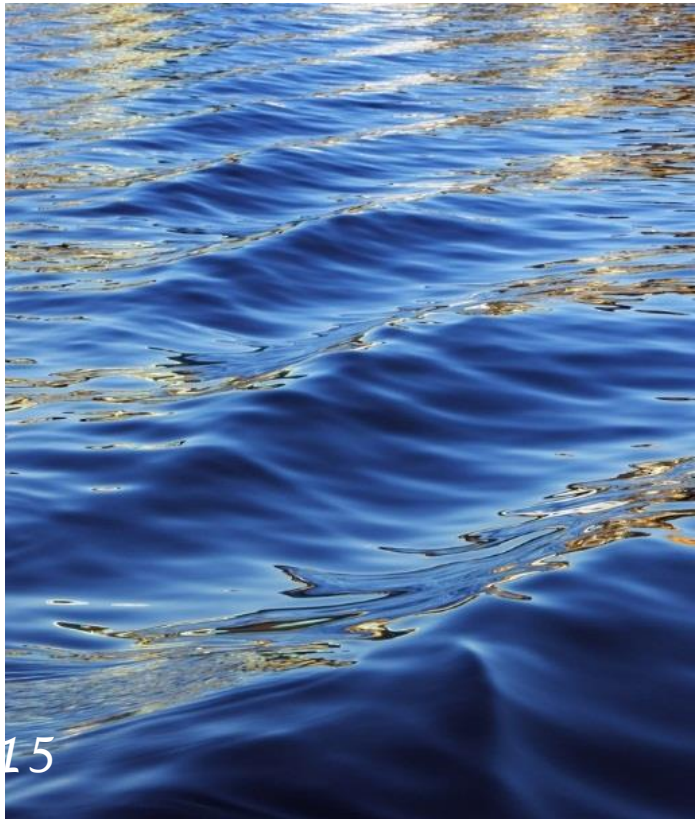
Multiple Group Analysis in PA & General SEM

NKBowen, Spring 2021



First, a reminder about nested models and chi square difference tests.

1. What is a nested model?
2. How do you compare the fit of nested models?
3. What happens to fit when you free more parameters?
4. What happens when you constrain more parameters?

Nested models

- A model "b" is nested in another model ("a") if "b" contains a subset of the freely estimated parameters of model "a," which means:
  - "a" has more freely estimated parameters than "b"
  - "b" has more parameters that are fixed (e.g., to 0, 1, or some other constant) *or* constrained to be equal (e.g., $l_1 = l_2$).
  - Model "b" is more restrictive than Model "a."
  - Model "b" has more degrees of freedom than "a."

Nested models

- Start by testing the less restrictive model (Model a) and obtaining the χ^2 value and df.
- Constraints are added and the new model (model b) is run.
  - χ^2 will get higher (worse), and df will go up (good): the question is, will fit get *significantly* worse
- The difference in χ^2 between the two models is evaluated for the difference in df between the two models.
  - If the change in χ^2 is greater than the critical ratio for the change in df, fit got *significantly worse* and you retain model a

Multiple Group Analysis

1. Path Analysis
2. General SEMs

(Multiple group testing in CFA is called measurement invariance testing)

Multiple Group Testing

How it works:

Two covariance matrices—one for each group

Two sets of parameters to be estimated—one set for each group

Sequence of restrictions (matrix by matrix) and χ^2 difference tests

Proceed to next step if χ^2 is not significantly higher (worse) with new constraints

Retain the previous model if χ^2 is significantly higher (worse), OR free individual parameters within the constrained matrix to see which one or more is statistically different across groups.

Multiple Group Analysis—Control over Specification

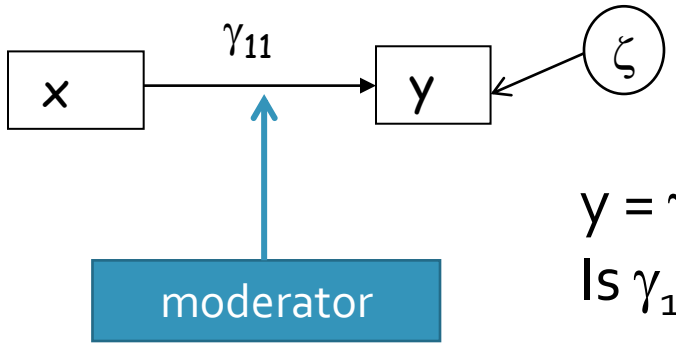
In previous analyses we have specified certain paths and variances to be fixed at 1 or 0

