models, the triggering mechanism for default is initiated by shareholders, leaving bondholders completely passive. In contrast, our model has a pre-specified boundary that determines when the firm defaults. Ideally, we would prefer to model explicitly the ability of bondholders to negotiate default and reorganization with shareholders, as in Gertner and Scharfstein (1991). However, doing so would be technically infeasible in the context of our model.

In models with the Leland-style default-triggering mechanism, higher asset volatility does lead to a lower ratio of debt to total capital. However, this ratio is not as sensitive to increases in asset volatility in the Leland-style models as in our model. For any quantity of debt, the optimal default level is negatively related to asset volatility increases in the Leland-style models (see Leland (1994) equation 14). This effect counteracts the relation between asset volatility and debt ratios, leading to a weaker negative relation between debt ratios and asset volatility in the Leland-style models than in our own.

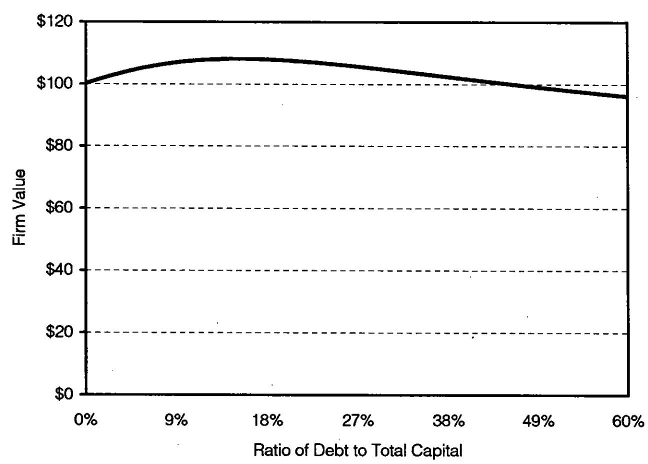
Constant Marginal Tax Rate. Finally, we assume that tax rates do not vary with leverage ratios. As Graham, Lemmon, and Schallheim (1998) emphasize, marginal tax rates decline with leverage in a tax system that has progressive rates because, for a given level of pre-tax income, the debt tax shields decrease taxable income. If this effect were incorporated into our model, it would make debt less attractive at the margin and the estimated ratios of debt to total capital would be even lower than those reported above. The assumption of a constant tax rate therefore results in an upward bias in the estimated debt ratios.

3. Measuring the Trade-Off between Tax Shields and Bankruptcy Costs

To implement a capital structure policy based on the notion of a trade-off between tax shields and bankruptcy costs, especially when there are transactions costs associated with issuing or retiring securities, one needs to know not just the securities associated with an optimal capital structure, but also the magnitude of the costs of deviating from it. Since we can calculate firm value for any capital structure, not just the optimal one, our approach provides a straightforward way to estimate these costs. To illustrate, Figure 1 presents a graph of firm value as a function of leverage, assuming that all other parameters are at their base case values.

FIGURE 1

Firm Value for Different Levels of Debt Financing



The values in Figure 1 are estimated using a dynamic model in which the firm refinances maturing debt that accounts for the impact of interest tax shields and bankruptcy costs on the total value of the levered firm.

Consistent with the numbers reported in Table 2, the value-maximizing leverage ratio in Figure 1 is 15.29%, implying a firm value of $108.13. Figure 1 illustrates that the relation between leverage and firm value is fairly flat around this optimal level. For example, with a leverage ratio of 11.0% or 20.3%, firm value is only 0.5% below the maximum of $108.13.[15] For ratios of debt to total capital between approximately 9.3% and 22.9%, firm value is still above $107. The value of a firm with leverage anywhere in the latter range would increase by less than 1.0% if that firm were to return to its optimal capital structure. Given transactions costs and the likely uncertainty about the precise leverage ratio that maximizes firm value, it is plausible that a reasonable capital structure policy would be to adjust the ratio of debt to total capital only infrequently.

