Ch. 7 Entropy

-Clausius Inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

- Valid for all cycles - reversible or irreversible
- Internally or totally reversible cycles:

$$\oint \frac{\delta Q}{T} = 0$$

T is the thermodynamic temperature at the boundary where transfer occurs

-Entropy (s)

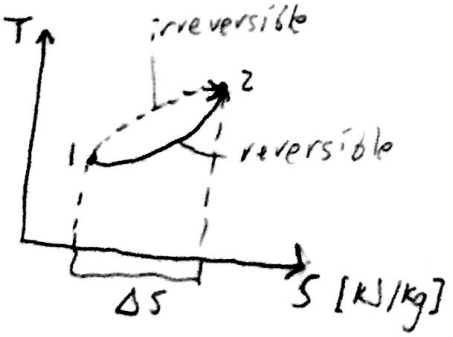
$$\left(\frac{\delta Q}{T}\right)_{\substack{\text{internally} \\ \text{reversible}}} = dS \quad [kJ/K]$$

- Extensive property
- Per unit mass: s $[kJ/kg\,K]$
- Change in system entropy during process:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int\,rev}$$

- Usually care about change - not the absolute value
- Assign $S = 0$ at some reference state
- Often choose $S_1 = 0$

- ΔS is the same between 2 states regardless of path
- Same ΔS between reversible and irreversible processes between the same two states



same ΔS for both

- Special Case: Internally Reversible Isothermal Heat Transfer

$$\Delta S = \frac{Q}{T_0}$$

T_0 = constant temp of system
Q = heat transfer

7-2 Increase of Entropy Principle

- The entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant.

$$\Delta S_{sys} = S_2 - S_1 \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

$$S_{gen} = \Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} \geq 0$$

$$S_{gen} \begin{cases} >0 & \text{irreversible} \\ =0 & \text{Reversible} \\ <0 & \text{impossible} \end{cases}$$

Key Remarks:

1. $S_{gen} \geq 0$ for a process to occur
   - Processes can only occur in a certain direction
   - Must comply with increase of entropy
2. Entropy is nonconserved
   - Only considered "conserved" for idealized reversible processes
   - Increases for <u>all</u> real processes
3. Entropy generation is a measure of magnitudes of irreversibilities in a process
   - Greater irreversibilities → ↑ΔS
   - Irreversibilities degrade engineering systems

7-3 Pure Substances



- Use tables to determine properties

① P_1, T_1 → $s_1 \equiv s_f$ at T_1

② T_2, x_2 → $s_2 = s_f + x_2 s_{fg}$

③ T_3, P_3 → s_3

- Closed system with mass m:

$$\Delta S = m\Delta s = m(s_2 - s_1)$$ s_2, s_1 per unit mass

7-4 Isentropic Processes

- Isentropic:
  - Internally reversible
  - Adiabatic

$\Delta s = 0$

$s_1 = s_2$

- Technically impossible in reality - idealized
- Isentropic process can still serve as a good model
- Can serve as a comparison point for calculation of efficiencies

T-S Property Diagrams

- 2 commonly used
  - Temperature- Entropy (T-S)
  - Enthalpy - Entropy (h-s)

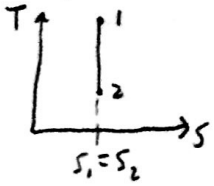
- T-S Diagram

$$Q_{int\,rev} = \int_1^2 T\,dS \quad [kJ]$$

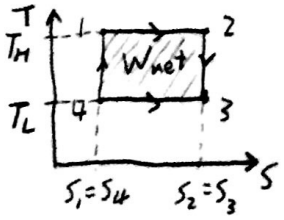
- Area under the process curve on a T-S diagram represents heat transfer during an internally reversible process



Isentropic:



- Carnot Cycle



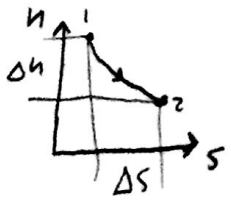
$W_{net} = Q_H - Q_L$

4-1 and 2-3 are isentropic

1-2 and 3-4 are reversible isothermal

- h-s Diagram

- Called Mollier diagram
- Valuable for steady-flow cases
- Shows measure of irreversibilities and primary property for first-law analysis



7-6 What is Entropy?