We have specified paths and variances (usually) to be freely estimated

Now we'll be specifying *equality constraints* for groups on path and variance estimates

MGA in Path Analysis

Test hypotheses about whether the effects of one variable on another vary for individuals with different scores on another variable



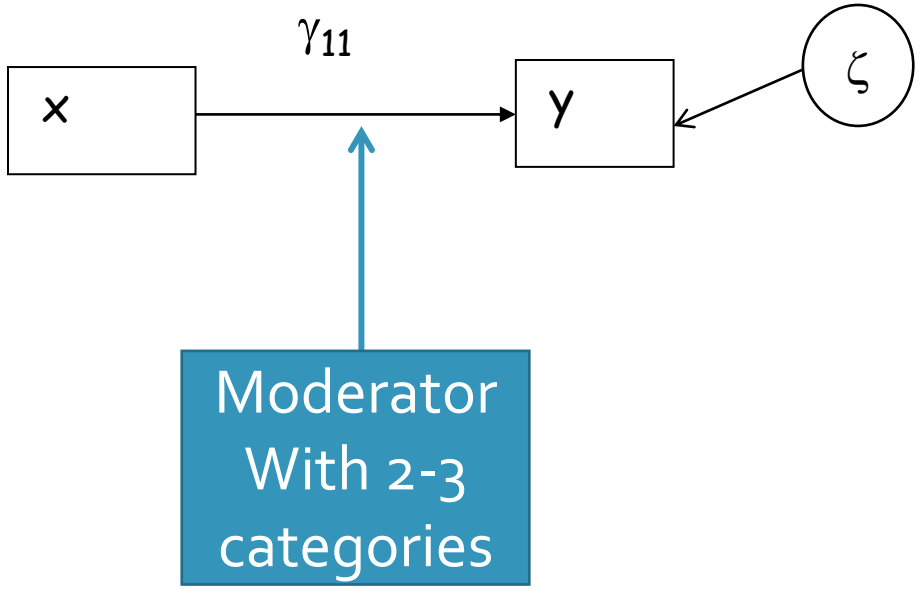
$y = \gamma_{11}(x) + \zeta$

Is γ_{11} statistically different for the groups?

Is the effect of x on y different for boys and girls, Latinos from Central America and Latinos from Spain, etc.

MGA in Path Analysis

Multiple group tests in path analysis are tests of moderation or interactions among variables. In a conventional regression, the interaction would be tested using a product term. MGA in SEM is for categorical moderators.



MGA in Path Analysis

The sequence of nested tests in path analysis recommended by Bollen (1989) is:

How is this model different than our usual single group model?

1. Specify your grouping variable. Test a model with parameters in the Beta, Gamma, Phi, and Psi matrices unconstrained across the groups. This model will have the best fit of all your models. It will have the most freely estimated parameters and the fewest df.

   If this initial model has poor fit, no need to continue. Fit will only decline with added constraints.

MGA in Path Analysis

If the first model has adequate fit:

2. Test the model in #1 except with the Betas (B) constrained to be equal across groups.
    --do a chi square diff test comparing 1 and 2
    --if the change in χ^2 per change in df is non-significant, proceed to next step
3. Test the model in #2 except with Gammas also constrained to be equal across groups. (B,Γ)
    --do a chi square diff test comparing model 2 and 3
    --if the change in χ^2 per change in df is non-significant, proceed to next step

MGA in Path Analysis

4. Test the model in #3 except with the Psi matrix also constrained to be equal across groups. (B,Γ,Ψ)
   --do a chi square diff test comparing 3 and 4
   --if the change in χ^2 per change in df is non-significant, proceed to next step

5. Test the model in #4 except with the Phi matrix also constrained to be equal across groups. (B,Γ,Ψ,Φ)
   --do a chi square diff test comparing 4 and 5
   --if the change in χ^2 per change in df is non-significant, the model is equivalent across the groups

MGA in Path Analysis

If at any step, chi square becomes statistically significantly worse, return to the previous model and constrain one path at a time until you find the one or more that are different across groups.

In path analysis, your theory determines whether you are hoping for invariance or differences in parameters. Usually, the betas and gammas are of primary interest.

