SIMULATING AND CORRECTING QUBIT LEAKAGE

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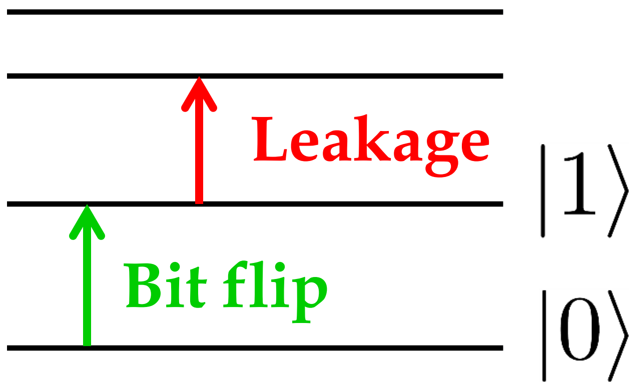
What is Qubit Leakage?

- Physical qubits are not ideal two-level systems and may leak out of the computational space



- With standard error correction techniques leaked qubits accumulate and spread errors

- This talk: simple model of leakage and comparisons of leakage reduction strategies

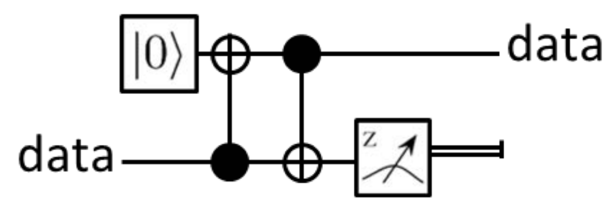
Leakage in the Literature

- First mentions of leakage detection

  (Gottesman, 1997, Preskill 1998)

- Analysis of leakage reduction units based on quantum teleportation, threshold theorem for concatenated codes

  (Aliferis, Terhal, 2005)



- Model of leakage for repetition code that labels leaked qubits (no quantum simulation)

  (Fowler, 2013)

Overview

I. Our leakage model

II. A few examples of leakage reduction circuits

III. Error decoding strategies

IV. Thresholds and error rates with leakage reduction

Abstract Model of Leakage: Erasures

- Leakage event is probabilistic erasure of qubit

$$\mathcal{E}_{\uparrow}(\rho) = (1 - p_{\uparrow})\rho + p_{\uparrow} \sum_{k=2}^{m+1} \frac{1}{m} |k\rangle\langle k|$$

- Leaked qubits may decay back to qubit space

$$\mathcal{E}_{\downarrow}(\rho) = (1 - p_{\downarrow})\rho + p_{\downarrow}\mathcal{E}_{\text{decay}}(\rho)$$

$$\mathcal{E}_{\text{decay}}(\rho) = A_1 \rho A_1^{\dagger} + \sum_{j=2}^{m+1} \sum_{k=2,3} A_{j,k} \rho A_{j,k}^{\dagger}$$

$$A_1 = |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$A_{2,k} = \frac{1}{\sqrt{2}} |1\rangle\langle k|$$

$$A_{3,k} = \frac{1}{\sqrt{2}} |0\rangle\langle k|$$

How Should Two-Qubit Gates Behave?

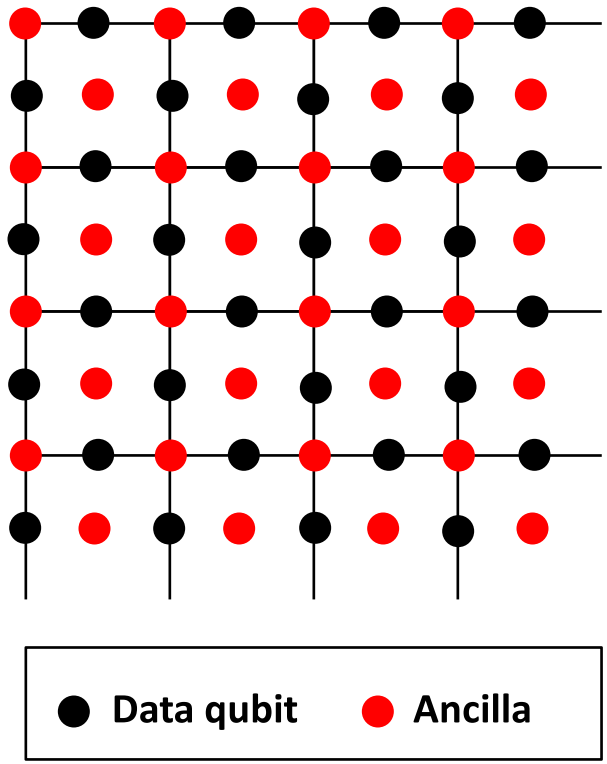
- Assume gates are direct sums of unitaries

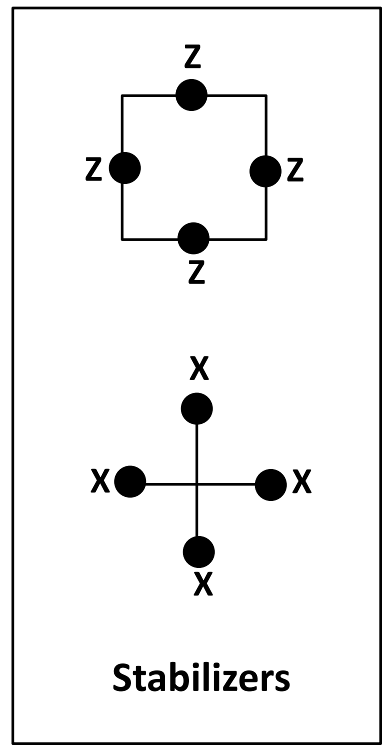
$$U \approx \begin{pmatrix} U_{2\otimes 2} & & & \\ & U_{2\otimes m} & & \\ & & U_{m\otimes 2} & \\ & & & U_{m\otimes m} \end{pmatrix}$$

- Assume unitaries of m⊗2 and 2⊗m blocks are maximally entangling and twirl over the L subsystem on these blocks after each gate
- If only one input leaks, this is equivalent to depolarizing the unleaked input

Model acts violently on the leakage subspace between gates.