- Measure of molecular disorder or randomness
- system becomes more disordered
  ↓
  Molecule positions less predictable
  ↓
  entropy increases
- Boltzmann Relation

  $S = k \ln W$

  with: k = Boltzmann's Constant ($1.3806485 \times 10^{-23}\ m^2 kg/s^2 K$)

  W = number of possible microstates relevant to system

- Gibbs formulation

  $S = -k \sum p_i \log p_i$

  with: p_i = non-uniform probability of microstates = $\frac{1}{W} << 1$

  - More general than Boltzmann
- Entropy increases with increase of thermal disorder
- 3rd Law of Thermodynamics:
  - Entropy of a pure crystalline substance at T = absolute zero is zero
  - Entropy relative to this reference = absolute entropy
- Entropy and Work
  - Work is organized form of energy
  - Free from disorder, randomness → free from entropy
  - There is no entropy transfer associated with energy transfer as work

- First law: quantity of energy is preserved during actual process
- Second law: quality of energy decreases during actual process

- Heat is disorganized energy
  - Some entropy flows with heat

7-7 The T ds Relations

$$ds = \frac{du}{T} + \frac{P\,dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$

v = specific volume

- Valid for reversible, irreversible
- Independent of process
- Find ΔS by integrating these equations
- May replace:

$$du = C_v\,dT$$
$$dh = C_p\,dT$$

7-8 Liquids and Solids

$$s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \approx C_{avg} \ln\left(\frac{T_2}{T_1}\right)$$

with: C_{avg} = average specific heat over given temp. interval of process

Isentropic: $s_2 - s_1 = C_{avg} \ln\left(\frac{T_2}{T_1}\right) = 0 \rightarrow T_2 = T_1$

- Therefore isentropic → isothermal for incompressible substance

7-9 Ideal Gases

$$ds = C_v \frac{dT}{T} + R\frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 C_v(T)\frac{dT}{T} + R\ln\left(\frac{v_2}{v_1}\right) \quad \text{or} \quad s_2 - s_1 = \int_1^2 C_p(T)\frac{dT}{T} - R\ln\left(\frac{P_2}{P_1}\right)$$

- Constant Specific Heats

- Approximated analysis

$$s_2 - s_1 = C_{v\,avg}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = C_{p\,avg}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

average specific heats over T_1 to T_2 range

- Variable Specific Heats

- Exact Analysis

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1}$$

with: $s^\circ = \int_0^T C_p(T)\frac{dT}{T}$

- function only of T
- zero at T = 0 Kelvin
- Use tables to find values

- Ideal Gas Isentropic Relations

s = constant $\quad k = C_p/C_v$ → Varies with T, use approximate average

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}$$

Tv^{k-1} = constant

or: $TP^{\frac{1-k}{k}}$ = constant

Pv^k = constant

- Ideal Gas Variable Specific Heats Isentropic Relations

$$s_2^\circ = s_1^\circ + R \ln\left(\frac{P_2}{P_1}\right)$$

- Relative Pressure and Specific Volume - Isentropic processes

- Relative Pressure: - Dimensionless quantity
  - Air: Table A-17

$$P_r = e^{s^\circ/R}$$

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}$$

- Relative specific volume:

$$v_r = \frac{T}{P_r}$$

$$\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}$$

7-10 Reversible Steady-Flow Work

$$W_b = \int_1^2 P\,dv$$ - Steady-flow devices

- Reversible work output:

$$w_{rev} = -\int_1^2 v\,dP - \Delta ke - \Delta pe \quad [kJ/kg]$$

$$\Delta ke \approx 0$$
$$\Delta Pe \approx 0$$

$$w_{rev} = -\int_1^2 v\,dP$$

- Work input:

$$w_{rev,in} = \int_1^2 v\,dP + \Delta ke + \Delta pe$$

- Incompressible:

$$w_{rev} = -v(P_2 - P_1) - \Delta ke - \Delta Pe$$

- Bernoulli Equation

- Internally reversible
- No work interactions
- Steady flow
- Incompressible

$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

(V: velocity of fluid)

- Key takeaways:
  - Larger specific volume → larger reversible work produced/consumed
  - With a reversible process, steady-flow devices deliver the most and consume the least work

7-11 Minimizing compressor work

$$W_{rev,in} = \int_1^2 v\,dP$$

- Isentropic: $Pv^k = \text{constant}$

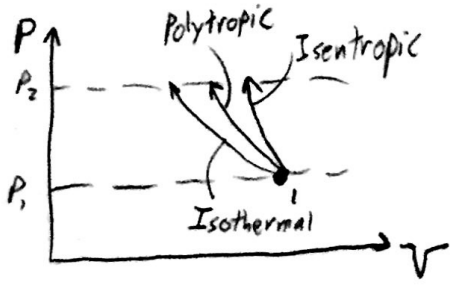
$$W_{comp,in} = \frac{kR(T_2 - T_1)}{k-1} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$

- Polytropic: $Pv^n = \text{constant}$

$$W_{comp,in} = \frac{nR(T_2 - T_1)}{n-1} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