These estimates help explain the empirical results of Welch (2004), who documents that firms do not regularly adjust their capital structures to maintain their target levels when equity values change. Figure 1 suggests that such behavior on the part of firms is consistent with the predictions of a simple trade-off model. It probably is optimal for a firm to adjust its capital structure only when the debt ratio diverges by a substantial amount from the target level. Over time, as individual firms delay adjusting their capital structures in response to idiosyncratic shocks to their equity values, we should observe cross-sectional differences in the capital

[15]If the transactions cost for issuing debt is 3.2% of face value, then the transactions cost for issuing the optimal quantity of debt equals 0.5% of total firm value in our base case. The transactions cost of issuing debt typically ranges from 1% to 4%.

structures of otherwise similar firms. Such differences, which are sometimes offered as evidence against theories of optimal capital structure, correspond exactly to what one would expect from a model such as our own.

B. Sensitivity of Optimal Capital Structure to Other Model Parameters

Since the major factor leading to a preference for debt is its tax-deductibility, we expect the model's results to be especially sensitive to tax rates. We compute optimal capital structures as a function of corporate tax rates in Panel A of Table 4. Not surprisingly, optimal debt to total capital ratios are positively related to the firm's tax rate. However, this relation appears to be nonlinear and is not as strong as one might expect. Holding the other model parameters at their base case values, we find that with a corporate tax rate of just 1%, the optimal leverage ratio equals 2.24%. This ratio rises 5.25 percentage points to 7.49% when the tax rate rises to 12%. In contrast, at higher tax rates the same 11 percentage point increase in tax rates (from 78% to 89%) leads to only a 3.96 percentage point increase in leverage, from 30.41% to 34.37%. Given the other base case parameters, even very high corporate tax rates do not lead to highly leveraged firms; a tax rate of 89% implies an optimal leverage ratio of only 34.37%.

TABLE 4

Model Output for Firms with Different Tax Rates, Bankruptcy Boundary Exponents, and Bankruptcy Cost Parameters

Panel A. Model Output for Firms with Different Tax Rates

	Tax Rate								
	1%	12%	23%	34%	45%	56%	67%	78%	89%
1. Debt/total capital	2.24%	7.49%	11.45%	15.29%	19.11%	22.89%	26.64%	30.41%	34.37%
2. Bankruptcy costs	$0.017	$0.784	$2.464	$5.110	$8.771	$13.518	$19.557	$27.398	$38.147
3. Tax benefits	$0.062	$2.352	$6.759	$13.238	$21.882	$32.840	$46.420	$63.271	$84.664

Panel B. Model Output for Firms with Different Bankruptcy Boundaries

	Exponent of the Bankruptcy Boundary Function (g)								
	0%	2%	4%	6%	8%	10%	12%	14%	16%
4. Debt/total capital	12.33%	13.88%	15.56%	17.41%	19.44%	21.72%	24.34%	27.54%	32.09%
5. Bankruptcy costs	$3.862	$4.512	$5.225	$5.999	$6.838	$7.755	$8.776	$9.972	$11.583
6. Tax benefits	$10.903	$12.141	$13.444	$14.808	$16.235	$17.737	$19.341	$21.123	$23.336

Panel C. Model Output for Firms with Different Bankruptcy Costs

	Bankruptcy Cost Parameter (α_{BC})								
	10%	15%	20%	25%	30%	35%	40%	45%	50%
7. Debt/total capital	38.75%	31.03%	26.29%	23.02%	20.62%	18.78%	17.31%	16.12%	15.13%
8. Bankruptcy costs	$6.736	$6.727	$6.533	$6.274	$6.003	$5.743	$5.501	$5.278	$5.075
9. Tax benefits	$22.351	$20.246	$18.603	$17.267	$16.160	$15.226	$14.428	$13.738	$13.135

An important element of our model is that the firm is assumed to default if it hits a pre-specified bankruptcy boundary. The idea underlying this assumption is that most publicly traded debt contains covenants enabling debtholders to force default when the value of the firm is sufficiently low. In our model, the exponent of the bankruptcy boundary function, g, defines the steepness of this boundary; a lower g increases the likelihood that the firm defaults given poor performance. Intuitively, g can be thought of as a negative function of the strength of the debt covenants. It is not clear conceptually how we should expect this variable to be relate to the shareholders' optimal leverage. Holding other factors constant, stronger debtholder rights make debt more valuable, enabling firms to issue debt at lower interest rates. Whether these lower interest rates are sufficient to compensate shareholders for the increased bankruptcy probabilities associated with stronger debtholder rights is not obvious.