Simulating Leakage for the Toric Code

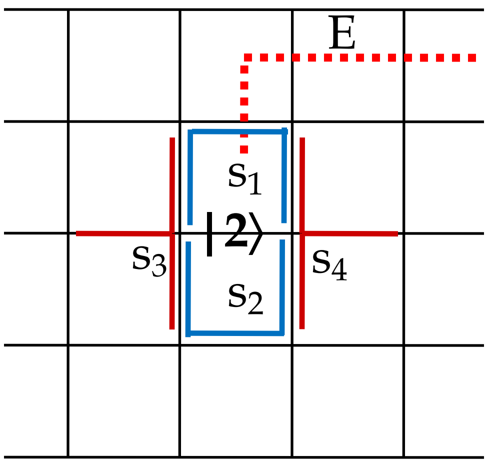




- Our label-based model: each qubit is in state I, X, Y, Z, or L

Simulating by Propagating Labels

- Our leakage model destroys syndrome correlations of less violent models



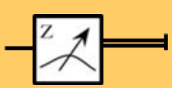
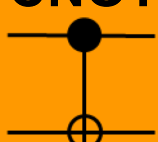
$$s_1 \oplus s_2 = 1$$

$$s_3 \oplus s_4 = 0$$

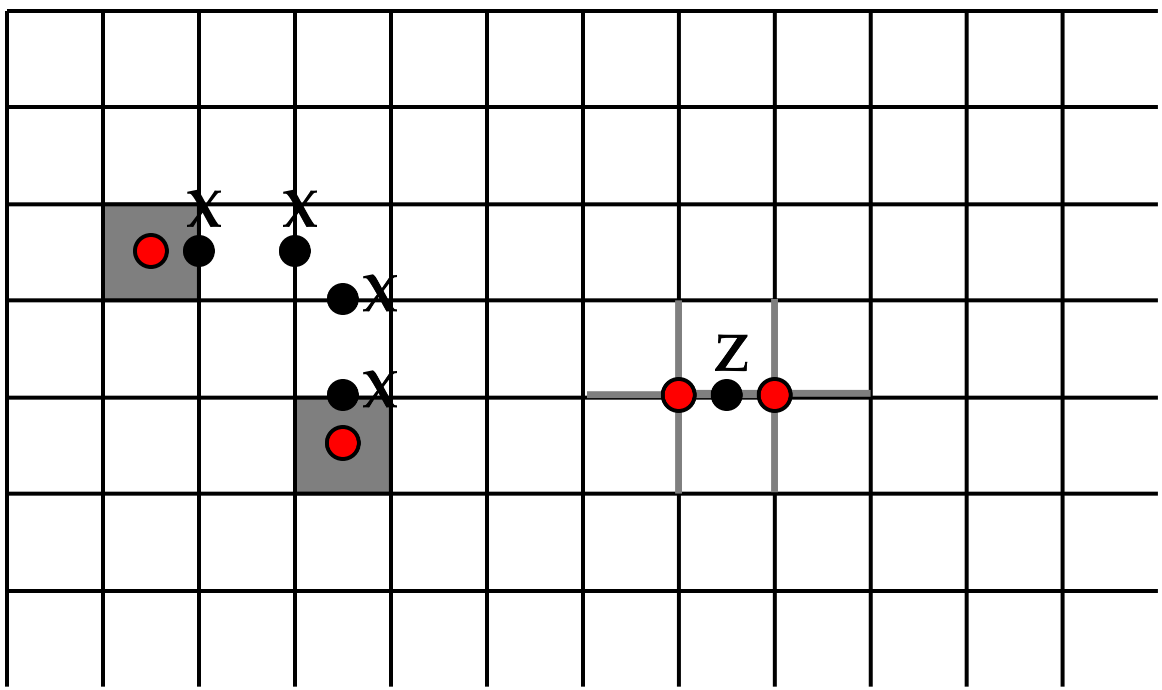
parity constraints violated with probability ½ since ancilla depolarized

- Does not appear necessary to retain quantum state in the simulation - conjecture propagating new error label faithfully simulates the model for surface code

Behavior of Gates

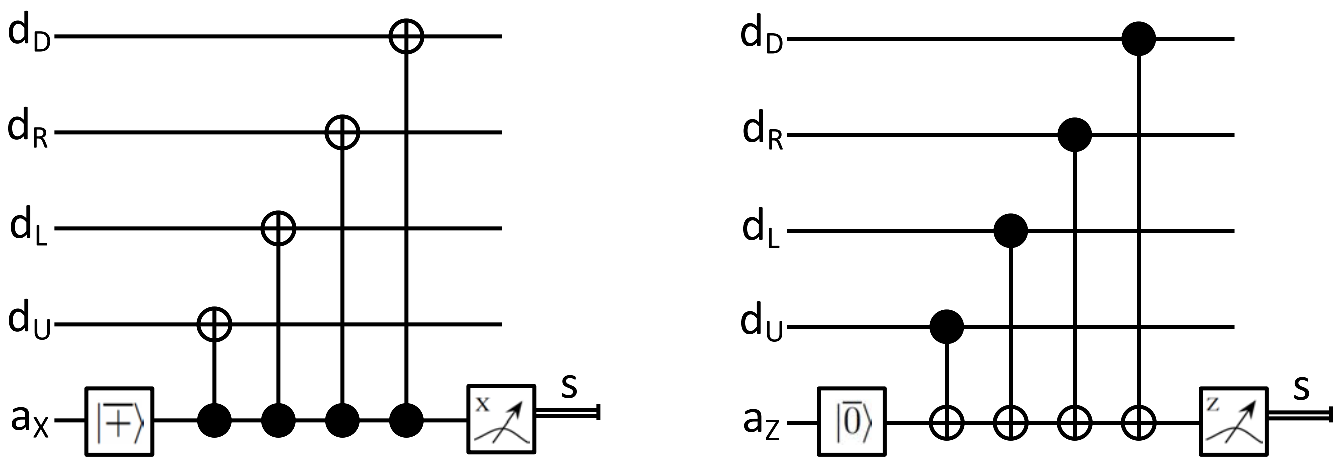
Gate	Possible Errors	Leakage Errors
Identity	X, Y, Z	if leaked relaxes w/ prob. pd, doesn't increase leakage
Preparation 	orthogonal state	leaks w/ prob. pu
Measurement 	incorrect	if leaked, always measures 1 (also consider leakage detection)
CNOT 	IX, XX, XZ, etc.	if leaked, applies random Pauli to the other qubit; leaks w/ prob. pu and relaxes w/ prob. pd

