Measuring Space-Time Prism Similarity Through Temporal Profile Curves

Harvey J. Miller, Martin Raubal and Young Jaegal

Abstract Space-time paths and prisms based on the time geographic framework model actual (empirical or simulated) and potential mobility, respectively. There are well-established methods for quantitatively measuring similarity between space-time paths, including dynamic time warping and edit-distance functions. However, there are no corresponding measures for comparing space-time prisms. Analogous to path similarity, space-time prism similarity measures can support comparison of individual accessibility, prism clustering methods and retrieving prisms similar to a reference prism from a mobility database. In this paper, we introduce a method to calculate space-time prism similarity through temporal sweeping. The sweeping method generates temporal profile curves summarizing dynamic prism geometry or semantic content over the time span of the prism's existence. Given these profile curves, we can apply existing path similarity methods to compare space-time prisms based on a specified geometric or semantic prism. This method can also be scaled to multiple prisms, and can be applied to prisms and paths simultaneously. We discuss the general approach and demonstrate the method for classic planar space-time prisms.

Keywords Activity space · Time geography · Similarity · Accessibility

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1 Introduction

Space-time paths and prisms based on the time geographic framework model actual and potential movement, respectively, of an object through geographic space with respect to time (Hägerstrand 1970). *Path similarity measures* quantitatively assess the resemblance between two space-time paths, with greater similarity indicating mobility and activity patterns that are more alike. Path similarity can provide insights into dynamic phenomena such as traffic congestion and crime (Yuan and Raubal 2014), identify similar patterns of environmental exposure (Briggs 2005; Sinha and Mark 2005), and analyze collective movement patterns (Gudmundsson et al. 2012). Other applications of path similarity measures include path clustering (finding groups of similar paths), path aggregation (forming composite representative paths), and mobile objects database (MOD) queries to find paths that resemble a reference path.

The *space-time prism* (STP) is the envelope of all possible space-time paths between two anchor locations with known departure and arrival times respectively. STPs are measures of space-time path uncertainty when a moving object's locations are undersampled with respect to time (Pfoser and Jensen 1999). They are also measures of individual accessibility and exposure within an environment and have been widely applied in human and ecological science (Espeter and Raubal 2009; Long and Nelson 2012). As extensions of space-time paths, STP similarity indicates similar patterns of accessibility and potential exposure within an environment. As with path similarity, we may wish to measure STP similarity to cluster or aggregate prisms as well as search for corresponding prisms within a MOD. However, no methods for measuring STP similarity exist.

This paper develops a time-based approach to measuring STP similarity. Our approach sweeps a STP with respect to time, recording its geometric and/or semantic properties at discrete moments. This allows us to construct one-dimensional temporal profile curves that summarize the properties with respect to time. These curves can be compared visually or quantitatively using existing path similarity measures. We restrict our attention to classic planar STPs in this paper but discuss how to extend these measures to other prism types, including network time prisms (Kuijpers and Othman 2009; Miller 1991) and field-based prisms (Miller and Bridwell 2009).

The next two sections of this paper provide background to our methods. Section 2 reviews existing approaches to measuring path similarity while Sect. 3 describes geometric and semantic properties of STPs that can be measured analytically. Section 4 presents our generic method for generating temporal profile curves. Section 5 provides an example using planar STPs. Section 6 identifies future research steps and Sect. 7 concludes the paper with a brief summary.