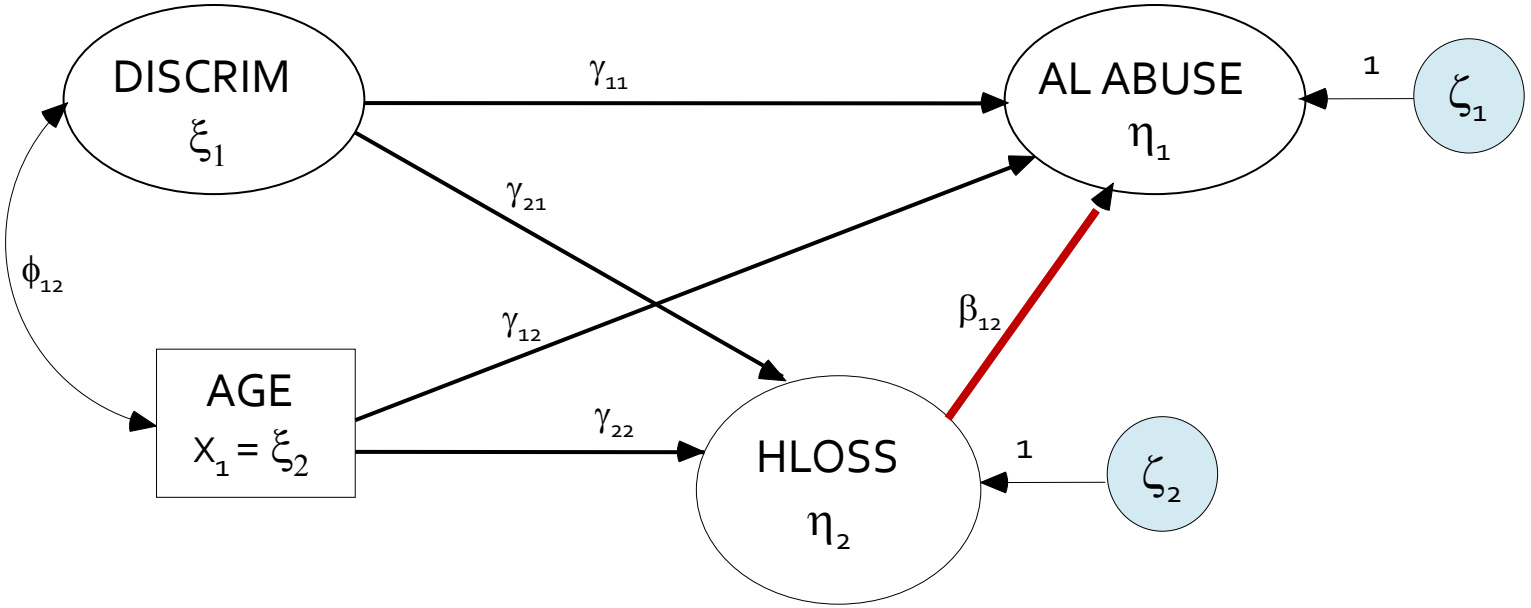
With Mplus, researchers can also test the equivalence of variable means across groups.

Multiple Group Analysis with General SEM

- As with path analysis, MGA with general structural models, is a test of the moderation of structural paths.
  - Are the relationships among variables in the model statistically the same for different groups?

- The measurement model's invariance must be established before structural paths are tested

H_B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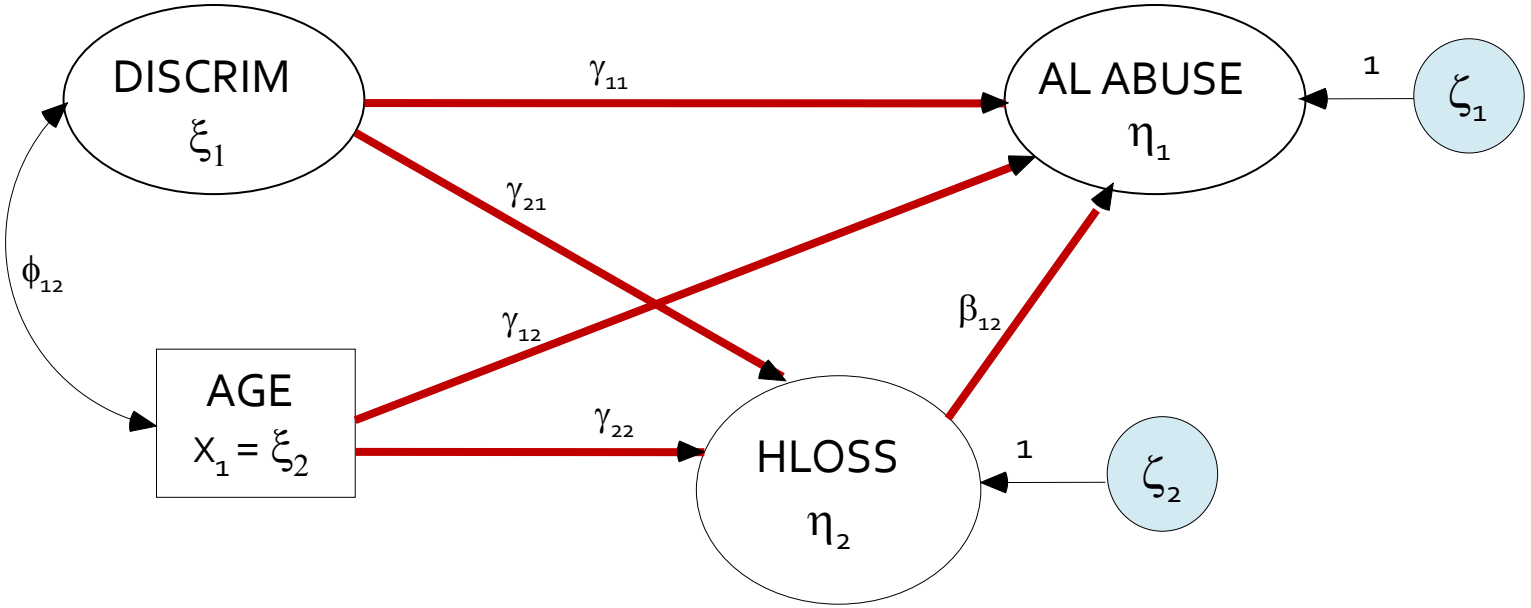
ζ_2