C++ Simulation Measures and Matches Error Syndromes



- Use minimum weight matching and correct errors between pairs of closest syndromes
- Circuit model simulates syndrome errors

Circuit Model of Syndrome Extraction



- Each gate in the circuit causes Pauli errors or leakage according to our model

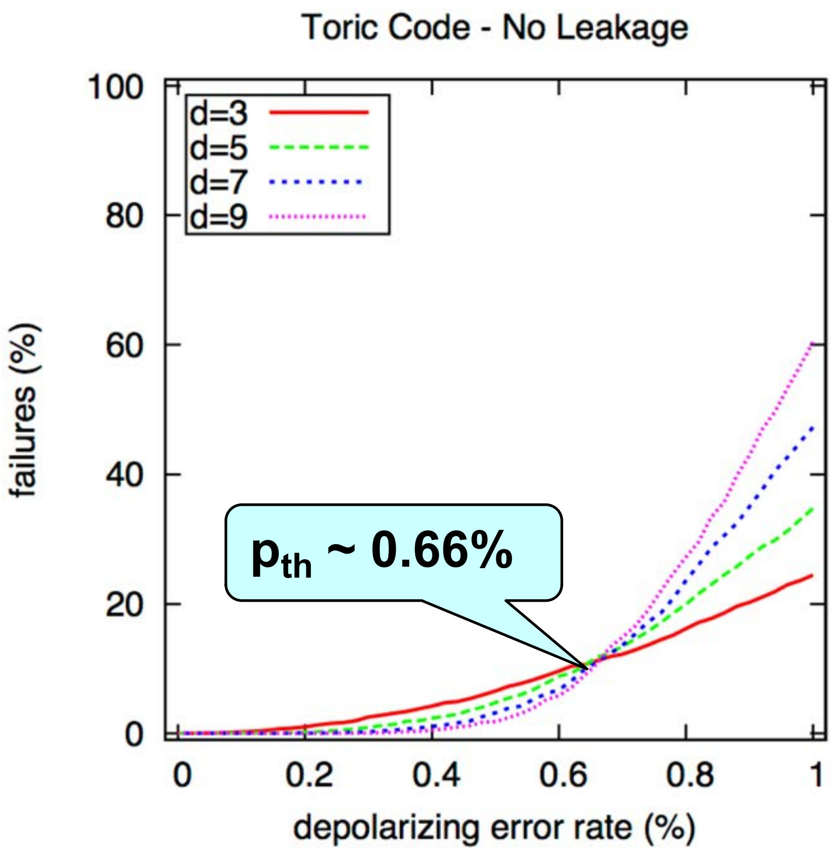
Leakage can Accumulate

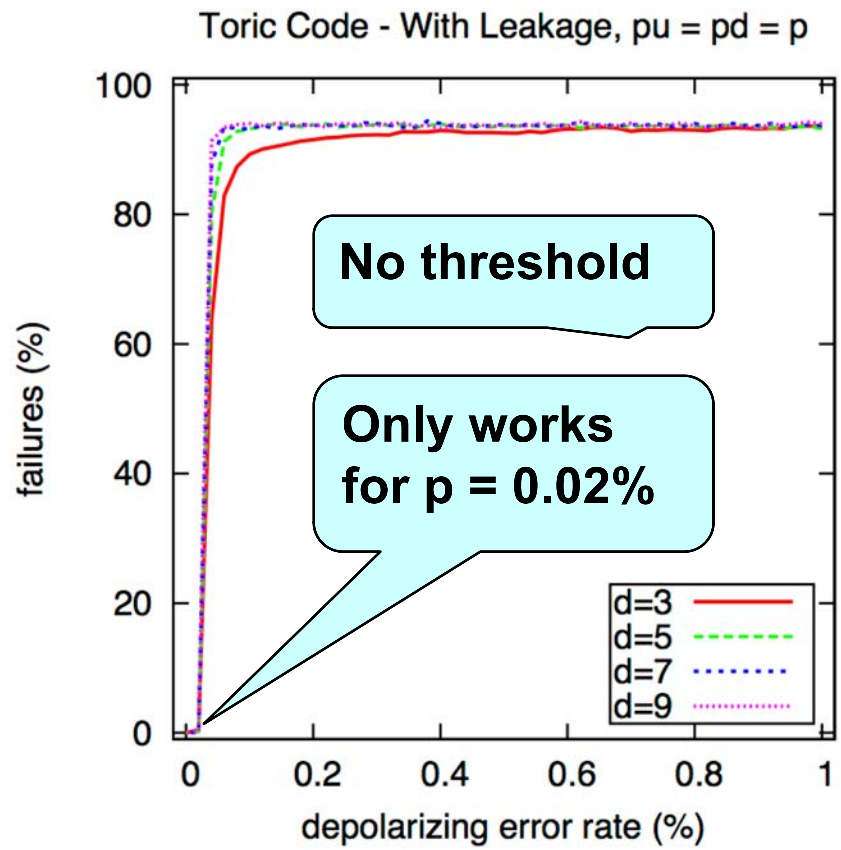
- Leakage accumulates on the data qubits
- Initialization of ancillas prevents accumulation
- Equilibrium leakage rate is a property of the circuit and its gates
  - Our circuit: $p_{eq} \approx \frac{4pu}{(4pu + 6pd)}$
  - 4pu: leakage caused by CNOTs
  - 6pd: leakage reduction of CNOTs and identities

Simulation Details

- Start simulation in equilibrium
  - A fraction of data qubits starts in L state
- A round of perfect leakage reduction at the end of each simulation
  - Leaked qubit replaced with I, X, Y, or Z
- We use d rounds of syndrome measurements, the last one is ideal

Success Probabilities





- Leakage reduction is necessary!