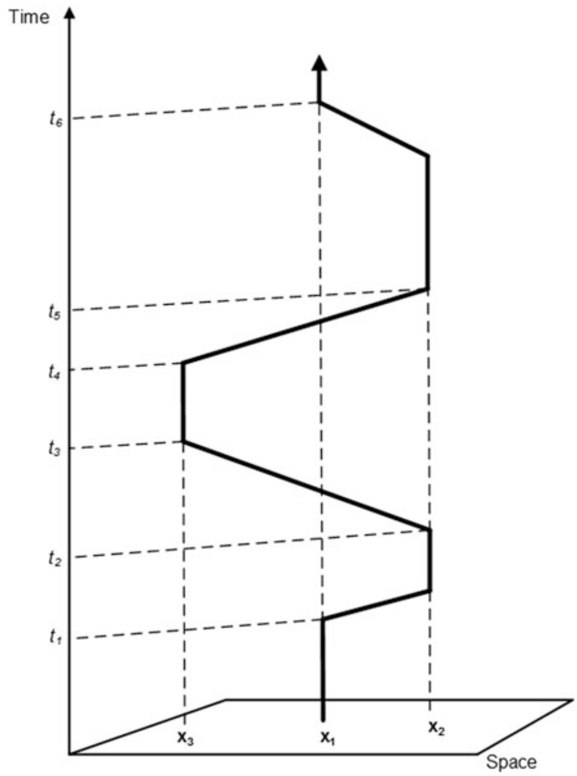
2 Space-Time Paths and Path Similarity

The space-time path represents a history of individual movement with visited locations, and sequence of the movement. Figure 1 shows a space-time path in two-dimensional space and time corresponding to a person moving among three locations with corresponding departure and arrival times at locations. The vertical line segments represent the duration that the person stayed at the same location for a certain amount of time. This integrated view of space and time provides an effective visual environment to understand human movement, activities and accessibility in space and time (Pred 1977).

In addition to visualization there is a wide range of analytical descriptions and summaries for space-time paths. Basic path measures include both *moment-based descriptors* (such as the time, location, direction and speed at any moment) and *interval-based descriptors* (such as the minimum, maximum and mean speed, the distribution and sequence of speeds and directions, and the geometric shape of the path over some time interval) (Andrienko et al. 2008).

Path similarity measures allow quantitative comparisons among space-time paths, particularly with respect to geometric similarity in space-time and with

Fig. 1 A space-time path in two-dimensional space and time



respect to semantics. Geometric similarity captures the resemblance of the mobile objects in terms of their patterns in space with respect to time. Semantic similarity (Janowicz et al. 2011) refers to the characteristics and activities of the moving object, such as commuting, shopping and socializing (in the case of humans) (Raubal et al. 2004), or foraging and migration (in the case of animals). Similarity measures also support *path clustering and aggregation methods* for identifying synoptic spatio–temporal patterns from large collections of mobile objects (Long and Nelson 2013).

Two types of path similarity measures are shape-based measures and time-based measures (Yuan and Raubal 2014). *Shape-based measures* focus on the geometry of the paths; these include the average Euclidean distance between corresponding locations on the paths, and the Hausdorff distance or maximum of the minimum distances between the paths. *Time-based measures* take into account the temporal aspects of paths by considering them as multidimensional time series data. They include synchronized Euclidean distance, Fréchet distance, dynamic time warping, longest common subsequences, and edit-distance functions. Fréchet distance is the shortest of the set of closest distances that connects the objects moving along their paths at any speed without backtracking.

The time-based measures discussed above focus on path geometry including time stamps of points and order, but more general time-based measures such as dynamic time warping, longest common subsequences and edit-distance functions can capture both geometry and semantics. Required is some coding that translates path geometry and/or semantics into an ordered and exhaustive sequence of states with their time durations (see, for example, Dodge et al. 2012). *Dynamic time warping* measures the similarity between two sequences or trajectories by local stretching or compressing to match the time series and compute the sum of the paired distances (Yuan and Raubal 2012). *Longest common subsequence* (LCSS) methods measure similarity based on the length of the longest common subsequence in a set of sequences (Nanni et al. 2008). *Edit-distance functions* generalize LCSS: these measure similarities between sequential patterns based on the cost of the insertion, deletion and substitution operations required to transform one sequence into the other. Such functions can also account for spatial and temporal information in their cost functions (Yuan and Raubal 2014).

3 Analytical Space-Time Prisms

A space-time prism (STP) is the envelope of all possible paths in space with respect to time between two anchoring locations and corresponding departure and arrival times, subject to a maximum travel speed and any stationary activity time. STP parameters are $\{\boldsymbol{x}_i, \boldsymbol{x}_j, t_i, t_j, s_{ij}, a_{ij}\}$ where $\mathbf{x}_i, \mathbf{x}_j$ are the first and second anchor locations with associated departure and arrival times t_i, t_j respectively, s_{ij} is the maximum travel speed and a_{ij} is the stationary activity time (if any). The spatial

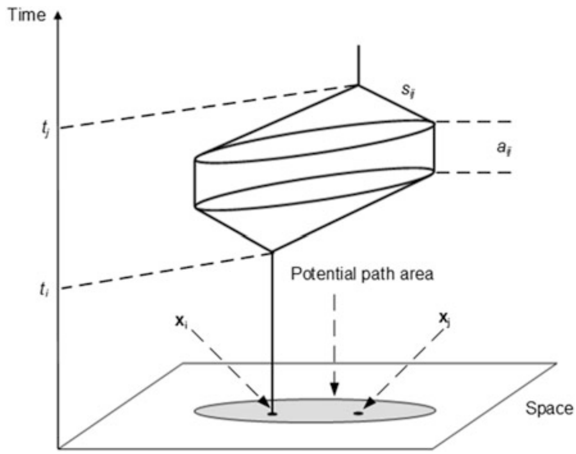


Fig. 2 A planar space-time prism

footprint of a STP is the *potential path area* (PPA): in general, this is an ellipse with the two anchors as foci. Figure 2 illustrates a general STP.

