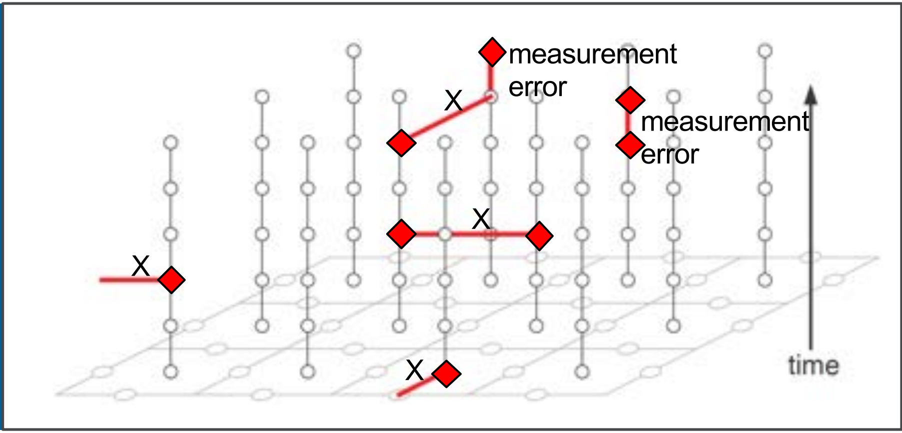
ARGONNE QUANTUM COMPUTING TUTORIAL

INTRODUCTION TO QUANTUM ERROR CORRECTION



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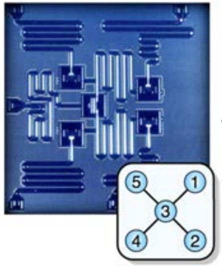
Lemont, IL

WHAT IS NEEDED TO MAKE QUANTUM COMPUTING RELIABLE

- Replace imperfect physical qubits by multiple logical ones and incur space and time overhead
- Must be able to correct many new types of errors:
  - Partial bit flip: $|0\rangle \rightarrow 0.99|0\rangle + 0.1|1\rangle$
  - Phase flip: $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$
  - Small shift: $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + e^{0.0001i}|1\rangle\right)$
  - Leakage: $|1\rangle \rightarrow |2\rangle$
- Must correct errors in a way to allow reliable information storage and computation with unreliable qubits

QUBIT DECOHERENCE TIME IS NOT THE ONLY CONSIDERATION

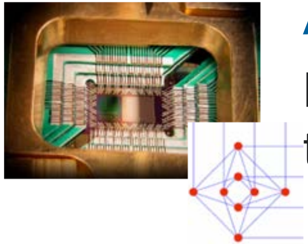


Superconducting qubits:

Josephson Junctions between superconducting electrodes

Ion traps:

Ions trapped in electromagnetic field, gates performed by applying lasers



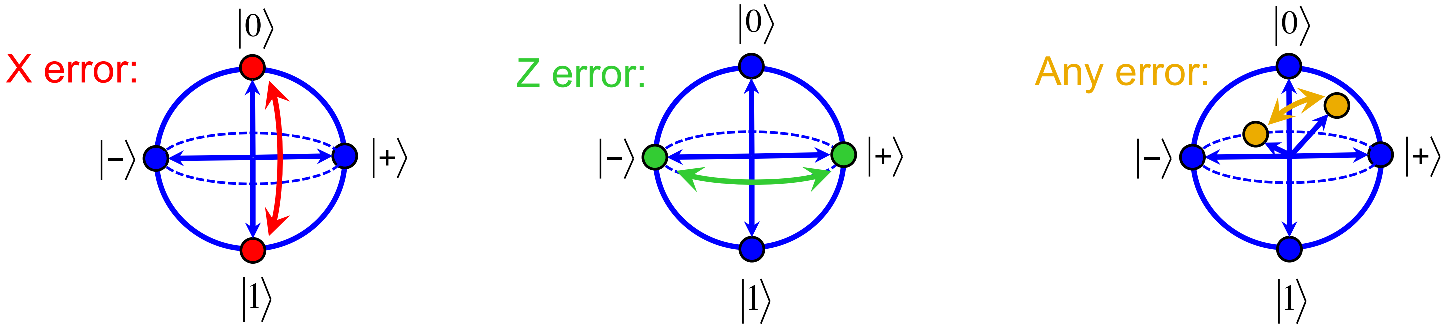
Adiabatic quantum computation:

Lattice of superconducting qubits that arrange themselves to solve an optimization problem

HOW QUANTUM ERROR CORRECTION WORKS

HOW QUANTUM ERROR CORRECTION WORKS

- Must satisfy the following conditions: (i) can't copy a qubit, (ii) can't measure a qubit without collapsing it, (iii) must correct arbitrarily small errors



- Use an entangled state inspired by the repetition code to correct X errors:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle \text{ (encoding)}$$

$$\rightarrow \alpha|000\rangle + \beta|111\rangle + \gamma|010\rangle + \delta|100\rangle + \ldots \text{ (errors)}$$

$$\rightarrow \alpha|000\rangle + \beta|111\rangle \text{ (decoding)}$$

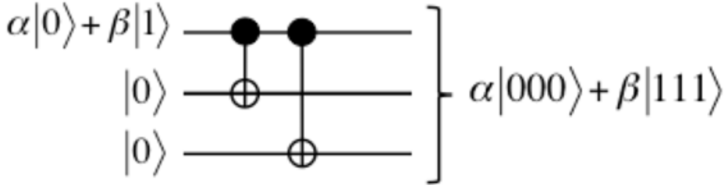
ENCODING QUBITS AND USING "PARITY CHECKS" ON GROUPS OF QUBITS

- What we need: $\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$ (encoding)

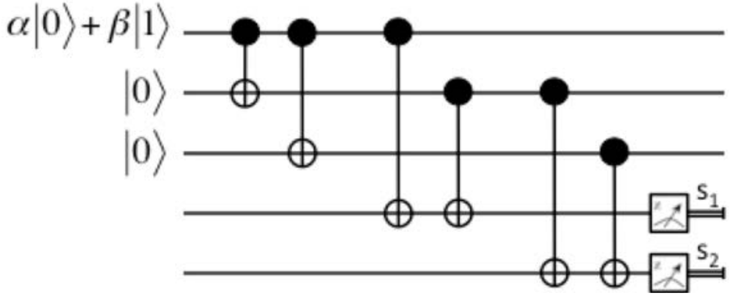
  $\rightarrow \alpha|000\rangle + \beta|111\rangle + \gamma|010\rangle + \delta|100\rangle + \ldots$ (errors)

