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Supply Chain Scheduling: Distribution Systems

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We study conflict and cooperation issues arising in a supply chain where a manufacturer makes products which are shipped to customers by a distributor. The manufacturer and the distributor each has an ideal schedule, determined by cost and capacity considerations. However, these two schedules are in general not well coordinated, which leads to poor overall performance. In this context, we study two practical problems. In both problems, the manufacturer focuses on minimizing unproductive time. The distributor minimizes customer cost measures in the first problem and minimizes inventory holding cost in the second problem. We first evaluate each party's conflict, which is the relative increase in cost that results from using the other party's optimal schedule. Since this conflict is often significant, we consider several practical scenarios about the level of cooperation between the manufacturer and the distributor. These scenarios define various scheduling problems for the manufacturer, the distributor, and the overall system. For each of these scheduling problems, we provide an algorithm. We demonstrate that the cost saving provided by cooperation between the decision makers is usually significant. Finally, we discuss the implications of our work for how manufacturers and distributors negotiate, coordinate, and implement their supply chain schedules in practice.

Key words: supply chain scheduling; distribution system; cooperation in decision making; algorithms.
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1. Introduction

The most active area within manufacturing research currently is *supply chain manufacturing.* A supply chain represents all the stages at which value is added to a manufacturing product, including the supply of raw materials and intermediate components, finished goods manufacturing, packaging, transportation, warehousing, and logistics. A central issue in supply chain management is *coordination* between the decisions made at different stages of the supply chain, e.g., between a manufacturer and a distributor. The different objectives of these decision makers and the data specific to each stage (costs, changeover times, customer due dates, and so on) define optimal schedules that, in general, differ between the stages. If each stage uses its optimal schedule, then the overall system performance is poor in many cases. The coordination problem is therefore to find schedules for both stages that provide better overall system performance. The mathematical modeling of scheduling coordination in supply chains is still a relatively undeveloped area of research. This paper studies coordinated decision making between a manufacturer and a distributor in two practical distribution systems.

General motivation for coordinated decision making in supply chains is provided by Banker and Khosla (1995). Karmarkar (1996) recommends an integrated approach to operations management and marketing decisions. In an extensive review paper, Thomas and Griffin (1996) identify several streams of supply chain management research, such as coordinated planning in inventory-distribution systems (Muckstadt and Roundy 1993), coordination in production-distribution systems (Chandra and Fisher 1994), and buyer-vendor coordination (Anupindi and Akella 1993). They also identify the need for research addressing *deterministic* and *operational* supply chain issues rather than stochastic and strategic ones. This paper addresses that need by considering operational supply chain issues with deterministic data. Sarmiento and Nagi (1999) survey the literature of integrated production and distribution models, pointing out that the recent trend towards reduced inventory levels creates a need for greater coordination between decisions at different stages of a supply chain.

The need for coordination between a manufacturer and a distributor arises in various practical situations. Two previous works, in particular, motivate the problems addressed in this study. Our first problem is motivated by Hurter and van Buer (1996), who study a problem of coordination between the printing (i.e., production) and distribution departments of a newspaper company. Here, the printing department prefers to produce jobs according to an ideal schedule that minimizes overall production time, whereas the distribution department prefers that products which will be shipped over longer distances are produced first. Hurter and van Buer's objective is to reduce the number of vans required to deliver the newspapers to drop-off points, while satisfying the vehicle capacity and time constraints. Their methodology is to solve several instances of the capacitated vehicle routing problem with time windows and then to design a production schedule that supports the resulting routes. They assume that the printing and distribution departments will cooperate on this schedule. They compare their results with the prevailing practice at a medium-sized newspaper company, and demonstrate a significant reduction in the number of vans required and the total distribution time. Our second problem is motivated by Blumenfeld et al. (1991), who consider a coordination problem in which one producer makes several products and has several customers, each cyclically receiving its specific product type. The production sequence is based on setup and in-process inventory costs; however, the distribution schedule depends on freight and in-transit inventory costs. Their objective is the minimization of overall inventory costs, and they analyze the trade-off between the benefits of coordination and increased management complexity.

Within traditional supply chain scheduling research, the literature that is most closely related to this paper considers production and distribution systems with multiple stages. Ow et al. (1988) discuss a multi-agent, or distributed, scheduling system. However, they do not evaluate the benefits of cooperation in detail. Lee and Chen (2001) consider the integration of transportation constraints, including time and capacity, into scheduling decisions. Chen and Vairaktarakis (2005) study an integrated scheduling model involving production and distribution operations, using an objective that considers both customer service level and distribution costs. Both of the last two papers study a variety of mathematical models, for which they provide either efficient algorithms, proofs of intractability, or heuristic analysis. None of these three papers, however, explicitly models conflict and coordination issues in a production and distribution system.

The study of coordination in scheduling decisions is called *supply chain scheduling*. The first study of supply chain scheduling, by Hall and Potts (2003), evaluates the benefits of coordinated decision making in a supply chain where a supplier makes deliveries to several manufacturers, who in turn supply several customers. They develop models which minimize the total scheduling and batch delivery costs. They demonstrate that coordination between a supplier and a manufacturer may reduce the total scheduling cost in this system by values in a range of at least 20% up to arbitrarily close to 100%, depending upon the scheduling objective being considered. Agnetis et al. (2006) study models for re-sequencing jobs, using limited buffer storage capacity, between a supplier and several manufacturers with different ideal sequences. They describe efficient solution procedures for a variety of objectives and indicate where coordination is or is not beneficial. Chen and Hall (2004) analyze a variety of models that arise when several suppliers deliver their products to the same assembly facility, depending upon the relative bargaining power of the various decision makers. They demonstrate that the conflicts between individual optimal schedules and the benefits of coordinated decision making are both substantial in many cases. Chang and Lee (2003) consider a two-stage scheduling problem that is similar to a flowshop, and analyze the worst case performance of the locally optimal decisions at each stage with respect to overall system performance. Li and Xiao (2004) focus on the coordination of lot streaming, i.e., lot splitting, decisions between a supplier and a manufacturer. They develop methods to structure costs and incentives in such a way that the objectives of both decision makers are aligned with the overall system objective. Kreipl and Pinedo (2004) study the interface of medium term planning and production scheduling within a supply chain and demonstrate their coordination at a brewer.

In this paper, we consider two problems in manufacturing and distribution. Problem 1 is a scheduling problem in which a manufacturer produces variations of a single perishable product, and a distributor must deliver the appropriate variation to each of several customers by specified times within a single day. A setup time occurs when the manufacturer changes from the production of one variation of the product to another. This problem arises in newspaper printing and distribution (Hurter and van Buer 1996). Conflict occurs because the manufacturer ("printer" in the case considered here) prefers a schedule with a minimum number of setups, whereas the distributor prefers a schedule in which all newspapers are delivered by a certain time.

Problem 2 is a scheduling problem in which a manufacturer produces two products in a single production line, and a distributor has to manage inventory and deliver specified combinations of the different products to several retailers within a planning horizon. There is a setup cost when the manufacturer changes from the production of one product to another. This problem arises, among other situations, in the manufacture and distribution of household cleansers (Lee and Tse 1992), plastic food containers (Bao-Ning Company 1999), soft drink bottling (Packaging World 2000), and candy bars (Masterfoods 2002). The manufacturer prefers a schedule that produces individual products in large batch sizes, in order to minimize total setup costs. However, in order to minimize its inventory holding cost, the distributor prefers a schedule that batches products which will be shipped together to retailers. We refer the reader to Potts and Kovalyov (2000) for an extensive discussion on models that integrate scheduling with batching decisions.

Both of these problems create a need for the manufacturer and the distributor to coordinate their decisions within the supply chain. In each, the manufacturer and the distributor may or may not be part of the same company. In some cities with two newspapers, one newspaper is printed on presses owned by the other; in this case, schedules and fees must be negotiated. Ideally, the manufacturer and the distributor can agree on a common schedule that minimizes overall system cost. Both problems are described in more detail in Section 2.

In each problem, we evaluate the benefits of cooperation from a detailed operational standpoint. This analysis is based on precise processing times or demands that have no closed form functional representation. Therefore, our results are derived as averages of relative percentages computed over a large number of instances.

This paper is organized as follows. In Section 2, we formally define the two problems considered and describe our notation and assumptions. Section 3 presents an evaluation of the conflict that results when one decision maker imposes its optimal schedule on the other. Section 4 considers the effect of different assumptions about the relative bargaining power of the manufacturer and the distributor. This requires solving the scheduling problem of the non-dominant decision maker under a constraint imposed by the dominant decision maker. In Section 5, we solve the overall system problems and evaluate the cost savings that result from coordination between the manufacturer and the distributor. We also discuss how manufacturers and distributors can negotiate, coordinate, and implement their supply chain schedules in practice. Finally, Section 6 contains a conclusion and some suggestions for future research.

2. Two Problems

Problem 1 is motivated by the production and distribution of a newspaper (Hurter and van Buer 1996). It is also applicable to various process industries, such as industrial chemicals (Lei et al. 2006). Consider a newspaper publisher, for example the *Dallas Morning News,* which serves a large metropolitan area. Each day, it produces an issue that contains five standard sections: *News, Metropolitan, Sports, Business* and *Living*. There are also supplements, such as community news, event listings, and advertisements, which are specific to zones within the metropolitan area, such as Plano, Irving, and so on. Each zone receives its own specific version of each day's issue. To formulate this problem for a general newspaper, let n be the number of different editions printed, one for each of n delivery zones. Zone Z_i, for $i = 1, \ldots, n$, is divided into ℓ_i subzones, each with a single local distribution center. The jth distribution center in zone Z_i is denoted by Z_{ij}.

Consider the manufacturing (i.e., printing) process. The printer generally does not begin processing until midnight so that it can include information (including zone-specific news) that is as up-to-date as possible. Let job J_{ij} denote the printing of enough copies of all sections and supplements required to serve distribution center Z_{ij}. Let schedule σ denote the order in which all jobs J_{ij}, for $i = 1, \ldots, n$, $j = 1, \ldots, \ell_i$, are processed non-preemptively in the production system. In Corollary 1 below, we show that there is no advantage for either the printer or the distributor to consider preemptive schedules. If the printer uses schedule σ, then the time at which the printing of job J_{ij} completes is denoted by $C_{ij}^M(\sigma)$. Before printing a job for a subzone within Z_i, the printing presses must be set up for zone Z_i, which requires time s_i. If jobs for two different subzones within the same zone are performed, then no additional setup is required between them.