$H_{B\Gamma}$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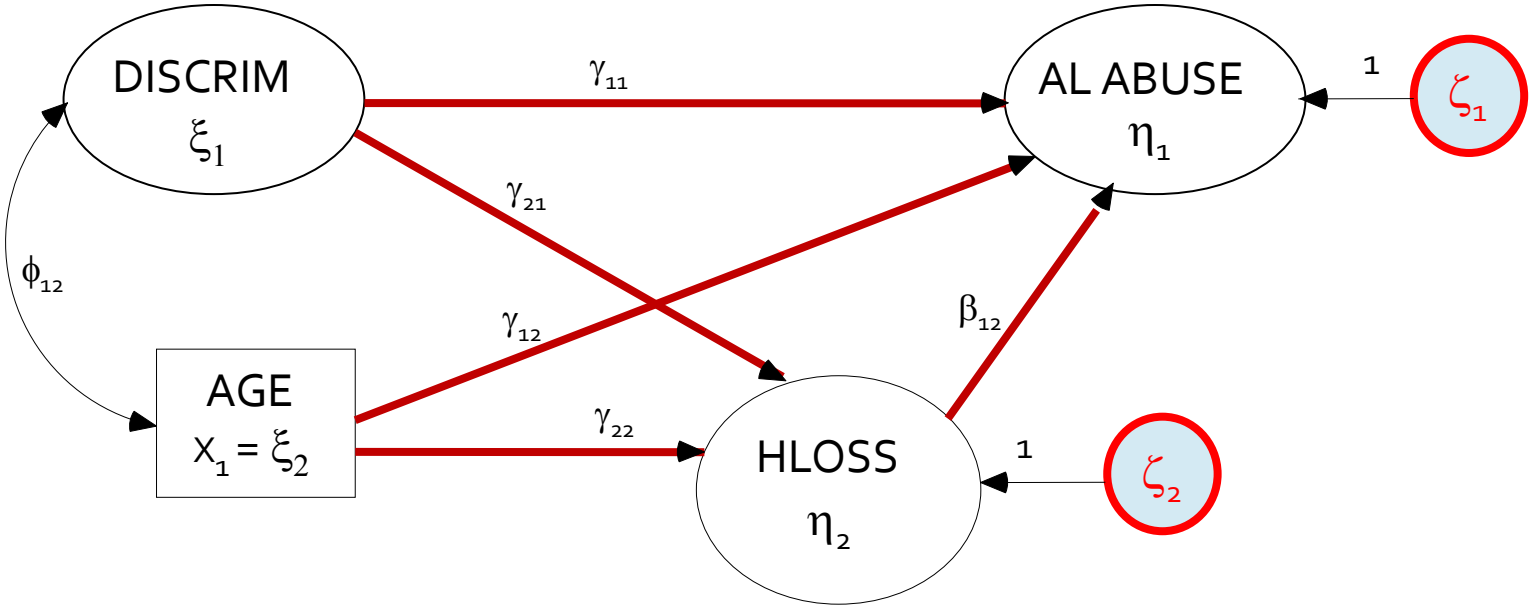
$H_{B\Gamma\Psi}$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$H_{B\Gamma\Psi\Phi}$