  $\rightarrow \alpha|000\rangle + \beta|111\rangle$ (decoding)

- Encoding circuit:



- Measure syndromes:



X error:	Syndrome:
top qubit	s_1=1, s_2=0
middle qubit	s_1=1, s_2=1
bottom qubit	s_1=0, s_2=1
no error	s_1=0, s_2=0

CORRECTING (PARTIAL) BIT FLIP AND PHASE FLIP ERRORS SIMULTANEOUSLY

- Theorem: correcting X errors with the 3-qubit bit flip code and then Z errors with the 3-qubit phase flip code corrects any arbitrary error affecting a single qubit

  The encoded state is $|\psi\rangle = \alpha|0_L\rangle + \beta|1_L\rangle$

  Noise can be described by a trace preserving operation that can be expanded in an operator sum: $\varepsilon(|\psi\rangle\langle\psi|) = \sum_i E_i|\psi\rangle\langle\psi|E_i^\dagger$

  E_i can be expanded as $E_i = e_{i0}I + e_{i1}X_1 + e_{i2}Z_1 + e_{i3}X_1Z_1$

  Measuring the superposition of state $E_i|\psi\rangle$ thus yields one of $|\psi\rangle$, $X_1|\psi\rangle$, $Z_1|\psi\rangle$, or $X_1Z_1 |\psi\rangle$ and recovery is performed by the error decoder

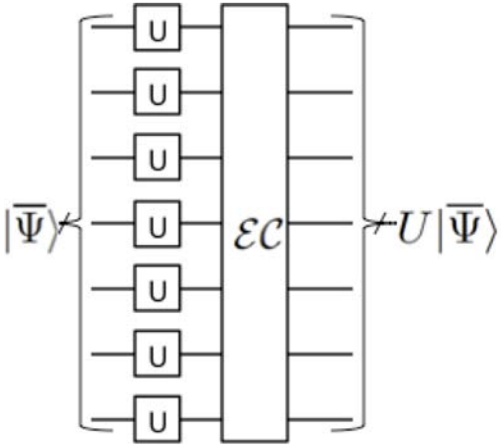
- The continuum of errors that may occur can be corrected by addressing only a discrete subset of these errors, namely the X and Z error on each qubit!

NEED THRESHOLD AND FAULT TOLERANCE: EXAMPLE – CONCATENATED CODES

- Threshold behavior of the repetition code when p is simple bit error probability:

Encoded 0 becomes	000	001, 010, or 100	110, 101, or 011	111
Probability	$(1-p)^3$	$3(1-p)^2p$	$3(1-p)p^2$	p^3

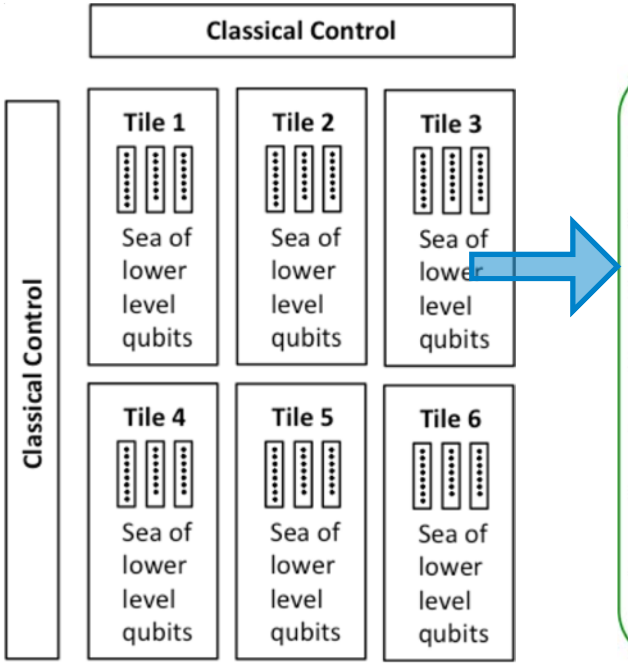
- Errors improve if $3p^2 - 3p^3 + p^3 \approx 3p^2 < p$
- Code threshold $p = 1/3$
- Can do arbitrarily well if errors are below this threshold by concatenating



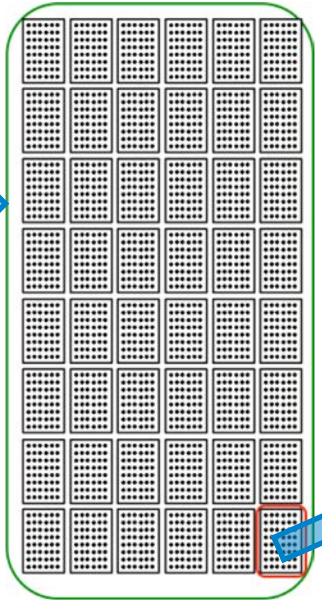
- Steane [[7,1,3]] concatenated code
  - 7 physical qubits encode 1 logical qubit
  - Fault tolerant computing with mostly transversal gates

CONCATENATED CODES ARE IMPRACTICAL (UNLESS ERROR RATES ARE VERY SMALL)

- Very expensive concatenations, non-transversal T gates, low threshold ≈ 10^{-5}

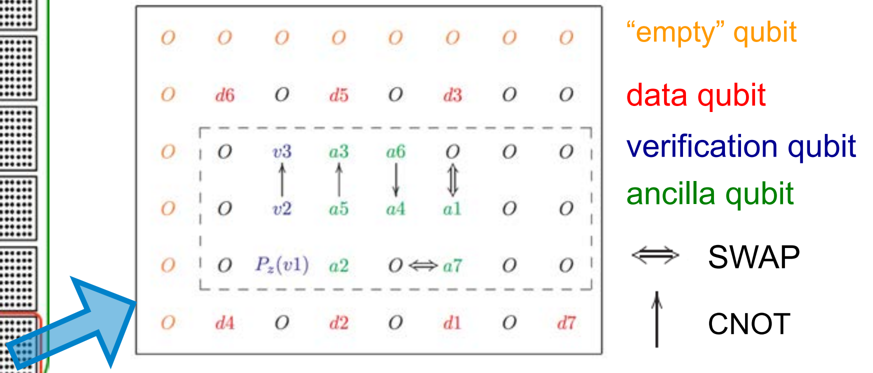


Inside each tile:



Optimized qubit layout for the Steane code:

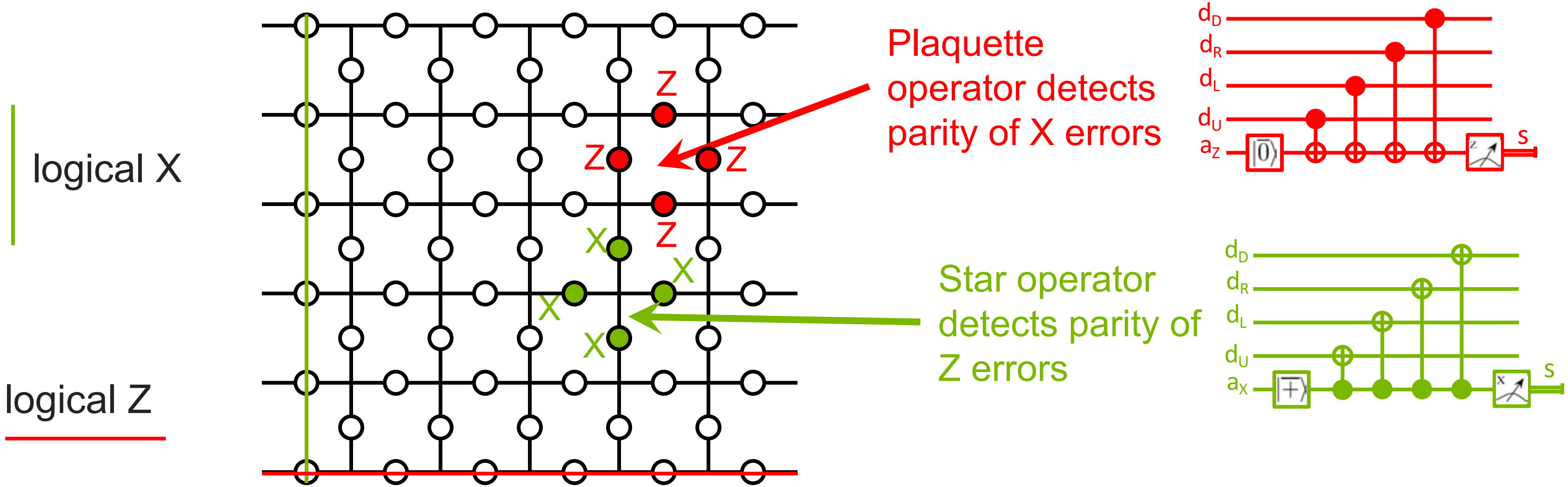
K. Svore et al.: Noise Threshold for a Fault-Tolerant Two-Dimensional Lattice Architecture, 2006



THE SURFACE ERROR CORRECTING CODE

SURFACE CODE – SYNDROME EXTRACTION

- n qubits on edges, here n=61, L=6

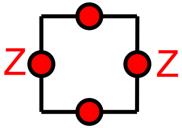


- Smallest [[13,1,3]] code encodes 1 qubit into 13 qubits and has distance 3
- The distance is the minimum weight of a logical operator

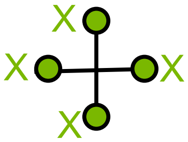
FEATURES OF THE SURFACE CODE

- Plaquette and star operators all commute and have eigenvalues ± 1

Plaquette operator A_p



Star operator B_q

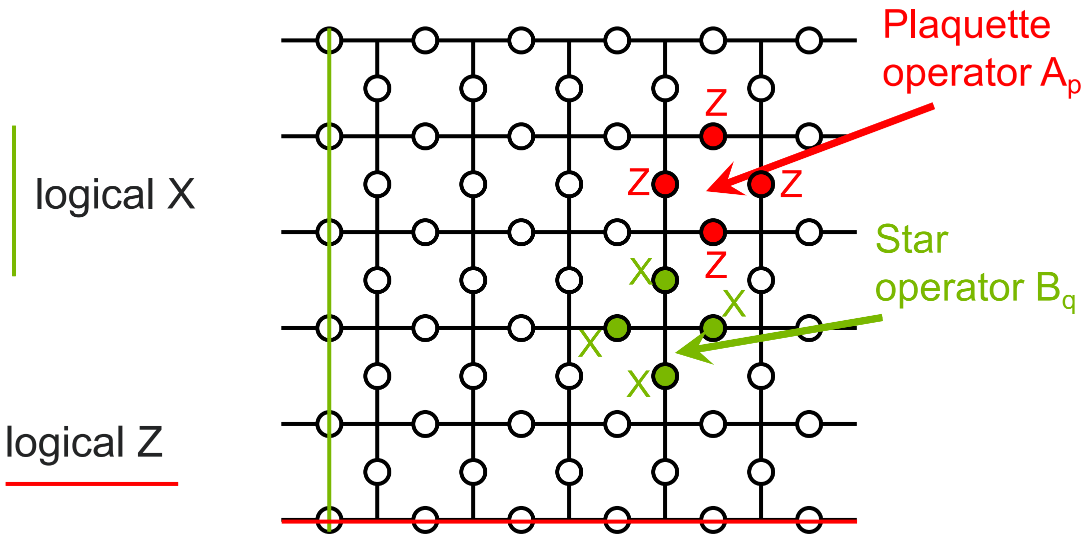


- Plaquette and star operators generate an Abelian group S, the stabilizer group
- The code space $C = \{|\varphi\rangle : A_p|\varphi\rangle = |\varphi\rangle, B_q|\varphi\rangle = |\varphi\rangle, \forall p, q\}$. Codespace is +1 eigenspace of group S.
- Dimension of code space for n qubits is 2^{n-k} where k is the number of linearly independent generators in S
- For surface code n-k=1, for toric code n-k=2