Now consider the distribution process. Newspapers are delivered by truck from the plant to the distribution centers. In large metropolitan areas, adult carriers using automobiles then pick up the newspapers from the distribution centers and deliver them to subscribers' homes (Hurter and van Buer 1999). Subscribers are allocated to carriers, and carriers are assigned to distribution centers so that each distribution center receives and distributes one truckload of newspapers. The travel time from the plant to distribution center Z_{ij} is denoted by t_{ij}. The delivery of newspapers to center Z_{ij} has due date d_{ij}, which is based on the time required by the carriers for delivery; for most distribution centers, this time is between 4:00 AM and 4:30 AM. If the printer uses schedule σ and the distributor uses schedule ν, then the time at which the delivery to center Z_{ij} is completed is denoted by $C_{ij}^{D}(\sigma, \nu)$.

Our assumption that each distribution center receives and distributes one truckload of newspapers is not restrictive. If multiple truckloads are demanded by a distribution center, then this assumption can be made without loss of generality, since we can treat that center's location as though it has multiple distribution centers, each of which demands one truckload. Hence, the time required for the printer to produce enough newspapers for any distribution center within zone Z_i is a constant, which we denote by p_i.

Since the printer does not deal directly with the customers, its goal is to minimize its overhead costs. Hence, its objective is to find a schedule σ^m that minimizes $C_{\max}^{M}(\sigma) = \max_{1 \le i \le n} \max_{1 \le j \le \ell_i}\{C_{ij}^{M}(\sigma)\}$ over all production schedules σ. The distributor wants to deliver each truckload of newspapers before its due date. This objective can be operationalized by using maximum lateness, number of tardy jobs, or total tardiness. We choose maximum lateness over number of tardy jobs because it is preferred to have a few subzones' newspapers delivered to customers slightly late (for example, 6:10 AM rather than 6:00 AM) rather than one or two subzones' newspapers delivered very late (for example, 7:30 AM or 8:00 AM). The total tardiness objective does not capture precisely the late deliveries to individual customers, and is, therefore, not an attractive choice. Moreover, the total tardiness problem is known to be NP-hard (Du and Leung 1990). Therefore, the distributor's objective is to minimize $L_{\max}^{D}(\sigma, \nu) = \max_{1 \le i \le n} \max_{1 \le j \le \ell_i}\{L_{ij}^{D}(\sigma, \nu)\}$ for a given σ, over all ν, where $L_{ij}^{D}(\sigma, \nu) = C_{ij}^{D}(\sigma, \nu) - d_{ij}$. The system problem here is to minimize $\alpha C_{\max}^{M}(\sigma) + (1 - \alpha) L_{\max}^{D}(\sigma, \nu)$, where α is a parameter, $0 \le \alpha \le 1$, over all schedules σ and ν.

In Problem 2, two closely related products P_1 and P_2 are manufactured on the same production line. For example, the products may be different brands of soap, such as *Mr. Clean* and *Spic and Span,* made by Procter and Gamble, Inc., in Toronto (Lee and Tse 1992). The products are to be distributed to n retailers R_i, $i = 1, \ldots, n$, by trucks, each with a fixed capacity C.

Define a *period* as the time required to produce one truckload of items. This time is Ct, where t is the fixed length of the interval between the completion of two consecutive items and t is the same for each product. After the production line starts producing a product, it restricts production to that product for a duration of time which is an integer multiple of Ct. This is done because a setup is required for changing the production from one product to the other. The cost of each setup is μ.

At the end of each period, the manufacturer transfers one truckload containing a single product to the distributor. Let the ratio of the demands for the two products for retailer R_i be $r_{1i} : r_{2i}$, where $r_{1i} + r_{2i} = 1$, $i = 1, \ldots, n$. For some retailer R_i of the distributor's choosing, the distributor sends a shipment containing $b_{1i} = r_{1i}C$ units of product P_1 and $b_{2i} = r_{2i}C$ units of product P_2. In this fashion, within time nCt, which represents a complete *distribution cycle,* each retailer receives exactly one truckload of the products. That is, the plant is operating at full capacity and the total demand from each retailer during each distribution cycle is at least C, so total demand is not necessarily satisfied.

If multiple truckloads are demanded by a retailer R_i, we assume that each delivered truckload is required to have a mix of the two products in the ratio $r_{1i} : r_{2i}$. This assumption is natural in that it matches supply with demand at the retailers as well as defines a simple truck loading policy for the distributor. Under this assumption, it can be assumed without any loss of generality that each retailer demands one truckload of products: if multiple truckloads are demanded by a retailer, then we can treat that retailer as multiple retailers, each of which demands one truckload.

The total number of periods devoted to producing product P_j during the distribution cycle, assuming that the inventory for each product at the beginning of the distribution cycle must equal the inventory at the end of the distribution cycle, is $k_j = \sum_{i=1}^{n} b_{ji}/C$. Hence, $k_1 + k_2 = n$. We assume k_1, k_2 to be integers. The distribution cycle's schedule is defined by a *production sequence* σ which specifies which product is produced during each of the n periods, and a *distribution sequence* ν which specifies which retailer is served at the end of each of the n periods. This cyclic schedule is then repeated until the end of the planning horizon. This model is suitable for products that have a stable demand over the planning horizon. For example, the demand for laundry products is typically stable during the winter period in North America. See a discussion by Lee and Tse (1992) of the winter demand pattern for brands of soap, such as *Mr. Clean* and *Spic and Span,* manufactured by Procter and Gamble, Inc., in Toronto.

Let $I_{j,s}$ denote the inventory level of product P_j, $j = 1, 2$, at the end of period s. At the beginning of each distribution cycle, some initial inventory $I_{j,0}$ of product P_j is kept in order to satisfy demand in the early periods, if needed; this amount depends on the production sequence and the delivery sequence. The inventory holding cost per production period for product P_j is h_j per unit. The holding cost for product P_j in a period s is computed using the period's average inventory: $h_j(I_{j,s-1} + I_{j,s})/2$, $j = 1, 2$.

We assume a make-to-order manufacturer. Since the manufacturer knows the exact demands of the two products during a distribution cycle, it does not need to hold any inventory. The manufacturer's problem is to minimize the total manufacturing cost (i.e., production cost + setup cost) per distribution cycle. Since the production cost during a distribution cycle is constant, minimizing the total manufacturing cost is equivalent to minimizing the total setup cost. Let $S(\sigma)$ denote the total setup cost in one distribution cycle for schedule σ. For the distributor, holding inventory is inevitable. This is because the production lots for both the products are in multiples of C, the fixed truck capacity. At the end of each period, one truckload of C units consisting entirely of one product is delivered to the distributor. However, in each period, the distributor has to supply a retailer, say R_i, with a mix of the two products with quantities $b_{1i} = r_{1i}C$ and $b_{2i} = r_{2i}C$, respectively, where $r_{1i} + r_{2i} = 1$. Thus, the distributor must incur inventory holding cost. Given σ, the distributor's problem is to find a distribution sequence $\nu(\sigma)$ that minimizes its total inventory cost, $T(\sigma, \nu(\sigma))$. The system problem here is to minimize $\alpha S(\sigma) + (1 - \alpha)T(\sigma, \nu(\sigma))$, where α is a parameter, $0 \le \alpha \le 1$, over all production sequences σ and distribution sequences ν.

3. Cost of Conflict

We consider the extent to which one decision maker's cost is larger than optimal when the other decision maker imposes its optimal schedule. This is the *cost of conflict.* Calculating the cost of conflict requires first solving the manufacturer's and the distributor's individual problems optimally. We then perform computational experiments to measure the average cost of conflict for both the manufacturer and the distributor in each problem. We show that this cost is significant in all cases, which motivates the subsequent sections' study of different power relationships and the benefits of cooperation.

3.1. Problem 1

The manufacturer (i.e., the printer), motivated by the objective of minimizing overhead cost, solves a makespan minimization problem. This problem can be written as

$$\min_{y_i \ge 1} C_{\max} = \sum_{i=1}^{n} \ell_i p_i + \sum_{i=1}^{n} s_i y_i, \qquad (1)$$

where the first term is a constant and $y_i \ge 1$ is a decision variable specifying the number of setups for the jobs in zone Z_i. This problem is easily solved, hence the result is stated without proof.

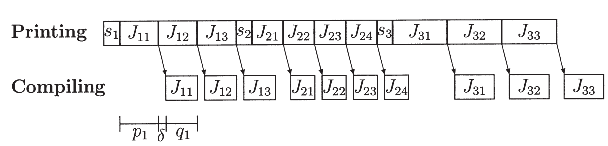
Proposition 1. *The printer's Problem 1 is solved by any schedule with* $y_1 = \cdots = y_n = 1$.

We refer to schedules that are characterized by Proposition 1 as *block family* schedules, since all jobs for the same zone are produced consecutively. In the newspaper context, this means that the jobs for all subzones of a particular zone are printed consecutively. It is important to note that the manufacturer's cost is unaffected by the order in which it processes the blocks.

To understand how the distributor can determine its optimal schedules for production and distribution, we first discuss how jobs are released to the distributor. After a job is printed, which requires time p_i for a job for zone Z_i, it is transferred to the packaging center. This transfer requires time δ. At the packaging center, the newspapers are collated, compiled with their inserts, and loaded into a truck. This process is performed in time q_i for each job for zone Z_i. Based on industry data (Hurter and van Buer 1996), we assume that $q_i \le p_i$ for all i, and $\max_{1 \le i \le n}\{q_i\} \le \min_{1 \le i \le n}\{s_i + p_i\}$; hence, no bottlenecks occur at the packaging center. We also assume that a sufficient number of trucks is available so that a compiled job can be dispatched immediately. This process is illustrated in Figure 1.