Panel B of Table 4 presents estimates of the optimal capital structure as a function of g. The results indicate that optimal leverage is a positive function of g. As the rights of debtholders to force default increase, firms find it optimal to use less leverage. Varying the g from 0% to 16% causes the optimal ratio of debt to total capital to increase from 12.33% to 32.09%. Thus, it appears that the direct effect of a lower g through increased bankruptcy probabilities is more than sufficient to offset the indirect effect of lower interest rates.

These results suggest that the decision of whether to make the boundary endogenously determined (as in the Leland-style models) or exogenously specified (as in our model) is not the only one that is important. For models in which the boundary is exogenously specified, the choice among reasonable sets of asset value levels at which the bondholders can force default also has a nontrivial impact on the optimal capital structure.

The bankruptcy cost parameter in our model, α_{BC}, represents the proportional value lost to bankruptcy costs conditional on hitting the default boundary. We examine the sensitivity of optimal capital structure to this parameter in Panel C of Table 4. Not surprisingly, leverage is negatively related to bankruptcy costs. However, the result is weaker than one might expect; when α_{BC} declines to 10%, the leverage ratio only increases to 38.75%.

The results in Panels B and C of Table 4 suggest that the threshold at which the debtholders can force the firm into bankruptcy is likely to be as important as the magnitude of the value that is consumed in the bankruptcy process. Financing decisions depend on expected bankruptcy costs at the time they are made, and bondholders' rights clearly affect expected bankruptcy costs through their impact on bankruptcy probabilities. Yet, in most discussions of the effect of bankruptcy on capital structure, incremental costs conditional on bankruptcy are discussed at length, while the rights of debtholders to force bankruptcy are not usually emphasized.[16]

C. Model Estimates for Individual Firms

In addition to estimating the model using parameters for a typical firm, we examine the model's ability to predict the capital structures observed in a sample of 15 actual firms, five firms from each of three industries—wholesale distribution, beer and wine manufacturing, and paper and allied products. The volatility of the underlying assets, σ, the exponent for the bankruptcy boundary function, g, and the debtholder bankruptcy recovery rate, $(1 - \alpha_{BC})$, are estimated for each

[16]To the extent that these rights are endogenous choices because of voluntarily adopted covenants, rather than exogenous consequences of the legal system, they should be modeled as a choice variable for the firm, instead of as exogenous parameter. Expanding the model in this way would be a useful direction for future research.

firm by computing the values for these three parameters that yield both the observed spread between each firm's approximate current cost of debt and the yield on Treasury bonds and an expected recovery rate equal to 45% of the face value of the debt. The value-weighted average maturity of the debt for each firm and estimates of each firm's marginal tax rate for 1999 (obtained from John Graham) are used in these calculations.

Table 5 reports the firm-specific model inputs, estimated asset volatility and bankruptcy parameters, and actual and estimated optimal debt to total capital ratios for each of the 15 sample firms. The estimated asset volatilities range from 27.67% to 71.98%, with a median value of 34.34%. For this sample of firms, the model appears to do a good job of predicting leverage for firms with relatively little to typical levels of shorter-term debt, such as Tessco Technologies, Audiovox, Grainger, and Kimberly-Clark. However, it substantially underestimates leverage for firms with large amounts of debt, such as Hughes Supply, Robert Mondavi, Golden State Vintners, and Boise Cascade. The fact that the model tends to underestimate rather than overestimate leverage for these individual firms is once again counter to the usual intuition that tax shields are far too large to be offset by bankruptcy costs. The cross-sectional variation in the observed differences between the actual and estimated ratios of debt to total capital reflects the fact that our estimates do not account for all factors that determine the capital structure choice. For example, our model does not incorporate factors such as strategic considerations and investment opportunities that influence capital structure decisions.