logical X logical Z

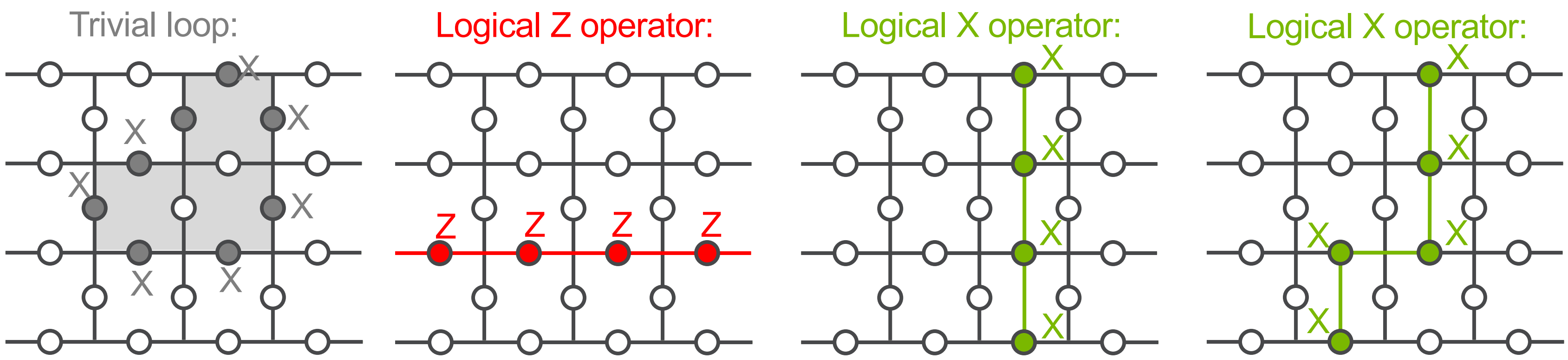
SURFACE CODE – LOGICAL OPERATORS

- n qubits on edges, here n=61, L=6



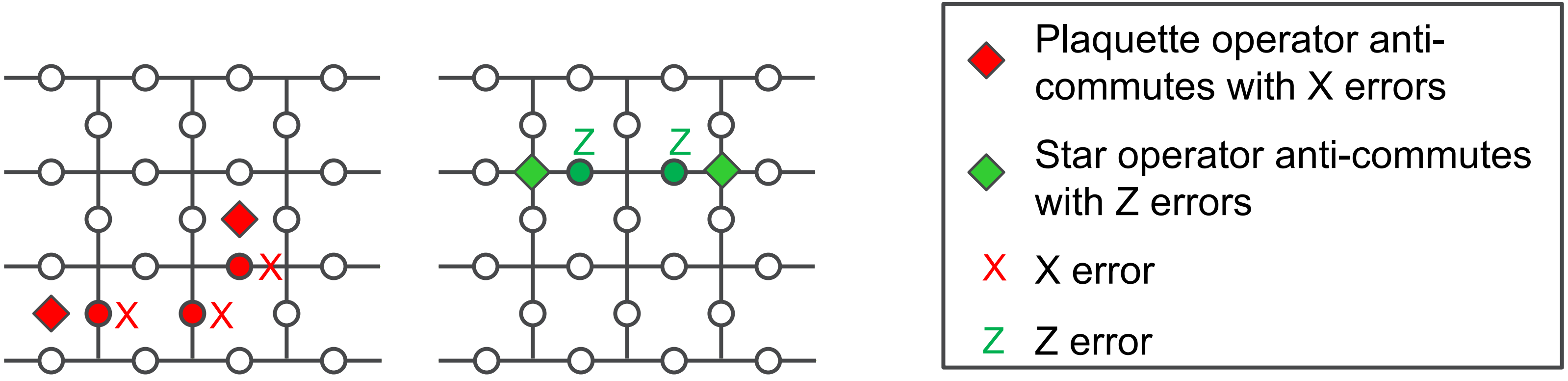
- Logical operators commute with all elements in the stabilizer group S but are not in S. If they are in S they act trivially.
- If they don't commute with S they map the codeword out of the codespace.

EXAMPLES OF LOGICAL OPERATORS



- Trivial loop made from plaquette operators.
- Logical X is a non-trivial loop which cannot be made from plaquette operators, i.e. this operator is not in S, but commutes with S
- Deformations: For φ in codespace, A_p φ= φ and B_q φ= φ so action of plaquette and star operator is trivial on codespace. Thus $\bar{X}$ B_q φ= $\bar{X}$ φ and we can deform a logical operator by multiplying with A_p and A_p.

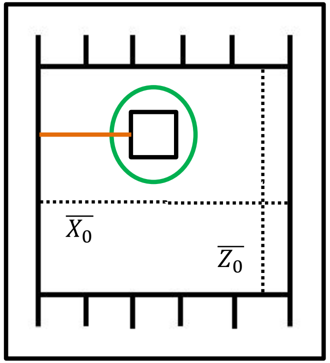
ERRORS



- Sufficient to consider Pauli errors and treat X and Z errors independently
- Pauli errors by definition anti-commute with at least one element of the stabilizer group
- If error E anti-commutes with some $s \in S$, then $s(E|\bar{\varphi}\rangle = -E|\bar{\varphi}\rangle)$ or codestate with error has -1 eigenvalue with respect to s.
- By measuring generators of S (plaquette & star operators), we get information about what errors occurred

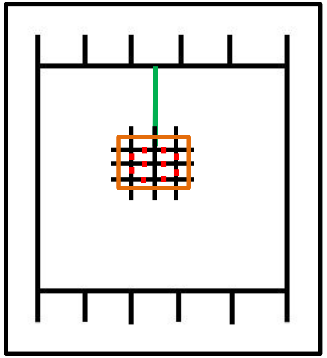
ENCODING MORE QUBITS IN SURFACE CODE

Smooth hole:



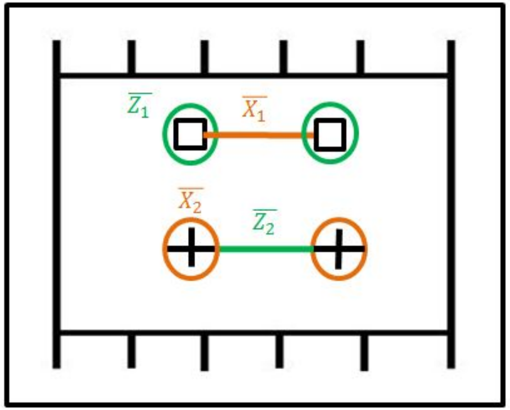
- What happens when we remove one plaquette from the stabilizer S?
- This plaquette becomes the logical operator $\overline{Z_1}$ of a new encoded qubit. Corresponding $\overline{X_1}$ is the orange X-string which connects the hole to the boundary.
- New qubit has distance 4, bad... make big hole