Job J_{ij} is released to the distributor at time $C_{ij}^M(\sigma) + \delta + q_i$, it arrives at distribution center Z_{ij} at time $C_{ij}^D(\sigma, \nu(\sigma)) = C_{ij}^M(\sigma) + \delta + q_i + t_{ij}$, and its lateness is $L_{ij}^D(\sigma, \nu(\sigma)) = C_{ij}^D(\sigma, \nu(\sigma)) - d_{ij}$. Conversely, since by assumption a truck is always available to make a delivery, the distributor's due dates d_{ij} can be translated into implied due dates d_{ij}^M for the printer, as follows:

Figure 1 **Operations of the Printer and the Packaging Center**



$$d_{ij}^M = d_{ij} - \delta - q_i - t_{ij}, \quad i = 1, \ldots, n; \quad j = 1, \ldots, \ell_i. \quad (2)$$

This discussion leads to the following lemma, which simplifies the search for the distributor's optimal production and distribution schedules.

LEMMA 1. *For a given printer's schedule σ^m, the distributor's cost $L_{\max}$ is minimized if it uses a schedule that is the same as the printer's schedule, i.e., $\nu(\sigma^m) = \sigma^m$.*

Proof. Since the printer uses a single production line to process the jobs, it releases the jobs to the distributor in the same sequence, σ^m, in which it begins them. Under the assumption that a truck is always available for delivery when a job is ready to be loaded, the distributor minimizes its maximum lateness by delivering each job as soon as possible, which is achieved by setting $\nu(\sigma^m) = \sigma^m$. □

Lemma 1 allows us to simplify our notation from $L_{ij}(\sigma, \nu(\sigma))$ to $L_{ij}(\sigma)$.

COROLLARY 1. *In either the printer's or distributor's Problem 1, there exists an optimal schedule in which no job is preempted during processing at the printer.*

Proof. We first consider the printer's problem. For any production schedule σ, $C_{\max}^M(\sigma)$ can be reduced only by reducing the number of setups. However, preemption cannot reduce the number of setups. Hence, there exists a nonpreemptive schedule σ^m that minimizes $C_{\max}^M(\sigma)$.

If we define the printer's maximum lateness to be $L_{\max}^M(\sigma) = \max_{1 \le i \le n} \max_{1 \le j \le \ell_i}\{C_{ij}^M(\sigma) - d_{ij}^M\}$, then it is easy to see from (2) that $L_{\max}^M(\sigma) = L_{\max}^D(\sigma)$, for any σ. To show the result for the distributor's problem it is therefore, sufficient to consider the printer's maximum lateness. Consider a schedule where a job is preempted into two parts. Modify this schedule by moving the first part of the preempted job to the position immediately precedent to the second part, and moving the jobs in between earlier to eliminate the resulting idle time. Clearly, this modification does not affect the completion time of the jobs prior to the first part. Also, the completion times of the preempted job, the jobs in between, and the jobs after the second part either decrease or remain unchanged. Thus, a preemptive schedule can be modified in a finite number of such steps to a nonpreemptive schedule without decreasing the maximum lateness. □

The implied due dates d_{ij}^M defined in (2) are used in the lemma and algorithm which follow. The algorithm finds a schedule that minimizes the distributor's maximum lateness over all schedules. It requires the following preliminary result.

LEMMA 2. *In the distributor's Problem 1, there exists an optimal schedule in which the jobs for zone Z_i are processed in earliest due date (EDD) order based on d_{ij}^M.*

Proof. For a given nonpreemptive schedule σ, the jobs for zone Z_i are released to the distributor at times $r_{ij} = C_{ij}^M + \delta + q_i$, $j = 1, \ldots, \ell_i$. This holds for any permutation of these jobs for zone Z_i. The distributor's objective function ($L_{\max}$), however, depends only on the values r_{ij}, $j = 1, \ldots, \ell_i$. Since each job for zone Z_i requires the same processing time p_i, the result follows from Jackson (1955). □

Lemma 2 motivates the following optimal algorithm for the distributor's problem of minimizing the maximum lateness in Problem 1.

Algorithm P1D

Input

Given d_{ij}, t_{ij} for $i = 1, \ldots, n$, $j = 1, \ldots, \ell_i$; p_i, q_i, s_i for $i = 1, \ldots, n$, and δ.

Indexing

Index the jobs for each zone Z_i such that $d_{i1}^M \le \cdots \le d_{i\ell_i}^M$, for $i = 1, \ldots, n$.

Value Function

$f(v_1, \ldots, v_n; u_1, \ldots, u_n; j)$ = minimum value of the maximum lateness for scheduling jobs $1, \ldots, v_i$ using $u_i \le v_i$ setups for zone Z_i, for $i = 1, \ldots, n$, where the last job scheduled is for zone Z_j if $j \ge 0$. Zone Z_0 is assumed to be a dummy zone that requires a zero setup time, and consists of a single dummy job that is scheduled at time zero. The dummy job has zero processing time and its due date is infinity.

Boundary Condition

$f(0, \ldots, 0; 0, \ldots, 0; 0) = -\infty$.

Optimal Solution Value

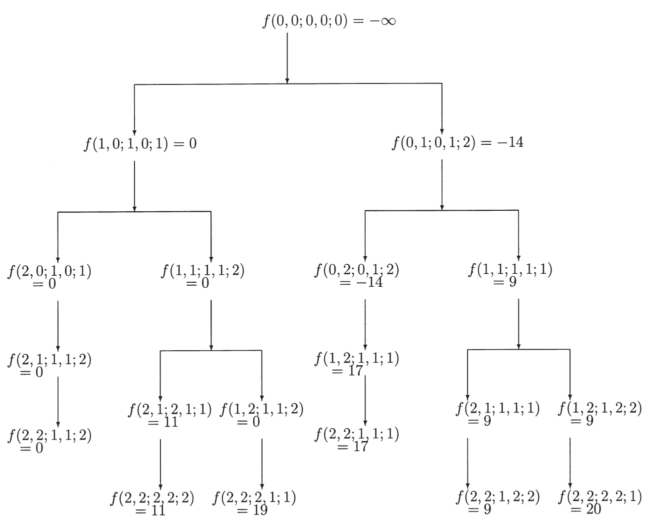
$\min_{1 \le j \le n, 1 \le u_1 \le \ell_1, \ldots, 1 \le u_n \le \ell_n}\{f(\ell_1, \ldots, \ell_n; u_1, \ldots, u_n; j)\}$.

Recurrence Relation

$$f(v_1, \ldots, v_n; u_1, \ldots, u_n; j)$$
$$= \min\left\{\begin{array}{l} \max\left\{\begin{array}{c} \sum_{i=1}^{n}(v_i p_i + u_i s_i) - d_{jv_j}^M, \\ f(v_1, \ldots, v_{j-1}, v_j - 1, v_{j+1}, \ldots, v_n; \\ u_1, \ldots, u_n; j) \end{array}\right\} \\ \min\limits_{1 \le i \le n, i \ne j}\left\{\max\left\{\begin{array}{c} \sum_{i=1}^{n}(v_i p_i + u_i s_i) - d_{jv_j}^M, \\ f(v_1, \ldots, v_{j-1}, v_j - 1, \\ v_{j+1}, \ldots, v_n; u_1, \ldots, u_{j-1}, \\ u_j - 1, u_{j+1}, \ldots, u_n; i) \end{array}\right\}\right\} \end{array}\right\}.$$

In the first term in the recurrence relation, no setup is needed because the zone served, Z_j, is the same as that for the previous job. In the second term, the choice of a job for a different zone necessitates a setup at the printer.

Figure 2 **Example of Algorithm P1D**



THEOREM 1. *Algorithm P1D finds an optimal schedule for the distributor's problem of minimizing the maximum lateness in Problem 1 in* $O(n^2 \prod_{i=1}^{n} \ell_i^2)$ *time.*

Proof. It follows from Lemma 2 that Algorithm P1D compares the cost of all possible optimal state transitions, and therefore finds an optimal schedule. We consider the time complexity of Algorithm P1D. Since u_i, $v_i \le \ell_i$ for $i = 1, \ldots, n$, and $j \le n$, the number of possible values for the state variables is $O(n \prod_{i=1}^{n} \ell_i^2)$. The second term in the recurrence relation requires the most computation, which is $O(n)$. Thus, the overall time complexity of Algorithm P1D is $O(n^2 \prod_{i=1}^{n} \ell_i^2)$, which is polynomial for fixed n. □

EXAMPLE. Consider the following two zone problem (i.e., $n = 2$) in which each zone consists of two subzones (i.e., $\ell_1 = 2$, $\ell_2 = 2$). Let $s_1 = 2$, $s_2 = 1$, $p_1 = 6$, $p_2 = 8$. The jobs corresponding to the subzones are J_{11}, J_{12}, J_{21}, and J_{22}; their due dates at the manufacturer are $d_{11}^M = 8$, $d_{12}^M = 14$, $d_{21}^M = 23$, $d_{22}^M = 31$.

Figure 2 shows the computations of the value function f required by Algorithm P1D. The optimal solution is $f(2, 2; 1, 1; 2) = 0$; the corresponding schedule can be obtained by tracing back the unique path to the root. Thus, an optimal schedule is J_{11}-J_{12}-J_{21}-J_{22} with one setup for each zone. □

The distributor's conflict arises when it must follow the printer's schedule. From Proposition 1, the printer is content to select a block family schedule randomly. From Lemma 2, we assume that within each block the jobs are ordered by EDD.

We define the relative increase in cost resulting from the distributor's conflict as

$$[L_{\max}^D(\sigma^m) - L_{\max}^D(\sigma^d)]/L_{\max}^D(\sigma^d), \tag{3}$$

where σ^d is a printer's schedule that allows the distributor to achieve minimum customer cost over all printer's schedules, and σ^m is a *randomly chosen* block family schedule for the printer. The use of a randomly chosen block family schedule models the case in which there is no cooperation between the parties. In Section 4.1 below, we find the manufacturer's block family schedule that minimizes the distributor's maximum lateness.

We evaluate (3) computationally by examining 100 random instances each for three different configurations. Each configuration has three zones and a total of 10 subzones. Different configurations are created by varying the number of subzones per zone. In the first configuration, there are four subzones in zone 1, three subzones in zone 2, and three subzones in zone 3. Similarly, in the second configuration, there are five, four, and one subzones in the three zones, respectively. Finally, in the third configuration, there are seven, two, and one subzones in the three zones, respectively. For the setup time, processing time and due date, each is generated from one of two distributions. Specifically, the setup times are randomly generated integers from a $U[1, 50]$ distribution or a $U[1, 100]$ distribution, the processing times are integers from a $U[1, 50]$ or a $U[1, 100]$ distribution, and the due dates are integers from a $U[400, 600]$ or a $U[200, 800]$ distribution. This variety creates $2 \times 2 \times 2 = 8$ different data sets for each configuration. These ranges are designed to be typical of those arising in a wide range of manufacturing problems.