TABLE 5

Individual Firm Estimates

	Firm-Specific Model Inputs			Estimated				
	Debt Maturity (T)	Cost of Debt Less Treasury Yield	Tax Rate (τ)	Volatility of Asset Value (σ)	Exponent of Bankruptcy Boundary Function (g)	Bankruptcy Cost Parameter (α_{BC})	Actual Debt/ Total Capital	Estimated Debt/Total Capital
Panel A. Wholesale Distribution Firms								
Hughes Supply Inc.	6.5	5.75%	35.0%	27.67%	3.95%	0.5004	56.36%	22.24%
Avnet, Inc.	2.0	3.00%	35.0%	33.52%	3.59%	0.5413	43.32%	22.90%
Tessco Technologies, Inc.	4.0	1.35%	34.0%	62.51%	2.71%	0.5384	7.37%	5.74%
Audiovox Corp.	1.5	0.50%	35.4%	70.60%	7.98%	0.5415	8.46%	8.69%
Grainger (W.W.) Inc.	1.5	0.50%	35.0%	71.98%	4.17%	0.5455	8.00%	8.19%
Panel B. Beer and Wine Manufacturing								
Robert Mondavi Corp.	8.0	2.25%	35.0%	33.69%	0.00%	0.5500	29.21%	15.07%
Willamette Valley Vineyards	13.0	2.25%	34.0%	33.03%	3.96%	0.4547	32.76%	22.04%
Pyramid Breweries	5.0	2.25%	34.2%	34.34%	3.75%	0.5239	31.74%	17.32%
Golden State Vintners	3.5	2.25%	35.0%	33.01%	3.65%	0.5338	35.52%	19.76%
Ravenswood Winery	20.0	1.35%	35.0%	46.95%	3.15%	0.4397	8.77%	25.84%
Panel C. Paper and Allied Products Manufacturing								
Boise Cascade Corp.	9.0	3.00%	12.2%	27.95%	3.92%	0.4886	48.19%	15.58%
Kimberley-Clark	10.0	1.35%	35.0%	53.87%	3.07%	0.5087	6.24%	7.57%
Mead	23.0	2.25%	35.0%	37.68%	3.81%	0.3438	30.75%	52.55%
P.H. Glatfelter Co.	3.0	2.25%	35.0%	32.80%	3.93%	0.5356	37.01%	20.82%
Wausau-Mosinee Paper Corp.	5.0	2.25%	29.5%	35.53%	3.79%	0.5238	30.90%	15.81%

V. Conclusion

This paper considers a model of optimal capital structure in which the major forces affecting a firm's financing decisions are corporate taxes and bankruptcy costs. As such, this model incorporates effects that have been discussed at great length in the corporate finance literature since Modigliani and Miller (1963). The model contains a number of features designed to capture key elements of the capital structure decision in a realistic way, including contingent claim valuation of tax shields, a bankruptcy boundary on firm value below which firms default, and a target capital structure at which the firm refinances its debt at maturity. We calculate closed-form solutions for important quantities in this model, we calibrate it using recent market data, and we solve for the optimal capital structures. Our model differs from the prior literature in that it uses a dynamic approach with a boundary on firm value below which firms default.

In contrast to most of the literature at least since Miller (1977), we find that the trade-off model does not predict that firms are underlevered. For a hypothetical firm constructed to be typical of large, publicly traded companies, the model predicts a leverage ratio less than the actual sample median—the predicted ratio of debt to total capital is 15.29% compared to a sample median of 22.62%. We also calibrate the model to reflect actual firms and find that the model's failures go in the opposite direction from what is usually presumed. Important factors that lead our model to predict lower debt to total capital ratios than the existing literature are our model's dynamic refinancing, the bankruptcy boundary, and our calibration procedure. The analysis suggests that a relatively straightforward model in which the only factors that affect capital structure are taxes and bankruptcy costs can lead to predicted leverage ratios consistent with the observed data.