Although it is the intersection of simple objects such as cones and cylinders (see Burns 1979), it is difficult to analytically describe the entire STP. However, it is easy to describe its spatial extent at a moment in time; this can serve as the basis for a wide range of prism analytics. At a moment in time $t \in (t_i, t_j)$, the spatial extent of a STP (denoted by $Z_{ij}(t)$) is the intersection of three convex spatial sets: (i) the *future disc* $f_i(t)$ comprising all locations that can be reached from the first anchor by time $t_i + t$; (ii) the *past disc* $p_j(t)$ encompassing all locations at time t that can reach the second anchor by time $t_j - t$; and, (iii) the *potential path area* g_{ij} that constrains the prism locations to account for any stationary activity time:

$$Z_{ij}(t) = \{f_i(t) \cap p_j(t) \cap g_{ij}\} \tag{1}$$

$$f_i(t) = \{\mathbf{x}\} | \|\mathbf{x} - \mathbf{x}_i\| \le (t - t_i)s_{ij} \tag{2}$$

$$p_j(t) = \{\mathbf{x}\} \left| \left\|\mathbf{x} - \mathbf{x}_j\right\| \le \left(t_j - t\right) s_{ij} \right. \tag{3}$$

$$g_{ij} = \{\mathbf{x}\} \left| \|\mathbf{x} - \mathbf{x}_i\| + \left\|\mathbf{x} - \mathbf{x}_j\right\| \le \left(t_j - t_i - a_{ij}\right) s_{ij} \right. \tag{4}$$

Figure 3 provides an illustration. This definition of the STP is not limited to two-dimensional space. In one-dimensional space, the sets described by Eqs. (2)–(4) are line segments. In two-dimensional space, the discs are circles and the geo-ellipse is an ellipse. In three-dimensional space, the discs are spheres and the geo-ellipse is a spheroid. There are scalable methods for calculating these objects and their intersections. Also, the intersection geometry of a STP at a moment in time never requires finding the intersection of all three spatial sets since the future and past disc change size and can be enclosed by the other two sets for part of the prism's existence. This will be discussed in more detail below (Miller 2005).

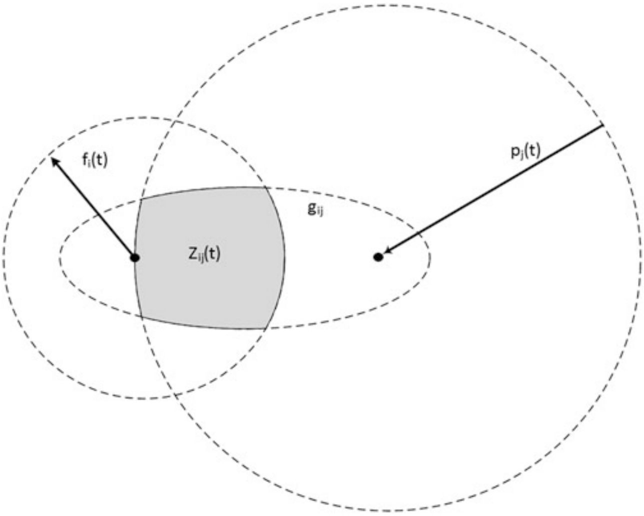


Fig. 3 Analytical construction of a STP at a moment in time

In addition to the prism boundaries, the prism interior also has *intrinsic* and *extrinsic* properties of interest. Intrinsic properties relate to the prism itself. The probabilities of the object visiting different locations within the prism interior are not equal: the object is more likely to visit locations near the axis connecting the two anchors relative to locations near the prism boundaries since there are more possible paths through the former rather than the latter (Winter and Yin 2010a). We can simulate the visit probability distribution within a prism interior using simple random walks or Brownian Bridges methods, truncated to account for STP constraints (Song and Miller 2014). Alternatively, the prism visit probability distribution at a moment in time can be approximated using a clipped bivariate normal distribution (Winter and Yin 2010b).

Related to the visit probability distribution within the prism interior is the distribution of possible speeds at each location at a moment in time. The speed distribution at a given location and time within the prism is Markovian in the sense that the history that precedes it is irrelevant: the location and remaining time determines the possible speeds. Locations near the prism interior tend to have a wider range of possible speeds while locations near the boundaries are more constrained, with locations on the prism boundary constrained to only the maximum speed.