Table 1 presents our results. The first two columns list the lower and upper bounds on setup times and processing times, respectively. Column 3 (respectively, Column 4) shows the relative percentage increase in distributor's cost (computed as the mean over 300 randomly generated instances, including 100 for each configuration) resulting from conflict for due dates generated from a $U[400, 600]$ (respectively, $U[200, 800]$) distribution. These results indicate that if the printer randomly chooses a block family schedule, then the overall average relative increase in maximum lateness from the distributor's conflict is 15.81%. Note that the relative lateness cost almost doubles for the more dispersed due dates.

The printer incurs a cost of conflict when it must produce according to a distributor's optimal schedule σ^d. If σ^m is a block family schedule, then we define the relative increase in cost that results from the printer's conflict as

$$[C_{\max}(\sigma^d) - C_{\max}(\sigma^m)]/C_{\max}(\sigma^m). \tag{4}$$

We evaluate expression (4) computationally using the same test data as for (3). The results are shown in Table 2.

Table 1 **Distributor's Cost of Conflict in Problem 1**

Setup times		Processing times		Due dates $l = 400$, $u = 600$	Due dates $l = 200$, $u = 800$
l	u	l	u	Conflict	Conflict
1	50	1	50	16.47%	20.16%
1	50	1	100	8.73%	22.83%
1	100	1	50	10.55%	18.87%
1	100	1	100	7.30%	21.59%
Overall averages				10.76%	20.86%

The mean relative increase in cost resulting from the printer's conflict is 5.45%. A 5.45% increase in the length of an eight-hour shift is almost thirty minutes. The resulting overtime pay for the entire production staff represents a significant cost to the printer. Therefore, each party in this production-distribution system faces a significant cost of conflict if forced to use the other's optimal schedule. This motivates our investigations of supply chain dominance in Section 4 and of the benefits of cooperation in Section 5.

3.2. Problem 2

As for Problem 1, the manufacturer's problem is solved by minimizing the number of setups. The result here is similarly stated without proof.

Proposition 2. *The manufacturer's Problem 2 is solved by any schedule that has one setup for each product.*

To determine a production sequence and a delivery schedule that minimize the distributor's overall inventory cost, we formulate that problem as the following assignment problem with side constraints. Recall that k_1 is the total number of periods during which product P_1 is produced. Let $y_s = 1$ if P_1 is produced during period s, and $y_s = 0$ otherwise. Let $x_{si} = 1$ if the distributor delivers to retailer i at the end of period s, and $x_{si} = 0$ otherwise. The objective is to minimize the total inventory holding cost. Since the average inventory level in period s for product j is $h_j(I_{j,s-1} + I_{j,s})/2$, the total inventory holding cost for a distribution cycle is $h_1[I_{1,0} + 2\sum_{s=1}^{n-1} I_{1,s} + I_{1,n}]/2 + h_2[I_{2,0} + 2\sum_{s=1}^{n-1} I_{2,s} + I_{2,n}]/2$. Because $I_{j,0} = I_{j,n}$, this reduces to $\sum_{s=1}^{n} (h_1 I_{1,s} + h_2 I_{2,s})$.

Minimize $\sum_{s=1}^{n} (h_1 I_{1,s} + h_2 I_{2,s})$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{si} = 1, \quad s = 1, \ldots, n \tag{5}$$

$$\sum_{s=1}^{n} x_{si} = 1, \quad i = 1, \ldots, n \tag{6}$$

$$I_{1,s} = I_{1,s-1} + Cy_s - \sum_{i=1}^{n} b_{1i} x_{si}, \quad s = 1, \ldots, n \tag{7}$$

$$I_{2,s} = I_{2,s-1} + C(1 - y_s) - \sum_{i=1}^{n} b_{2i} x_{si}, \quad s = 1, \ldots, n \tag{8}$$

$$I_{1,0} = I_{1,n} \tag{9}$$

$$I_{2,0} = I_{2,n} \tag{10}$$

$$\sum_{s=1}^{n} y_s = k_1 \tag{11}$$

$$I_{1,s} \geq 0, \quad s = 1, \ldots, n \tag{12}$$

$$I_{2,s} \geq 0, \quad s = 1, \ldots, n \tag{13}$$

Table 2 **Manufacturer's Cost of Conflict in Problem 1**

Setup times		Processing times		Due dates $l = 400$, $u = 600$	Due dates $l = 200$, $u = 800$
l	u	l	u	Conflict	Conflict
1	50	1	50	6.07%	3.18%
1	50	1	100	1.57%	5.46%
1	100	1	50	2.85%	14.61%
1	100	1	100	1.24%	8.65%
Overall averages				2.93%	7.97%

$$y_s \in \{0, 1\}, \quad s = 1, \ldots, n \tag{14}$$

$$x_{si} \in \{0, 1\}, \quad i = 1, \ldots, n, \quad s = 1, \ldots, n. \tag{15}$$

Constraints (5) and (6) ensure that exactly one retailer is served at the end of each period. Constraints (7) and (8) define each period's ending inventory level for each product. Constraints (9) and (10) ensure that the distribution cycle's ending inventory level equals its starting inventory level for each product. Constraint (11) ensures that the required amount of each product is manufactured.

As in Problem 1, the distributor's conflict arises when the manufacturer solves its problem independently from the distributor. We perform a computational study to determine the cost of the distributor's conflict. We compare its costs under the following two scenarios.

1. The manufacturer uses its optimal schedule σ^m, and the distributor then determines its best schedule $\nu(\sigma^m)$, given the manufacturer's schedule. The distributor's inventory holding cost in this scenario is denoted by $T(\sigma^m, \nu(\sigma^m))$.
2. The manufacturer produces according to a schedule σ^d preferred by the distributor, which then uses an optimal delivery schedule $\nu(\sigma^d)$. That is, the distributor chooses σ^d for the manufacturer such that $\nu(\sigma^d)$ is optimal for the distributor. The distributor's inventory holding cost in this scenario is denoted by $T(\sigma^d, \nu(\sigma^d))$.

The data set includes two products delivered to six retailers; the truck capacity is 100 units. The manufacturer, therefore, produces 100 units of one product during each of the six periods, and each retailer receives a total of 100 units, mixed according to its demand ratio. The demand ratios across all retailers sum to 50:50. To simulate this, the demand for product P_1 is a randomly generated integer from $U[1, 99]$ for each of the first five retailers, and $b_{16} = 300 - \sum_{i=1}^{5} b_{1i}$. Infeasible instances where $b_{16} \leq 0$ or $b_{16} \geq 100$ are discarded. Also, $b_{2i} = 100 - b_{1i}$, for $i = 1, \ldots, 6$. One set of these six pairs of demands defines an instance. Each of 100 instances is tested for several combinations of holding costs for product P_1 ($h_1 \in \{5, 10, 20, 40\}$) and product P_2 ($h_2 \in \{1, 5, 10, 20, 40\}$). The distributor's cost of conflict for an instance is computed as $[T(\sigma^m, \nu(\sigma^m)) - T(\sigma^d, \nu(\sigma^d))]/T(\sigma^d, \nu(\sigma^d))$, i.e., the percentage increase in the distributor's inventory cost if the manufacturer uses its optimal schedule. For each combination of holding costs, the mean cost of conflict over 100 instances is computed. The distributor's problem (i.e., the assignment problem with side constraints) is solved as a linear mixed integer program using CPLEX (version 8.1). The results are shown in Table 3.

Note that the distributor's cost in Table 3 is symmetric with respect to holding costs. This is discussed below in Corollary 2, Section 4.2.1.

The manufacturer incurs a cost of conflict when it produces according to a schedule chosen by the distributor. This can increase the number of setups from the manufacturer's optimal number (which is the number of products, two) to as many as the number of periods, n. This would increase the manufacturer's cost from 2μ to $n\mu$. Since the number of setups required by the distributor's optimal schedule is $S(\sigma^d)/\mu$, the manufacturer's cost of conflict is computed as $[S(\sigma^d)/\mu - 2]/2$, the relative increase in the number of setups.

Our computational study includes two products and six retailers, with 1000 random instances of demand ratios generated as described above. In none of the instances tested does the distributor's optimal schedule allow the manufacturer to have only two setups, i.e., the schedule $(P_1P_1P_1P_2P_2P_2)$. In 24.6% of these instances, the manufacturer performs four setups, i.e., $(P_1P_1P_2P_1P_2P_2)$ or $(P_1P_1P_2P_2P_1P_2)$. In the other 75.4% of the instances, the manufacturer performs six setups, i.e., $(P_1P_2P_1P_2P_1P_2)$. These data show that the mean number of setups increases from 2.00 to 5.51; therefore, the manufacturer's mean cost of conflict is 175.5%.

4. Supply Chain Dominance

In this section, we consider two alternative scenarios for each problem with respect to supply chain dominance. In the first scenario, the manufacturer is dominant and insists on a schedule that serves as a constraint for the distributor. In the second scenario, the roles are reversed. In each scenario, we discuss how the decision maker which is not dominant can minimize its cost under the constraint imposed by the dominant decision maker.

Table 3 Distributor's Cost of Conflict in Problem 2

	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
$h_1 = 5$	87.96%	89.28%	88.26%	87.94%	88.10%
$h_1 = 10$	88.19%	88.26%	89.28%	88.26%	87.94%
$h_1 = 20$	88.46%	87.94%	88.26%	89.28%	88.26%
$h_1 = 40$	88.66%	88.10%	87.94%	88.26%	89.28%

4.1. Problem 1

We have seen in Section 3.1 how the printer and the distributor each optimize their individual problems. We now discuss their respective scheduling strategies if they do not have negotiating power within the supply chain.

4.1.1. Printer Dominates. Since there is no difference in the printer's cost between all block family schedules, the printer may be willing to use whichever block family schedule the distributor requests. Thus, we now determine which block family schedule is best for the distributor. Recall that the distributor's due dates can be translated into printer's due dates d_{ij}^M using (2). Consequently, the distributor's minimum maximum lateness can be found by solving a maximum lateness scheduling problem at the printer.

