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PARADOXES OF PURE CURIOSITY

ABSTRACT. We consider how a rational decision theorist would justify committing resources to an investigation designed to satisfy pure curiosity. We derive a strange result about the need to be completely open-minded about the outcome.

Keywords: paradox, epistemic value, resource allocation, SETI.

1. PURELY INTELLECTUAL CURIOSITY, SUBJECTIVE PROBABILITY, AND EPISTEMIC VALUE

Among existential questions and discoveries, I want to distinguish between what one might call the purely intellectual ones, and the theoretical ones. The purely intellectual discoveries are ones which, we are certain, cannot lead to tangible benefits of a technological or therapeutic kind. The theoretical discoveries, on the other hand, might do so. Existential questions are likewise classified by the discoveries that would answer them positively. I am not claiming that this is a watertight distinction; indeed, it cannot be, since subjective probabilities are built into its very definition. But it is a handy distinction to advance the discussion.¹

I shall understand by 'curiosity' a curiosity of the kind that leads us to expend effort and resources to satisfy it. The curiosity must consist in more than simply mouthing an existential question.

'Subjective' probabilities² feature in two ways in our intellectual and emotional and deliberative lives.

First, there is the epistemic value of a discovery of some important fact. Such facts may be many and various: that DNA is the mechanism of heredity; that there is a Loch Ness monster; that there is a cure for AIDS; that there is extraterrestrial intelligence. The epistemic value of a discovery consists in a dramatic raising of the subjective probability attached to a statement. Usually it is an existential statement, to the effect there are *F*'s, that *F*'s exist. The prior probability has to be much smaller than the posterior probability in order for the discovery to have epistemic value. Crudely put, the difference between the prior and the posterior probabilities is roughly proportional to the epistemic value, or 'surprise value', of the discovery. Insofar as purely intellectual curiosity is concerned, the epistemic value of an investigation must be determined at least in part by the epistemic value of the discovery that might result. The more certain we are, in advance, that things of a certain kind exist, the less epistemic value would attach to the discovery, with certainty or near-certainty, that things of that kind do indeed exist. Likewise, the less certain we are, in advance, that such

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things exist, the greater the epistemic value we would attach to such discovery. Note that I am here talking about epistemic value – not value *tout court*. As to the epistemic value of the investigation itself – that will depend on the epistemic value of discovery in a way to be made clear below.

Secondly, there is the role played by subjective probabilities in individual and group decision-making. For the reader unfamiliar with the basics of rational decision theory, I shall illustrate the general principles with an example.

2. RATIONAL DECISION THEORY

Suppose the problem is to decide between two courses of action, *A* and *B*. Suppose we envisage three possible outcomes for action *A* – let us call them *A1*, *A2* and *A3*; but only two possible outcomes for action *B* – let us call them *B1* and *B2*. The problem, note, is to choose between *A* and *B* under conditions of uncertainty as to which of outcomes *A1*, *A2* and *A3* action *A* would lead to, and as to which of outcomes *B1* and *B2* action *B* would lead to. Thus we have to use our subjective probabilities concerning the links between action and outcomes. So let p_1 , p_2 , p_3 (whose sum is 1) be the respective probabilities for outcomes *A1*, *A2*, *A3*, given that action *A* is undertaken; and let q_1 , q_2 (whose sum is 1) be the respective probabilities for outcomes *B1*, *B2*, given that action *B* is undertaken. Moreover, we have to take into account not only these probabilities of outcomes of whichever action might be chosen, but also the utility or payoff of each outcome. In calculating each utility we must take into account the cost expended in undertaking the action that leads to the outcome in question. That is, the utility will be the net value after the cost-of-action has been subtracted. So suppose *A1*, *A2*, *A3* have utilities U_1 , U_2 , U_3 , respectively, and *B1*, *B2* have utilities V_1 , V_2 , respectively. Then the expected utility of action *A* is simply

$$p_1.U_1 + p_2.U_2 + p_3.U_3$$

while the expected utility of action *B* is, likewise,

$$q_1.V_1 + q_2.V_2$$

If the former weighted sum is greater, perform action *A*; if the latter sum is greater, perform action *B*. This, quintessentially, is the widely accepted model of rational decision making.