The magnitudes of the predicted optimal leverage ratios we report are strongly influenced by the dynamic nature of our model. Having the flexibility to rebalance the capital structure as the value of a firm's assets changes enables the firm to benefit from larger tax shields. In a dynamic model, the firm strategically lowers its initial leverage ratio to avoid bankruptcy so that it can optimally lever the assets when the existing debt matures. In contrast, in static models, the firm issues debt more aggressively because, while the value of its assets is expected to increase in the future, it is not able to rebalance its capital structure to reflect this increase. For example, in our analysis the optimal leverage ratio increases from 15.29% when debt matures in 10 years to 29.01% when debt matures in 20 years, suggesting that it is important to account for the ability of firms to rebalance their capital structures in models that examine optimal capital structure choice.

The way in which the bankruptcy boundary is modeled also affects the optimal debt ratio. The impact of changes in the value of the exponent of the bankruptcy boundary is relatively modest. In our base case, increasing this exponent approximately 4.00%, from 3.69% to 8.00%, increases the optimal debt to total capital ratio from 15.29% to 19.44%, reducing the exponent a similar amount, to zero (so that the bankruptcy boundary is a constant, horizontal line), causes the predicted leverage ratio to decline to 12.33% (Table 4). In addition, the fact that the bankruptcy boundary is fixed, rather than endogenous, as in the

Leland-style models, also causes the predicted optimal debt to total capital ratio to be lower.

Our approach allows the computation not only of the optimal capital structure, but also of the cost to a firm of any deviation from the optimum. Our estimates indicate that these costs are relatively small, less than 0.5% in value for about a nine-percentage point deviation in leverage. This finding is consistent with recent evidence that adjustments to capital structure to maintain a long-run target are relatively rare (Welch (2004)).

By focusing on the trade-off between tax shields and bankruptcy costs, we do not mean to downplay the importance of other factors. Clearly, the literature has identified agency and information issues as key factors that must be considered in financing decisions. Rather, our message is that the simple trade-off framework actually does much better at predicting typical leverage levels than is generally recognized, and it should not be dismissed lightly as a first-pass way of understanding a firm's financing choices.

We also want to emphasize the usefulness of formal models calibrated using market data in the study of corporate finance. This quantitative approach has been usefully applied in other branches of economics, notably macroeconomics. Its main appeal is that it allows for quantitative comparisons between alternative theories. Given the many theories in corporate finance, together with the limited exogenous variation across firms with which empirical researchers must contend, the application of numerical methods to well-constructed formal models is likely to help us better understand the relative importance of our theories.

References

Black, F., and J. Cox. "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions." *Journal of Finance*, 31 (1976), 351–367.

Brennan, M., and E. Schwartz. "Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure." *Journal of Business*, 51 (1978), 103–114.

Fan, H., and S. M. Sundaresan. "Debt Valuation, Renegotiation, and Optimal Dividend Policy." *Review of Financial Studies*, 13 (2000), 1057–1099.

Fischer, E.; R. Heinkel; and J. Zechner. "Dynamic Capital Structure Choice: Theory and Tests." *Journal of Finance*, 44 (1989), 19–40.

Francois, P., and E. Morellec. "Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures." *Journal of Business*, 77 (2004), 387–411.

Gertner, R., and D. Scharfstein. "A Theory of Workouts and the Effects of Reorganization Law." *Journal of Finance*, 46 (1991), 1189–1222.

Goldstein, R.; N. Ju; and H. Leland. "An EBIT-Based Model of Dynamic Capital Structure." *Journal of Business*, 74 (2001), 483–512.

Graham, J. R. "Proxies for the Corporate Marginal Tax Rate." *Journal of Financial Economics*, 42 (1996), 187–221.

Graham, J. R. "How Big Are the Tax Benefits of Debt?." *Journal of Finance*, 55 (2000), 1901–1941.

Graham, J. R., and C. Harvey. "The Theory and Practice of Corporate Finance: Evidence From the Field." *Journal of Financial Economics*, 60 (2001), 187–243.

Graham, J. R.; M. H. Lang; and D. A. Shackelford. "Employee Stock Options, Corporate Taxes and Debt Policy." *Journal of Finance*, 59 (2004), 1585–1618.