To study this problem, we need a measure to compare the lateness of jobs within a block. Index the jobs such that $d_{i1}^M \leq \cdots \leq d_{i\ell_i}^M$, for $i = 1, \ldots, n$. Thus, $C_{i\ell_i}^M(\sigma)$ denotes the time at which the last job for zone Z_i is completed by the printer. Since jobs are ordered by EDD within each block, for each $j \in \{1, \ldots, \ell_i\}$, $C_{ij}^M(\sigma) - C_{i\ell_i}^M(\sigma)$ is constant for all block family schedules. For each zone Z_i, we define the constant

$$X_i = \max_{1 \leq j \leq \ell_i} \{C_{ij}^M(\sigma) - C_{i\ell_i}^M(\sigma) - d_{ij}^M\}.$$

We are now ready to solve the maximum lateness block scheduling problem.

THEOREM 2. *Among all of the printer's block family schedules, one which minimizes the maximum lateness is given by sequencing the blocks $i = 1, \ldots, n$, in nonincreasing order of X_i.*

Proof. Use adjacent pairwise interchange. Consider scheduling the blocks in reverse order, i.e., starting at $C_{\max}$ and moving backward. Suppose we have just scheduled a block whose starting time is B. Assume that block i is selected at this point, and then block k is selected to precede block i, where $X_k \leq X_i$, as shown in Figure 3. Here, the maximum lateness within blocks i and k is $\max\{X_i + B, X_k + B - \ell_i p_i - s_i\} = X_i + B$. However, if we interchange blocks i and k (Figure 4), then the maximum lateness within those blocks is $\max\{X_i + B - \ell_k p_k - s_k, X_k + B\} \leq X_i + B$. □

We perform a computational study similar to the one in Section 3.1 which evaluates the distributor's conflict for a randomly chosen block family schedule. Here, however, we evaluate the distributor's conflict from using a block family schedule σ_2^m that satisfies Theorem 2, which is $[L_{\max}(\sigma_2^m) - L_{\max}(\sigma^d)]/L_{\max}(\sigma^d)$. We also evaluate the *improvement* in the distributor's maximum lateness from the use of such a block family schedule rather than a randomly chosen one, denoted by σ_r^m, which is $[L_{\max}(\sigma_r^m) - L_{\max}(\sigma_2^m)]/L_{\max}(\sigma_r^m)$. The results are shown in Table 4. The mean reduction in the distributor's maximum lateness that results from the printer using the most favorable block family schedule is 9.08%, which still leaves an average distributor's conflict of 4.17%. Again, the distributor's cost is significantly larger for the case of more dispersed due dates.

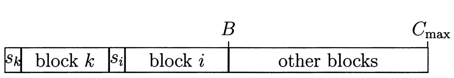


Figure 3 **Schedule where Block *k* Precedes Block *i***

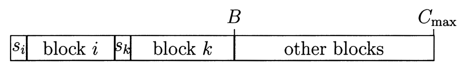


Figure 4 **Schedule Where Block *i* Precedes Block *k***

These results suggest that roughly one-third of the distributor's conflict computed in Section 3.1 arises from the printer's use of a block family schedule, and the remainder comes from the choice of which block family schedule the printer uses.

4.1.2. Distributor Dominates. If the distributor has dominant power in the supply chain, then it may impose a limit, Δ, on the maximum lateness that results from the decisions of the printer. The printer then minimizes its cost, subject to this constraint. We now show how this can be modeled.

LEMMA 3. *In the printer's problem of minimizing $C_{\max}$ subject to the constraint $L_{\max} \leq \Delta$, there exists an optimal schedule in which the jobs for zone Z_i are processed in EDD order based on d_{ij}^M.*

Proof. Similar to Lemma 2. □

Algorithm P1MC finds a schedule that minimizes the printer's makespan while satisfying the distributor's requirement that $L_{\max} \leq \Delta$. It differs from Algorithm P1D only in the Optimal Solution Value step.

Algorithm P1MC

Optimal Solution Value

$\min_{1 \leq j \leq n, 1 \leq u_1 \leq \ell_1, \ldots, 1 \leq u_n \leq \ell_n} \{\sum_{i=1}^n u_i s_i | f(\ell_1, \ldots, \ell_n; \; u_1, \ldots, u_n; j) \leq \Delta\}$.

THEOREM 3. *Algorithm P1MC finds an optimal schedule for the printer's version of Problem 1 with the $L_{\max} \leq \Delta$ constraint in $O(n^2 \prod_{i=1}^n \ell_i^2)$ time.*

Proof. Similar to Theorem 1. □

In order to evaluate the cost incurred by the printer if it must satisfy a maximum lateness constraint imposed by the distributor, we compute the relative cost increase resulting from the printer's conflict. If σ^m denotes any block family schedule, and $\sigma^d(\Delta)$ denotes the printer's schedule derived from Algorithm P1MC, then we define the relative increase in cost resulting from the printer's conflict as

Table 4 Distributor's Cost of Conflict and its Improvement from Theorem 2

Setup times		Processing times		Due dates			
				l = 400, u = 600		l = 200, u = 800	
l	u	l	u	Conflict	Improvement	Conflict	Improvement
1	10	1	50	3.54%	10.06%	6.73%	10.25%
1	10	1	100	0.58%	6.82%	7.39%	11.45%
1	50	1	50	1.68%	7.33%	6.39%	9.68%
1	50	1	100	0.64%	5.71%	6.38%	11.38%
Overall averages				1.61%	7.48%	6.72%	10.69%

$$[C_{\max}(\sigma^d(\Delta)) - C_{\max}(\sigma^m)]/C_{\max}(\sigma^m). \qquad (16)$$

We evaluate this expression computationally, as for (4) in Section 3.1. The results appear in Table 5. For each combination of setup times, processing times and due dates, we evaluate (16) for three values of Δ. Let $L^*_{\max}$ be the optimum $L_{\max}$ value from Algorithm P1D and $Z = L_{\max}(\sigma^m) - L^*_{\max}$. The three values of Δ for which each instance is tested are $\Delta = L^*_{\max} + .25Z$, $\Delta = L^*_{\max} + .5Z$ and $\Delta = L^*_{\max} + .75Z$. Recall that values for $\Delta = L^*_{\max}$ can be found in Table 2, whereas for $\Delta = L_{\max}(\sigma^m)$ the cost of conflict is zero.

The results in Table 5 show that most of the printer's conflict is incurred when the weakest constraint, $L_{\max} \leq \Delta = L^*_{\max} + .75Z$, is imposed by the distributor. In fact, by comparing Table 5 with Table 2, we see that on average 68.0% of the printer's total conflict is incurred as a result of this first marginal reduction in allowable $L_{\max}$ value from $L_{\max}(\sigma^m)$ to $L^*_{\max} + .75Z$. It follows that the largest marginal reduction in conflict occurs when the printer first uses a schedule that is not a block family schedule.

4.2. Problem 2

We have seen in Section 3.2 how the manufacturer and the distributor each optimize their individual problems. We now discuss their respective scheduling strategies if they do not have negotiating power within the supply chain.

4.2.1. Manufacturer Dominates. The manufacturer produces so that it has only two setups per distribution cycle, one for each product. Given this production sequence, the distributor seeks a distribution sequence that minimizes its cost over all possible distribution sequences. This distribution sequence is then repeated throughout the planning horizon.

The manufacturer produces product P_1 for k_1 consecutive periods and then produces product P_2 for k_2 consecutive periods. Let $\nu = (\nu(1), \ldots, \nu(n))$ denote the delivery sequence. We first explain how the inventory level of product P_j, for $j = 1, 2$, can be computed over the distribution cycle time nCt, and then specify the distributor's best distribution sequence for this production sequence. Let the total inventory holding cost from period 1 through period k_1 for product P_1 be denoted T^1_{1,k_1}, and the total inventory holding cost for both products from period 1 through period n be denoted $T_{1,n}$.

LEMMA 4. *For production sequence* $(P_1 \cdots P_1 P_2 \cdots P_2)$, *the distributor's inventory cost in Problem 2 is*

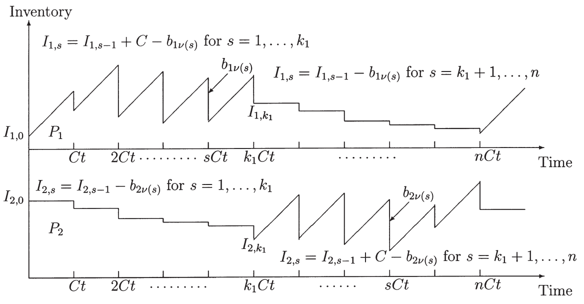
$$T_{1,n} = nI_{1,0}h_1 + nI_{2,k_1}h_2 + \left(\frac{k_1^2}{2} + k_1k_2 + \frac{k_1}{2}\right)h_1C + \left(\frac{k_2^2}{2} + k_1k_2 + \frac{k_2}{2}\right)h_2C - \sum_{s=1}^{n}(n-s+1) \times (h_1b_{1\nu(s)} + h_2b_{2\nu(k_1+s)}),$$

where $b_{2\nu(s)} = b_{2\nu(n+s)}$, $s = 0, 1, \ldots, n$.

Table 5 Printer's Cost of Conflict in Problem 1 as Δ Varies

Setup times		Processing times		Due dates					
				l = 400, u = 600			l = 200, u = 800		
				Maximum allowable lateness (Δ)			Maximum allowable lateness (Δ)		
l	u	l	u	$L^*_{\max}$ + .25Z	$L^*_{\max}$ + .5Z	$L^*_{\max}$ + .75Z	$L^*_{\max}$ + .25Z	$L^*_{\max}$ + .5Z	$L^*_{\max}$ + .75Z
1	50	1	50	5.31%	4.42%	3.99%	2.64%	1.94%	1.79%
1	50	1	100	1.46%	1.28%	1.24%	4.46%	3.49%	3.11%
1	100	1	50	2.77%	2.70%	2.63%	12.15%	9.59%	8.69%
1	100	1	100	1.24%	1.20%	1.18%	6.97%	5.79%	5.22%
Overall averages				2.70%	2.40%	2.26%	6.56%	5.20%	4.70%

Figure 5 **Inventory Level for Production Sequence ($P_1 \cdots P_1P_2 \cdots P_2$) and Distribution Sequence ν**



Proof. Figure 5 shows the inventory of both products during the distribution cycle.