Overview

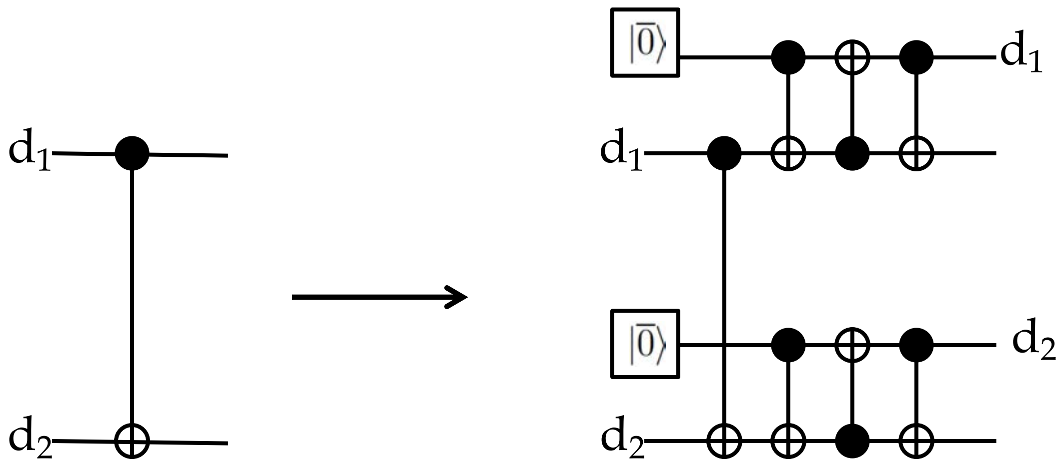
I. Our leakage model

II. A few examples of leakage reduction circuits

III. Error decoding strategies

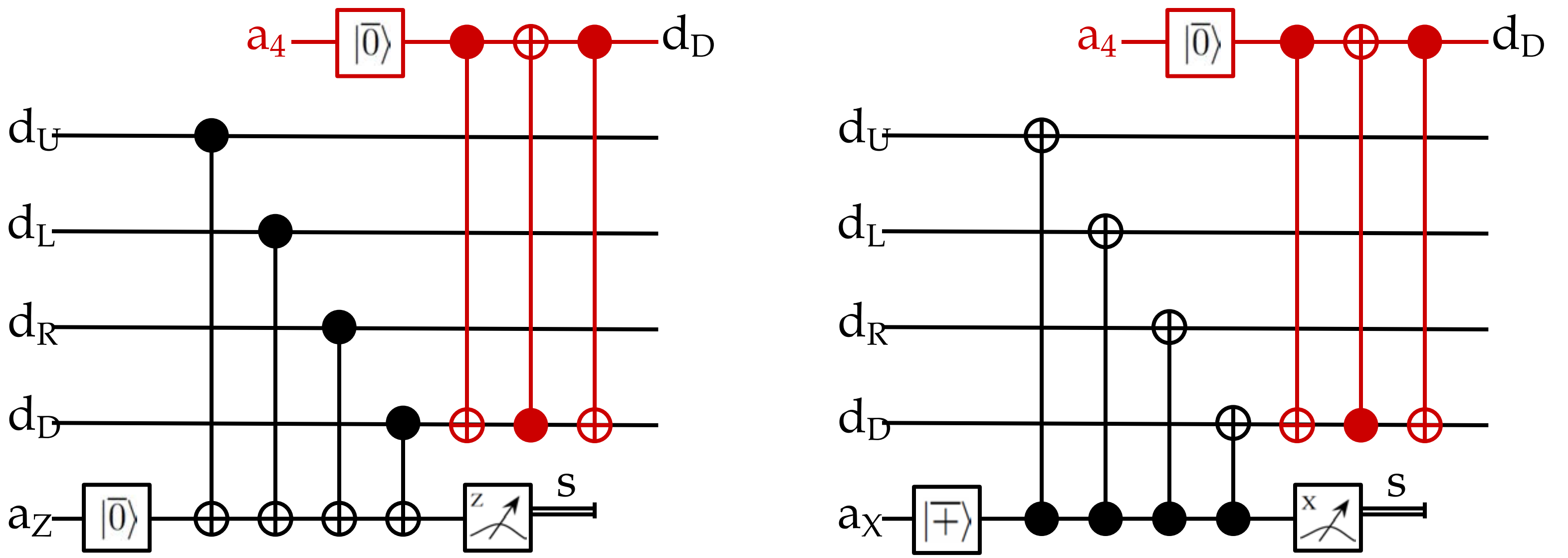
IV. Thresholds and error rates with leakage reduction

Full-LRU Circuit



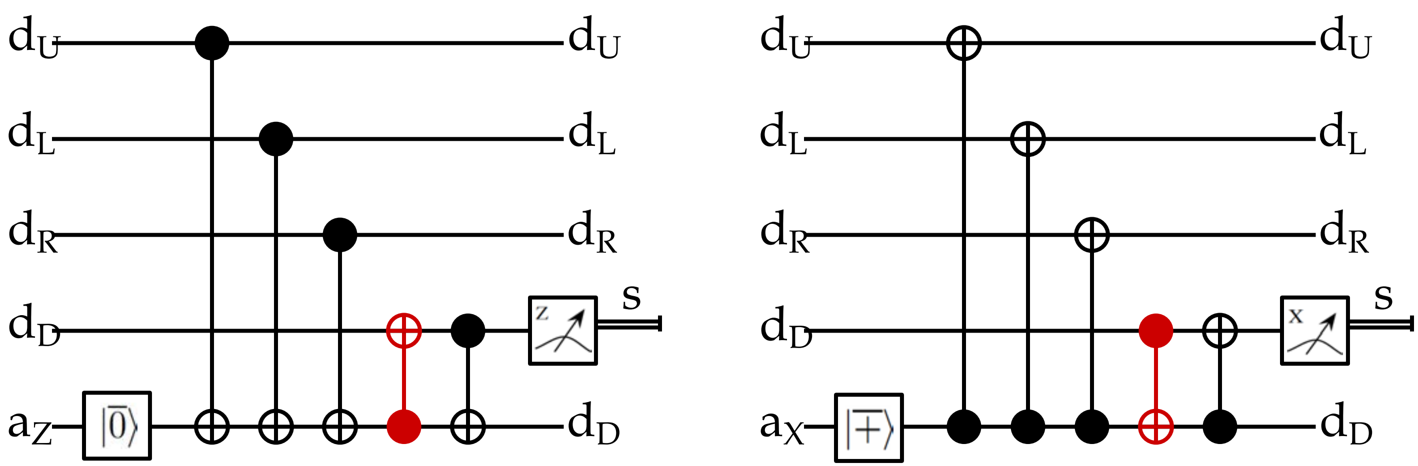
- Swap with a newly initialized qubit after each gate
- Slow and expensive