Let us now focus on outcome A_i of action *A*. If the probability p_i is very low, this diminishes the value that U_i contributes to the overall value of action *A*; and the value U_i would accordingly have to be much higher in order for the possible (but improbable) outcome A_i to contribute much to the overall value of the contemplated action *A*. Likewise for any of the outcomes for action *B*.

3. ON SATISFYING PURELY INTELLECTUAL CURIOSITY

How is this relevant to investigations aimed at satisfying purely intellectual curiosity? The answer is that science is a communal project, consuming resources provided by the community. Therefore it comes fairly within the purview of any ethics of resource allocation that might be brought to bear by the community. Think of the action 'engaging in the search for extraterrestrial intelligence'³ at the cost of 100 megadollars over a decade'. Call it *B*. These are the possible outcomes:

*B*1 – an ETI is discovered (with probability q)⁴

*B*2 – no ETI is discovered (with probability $1 - q$)

The *social* utility – *S*, say, will be negative, because of the cost of the search – unless, of course, some beneficial spin-offs offset it. Let us suppose that they do not, so that *S* lies somewhere between 0 and 100 megadollars. Noting that there might be beneficial spin-offs from the basic research that would go into supporting SETI, and that there might be economic benefits (under the multiplier effect) in providing the SETI community with salaried employment, let us try to focus just on the expected epistemic value of *B*1 and *B*2.

The latter value could be worked out very easily by a pessimist. The pessimist could say that q^2 is simply $1 - q$, V^2 is 0 (zero), or something infinitesimally close to 0, so $q^2.V^2$ is (infinitesimally close to) 0.

It might be objected here against such a pessimist that there would be some non-negligible value attaching even to a failed search, since failure might scotch some otherwise plausible *a priori* hypothesis and help orient renewed search (should it ever again be undertaken) in a new and possibly more profitable direction. Let us relax our earlier pessimism and enquire after the epistemic utility, if any, of 'failed' search.⁵

We are looking, then, for a term for empty-handedness in our eventual formula for expected utility. Let us consider the two extremes, namely the committed skeptic, for whom $q = 0$, and the gung-ho search advocate, for whom $q = 1$. In the event of 'failure' or empty-handedness, the committed skeptic's response will be "I told you so". Failure will accord fully with her expectations. There will be no epistemic utility, for her, apart from possibly smug satisfaction. But that would probably be offset by her *chagrin* at such an expensive search having been undertaken with her tax money by others in the first place!⁶ I think it is safe to say that when $q = 0$ the epistemic payoff of 'failed' search is 0.

What is the response of the gung-ho advocate in the event of 'failure' or empty-handedness? For the gung-ho the epistemic payoff might be slightly positive. It *might*, of course, be 0: he might say "Look, we've only searched a tiny fraction of the space they might inhabit. They might just be elsewhere; we'll have

to keep searching.” On the other hand, he might say “Mmm, we should have had a reasonable chance with these recent search methods we’ve been employing. So if they’ve not turned up anything, maybe the discovery’s not to be had by these search methods. Maybe we should try searching by these *other* methods . . .”⁷ We might concede here that from the gung-ho point of view, empty-handed search has brought *some* epistemic payoff. Call it δ . For values of q between 0 and 1, I shall assume that the epistemic payoff of empty-handedness is linear: $V_2 = \delta q$. Thus the expected contribution, to expected utility of search, of ‘failed’ or empty-handed search, is $(1 - q)\delta q$. This is the sought term for empty-handedness in our formula for expected utility. Having this term instead of 0 is what I shall call the *concession to empty-handedness*.

What about the former case? What is the expected epistemic utility of *successful* search? To answer this, we have to enquire after the subjective probability q and the epistemic payoff V_1 . q is the probability that, if we search for ETI, we’ll find it. The smaller this probability, the bigger the surprise if and when we find ETI. That is to say, if q is small, the epistemic value V_1 will be correspondingly greater. Let us, then, set V_1 equal to $(1 - q)V$, where V is a constant of proportionality.⁸ So the product $q \cdot (1 - q) \cdot V$ reflects the additive contribution, to expected epistemic utility of SETI, of successful SETI.