Initially, the inventory of product P_1 is $I_{1,0}$. At time Ct retailer $R_{\nu(1)}$ is served, the total inventory of product P_1 is $I_{1,0} + C - b_{1\nu(1)}$, and the inventory cost for P_1 for the first period is $(I_{1,0} + C - b_{1\nu(1)})h_1$. At time $2Ct$, retailer $R_{\nu(2)}$ is served, the total inventory of product P_1 is $I_{1,0} + 2C - b_{1\nu(1)} - b_{1\nu(2)}$, and the inventory cost for P_1 over the first two periods is $(I_{1,0} + 2C - b_{1\nu(1)} - b_{1\nu(2)})h_1$. Finally, at time k_1Ct, when a delivery has been made to retailer $R_{\nu(k_1)}$, the inventory of product P_1 is $I_{1,0} + k_1C - \sum_{s=1}^{k_1} b_{1\nu(s)}$. Thus,

$$T^1_{1,k_1} = h_1\left[k_1 I_{1,0} + \frac{k_1^2}{2}C + \frac{k_1}{2}C - \sum_{s=1}^{k_1}(k_1 - s + 1)b_{1\nu(s)}\right], \tag{17}$$

where $I_{1,k_1} = I_{1,0} + k_1C - \sum_{s=1}^{k_1} b_{1\nu(s)}$. Similarly, we have

$$\begin{aligned} T^1_{k_1+1,n} &= h_1\left[k_2 I_{1,k_1} - \sum_{s=k_1+1}^{n}(n - s + 1)b_{1\nu(s)}\right] \\ &= h_1\left[k_2 I_{1,0} + k_1k_2C - (n - k_1)\sum_{s=1}^{k_1} b_{1\nu(s)} \right. \\ &\qquad \left. - \sum_{s=k_1+1}^{n}(n - s + 1)b_{1\nu(s)}\right]. \end{aligned} \tag{18}$$

From (17) and (18), the inventory holding cost for period 1 to period n for product P_1 is

$$T^1_{1,n} = h_1\left[nI_{1,0} + \frac{k_1^2}{2}C + k_1k_2C + \frac{k_1}{2}C - \sum_{s=1}^{n}(n - s + 1)b_{1\nu(s)}\right]. \tag{19}$$

Similarly, the inventory holding cost for period 1 to period n for product P_2 is

$$T^2_{1,n} = h_2\left[nI_{2,k_1} + \frac{k_2^2}{2}C + k_1k_2C + \frac{k_2}{2}C - \sum_{s=1}^{n}(n - s + 1)b_{2\nu(k_1+s)}\right]. \tag{20}$$

The result then follows from (19) and (20). □

We can now find the distributor's best delivery sequence if the manufacturer uses its optimal schedule.

Theorem 4. *Suppose that $h_1 \geq h_2$ and that the retailers are indexed such that $b_{11} \geq \cdots \geq b_{1n}$. For production sequence ($P_1 \cdots P_1P_2 \cdots P_2$), the distributor's inventory cost in Problem 2 can be minimized by using the delivery sequence $\nu = (1, \ldots, n)$.*

Proof. First, observe that since the distribution sequence is cyclic,

$$\sum_{s=1}^{n}(n - s + 1)h_2 b_{2\nu(k_1+s)} = \sum_{s=1}^{k_1}(k_1 - s + 1)h_2 b_{2\nu(s)} + \sum_{s=k_1+1}^{n}(n + k_1 - s + 1)h_2 b_{2\nu(s)}.$$

Suppose that ν_o is a distribution schedule in which the retailers' demands are not served in nonincreasing order. Then there must be two adjacent periods i and $j = i + 1$ in ν_o, such that $b_{1\nu_o(i)} < b_{1\nu_o(j)}$. We perform a pairwise interchange on the demands served in those periods, and thus obtain a new schedule ν'. We show that $T_{1,n}(\nu') - T_{1,n}(\nu_o) \leq 0$. If $2 \leq j \leq k_1$, then

$$\begin{aligned} &T_{1,n}(\nu') - T_{1,n}(\nu_o) \\ &\quad = -h_1[(n - i + 1)(b_{1\nu_o(j)} - b_{1\nu_o(i)}) + (n - j + 1)(b_{1\nu_o(i)} \\ &\qquad - b_{1\nu_o(j)})] - h_2[(k_1 - i + 1)(b_{2\nu_o(j)} - b_{2\nu_o(i)}) \\ &\qquad + (k_1 - j + 1)(b_{2\nu_o(i)} - b_{2\nu_o(j)})] \\ &\quad = -h_1(j - i)(b_{1\nu_o(j)} - b_{1\nu_o(i)}) - h_2(j - i)(b_{2\nu_o(j)} - b_{2\nu_o(i)}) \\ &\quad = (h_2 - h_1)(b_{1\nu_o(j)} - b_{1\nu_o(i)}) \leq 0. \end{aligned}$$

Thus, any distribution schedule can be rearranged into nonincreasing order of the b_{1i} values without increasing the total inventory cost. The proof is similar for $k_1 + 1 \leq j \leq n$. □

This result leads to the symmetry with respect to holding costs observed in Table 3. The distributor's optimal inventory cost remains the same if the holding costs of the two products are interchanged.

COROLLARY 2. *For the production sequence* $(P_1 \cdots P_1 P_2 \cdots P_2)$, *the distributor's optimal inventory cost in Problem 2 is symmetric with respect to the holding costs* h_1 *and* h_2.

Proof. Theorem 4 implies that if $h_2 \geq h_1$, then for production sequence $(P_1 \cdots P_1 P_2 \cdots P_2)$, the distributor's inventory cost in Problem 2 is minimized by using delivery order $\nu' = (k_1, k_1 - 1, \ldots, 1, n, n - 1, \ldots, k_1 + 1)$.

We consider two different problems. In each, the production sequence is $(P_1 \cdots P_1 P_2 \cdots P_2)$. In the first, $h_1 \geq h_2$ and the distributor uses delivery order $\nu = (1, 2, \ldots, n - 1, n)$. Inventory levels are represented by $I_{j,s}$. In the second, $h_2 \geq h_1$ and the distributor uses delivery order $\nu' = (k_1, k_1 - 1, \ldots, 1, n, n - 1, \ldots, k_1 + 1)$. Inventory levels are represented by $I'_{j,s}$. We prove Corollary 2 by showing that $I_{1,s} = I'_{2,k_1 - s}$ for $s = 0, \ldots, k_1$, and $I_{1,s} = I'_{2,n+k_1-s}$ for $s = k_1, \ldots, n - 1$. We have

$$I_{1,0} = I'_{2,k_1} = 0$$

$$I_{1,1} = I_{1,0} + C - b_{1,\nu(1)} = I'_{2,k_1} + C - b_{1,\nu'(k_1)} = I'_{2,k_1} + b_{2,\nu'(k_1)} = I'_{2,k_1 - 1}$$

$$\vdots$$

$$I_{1,k_1} = I_{1,0} + k_1 C - \sum_{i=1}^{k_1} b_{1,\nu(i)}$$

$$= I'_{2,k_1} + \sum_{i=1}^{k_1} (C - b_{1,\nu'(i)})$$

$$= I'_{2,k_1} + \sum_{i=1}^{k_1} b_{2,\nu'(i)} = I'_{2,0}$$

$$I_{1,k_1+1} = I_{1,k_1} - b_{1,\nu(k_1+1)}$$

$$= I'_{2,0} - C + b_{2,\nu'(n)} = I'_{2,n-1}$$

$$\vdots$$

$$I_{1,n-1} = I_{1,k_1} - \sum_{i=k_1+1}^{n-1} b_{1,\nu(i)}$$

$$= I'_{2,0} - \sum_{i=k_1+2}^{n} b_{1,\nu'(i)}$$

$$= I'_{2,0} - \sum_{i=k_1+2}^{n} (C - b_{2,\nu'(i)}) = I'_{2,k_1+1}. \qquad \square$$

Recall that for Problem 1 in Section 4.1.1, we show how the distributor can benefit by finding a production sequence that improves $L_{\max}$ and does not degrade $C_{\max}$. However, since the manufacturer's optimal schedule in Problem 2 is simply $(P_1 \cdots P_1 P_2 \cdots P_2)$, no such improved schedule exists in this case. Hence, no computational study is needed here. Therefore, the contribution of this section is to show that the problem of finding the best distribution schedule, given a manufacturer's schedule $(P_1 \cdots P_1 P_2 \cdots P_2)$, is polynomially solvable via a sorting algorithm.

4.2.2. Distributor Dominates. If the distributor has dominant power in the supply chain, then it may impose a limit, I^*, on the total inventory holding cost that results from the decisions of the manufacturer. The manufacturer then tries to minimize its cost, subject to this constraint, by solving a variant of the assignment problem with side constraints in Section 3.2. We let $g_s = 1$ if a setup is required in period s; otherwise, $g_s = 0$.

Minimize $g_2 + g_3 + \cdots + g_{n+1}$

$$\text{s.t.} \quad \sum_{s=1}^{n} (h_1 I_{1,s} + h_2 I_{2,s}) \leq I^* \tag{21}$$

$$y_s - y_{s-1} \leq g_s, \quad s = 2, \ldots, n + 1 \tag{22}$$

$$y_{s-1} - y_s \leq g_s, \quad s = 2, \ldots, n + 1 \tag{23}$$

$$g_s \in \{0, 1\}, \quad s = 2, \ldots, n + 1$$

$$\text{and } (5) - (15).$$

Constraint (21) enforces the inventory holding cost limitation. Constraints (22) and (23) determine whether a setup is required in period s.