Partial-LRU Circuit



- Swaps each data qubit with a fresh one during ancilla measurement
- Requires 3 CNOTs

Quick Leakage Reduction Circuit



- Swaps data qubits and ancillas
- Sufficient to add a single CNOT gate

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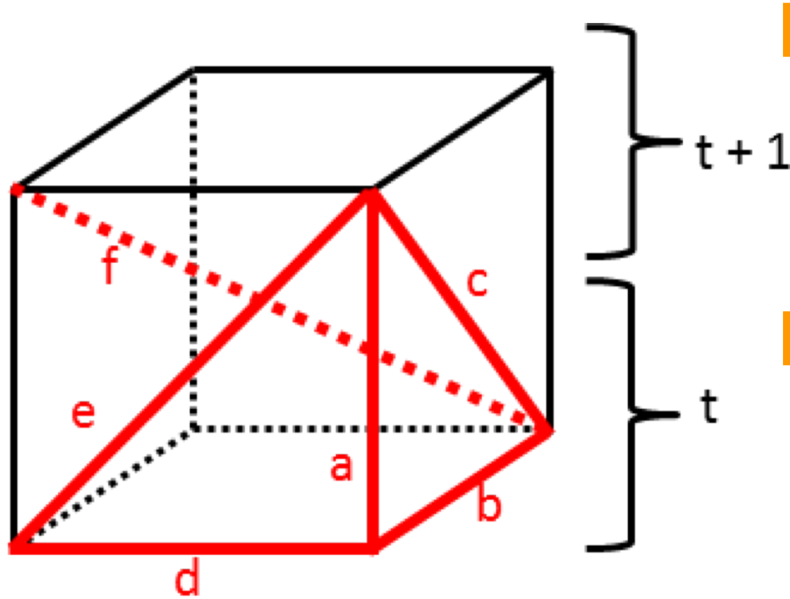
The Standard and Heralded Leakage (HL) Decoders

- **Standard Decoder** only relies on syndrome history to decode errors

- **HL Decoder** uses leakage detection when qubits are measured
  - Partial information about leakage locations
  - Error decoder must be modified

Standard Decoder for the Toric Code



- Need to correct error chains between pairs of syndromes
- Decoding graphs for X and Z errors built up using this unit cell

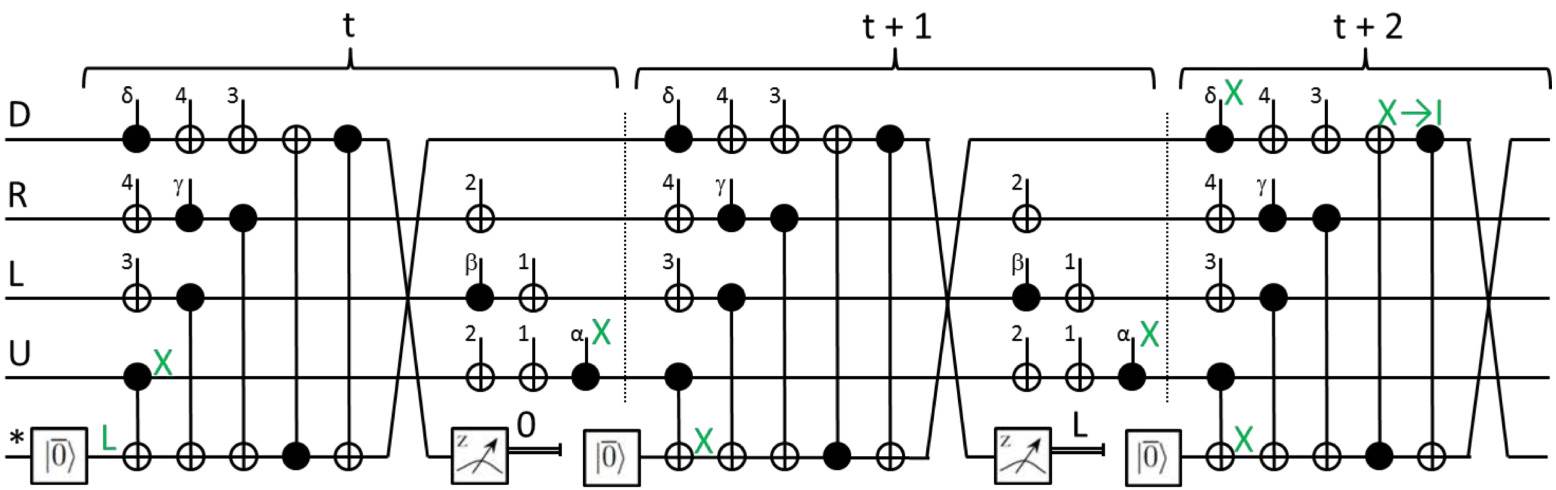
(Fowler 2011)

- Need to adjust edge weights for each leakage suppressing circuit (Full-LRU, Partial-LRU, Quick circuit)

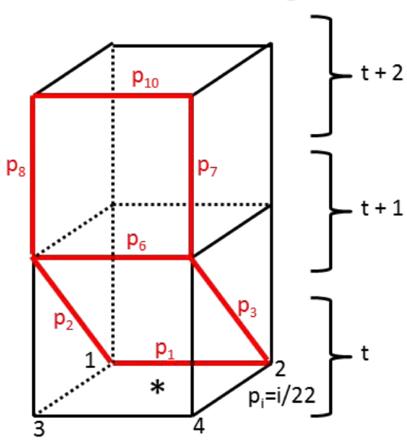
Standard Decoder – Adjustment of Edge Weights