- Isothermal: $Pv = \text{constant}$

$$W_{comp,in} = RT\ln\left(\frac{P_2}{P_1}\right)$$



-Multistage Compression With Intercooling

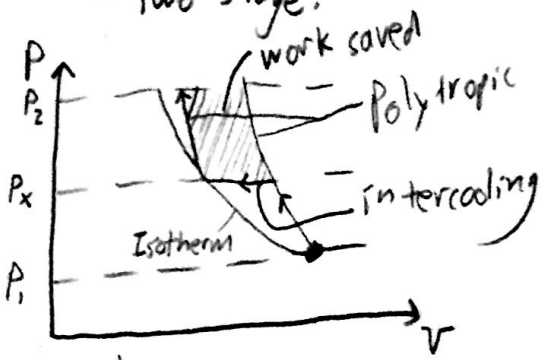
- Cooling gas in compression reduces work required

Compresser → cooled → compressed → cooled → compressed
(in intercooler)

- Attractive for very high pressure

$$P_x = (P_1 P_2)^{1/2} \rightarrow \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

- Two stage:



- Minimize compression work during 2-stage compression: pressure ratio across each stage must be the same

7-12 Steady-Flow Device Isentropic Efficiencies

- Turbine

$$\eta_T = \frac{\text{actual work}}{\text{Isentropic work}} = \frac{w_a}{w_s} \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

(h_{2a}: actual enthalpy; h_{2s}: isentropic enthalpy)

- Compressors

$$\eta_c = \frac{\text{actual work}}{\text{Isentropic work}} = \frac{w_s}{w_a} \approx \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

Isothermal: model cooling:

$$\eta_c = \frac{w_t}{w_a} = \frac{\text{Isothermal work}}{\text{actual work}}$$

- Pumps

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

- Nozzles

$$\eta_N = \frac{\text{Actual exit KE}}{\text{Isentropic exit KE}} = \frac{V_{2a}^2}{V_{2s}^2}$$

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

7-13 Entropy Balance

- Increase of Entropy Principle:

$$\left[\begin{matrix}\text{Total}\\ \text{Entropy}\\ \text{In}\end{matrix}\right] - \left[\begin{matrix}\text{Total}\\ \text{Entropy}\\ \text{out}\end{matrix}\right] + \left[\begin{matrix}\text{Total}\\ \text{Entropy}\\ \text{Generated}\end{matrix}\right] = \left[\begin{matrix}\text{Change in total}\\ \text{entropy of system}\end{matrix}\right]$$

- ΔS_{system}

$$\Delta S_{system} = S_{final} - S_{initial} = S_2 - S_1$$

- Entropy Transfer Mechanisms

- Heat Transfer

$$S_{heat} = \frac{Q}{T} \qquad T = \text{constant}$$

$$S_{heat} = \int_1^2 \frac{\delta Q}{T} \approx \sum \frac{Q_k}{T_k} \qquad T = \text{not constant}$$

- Work

$$S_{work} = 0$$

- Mass Flow

$$S_{mass} = ms \qquad s \text{ in } kJ/kgK$$

Changing Properties:

$$\dot{S}_{mass} = \int_{A_c} s \rho V_n \, dA_c \qquad \text{with } A_c = \text{Flow cross-sectional area}$$

$$S_{mass} = \int s \, \delta m = \int_{\Delta t} \dot{S}_{mass} \, dt$$

- Entropy Generation

$$S_{in} - S_{out} + S_{gen} = \Delta S_{system} \quad [kJ/k]$$

Rate form:

$$\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen} = \frac{dS_{system}}{dt} \quad [kW/k]$$

-Closed Systems

$$\sum \frac{Q_k}{T_k} + S_{gen} = \Delta S_{sys} = S_2 - S_1$$

Adiabatic: $Q = 0$

$$S_{gen} = \Delta S_{adiabatic\ system}$$

Surroundings:

$$S_{gen} = \sum \Delta S = \Delta S_{system} + \Delta S_{surroundings}$$

$$\Delta S_{system} = m(s_2 - s_1)$$

$$\Delta S_{surroundings} = \frac{Q_{surr}}{T_{surr}}$$

Control Volumes:

$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{gen} = (S_2 - S_1)_{CV} \quad [kJ/K]$$

Rate form:

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{gen} = \frac{dS_{CV}}{dt} \quad [kW/K]$$

Steady-Flow:

$$\dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

Steady + Single-Stream:

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$$

Steady + Single-Stream + adiabatic:

$$\dot{S}_{gen} = \dot{m}(s_e - s_i)$$

Reversible, adiabatic:

$$s_e = s_i$$