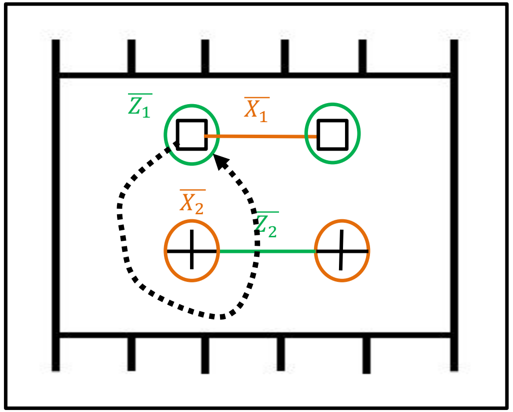
Rough hole:



- Similarly, one can make a rough hole by removing a cluster of star operators. It will have a rough boundary.
- Distance is minimum of distance to boundary or circumference of hole

THE CNOT GATE IN THE SURFACE CODE





- Data qubits are encoded into smooth / rough qubits of sufficiently large area and sufficiently far apart
- CNOTs can be done between smooth and rough qubits
- The smooth qubit is the control and the rough qubit the target
- We move one of the holes of the smooth qubit around a rough hole causing a deformation of the logical operators of the two qubits
- The CNOT gate acts on the Pauli's $UPU^\dagger = P'$ as follows:

$$X_1 \otimes I_2 \rightarrow X_1 \otimes X_2$$
$$I_1 \otimes X_2 \rightarrow I_1 \otimes X_2$$
$$Z_1 \otimes I_2 \rightarrow Z_1 \otimes I_2$$
$$I_1 \otimes Z_2 \rightarrow Z_1 \otimes Z_2$$

After some work can show deformation causes this action

UNDERSTANDING PERFORMANCE OF THE SURFACE CODE WITH NUMERICAL SIMULATIONS

THE ERROR MODEL AND PERFORMANCE OF THE DECODER

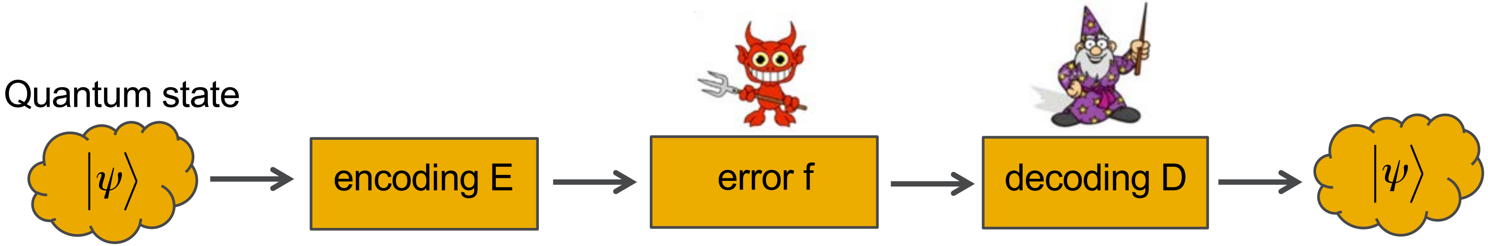
- Typically use depolarizing noise model is simulation and estimation of noise thresholds: $\rho \rightarrow (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

Pauli Errors:

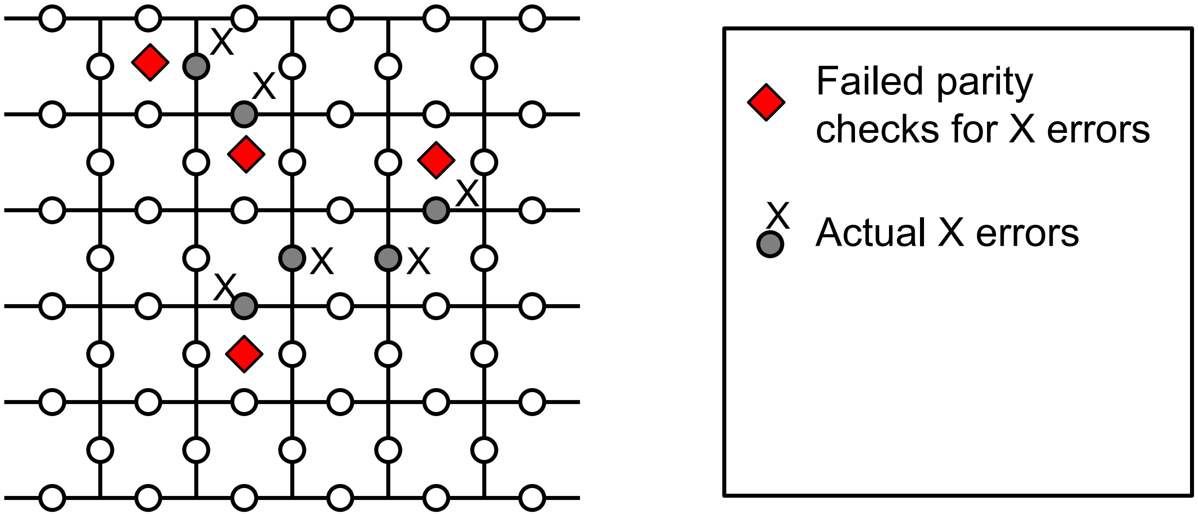
I	Perfect transmission
X	Bit flip
Z	Phase flip
Y	Bit and phase flip

$\alpha|0\rangle + \beta|1\rangle \rightarrow$

- $\boxed{I}$ $\alpha|0\rangle + \beta|1\rangle$
- $\boxed{X}$ $\alpha|1\rangle + \beta|0\rangle$
- $\boxed{Z}$ $\alpha|0\rangle - \beta|1\rangle$
- $\boxed{Y}$ $\alpha|1\rangle - \beta|0\rangle$