So far, apart from a brief mention earlier of social cost, I have been focusing on epistemic utilities. For those, however, who are inclined to reckon in (if not concentrate exclusively on) the social cost of SETI, we must look at the value S . The genuine expected utility, remember, is the result of ‘netting’ for cost of actions. The cost of action B – SETI itself – is what remains to be determined.

We have agreed to call the ceiling to this cost S , lying somewhere between 0 and 100 megadollars. Here one might take the cost of search to be independent of the prior probability q of success given that the search is undertaken. This would be a reasonable assumption for the most likely circumstances, where the initial capital costs of dedicated equipment far outweigh the subsequent overheads in deploying it. In other circumstances, where the accrued cost of search is proportional, say, to its duration, one may say that the expected total cost varies inversely with q .⁹ For, the lower the value of q , the longer we may expect the search to last. If we have an upper limit, namely the value S above, budgeted for the search, then even one for whom the subjective probability q was very close to 1 may nevertheless reasonably expect all of the budget S to be expended before the search meets with success. And even those for whom $q = 1$ may expect a significant portion σ of S to be expended before the search meets with success.

Thus as a function of q the expected cost of the search will be S (or very close to S) for all values of q from 0 to some point just less than 1, and thereafter decrease to σ at $q = 1$. The most magnanimous concession one could make to the advocate of faith-induced savings¹⁰ in the likely cost of search would be to set the cost of search equal to $(\sigma - S)q + S$. I shall call this the *linear concession*.

A (to my mind) more realistic concession would be to set the cost of search equal to $2S - S(2S - \sigma)/((2S - \sigma) + (\sigma - S)x)$. This hyperbolic curve passes through the points $(0, S)$ and $(1, \sigma)$ and stays higher for longer than does the linear graph, as q increases in value from 0 to 1. I shall call this the *hyperbolic concession* to the advocate of faith-induced savings.¹¹

We are now in a position to lay out the formula for expected utility of search. It combines both social utility and epistemic utility. The social utility is based, generously, on the linear concession to those whose faith induces them to believe in savings. The epistemic utility incorporates concession of positive value even to empty-handed search.

$$U(q) = -S - (\sigma - S)q + q(1 - q)V + (1 - q)\delta q$$

The first two terms cover social utility, the second of which derives from the linear concession. The second two terms cover epistemic utility, the second of which derives from the concession of positive value to empty-handed search.

We see that epistemic utility is proportional to $q(1 - q)$ and thus achieves its maximum at $q = 1/2$. What this tells us is that, before we know whether SETI is successful, the contribution of success to the expected *epistemic* utility of SETI is greatest for those with *completely open minds* on the question of whether ETI will be discovered!¹²

We see also that the equation would be considerably simplified if we did not make the two concessions mentioned:

$$U(q) = -S + q(1 - q)V$$

Call this the equation for the *bare position*. In the bare position we have $S = \sigma$ and $\delta = 0$.

Our more complicated equation above, involving the two concessions, can be re-cast in the form

$$U(q) = -S + (\Sigma + W)q - Wq^2$$

where $\Sigma = S - \sigma$, $W = V + \delta$. (In the bare position we have $\Sigma = 0$ and $W = V$.)

This form of the question blurs the distinction between social and epistemic utilities, leaving a quadratic in q amenable to straightforward analysis. Assuming V is positive, $U(q)$ is maximized, on the interval $(0, 1)$, at the value $q = 1/2 + \Sigma/2W$, and its maximum is $-S + (\Sigma + W)^2/4W$. (So in the bare position, $U(q)$ attains its maximum value $-S + V/4$ at $q = 1/2$.) We are assuming here that Σ is less than W , which seems perfectly in order. Σ is no greater than S ; and W is no smaller than V .

But now we have a problem, most easily expressed in the context of the bare position as follows: are not S and V incommensurable? This is a judgement call

for wiser souls than I. What we *can* observe, however, is that the problem of comparing S and V only arises for rationally contemplatable search.