Extrinsic properties of the prism interior relate to type of activity locations, resources, environmental features and other individuals encompassed by the spatial region. This external content describes the possible activities and experiences for the individual and therefore supports derivation of accessibility semantics. At a moment in time these properties comprise a two-dimensional spatial distribution that can be described using spatial statistics. Point pattern measures including density-based approaches or distance-based measures, such as the K function that is based on counting points within a series of distances of each point (O'Sullivan and

Unwin 2010), can be applied to a spatial distribution of activity locations such as restaurants or movie theaters. With entropy measures such as spatial entropy, one can quantitatively determine the uncertainties about the structure of two-dimensional spatial distributions (Batty 2010). Furthermore, spatial autocorrelation can be measured for these distributions, for example, through indices such as Moran's I, which is usually applied to areal units representing environmental or epidemiological data (O'Sullivan and Unwin 2010).

Other properties of a prism include its relationship with other space-time paths and prisms. Calculating the binary query if a path lies in a prism at a given moment in time only requires testing if a point (the path at a moment in time) lies within a disc, ellipse, or a disc–ellipse intersection. We also can easily test if a prism-prism intersection exists at a given moment in time by evaluating a small set of linear inequalities. More complicated but still tractable is solving for the intersection region at a given moment in time: this requires solving for the intersection of two, three, or four simple spatial sets based on the prisms' morphologies at that moment. The worse-case for two prisms is a four-set intersection involving two discs and two ellipses (Miller 2005). Finally, it is also possible to solve for the Euclidean, Hausdorff and other distances between two prisms at a moment in time since these are simple spatial sets.

4 Measuring Space-Time Prism Similarity Using Temporal Profile Curves

The basic idea behind our method is to reduce the dimensionality of the space-time prism by sweeping it with respect to time and summarizing its geometric and/or semantic properties at discrete moments in time using the methods outlined in general above. This generates a temporal profile curve for the given attribute. These

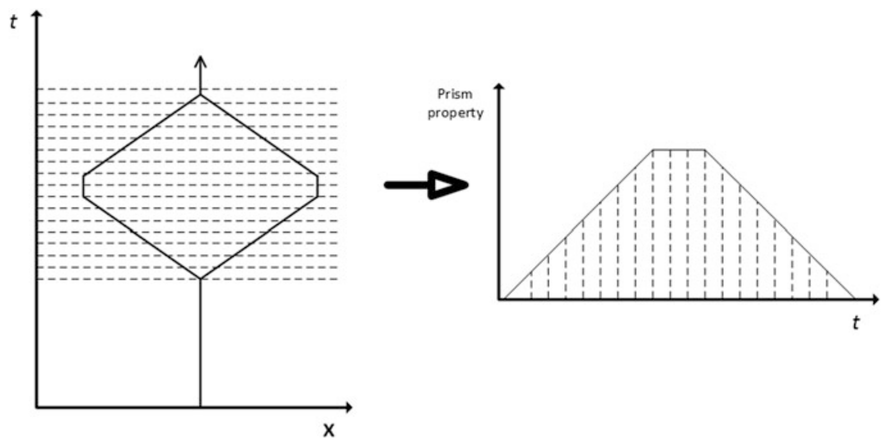


Fig. 4 Temporally sweeping a space-time prism

profile curves can be compared visually. We can also apply existing path similarity measures to determine their resemblance based on the chosen parameter, as well as apply clustering and aggregation methods based on these curves. Figure 4 illustrates the core idea for a single STP. It is also possible to sweep multiple STPs simultaneously to compare their properties within the same time frame. We could also sweep multiple STPs independently and compare the prisms after normalizing the time horizons of each prism.

As noted above, the intersection geometry of a STP at a moment in time never requires finding the intersection of all three spatial sets since the future and past disc change size and may be enclosed by the other two sets during subintervals of the prism's existence. With a general prism as in Fig. 2 we only need to solve (in the following order) the future disc alone, the intersection of the future disc with the potential path ellipse, the potential path ellipse alone, the intersection of the potential path ellipse with the past disc intersection and finally the past disc alone (Miller 2005). Figure 5 illustrates these subintervals for a general prism. The temporal subinterval boundaries are:

$$t^0 = \frac{\left(t_i + t_j - t^*_{ij} - a_{ij}\right)}{2} \tag{5}$$

$$t' = \frac{\left(t_i + t_j - t^*_{ij}\right)}{2} \tag{6}$$

$$t'' = \frac{\left(t_i + t_j + t^*_{ij}\right)}{2} \tag{7}$$

$$t''' = \frac{\left(t_i + t_j + t^*_{ij} + a_{ij}\right)}{2} \tag{8}$$

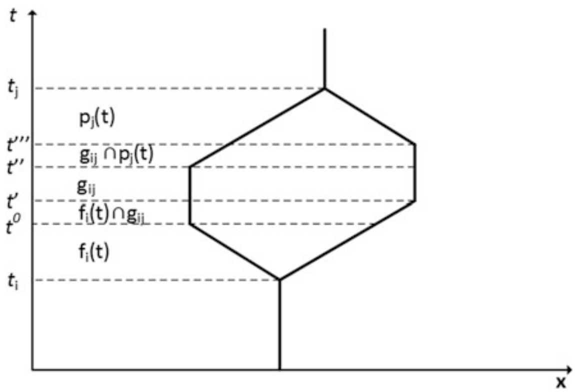


Fig. 5 Temporal subintervals for analytically calculating planar STPs at a moment in time

where t^*_{ij} is the minimum travel time between the anchors. If stationary activity time a_{ij} is zero then Eqs. (5) and (8) are irrelevant and the prism simplifies to only three subintervals that require solving for the future disc, future disc-past disc intersection and the past disc, respectively. Forming these sets and intersections are simple operations that can be performed using standard buffering and overlay techniques available in most GIS software shortcuts. In the examples below, we calculate these intersections and their properties at discrete moments in time using off-the-shelf overlay tools available in ArcGIS 10.1.

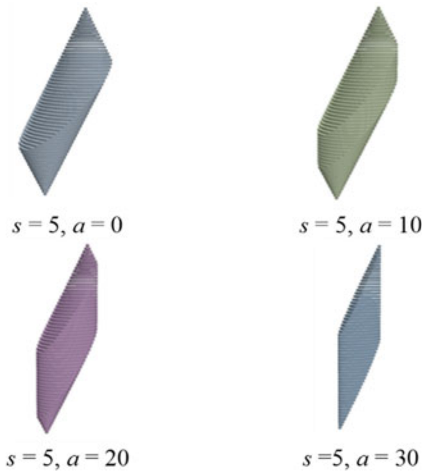
5 Examples

This section illustrates the temporal sweeping method for planar STPs. We first demonstrate the technique for generating temporal profile curves for summarizing geometric prism properties, specifically, prism area. We then demonstrate the technique for summarizing prism semantics for an empirical example.

Geometric similarity. Figure 6 provides four prisms with varying activity times but speed limits fixed at s = 5. Figure 7 provides four prisms with varying speed limits but with stationary activity times fixed at a = 10. Stationary activity times refer to minimum times required for immobile activity, such as shopping or dining. All prisms have anchors at (0, 0) and (100, 100) and time budgets of 60. Also note that the prism with s = 5 and a = 10 is the same in both figures.

Figures 8 and 9 provide the corresponding temporal profile curves for the prism areas in Figs. 6 and 7, respectively. As expected, lower activity times and higher speeds correspond to larger prism areas and shifts in the locations of the curves positively with respect to the y-axis. Changes in STP speed limit have a larger impact on the locations of the curves relative to changes in activity time (note the

Fig. 6 Four prisms with varying activity times



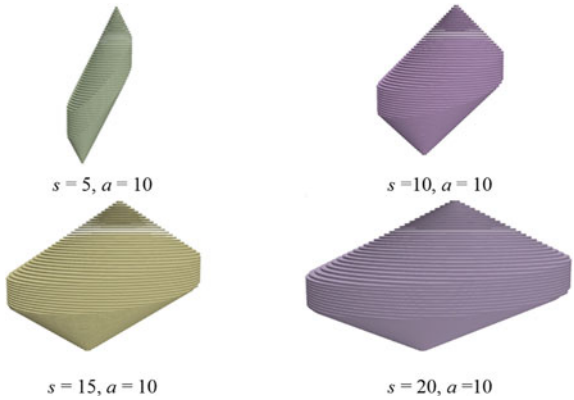


Fig. 7 Four prisms with varying speed limits

difference in y-axis scale between the two curves). This corresponds to the more dramatic changes in STP size evident from comparing Figs. 6 and 7, as well as time geographic theory that suggests relaxing speed limits has a bigger impact on accessibility than reducing stationary activity time (see Burns 1979). However, in addition to shifts in the curve locations there are changes in the curve morphology. Figure 8 indicates that a lower activity time results in the shapes of the curves to become more rounded. In contrast, Fig. 9 indicates that higher speed limits correspond to the curves becoming more peaked. These differences in profile curve locations and morphology can be exploited when measuring prism similarity or performing related analysis such as prism clustering and aggregation.