We perform a computational study to evaluate how different values of I^* affect the manufacturer's cost. The same data generated for the study of the manufacturer's conflict in Section 3.2 are used to compute the mean number of setups, $S(\sigma(I^*))/\mu$, required for the manufacturer to meet the given inventory cost constraint and to compute the mean relative cost increase, $[S(\sigma(I^*))/\mu - 2]/2$. This is performed for five values of I^*. Let $T(\sigma^m, \nu(\sigma^m))$ be the distributor's total inventory holding cost for the manufacturer's optimal production sequence, $T(\sigma^d, \nu(\sigma^d))$ be the distributor's optimal total inventory holding cost, and $Y = T(\sigma^m, \nu(\sigma^m)) - T(\sigma^d, \nu(\sigma^d))$. The five values of I^* for which the 100 instances are tested are $I^* = T(\sigma^d, \nu(\sigma^d))$, $I^* = T(\sigma^d, \nu(\sigma^d)) + .25Y$, $I^* = T(\sigma^d, \nu(\sigma^d) + .5Y$, $I^* = T(\sigma^d, \nu(\sigma^d)) + .75Y$, and $I^* = T(\sigma^m, \nu(\sigma^m))$. Holding cost values $h_1 = 5$ and $h_2 \in \{5, 10, 20, 40\}$ are used. Mean results over the 100 instances are shown in Table 6.

The results in Table 6 show that 57.8% of the manufacturer's cost of conflict is incurred when the weak-

Table 6 Manufacturer's Cost of Conflict in Problem 2 as I^* Varies

	$T(\sigma^d, \nu(\sigma^d))$		$T(\sigma^d, \nu(\sigma^d)) + .25Y$		$T(\sigma^d, \nu(\sigma^d)) + .5Y$		$T(\sigma^d, \nu(\sigma^d)) + .75Y$		$T(\sigma^m, \nu(\sigma^m))$	
	Setups	Cost	Setups	Cost	Setups	Cost	Setups	Cost	Setups	Cost
$h_2 = 5$	5.26	163%	4.32	116%	4.00	100%	4.00	100%	2.00	0%
$h_2 = 10$	5.62	181%	4.36	118%	4.00	100%	4.00	100%	2.00	0%
$h_2 = 20$	5.50	175%	4.38	119%	4.00	100%	4.00	100%	2.00	0%
$h_2 = 40$	5.45	172%	4.43	121%	4.00	100%	4.00	100%	2.00	0%
Averages	5.46	173%	4.37	119%	4.00	100%	4.00	100%	2.00	0%

est constraint, $I^* = T(\sigma^d, \nu(\sigma^d)) + .75Y$, is imposed by the distributor.

5. Benefit of Cooperation

In this section, we evaluate the benefit of cooperation between the manufacturer and the distributor. More specifically, we compare the total system cost in a situation where the manufacturer and the distributor cooperate fully to the total system cost that results if either party makes an independent decision. For each problem, the combined objective function is a convex combination of the individual objectives. This model is chosen for generality; we evaluate the objective for different values of the combination parameter α. In any practical system consisting of a manufacturer and a distributor, the value of α is defined either by an executive decision if they are in the same company or by the two parties' relative negotiating power otherwise. We close this section by briefly discussing methods of encouraging and implementing cooperation.

5.1. Problem 1

For the newspaper example, various reasonable objectives for the combined problem can be formulated. In practice, the distributor's goal may not be to minimize $L_{\max}$, but rather to ensure that $L_{\max} = 0$. In this case, the discussion in Section 4.1.2 and the results in Table 5 are applicable. Similarly, the printer may not want to minimize $C_{\max}$ as much as requiring that $C_{\max} \leq C^*$, where C^* is a value which ensures that production can be completed without overtime labor. Here, the printer can offer the distributor the opportunity to design and request a production sequence with a fixed number of extra setups allowed. Each of these scenarios is more likely if the printer and the distributor are in the same company, which is typical but not ubiquitous in the newspaper industry.

If the printer and distributor cooperate, then they select a schedule σ^c that minimizes the convex combination $\alpha C^M_{\max}(\sigma) + (1 - \alpha) L^D_{\max}(\sigma)$, for some fixed α. Given α, Algorithm P1C, which differs from Algorithm P1D only in the Input and Optimal Solution Value steps, finds such a schedule.

If the distributor agrees to a certain value of $L_{\max}$, then it is important for the distributor to assess the cost implication of choosing this value. We now describe an approach to assess this cost in practice. The demand of each distribution center Z_{ij} typically consists of demand from individual subscribers and from newsstands. Late deliveries may affect these demands; the demand at newsstands is particularly time sensitive. Using historical data, each distribution center Z_{ij} can express the dollar value of lost sales, $G_{ij}(L_{ij})$, as a function of the lateness L_{ij}. Typically, $G_{ij}(L_{ij})$ is an increasing function of L_{ij}. Thus, by agreeing to a specific value of $L_{\max}$, the distributor may incur a maximum total cost of $\sum_{i=1}^{n} \sum_{j=1}^{\ell_i} G_{ij}(L_{\max})$. Note that this is a worst-case scenario in which the lateness of each distribution center is $L_{\max}$. In practice, when a solution with $L_{\max}$ is implemented, the distributor's cost will be much less than the maximum committed (i.e., $\sum_{i=1}^{n} \sum_{j=1}^{\ell_i} G_{ij}(L_{\max})$).

Algorithm P1C

Input

Given d_{ij}, t_{ij} for $i = 1, \ldots, n$, $j = 1, \ldots, \ell_i$; p_i, q_i, s_i for $i = 1, \ldots, n$; δ and α.

Optimal Solution Value

$\min_{1 \leq j \leq n, 1 \leq u_1 \leq \ell_1, \ldots, 1 \leq u_n \leq \ell_n} \{\alpha \sum_{i=1}^{n} (\ell_i p_i + u_i s_i) + (1 - \alpha) f(\ell_1, \ldots, \ell_n; u_1, \ldots, u_n; j)\}$.

THEOREM 5. *Algorithm P1C finds an optimal schedule for the combined version of Problem 1 in $O(n^2 \prod_{i=1}^{n} \ell_i^2)$ time.*

Proof. Similar to Theorem 1. □

In order to evaluate the benefit of cooperation, we compare the schedule derived by Algorithm P1C to the printer's optimal schedule and to the distributor's optimal schedule. As in Section 3.1, we use eight combinations of ranges for the setup times, processing times, and due dates. For each combination, we generate 100 random instances for each of the three configurations defined in Section 3.1 and for $\alpha \in \{.2, .4, .6, .8\}$. Then we compute:

1. A random block family schedule σ^m and its cost $\Gamma^m = \alpha C_{\max}(\sigma^m) + (1 - \alpha) L_{\max}(\sigma^m)$,

2. The schedule σ^d found by Algorithm P1MC, where the printer is required to satisfy $L_{\max} \leq \Delta$

Table 7 Benefit of Cooperation versus Printer's Optimal Schedule in Problem 1

Setup times		Processing times		Due dates		Savings over manufacturer's optimal schedule			
l	*u*	*l*	*u*	*l*	*u*	$\alpha = .2$	$\alpha = .4$	$\alpha = .6$	$\alpha = .8$
1	50	1	50	400	600	9.69%	7.49%	5.22%	2.56%
1	50	1	100	400	600	6.81%	5.06%	3.31%	1.77%
1	100	1	50	400	600	7.86%	5.48%	3.78%	2.09%
1	100	1	100	400	600	4.94%	3.93%	2.54%	1.36%
1	50	1	50	200	800	13.64%	10.07%	7.36%	3.59%
1	50	1	100	200	800	14.04%	10.25%	6.77%	3.63%
1	100	1	50	200	800	12.30%	9.79%	6.81%	3.96%
1	100	1	100	200	800	12.63%	9.72%	6.09%	3.34%
				Averages		10.24%	7.72%	5.24%	2.79%

$= L^*_{\max}$, and its cost $\Gamma^d = \alpha C_{\max}(\sigma^d) + (1-\alpha)L_{\max}(\sigma^d)$, and

3. The optimal system schedule σ^c found by Algorithm P1C and its cost $\Gamma^c = \alpha C_{\max}(\sigma^c) + (1 - \alpha)L_{\max}(\sigma^c)$.

The benefit of cooperation relative to the cost of the printer's optimal schedule, $(\Gamma^m - \Gamma^c)/\Gamma^m$, is shown in Table 7. The corresponding information, $(\Gamma^d - \Gamma^c)/\Gamma^d$, for the distributor's optimal schedule is shown in Table 8.

Our results show that the mean benefit of cooperation relative to using the printer's optimal schedule is 6.50%, and relative to using the distributor's optimal schedule is 1.28%. The parameter that has the greatest effect on the benefit of cooperation is due dates. For more dispersed due dates (200–800 versus 400–600), the benefit of cooperation is 81.35% greater (8.37% versus 4.62%) when compared to the printer's optimal schedule, and it is 206.43% greater (1.92% versus 0.63%) when compared to the distributor's optimal schedule. These results imply that cooperation is more valuable when due dates are more variable.

5.2. Problem 2

Whereas in Problem 1 the decision makers minimize time, in Problem 2 they minimize cost. Therefore, there is no obvious scenario in which one party will freely relinquish its optimal schedule without compensation, even if the parties are within the same company. We again use a convex combination of the individual objectives to define a combined objective function.

If the manufacturer and the distributor cooperate, then they can find the manufacturer's schedule σ^c and the distributor's schedule $\nu(\sigma^c)$ that jointly minimize $\alpha S(\sigma) + (1-\alpha)T(\sigma, \nu(\sigma))$ by solving an assignment problem with side constraints that is similar to that in Section 4.2.2. The only changes are to remove the first constraint and to change the objective function to

$$\text{Minimize } \alpha(g_2 + g_3 + \cdots + g_{n+1}) \times \mu + (1-\alpha)\sum_{s=1}^{n}(h_1 I_{1,s} + h_2 I_{2,s}).$$

We perform a computational study to determine the relative cost saving to the supply chain that results from cooperation between the two parties. The same data generated for the study of the manufacturer's conflict in Section 3.2 are used, under three different scenarios:

1. The manufacturer uses its optimal schedule σ^m, and then the distributor uses Theorem 4 to determine its best schedule $\nu(\sigma^m)$. We denote the total cost of this scenario by $\Gamma^m = \alpha S(\sigma^m) + (1-\alpha)T(\sigma^m, \nu(\sigma^m))$.