Now our formula for the maximum expected utility in the bare position does allow us to state a minimal rationality requirement on those contemplating whether to embark on a search. This is that the maximum expected utility should at least be positive! Thus in the bare position V , the proportionality constant, must exceed $4S$. This is needed to make it worth taking a look. In the light of the well-known Drake equation employed by SETI-advocates, I cannot resist the temptation to call this the *bare Gander Condition*. It is the bare Gander Condition that assures us that $U(q)$ will achieve its maximum on the interval $(0, 1)$.

The bare Gander Condition is a special case of the concessionary Gander Condition, which is that the maximum of expected utility, when the concessions are made, be positive. This is the condition that $(\Sigma + W)^2/W$ should exceed $4S$; that is, that $(S - \sigma + V + \delta)^2/(V + \delta)$ exceed $4S$. It is in general a little easier for V to meet the concessionary Gander condition rather than the bare Gander condition. But in my view both $S - \sigma$ and δ will be negligible; and it is easier to work with the cleaner, bare Gander condition.

The bare Gander condition has another constraining consequence. When V exceeds $4S$, then W exceeds $4S + \delta$; whence the term $\Sigma/2W$ cannot exceed $(S - \sigma)/(8S + 2\delta)$, which is less than $1/8$. Thus expected utility is constrained to be maximised in the interval $[1/2, 5/8]$.

If we are concerned with the ethics of resource allocation, the maximum expected utility $-S + (S - \sigma + V + \delta)^2/4$ of engaging in the search has to be weighed against the expected utility of not engaging in the search, and devoting 100 megadollars instead to other projects and causes. On behalf of the SETI advocate, let us imagine for the time being that the former expected utility is the greater,¹³ so that we choose to engage in the search. All parties to the considerations would now wish to see that particular value achieve a maximum, so as to be happy with the decision to expend effort and resources on the search.

4. THE PARADOX OF JUGGLED PROBABILITIES

The problematic tension that I want to point out, however, is this. For those of skeptical bent (those with very low q) the social cost rapidly outweighs the expected epistemic utility of SETI, no matter how high a value of V might be urged upon them. And the SETI community tends to respond with arguments that are, or might be, pragmatically self-defeating.¹⁴ They try to give plausibility arguments from cosmology, organic chemistry and evolutionary biology, designed to raise the subjective probability q in order to avoid this disastrous swamping effect of very low q . And in doing so, they go in for overkill. Their rehearsals of the fundamentals of cosmic and biological evolution, complete with slide-shows and impressive figures, seem designed to dispel any doubt whatsoever that ETI's

exist; even, to dispel any doubt that some of them would be detectably saying hello across interstellar space. They appear to me to make the mistake of overprobabilifying both the existence, and the detectable existence, of ETI. They ought, however, (on this analysis at least) to be very careful not to represent themselves as having a value of q much greater than $1/2$, and certainly not more than $5/8$! For otherwise a symmetrical swamping effect occurs, as can be ascertained from the graph of the curve $q \cdot (1 - q)$. Their rhetorical strategy ought, it seems to me, to be to dispel skepticism only to the point where one has a reasonably open mind about whether we shall discover ETI or no. Any amount of skepticism, and any amount of over-confidence in the matter, diminishes the expected utility of SETI! I propose to call this the paradox of juggled probabilities.

The nasty twist is not yet complete. Remember that q is the probability that we will find ETI if we look for it. Unless we are crazy enough to believe our search strategy one hundred percent effective, this means that the probability d of mere existence of signalling ETI has to be held higher than one-half if q itself is to be exactly one-half. Arguably, the epistemic or surprise value of discovery should then be $(1 - d) \cdot V$ rather than $(1 - q) \cdot V$. Now q will be $d \cdot e$, where e (between 0 and 1) is the effectiveness of search. So the epistemic value of discovery is $(1 - q/e) \cdot V$. The family of curves we get for varying e (between 0 and 1) is most interesting. The parabolic hump above the horizontal q -axis shrivels down and to the left, always passing through the origin, as e decreases from 1 to 0. The lower the value of e , the lower the value of q at which the epistemic value is maximized, and the lower too the maximum itself. Paradoxically: the less effectively we knowingly search, the less valuable a discovery would be; and the less effectively we knowingly search, the less confident we would have to be of finding what we were looking for, in order to make the most valuable possible discovery! This is disconcerting, to say the least. Rational decision theory seems to enjoin us, if we cannot search with one hundred percent effectiveness, to search diffidently (but not overly diffidently!), if search there must be. I cannot quite fathom this: to hold out the best prospects for a non-perfect search, one has to be ever so cool about the outcome. Maybe that's why SETI is headquartered in California.¹⁵