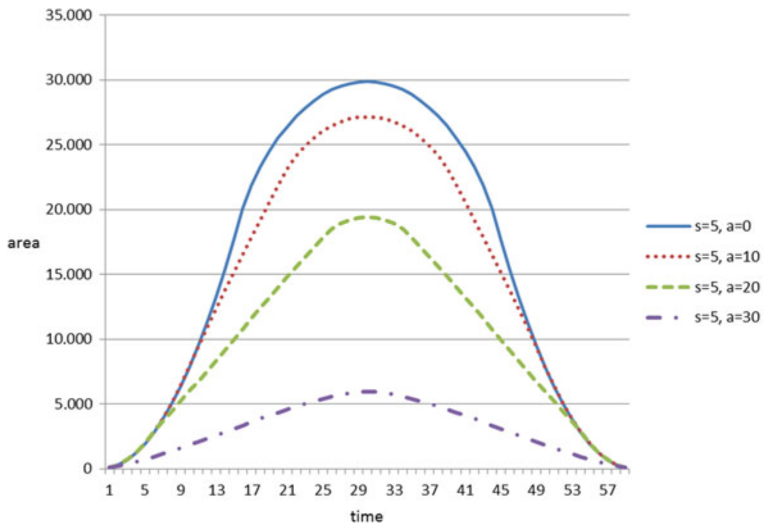


Fig. 8 Profile *curves* for the varying activity time prisms in Fig. 6

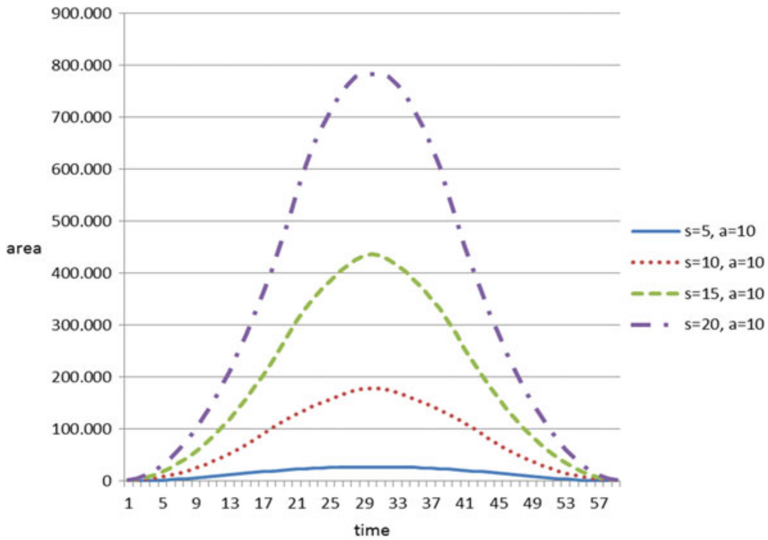


Fig. 9 Profile *curves* for the varying speed limit prisms in Fig. 7

Table 1 provides a summary of inter-curve distances calculated via the dynamic time warping (DTW) procedure in R using the Sakoe-Chiba band window with a maximum time deviation between matched pairs of 4 (Giorgino 2009; Sakoe and Chiba 1978). We normalized these distances using the symmetric2 procedure in R: this gives a higher weight to diagonal transition relative to horizontal or vertical transitions (see Giorgino 2009). As Table 1 indicates, the DTW distances distinguish between STPs with different activity times and speed limits. However, distances between STPs with different speed limits are greater than STPs with different activity times due to the greater effect of speed limit than activity time on STP area. This is not necessarily true of all STP geometric properties; an open research question is to identify and assess STP geometric properties with respect to changes in the STP parameters (see Burns 1979). Also, DTW captures differences in curve locations better than curve morphology; another open research question is determining curve similarity measures that can capture morphological differences.

Table 1 DTW inter-curve distances

Speed limit changes	Activity time changes			
	s = 5, a = 0	s = 5, a = 10	s = 5, a = 20	s = 5, a = 30
s = 5, a = 10	528.4	–	2,501.2	9,441.9
s = 10, a = 10	51,092.3	53,026.6	58,132.1	65,820.8
s = 15, a = 10	145,217.3	147,151.6	152,264.4	160,036.4
s = 20, a = 10	277,258.9	279,193.2	284,306.2	292,114.5