Graham, J. R.; M. Lemmon; and J. Schallheim. "Debt, Leases, Taxes, and the Endogeneity of Corporate Tax Status." *Journal of Finance*, 53 (1998), 131–161.

Hamilton, D. T.; G. Gupton; and A. Berhault. "Default and Recovery Rates of Corporate Bond Issuers: 2000." New York, NY: Moody's Investors Service (2001).

Huang, J., and M. Huang. "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?: A New Calibration Approach." Unpubl. Working Paper, Penn State Univ. and Stanford Univ. (2003).

Jensen, M. C., and W. H. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics*, 3 (1976), 305–360.

Ju, N. "Essays in Corporate Finance and Derivatives Pricing." Unpubl. Dissertation, Univ. of California at Berkeley (1998).

Ju, N. "Dynamic Optimal Capital Structure and Maturity Structure." Unpubl. Working Paper, Univ. of Maryland (2001).

Kane, A.; A. J. Marcus; and R. L. McDonald. "How Big is the Tax Advantage to Debt?" *Journal of Finance*, 39 (1984), 841–852.

Kane, A.; A. J. Marcus; and R. L. McDonald. "Debt Policy and the Rate of Return Premium to Leverage." *Journal of Financial and Quantitative Analysis*, 20 (1985), 479–499.

Leland, H. E. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure." *Journal of Finance*, 49 (1994), 1213–1252.

Leland, H. E. "Agency Costs, Risk Management, and Capital Structure." *Journal of Finance*, 53 (1998), 1213–1244.

Leland, H. E., and K. Toft. "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads." *Journal of Finance*, 51 (1996), 987–1019.

Longstaff, F. A., and E. S. Schwartz. "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt." *Journal of Finance*, 50 (1995), 789–819.

Mello, A. S., and J. E. Parsons. "Measuring the Agency Cost of Debt." *Journal of Finance*, 47 (1992), 1887–1904.

Merton, R. C. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance*, 29 (1974), 449–470.

Miller, M. "Debt and Taxes." *Journal of Finance*, 32 (1977), 261–275.

Modigliani, F., and M. H. Miller. "Corporate Income Taxes and the Cost of Capital: A Correction." *American Economic Review*, 53 (1963), 433–443.

Morellec, E. "Can Managerial Discretion Explain Observed Leverage Ratios?" *Review of Financial Studies*, 17 (2004), 257–294.

Morellec, E., and C. W. Smith, Jr. "Investment Policy, Financial Policies, and the Control of Agency Conflicts." Unpubl. Working Paper, Univ. of Rochester (2003).

Myers, S. C. "Determinants of Corporate Borrowing." *Journal of Financial Economics*, 5 (1977), 147–175.

Myers, S. C. "The Capital Structure Puzzle." *Journal of Finance*, 39 (1984), 575–592.

Myers, S. C., and N. S. Majluf. "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have." *Journal of Financial Economics*, 13 (1984), 187–221.

Parrino, R., and M. S. Weisbach. "Measuring Investment Distortions Arising from Stockholder-Bondholder Conflicts." *Journal of Financial Economics*, 53 (1999), 3–42.

Parrino, R.; A. M. Poteshman; and M. S. Weisbach. "Measuring Investment Distortions When Risk-Averse Managers Decide Whether to Undertake Risky Projects." Unpubl. Working Paper, Univ. of Illinois and Univ. of Texas (2003).

Rajan, R. G., and L. Zingales. "What Do We Really Know About Capital Structure? Evidence From International Data." *Journal of Finance*, 50 (1995), 1421–1460.

Titman, S., and S. Tsyplakov. "A Dynamic Model of Optimal Capital Structure." Unpubl. Working Paper, Univ. of Texas at Austin and Univ. of South Carolina (2001).

Titman, S., and R. Wessels. "The Determinants of Capital Structure." *Journal of Finance*, 43 (1988), 1–19.

Welch, I. "Capital Structure and Stock Returns." *Journal of Political Economy*, 112 (2004), 106–131.