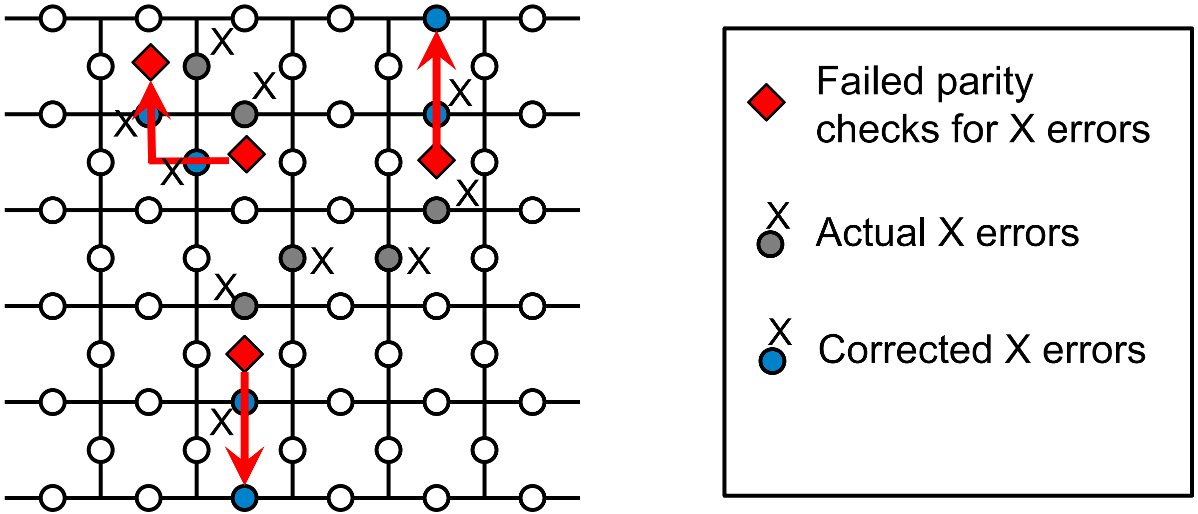


NUMERICAL SIMULATION – STEP 1: INJECT ERRORS AND MEASURE SYNDROMES



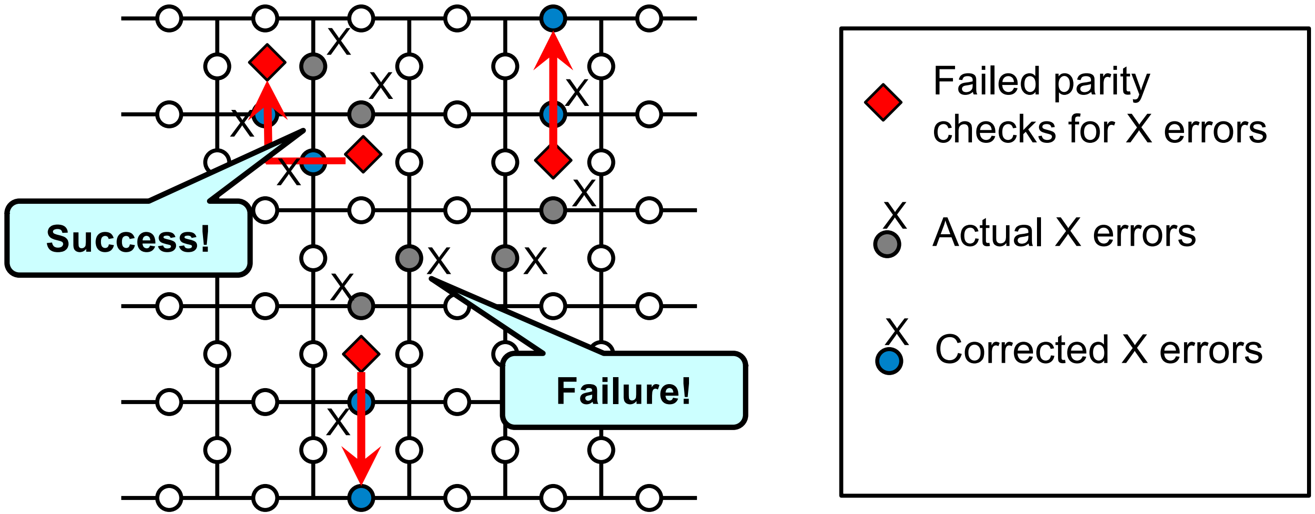
- Plaquette syndromes indicate presence of odd weight X errors in the vicinity
- Only illustrate X errors here, Z errors are corrected similarly with site syndromes

NUMERICAL SIMULATION – STEP 2: GUESS ERRORS AND DETERMINE OUTCOME



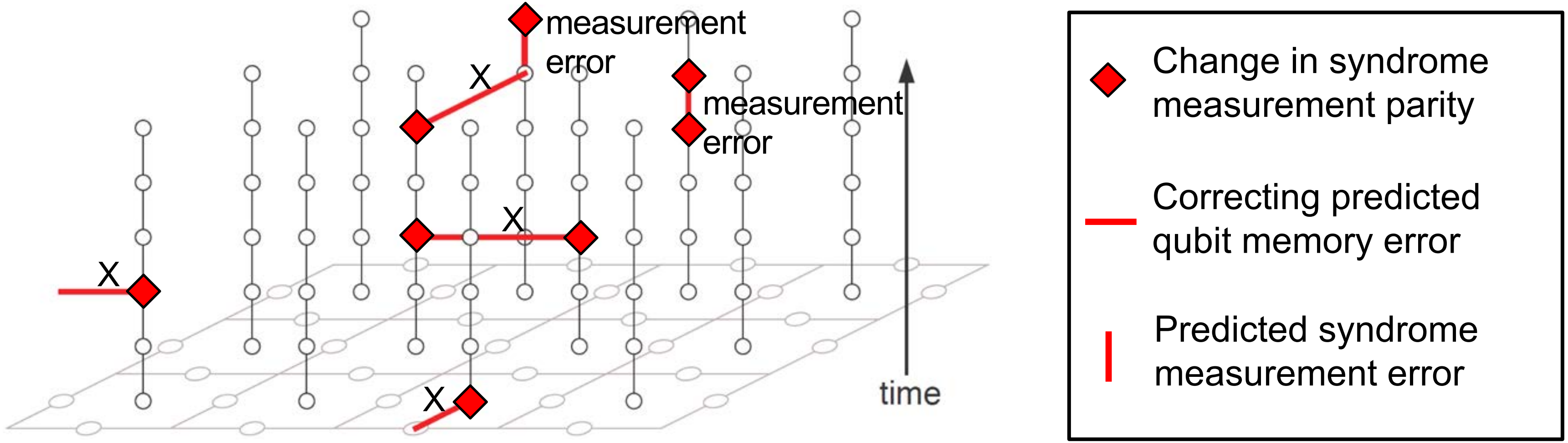
- Minimum Weight Matching heuristic corrects errors on chains between matched syndromes or syndromes and the boundary

NUMERICAL SIMULATION – STEP 2: GUESS ERRORS AND DETERMINE OUTCOME



- Minimum Weight Matching heuristic corrects errors on chains between matched syndromes or syndromes and the boundary

WHAT IF SOME SYNDROME MEASUREMENTS FAIL? REPEAT SYNDROME MEASUREMENTS!