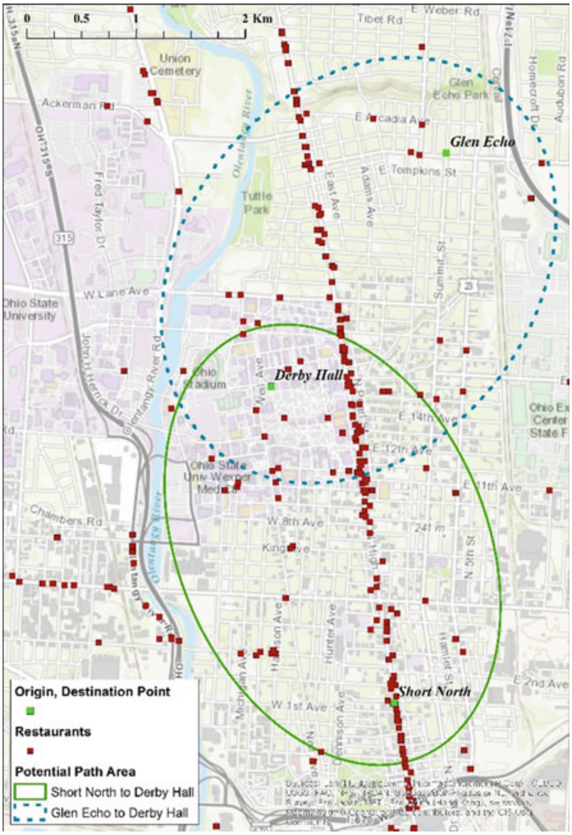


Fig. 10 Example for semantic similarity of two prisms (i) Short North to Derby Hall; (ii) Glen Echo to Derby Hall

Semantic similarity. Figure 10 provides an empirical example for calculating semantic similarity: walking to Derby Hall on The Ohio State University campus from two different neighborhoods in Columbus, Ohio, USA. The Short North neighborhood to the south is a denser urban setting while Glenn Echo to the northeast is a more suburban neighborhood. Figure 10 shows the potential path areas for two prisms with origin anchors in the centers of the Short North and Glen Echo neighborhoods but with a common destination anchor at Derby Hall. Both prisms reflect a speed limit of 6.4 kph (a brisk walk) with a total time budget of one hour, and stationary activity time of 30 min for stopping at a restaurant. The map also shows the location of all restaurants in the respective areas.

Figures 11 and 12 show the prisms' profile curves for two semantic properties: the restaurant density (relative to prism area at each moment in time) and the average nearest neighbor ratio calculated as the observed average distance divided by the expected average distance based on a null model of complete spatial randomness. In Fig. 12, values less than 1 indicate spatial clustering while values greater than 1 indicate spatial dispersion.