Circuit	a	b	c	d	e	f
No-LRU	11/5p + q	28/15p	16/15p	52/15p	8/15p	8/15p
Quick circuit	7/3p + q	32/15p	4/3p	4p	8/15p	32/15p
Full-LRU	103/15p + q	52/15p	88/15p	172/15p	32/15p	8/15p
Partial-LRU	31/15p + q	52/15p	16/15p	76/15p	8/15p	8/15p

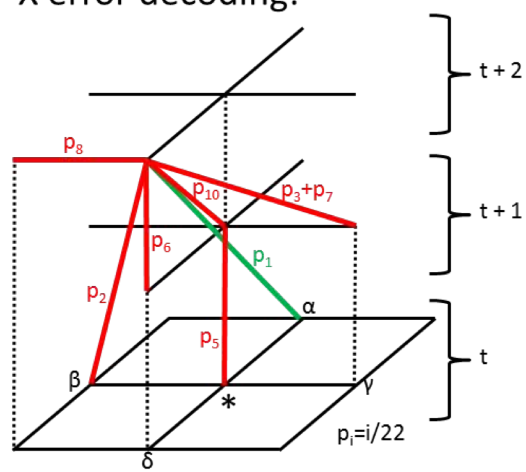
HL Decoder: Quick Circuit (11 leakage locations)

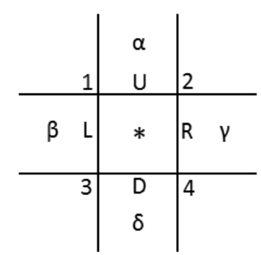


Z error decoding:

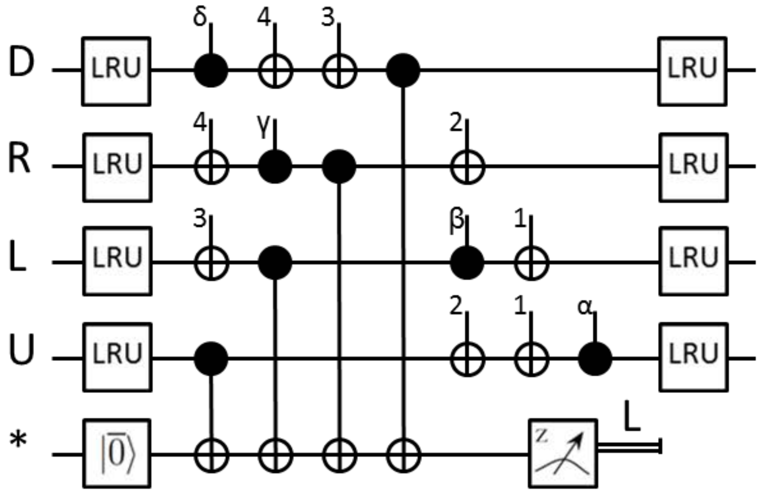


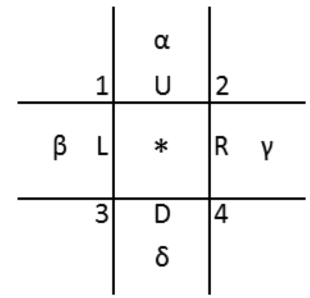
X error decoding:



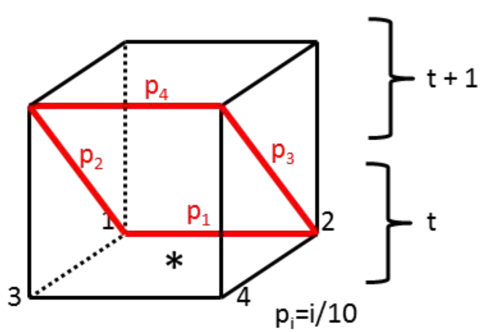


HL Decoder: Partial-LRU Circuit (5 ancilla leakage locations)

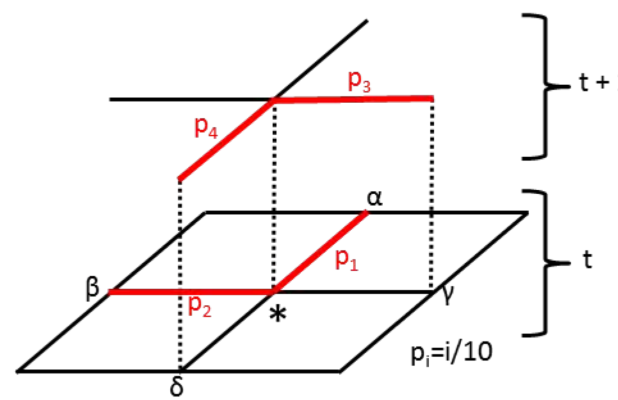




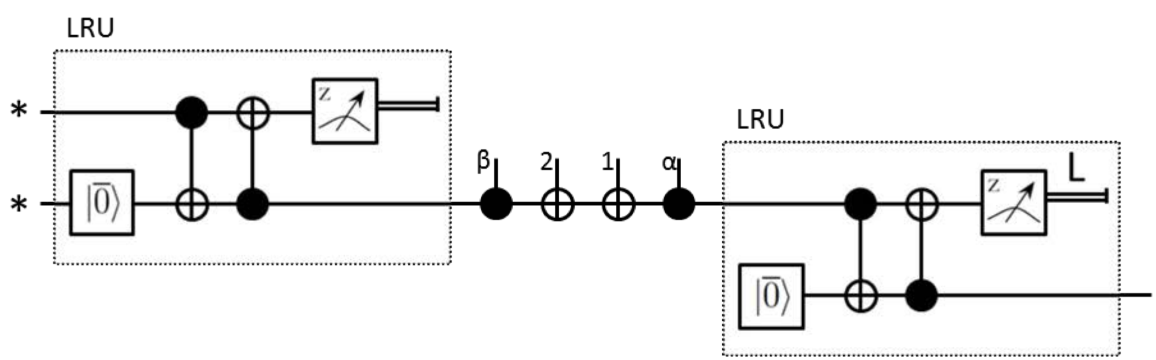
Z errors:

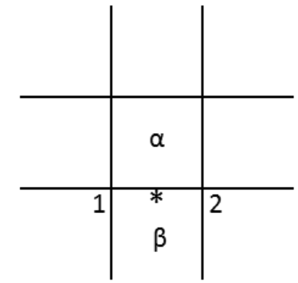


X errors:

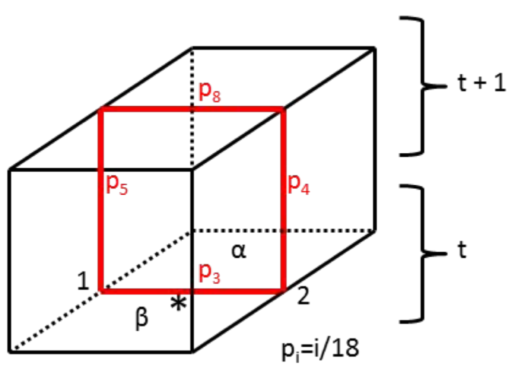


HL Decoder: Partial-LRU Circuit (9 data leakage locations)

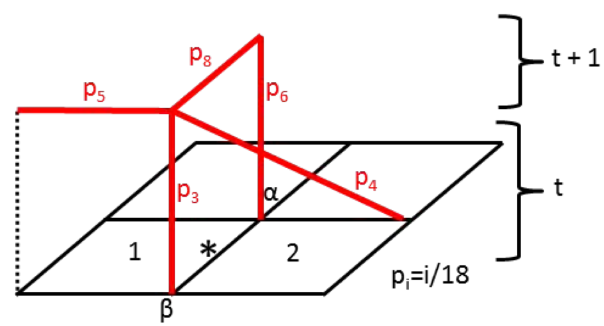




Z errors:



X errors:



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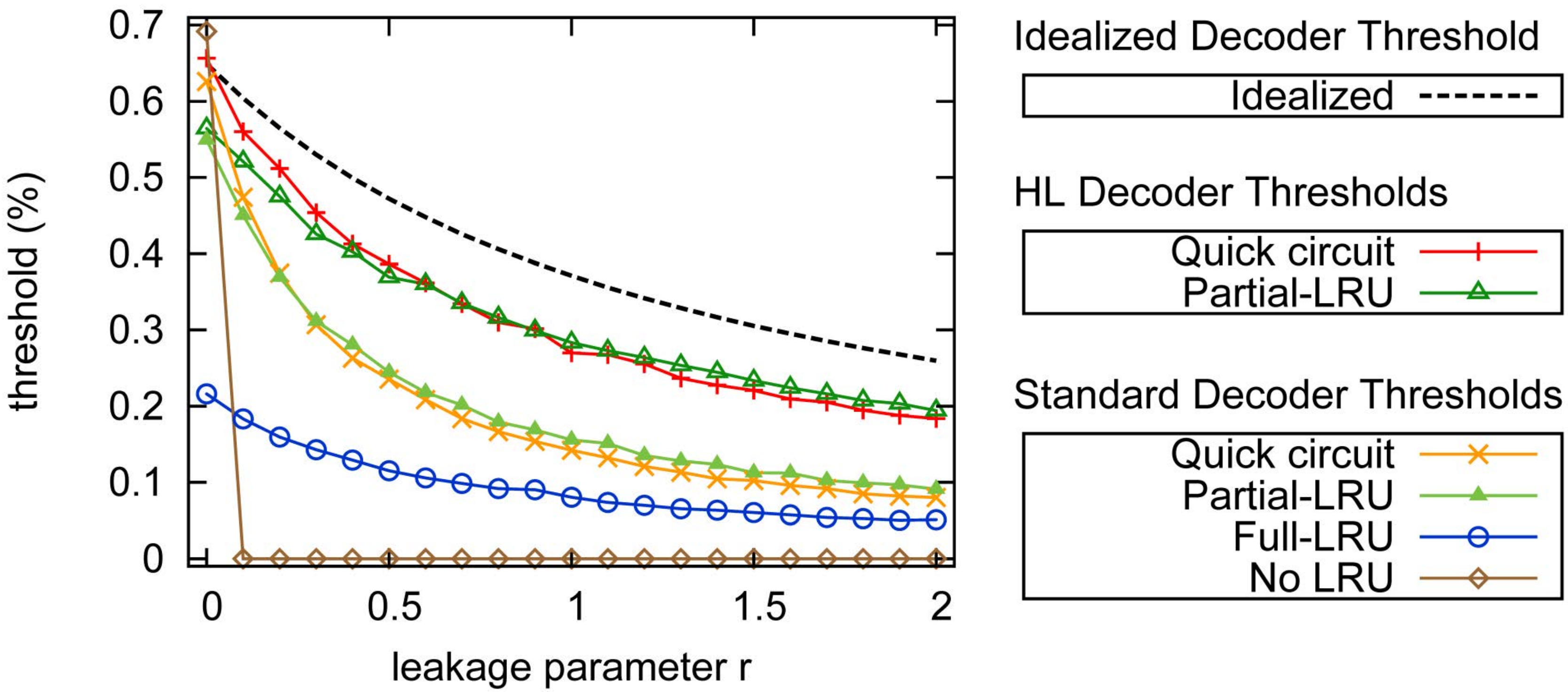
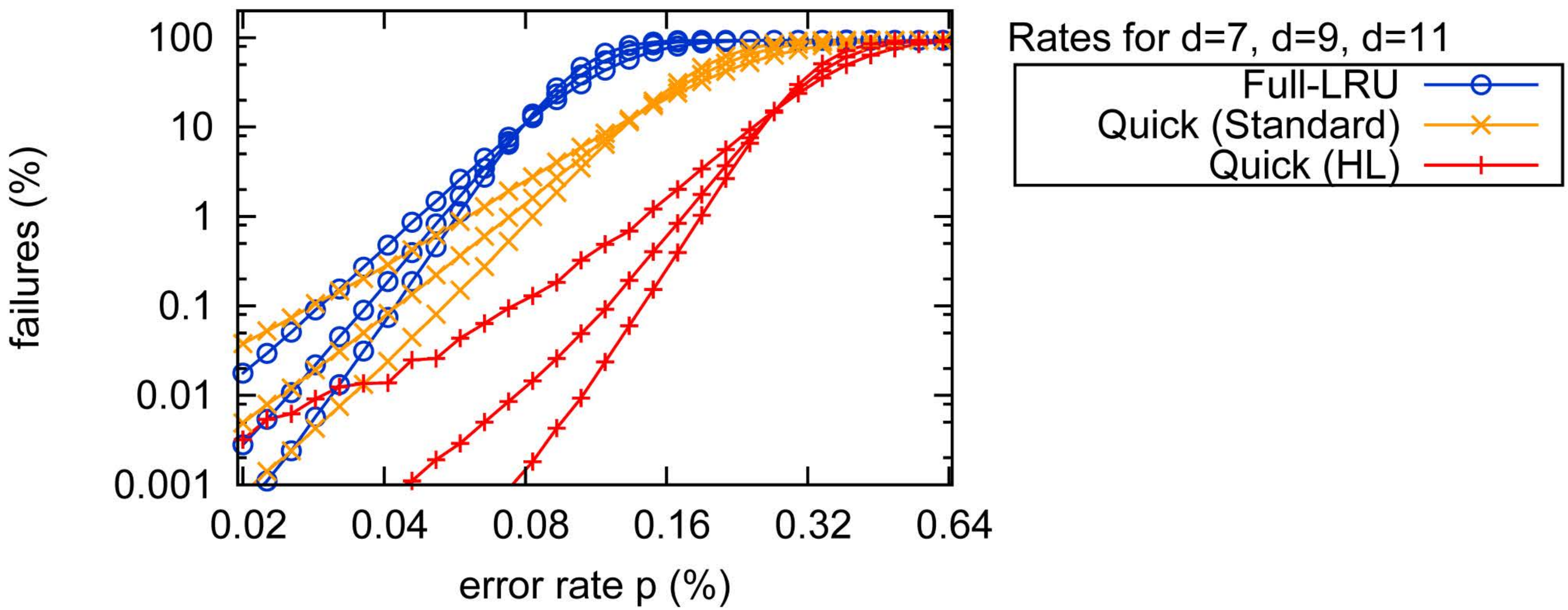
Threshold Comparison