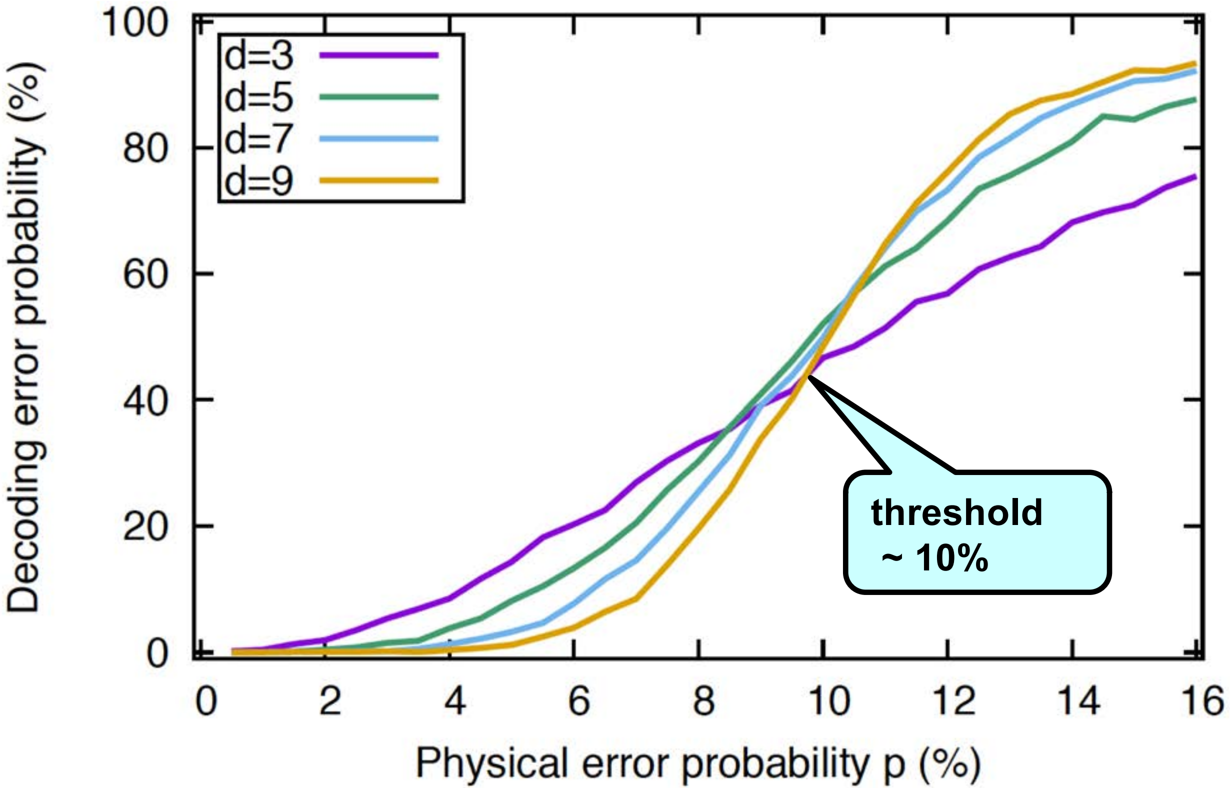
- Syndromes are measured repeatedly, parity changes are marked as defects
- Decoder matches pairs of defects and corrects errors in 2D on the physical qubits

SIMULATIONS WITH THREE DIFFERENT ERROR MODELS

Error model:	Data qubit errors:	Measurement gate errors:	Syndrome extraction circuit errors:
Code capacity	yes	no	no
Phenomenological	yes	yes	no
Circuit	yes	yes	yes

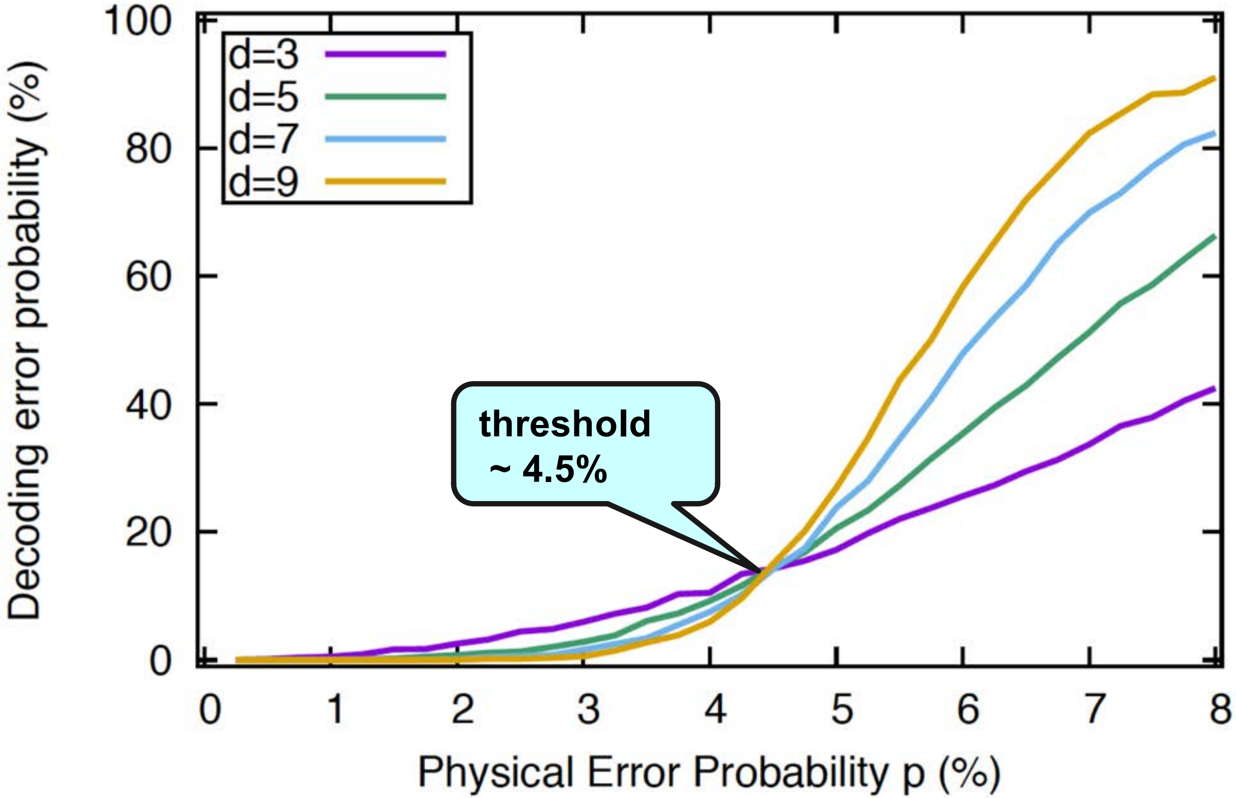
- Monte Carlo simulation in C++
  - Repeatedly generate and correct random errors
  - Ability to simulate memory errors, quantum gate errors, and propagation of X, Y, and Z errors in syndrome extraction circuits