Being a philosopher based on Ohio, I would like to have a completely open mind on the question of whether we would discover ETI if we looked for it. My problem, as a philosopher, however, is that I simply cannot keep the relevant probability q anywhere near one-half. For me, the relevant probability q is not the one we have been talking about here – the probability of our finding ETI if we look for it. This latter probability is irrelevant to me because I cannot imagine it having any impact on human life in respects that matter. Show me a person already capable of setting this probability at one-half or higher and I will show you a person already mentally adjusted to the possibility of ETI, and therefore already willing to adopt the appropriately relativized cosmic perspective on

terrestrial life. No further Copernican turn is necessary or possible for such a mind. Far more relevant, for the purposes of deciding whether to search for ETI, is the probability that, if we find ETI via their signalling, then we shall be able to learn more than merely that ETI exists – that we shall be able to understand what they are trying to tell us, and learn from it.¹⁶

5. ON UNFORESEEABLE SPIN-OFF

My reflections so far have been based on the assumption that we are dealing with a search for such *F*'s as could hold out, demonstrably, no promise at all of unforeseen benefits, to human and other sentient beings,¹⁷ of a technological or therapeutic kind. The existence of *F*'s, that is, is a matter for purely intellectual, not theoretical, discovery. I want to discount the debatable value of the wondrousness of such a discovery – the suggestion that it might make much more broad-minded, enlightened and less anthropocentric creatures of us all – by means of the foregoing considerations about a mind already primed for the possibility of there being *F*'s. So the question to face now is whether we can ever be sure in advance that the satisfaction of purely intellectual curiosity might not lead, in ways impossible to predict at the time investigation is undertaken, to benefits of a tangible kind.

In asking this question about potential spin-offs, we are not to consider the sorts of spin-offs that might result from the basic science developed to help with the search itself. Thus for example, I shall not admit for such consideration spin-offs, from SETI, such as possibly improved aircraft navigation systems resulting from engineering advances in connection with a radio sky-survey. This is for the simple reason that research funds and human effort could be targeted precisely to such projects, so that they wouldn't have to be unforeseen spin-offs of some larger project of pure curiosity. And in targeting funds in this way, one need not be inimical to the pure research (in, say, pure mathematics and theoretical physics, information theory, complexity theory etc.) that could conceivably contribute to producing the tangible benefit. One just wouldn't go looking for *F*'s to get these other tangible benefits. One would get them more directly (and probably less expensively), but still in an intellectually enlightened way, fostering the relevant pure research to do so.

The question posed above, subject to the caveats just explained, seems to me to be both important and, with our present level of understanding of such matters, intractable. We have at present no theory of when and how a new discovery might connect, logically and nomologically, with other things and propositions known, so as to produce matters of great moment for human and cultural development. Pure curiosity might have to take refuge in this lack of general theory about the pragmatic efficacy of pure curiosity – for the time being, at least. To take refuge

in this way forever would be to invite a further pragmatic paradox about justifiable curiosity.

I am (I hope justifiably) curious about the (apparently purely intellectual) question as to whether there is justifiable curiosity about purely intellectual questions. Can there be any purely intellectual questions? If so, is curiosity about them justifiable? Are these themselves purely intellectual questions? If so, am I justifiably curious about them?¹⁸

NOTES

¹ Note that nowadays even questions about Hilbert space theory would count as theoretical, rather than purely intellectual, because of the applications made of that theory in quantum mechanics.

² This is not a pejorative label. They may be probabilities estimated by rational individuals or by groups of rational individuals; they may be probabilities based on actuarial and statistical data, and not just private hunches unfounded on any objective evidence.