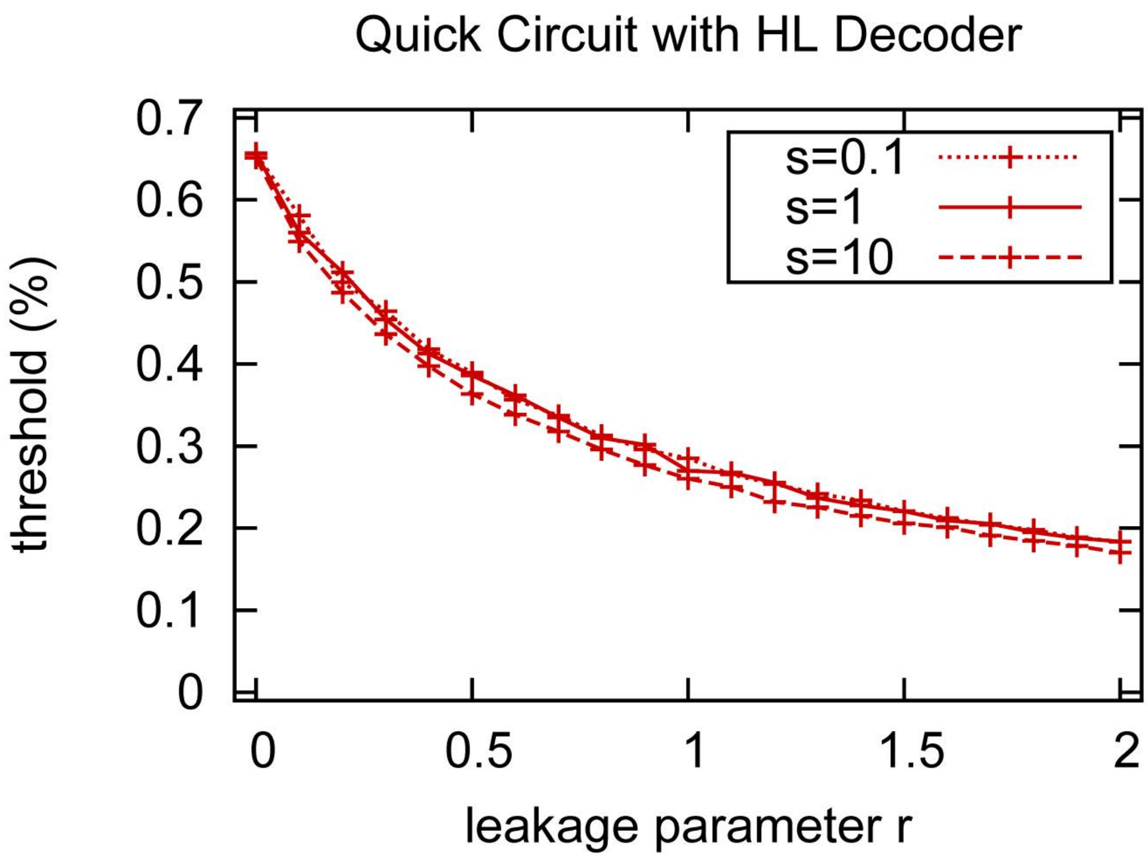
- More complicated circuits have lower threshold
- HL decoder helps boost the threshold

Decoding Failure Rates



- Full-LRU performs well at low error rates

Effect of the Leakage Relaxation Rate (Quick circuit)



- Leakage relaxation rate small compared to the leakage suppression capability of the circuits

Conclusion

- Leakage reduction is necessary
- Model of leakage that allows efficient simulation
- A simple leakage reduction circuit that only adds a single CNOT gate and new decoders
- Systematic exploration of error correction performance
- Available as arXiv 1410.8562

Thank You!