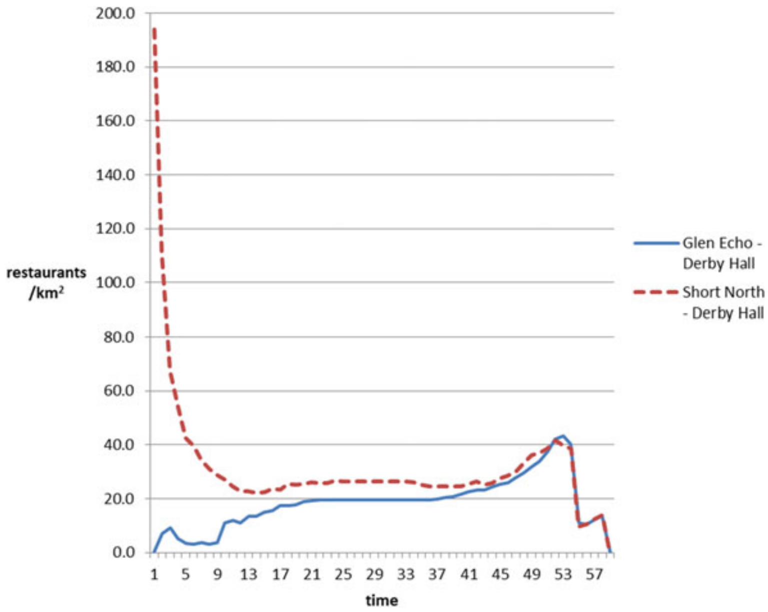


Fig. 11 Prism profile *curves* for restaurant density

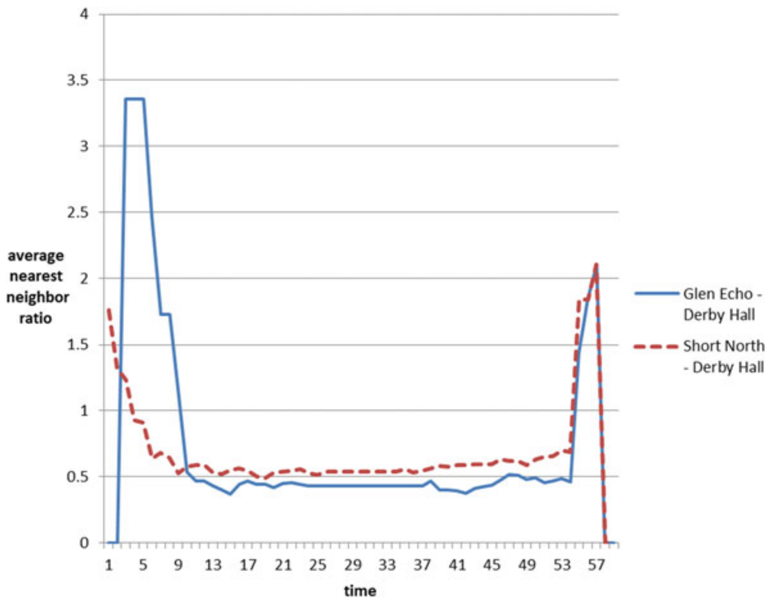


Fig. 12 Prism profile *curves* for average nearest neighbor ratio

Figures 11 and 12 suggest that the profile curves distinguish well between the semantic content of the two prisms. The profile curve for the Short North prism suggests an accessible environment with initially a high density of spatially clustered restaurants that becomes less dense and clustered with time and movement towards the destination. In contrast, the profile curve for the Glen Echo prism suggests an accessible environment with initially a low density of spatially dispersed restaurants that becomes denser and more clustered with time and movement towards the destination. In the last stages of both prisms the profile curves converge as the two prisms converge on the common destination. However, except for an early spike in the Glenn Echo nearest neighbor curve, the nearest neighborhood profiles appear more similar than the density profiles for both prisms. Normalized DTW distances for the profile curves of 10.1 and 0.2 for density and nearest neighbor, respectively, support this qualitative result.

6 Future Steps

The preliminary analysis in this paper suggests the value of the temporal profile curves for distinguishing among STPs. With respect to geometry, the STP area profiles indicate that changes in prism morphology can be reflected in changes in both the locations and shape of the profile curves. The semantic example of restaurant density and spatial clustering also generated profile curves that distinguish between the content of prisms. A next step is to explore different geometric and semantic STP indicators, such as shape measures (e.g., compactness versus elongation), physical properties such as average speed, and spatial statistics describing prism content at a moment in time, and assess their effectiveness at distinguishing among STPs under different conditions.

As noted earlier in this paper, similarity measures can facilitate the analysis of prism collections by supporting summarization methods. A next step after determining appropriate geometric and semantic indicators is to develop scalable STP clustering and aggregation methods. STP clustering is more straightforward than STP aggregation since the latter requires procedures for generating a composite STP that reflects the common properties of the disaggregated STPs. A simple approach for space-time paths is to use vector averaging: treat each segment of the polyline as a vector and find the average of the corresponding vectors (see Kobayashi and Miller 2014). Additional investigation is required to determine aggregation methods for the more complex case of STPs.

We restrict our attention to classic planar STPs in this paper. A longer term research task is to develop similar methods for the important case of network time prisms (NTPs). These methods can be based on graph theoretical measures of the spatial footprint at each moment in time in a NTP. Another possibility is to calculate geometric properties for the NTP using its trapezoidal and triangular regions in space and time (see Kuijpers and Othman 2009). With respect to NTP semantics, the profile curves can exploit address-matched and other network-referenced data combined with network-based spatial analytical methods (Okabe and Sugihara 2012). Another future step is to develop methods for the more complex geometry of field-based prisms where speeds vary continuously in space. These methods can exploit properties of the lattice approximation for field-based prisms (Miller and Bridwell 2009).

7 Conclusion

This paper develops an approach to measuring space-time prism (STP) similarity in a manner similar to methods for measuring path similarity. Our method reduces the dimensionality of a STP by temporally sweeping it to generate one-dimensional profile curves that summarize changes in geometric and/or semantic properties with respect to time. We demonstrate this approach using the example of prism area under varying activity times and speed limits, as well as the semantic content for an empirical example of a travel and activity episode. Preliminary results suggest that this approach is promising: the locations and morphologies of the profile curves reflect changes in prism geometry and semantics, and differences between the curves can be summarized effectively using distance measures such as Dynamic Time Warping. We outline several next steps to continue this research, including determining effective geometric and semantic indicators for STPs, developing STP clustering and aggregation methods that exploit these profile curves and distance measures, and extending these methods to other types of prism, such as network time prisms and field-based prisms.

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