³ Now known by the acronym SETI. For the rest of the paper, I shall focus on this example. But the points being made are entirely general, and concern any investigation at all which is designed simply to satisfy purely intellectual curiosity. So we could talk of SF, the Search for *F*'s of any kind, instead.

⁴ Note that this probability *q* is the probability of discovery given that a search is undertaken (action *B*), not the probability of existence, which would exceed *q*. ETI may exist without signalling; and even signalling ETI may go undetected.

⁵ I say 'failed', in scare quotes, because there is a sense in which, if the truth be that there is no ETI, then not finding it after searching for it is not a genuine failure. Thus I shall speak more neutrally of *empty-handedness*.

⁶ Admittedly, this is hardly an offsetting *epistemic* disutility; it is, rather, a personal one, to be subsumed into the *social* disutility of 'failed' search.

⁷ Thus, for example, if the expensive microwave sky survey project fails after ten years, SETI researchers might turn instead to the relatively low-cost methods of optical SETI, that can be employed by amateur astronomers operating out of their own back yards. See S. Kingsley, 'The Search for Extraterrestrial Intelligence (SETI) in the Optical Spectrum: A Review', and 'Amateur Optical SETI', in S. Kingsley (ed.), *Proceedings of the SPIE Symposium on the Search for Extraterrestrial Intelligence in the Optical Spectrum*, OE LASE '93, Vol. 1867, Los Angeles, 1993.

⁸ In general, we are looking for some utility-function $f(p)$. To make matters simple, I have chosen $f(p)$ to be proportional to $1 - p$. The only constraints so far on the function $f(p)$ are that it should be monotonic decreasing, passing through the points $(0, V)$ and $(1, 0)$. We could consider other curves besides a straight line which do this. We could consider $V \cos(p\pi/2)$, or (V times) the arc of a circle with unit radius and centre $(0, 0)$, or (V times) the arc of a circle with unit radius and centre $(1, 1)$, The general point being made in what follows is invariant across these alternatives. There will be some definite probability value that it is optimal to attach to the prospect that searching for *F*'s will turn up *F*'s.

⁹ This will be more nearly the case when SETI piggybacks on existing technologies whose primary users see some benefit in allowing SETI researchers to use them. For example, their instruments might come to be better calibrated; or there might be serendipitous side-effects, in the primary area, from SETI research. (One such contrary-to-fact serendipity might have been a SETI researcher discovering pulsars. As it happens, the astrophysicist who did discover pulsars, the Nobel Laureate Professor Anthony Hewish, did toy for a while with the hypothesis that the pulses came from an ETI.)

¹⁰ "Blessed be they who believe, for unto them shall it be given."

¹¹ Ironically, it is the linear concession which exaggerates the extent of the cost-reduction expected with higher *q*! The hyperbolic concession is named after the hyperbola rather than any hyperbole involved.

¹² *Not whether ETI exists*. As noted earlier, *q* is the probability of discovery, not the probability of

existence. The latter would have to exceed q unless we had the hubris to believe that our search would be one hundred percent effective. This higher probability (call it d) that (signalling) ETI exists (within our galaxy) is what is calculated by the well-known Drake equation. See below for further discussion.

¹³ This is to assume a lot more than just the Gander Condition.

¹⁴ Bearing in mind that SETI is but one example of SF as previously defined, the cautionary considerations to follow would apply to advocacy of SF for any F .

¹⁵ SETI Institute, 2035 Landings Drive, Mountain View, CA94043, for the curious.

¹⁶ For a skeptical argument against this possibility, see my paper 'The Decoding Problem: Do we need to Search for Extra Terrestrial Intelligence to Search for Extraterrestrial Intelligence?', in S. Kingsley (ed.), *Proceedings of the SPIE Symposium on the Search for Extraterrestrial Intelligence in the Optical Spectrum*, OE LASE '93, Vol. 1867, Los Angeles, 1993.

¹⁷ I would want to include here any beings that fall within the scope of moral concern.

¹⁸ I would like to thank the referee for comments that provoked my treatment of the two kinds of concession discussed above. Work on this paper was partially supported by an Overseas Fellowship at Churchill College, Cambridge.

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