Table 8 Benefit of Cooperation versus Distributor's Optimal Schedule in Problem 1

Setup times		Processing times		Due dates		Savings over distributor's optimal schedule			
l	*u*	*l*	*u*	*l*	*u*	$\alpha = .2$	$\alpha = .4$	$\alpha = .6$	$\alpha = .8$
1	50	1	50	400	600	0.08%	0.45%	1.30%	2.54%
1	50	1	100	400	600	0.03%	0.17%	0.40%	0.79%
1	100	1	50	400	600	0.12%	0.50%	1.03%	1.72%
1	100	1	100	400	600	0.02%	0.11%	0.28%	0.51%
1	50	1	50	200	800	0.14%	0.66%	1.92%	4.55%
1	50	1	100	200	800	0.05%	0.38%	1.23%	2.95%
1	100	1	50	200	800	0.30%	1.37%	3.55%	6.86%
1	100	1	100	200	800	0.15%	0.77%	1.99%	3.92%
				Averages		0.11%	0.55%	1.46%	2.98%

Table 9 Benefit of Cooperation versus Manufacturer's Optimal Schedule in Problem 2

	$\alpha = .2$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	30.10%	36.44%	37.83%	39.73%	41.32%
10	35.35%	37.83%	39.93%	40.49%	41.50%
20	39.05%	39.73%	40.49%	41.79%	41.92%
40	41.29%	41.32%	41.50%	41.92%	42.74%
	$\alpha = .4$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	15.64%	26.72%	30.31%	34.32%	37.96%
10	25.15%	30.31%	34.33%	36.16%	38.58%
20	32.60%	34.32%	36.16%	38.72%	39.58%
40	37.56%	37.96%	38.58%	39.58%	41.17%
	$\alpha = .6$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	1.86%	13.13%	18.94%	25.78%	32.17%
10	11.33%	18.94%	25.04%	28.98%	33.34%
20	22.54%	25.78%	28.98%	33.30%	35.36%
40	31.22%	32.17%	33.34%	35.36%	38.13%
	$\alpha = .8$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	0.07%	0.00%	1.72%	9.21%	19.19%
10	0.10%	1.72%	7.49%	13.73%	21.49%
20	5.31%	9.21%	13.73%	20.50%	25.28%
40	17.11%	19.19%	21.49%	25.28%	30.34%

2. The manufacturer uses a schedule σ^d, derived from the assignment problem with side constraints in Section 4.2.2, that enables the distributor to achieve its overall minimum total inventory holding cost. We denote the total cost of this scenario by $\Gamma^d = \alpha S(\sigma^d) + (1 - \alpha)T(\sigma^d, \nu(\sigma^d))$.

3. A production schedule σ^c that minimizes the overall system cost is used. We denote the total cost of this scenario by $\Gamma^c = \alpha S(\sigma^c) + (1 - \alpha)T(\sigma^c, \nu(\sigma^c))$.

The setup cost is $\mu = 200$ throughout. The relative gain from using the cooperative schedule over the manufacturer's optimal schedule is computed as $(\Gamma^m - \Gamma^c)/\Gamma^m$. The mean benefit of cooperation for various combinations of holding costs and different values of α is shown in Table 9.

The relative gain from using the cooperative schedule over the distributor's optimal schedule is computed as $(\Gamma^d - \Gamma^c)/\Gamma^d$. The mean benefit of cooperation for various combinations of holding costs and different values of α is shown in Table 10.

As in Table 3, the results in Tables 9 and 10 are approximately symmetric with respect to holding costs. Here, the manufacturer does not use production sequence $(P_1 \cdots P_1 P_2 \cdots P_2)$ in all instances, however analogs to Corollary 2 can be similarly proven for the different production sequences.

The average improvement over the manufacturer's optimal schedule is 25.26%, whereas the average improvement over the distributor's optimal schedule is 6.02%. Additional computational results show that the improvement over the manufacturer's optimal schedule decreases and the improvement over the distributor's optimal schedule grows as μ increases. This is to be expected because a larger setup cost gives more weight to the manufacturer's cost in the combined objective. Similar reasoning applies to other observations from the results: (i) as holding costs increase, the benefit of cooperation relative to the manufacturer's optimal schedule increases, and the benefit relative to the distributor's optimal schedule decreases, (ii) as α increases, the benefit of cooperation relative to the manufacturer's optimal schedule decreases, and the benefit relative to the distributor's optimal schedule increases.

5.3. Implementing Cooperation

In both the problems considered, a benefit of cooperation arises when the dominant player agrees not to use its individually optimal schedule. Note that unless

Table 10 Benefit of Cooperation versus Distributor's Optimal Schedule in Problem 2

	$\alpha = .2$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	1.92%	0.23%	0.46%	0.18%	0.03%
10	0.70%	0.46%	0.01%	0.12%	0.05%
20	0.24%	0.18%	0.12%	0.00%	0.04%
40	0.08%	0.03%	0.05%	0.04%	0.00%
	$\alpha = .4$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	7.21%	2.03%	2.38%	0.87%	0.26%
10	3.57%	2.38%	0.57%	0.74%	0.28%
20	1.41%	0.87%	0.74%	0.04%	0.22%
40	0.47%	0.26%	0.28%	0.22%	0.00%
	$\alpha = .6$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	17.83%	6.81%	6.71%	3.43%	1.32%
10	9.42%	6.71%	2.46%	2.80%	1.06%
20	4.53%	3.43%	2.80%	0.73%	0.91%
40	1.86%	1.32%	1.06%	0.91%	0.07%
	$\alpha = .8$				
h_1	$h_2 = 1$	$h_2 = 5$	$h_2 = 10$	$h_2 = 20$	$h_2 = 40$
5	40.24%	23.44%	18.71%	11.16%	5.60%
10	27.53%	18.71%	9.66%	9.19%	4.96%
20	13.99%	11.16%	9.19%	3.86%	4.12%
40	6.48%	5.60%	4.96%	4.12%	1.25%

the distributor can assert itself as dominant, the manufacturer is dominant because it chooses its schedule first i.e., it is a Stackelberg leader (Frank and Parker 2004). When the supply chain switches from the dominant party's schedule to the overall optimal schedule, the dominant player's cost increases, but the non-dominant party's cost decreases by a larger amount. We note in Section 5.1 that there are situations in which the dominant party may accept a sub-optimal schedule. However, in general the nondominant party must use some of its cost saving to compensate the dominant party for changing its schedule. Classical game theory (Nash 1953) states that for the dominant party to agree to cooperate, this compensation should exceed the amount by which its cost increases in the overall optimal schedule.

More formally, suppose $S(\pi)$ is the manufacturer's cost and $T(\pi)$ is the distributor's cost if schedule π is used. Suppose the manufacturer is dominant, and σ is its preferred schedule. The total cost of the non-coordinated schedule is thus $S(\sigma) + T(\sigma)$. Let σ^c denote an optimal coordinated system schedule. The total cost savings from implementing the coordinated schedule is thus $p^* = [S(\sigma) + T(\sigma)] - [S(\sigma^c) + T(\sigma^c)]$. Note that the dominant party's *increase* in cost from agreeing to use the coordinated schedule is $p = S(\sigma^c) - S(\sigma) < p^*$, so the non-dominant party's *decreases* in cost is $p^* + p$. A possible mechanism for cooperation is for the dominant party to receive a lump sum side payment of $p + \beta p^*$, where $0 < \beta < 1$; the value of β typically depends on the relative negotiating power of the two parties. Note that under this mechanism both parties have an incentive to cooperate since the dominant party's net cost is $S(\sigma^c) - p - \beta p^* = S(\sigma) - \beta p^*$ and the nondominant party's net-cost is $T(\sigma^c) + p + \beta p^* = T(\sigma) - (1- \beta)p^*$. A similar argument applies to the case in which the distributor is dominant.

The distribution of the surplus between the two parties is determined by the value of β. Marketing research (Corfman and Lehman 1993; Lehman 2001) and management research (Williamson 1975; Ouchi 1980) suggest that the surplus should be divided equitably to ensure continued cooperation. A perfectly equitable split of the surplus, however, would require that the parties accurately and continuously share all cost data in a verifiable way.

An alternate approach is one that is more market-oriented. For either problem, if the manufacturer dom-

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inates, then it can negotiate a price to be paid by the distributor for each additional setup that the manufacturer performs. This price should be at least the manufacturer's cost for the additional setup and at most the distributor's benefit from the resulting increase in scheduling flexibility. Because the result of this negotiation is an easily verified job release schedule to which compensation is tied, the manufacturer is unable to cheat without being detected. If the distributor dominates, then a similar negotiation determines the price paid by the manufacturer for the right to impose on the distributor a larger maximum lateness in Problem 1 or a larger inventory cost in Problem 2. Again, the results are easily verified. In both situations, the burden of initiating cooperation and of determining an alternative schedule is borne by the nondominant party.

6. Concluding Remarks

In this paper, we consider two manufacturing environments in which a manufacturer and a distributor naturally solve different problems and, by doing so, choose solutions that are typically not well coordinated. We evaluate the additional costs which are incurred by each decision maker when the other imposes its optimal schedule. Since these costs are substantial, the issue of cooperation between the manufacturer and distributor requires detailed investigation. In order to understand the importance of power relationships within the supply chain, we analyze the scheduling problems faced by each decision maker when the other imposes a constraint on the schedule. Next, we consider the system problem that minimizes a combination of the manufacturer's and the distributor's costs, which enables us to evaluate the cost saving that cooperation between the decision makers provides. Finally, incentives and mechanisms for cooperation between the manufacturer and the distributor are discussed.

Several important research issues remain open for future investigation. First, while we study various objectives which a manufacturer and a distributor may use, there are also other customer-related objectives that are relevant. These include minimization of the number of late jobs and the total tardiness of the jobs. A second important research topic is to extend our cooperation mechanisms to multiple-stage supply chains. We also encourage the development of models of the bargaining process between a manufacturer and a distributor that generalize our models to consider imperfect or asymmetric information. Another important issue is how to maintain cooperation between the parties. Finally, our methods for evaluating the benefits of cooperation between a manufacturer and a distributor should be incorporated into models which evaluate the benefits and costs of vertical supply chain integration.

In conclusion, we expect that this study will help motivate supply chain management research on operational issues in deterministic environments, since the tradeoffs that arise in supply chains can be evaluated most effectively there.

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