CODE CAPACITY ERROR MODEL:



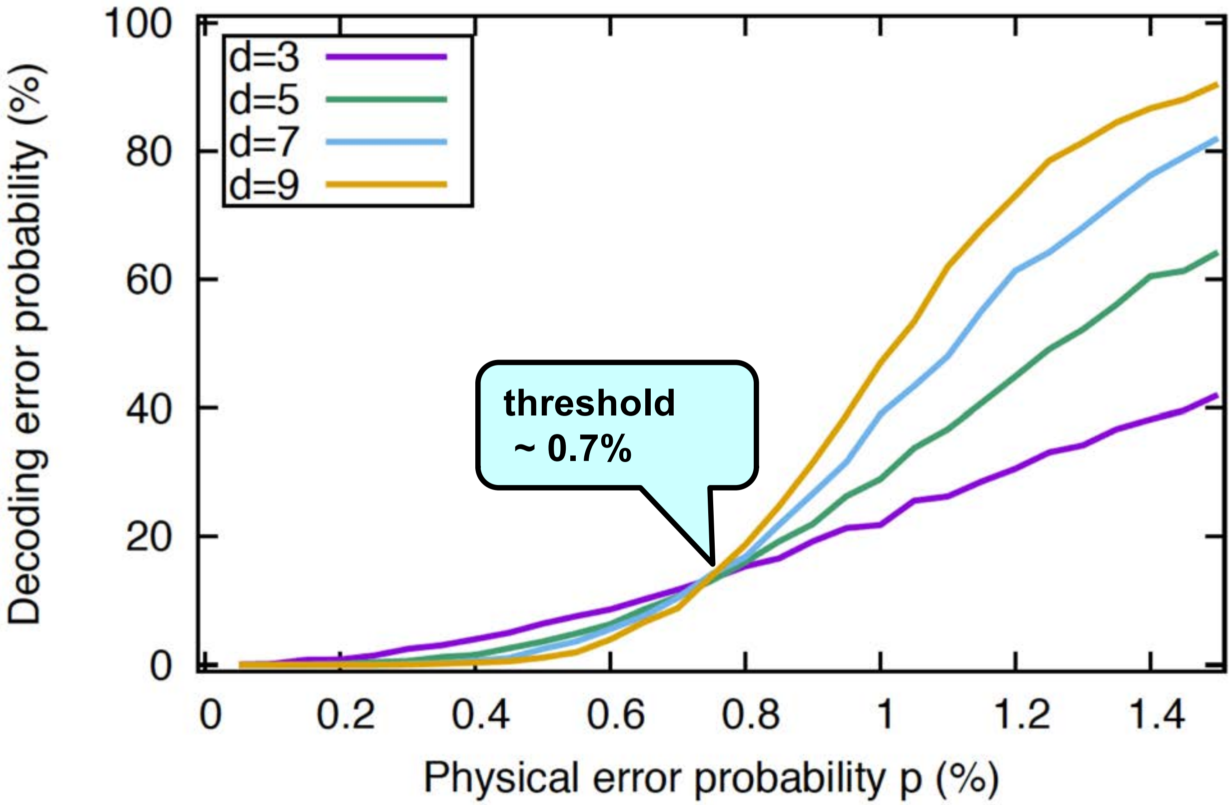
- Data qubit errors only

PHENOMENOLOGICAL ERROR MODEL:



- Data qubit errors
- Measurement gate errors

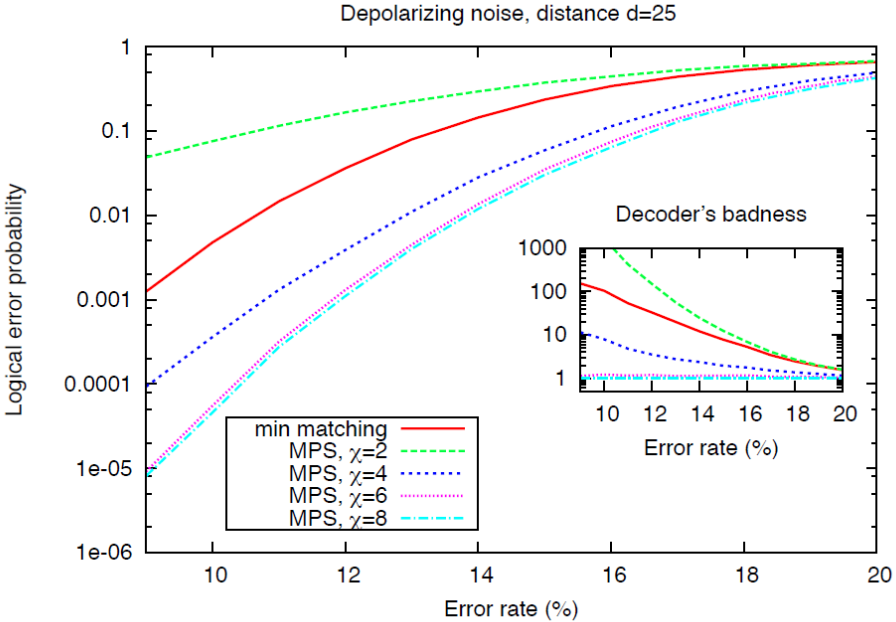
CIRCUIT ERROR MODEL:



- ➢ Data qubit errors
- ➢ Measurement gate errors
- ➢ Syndrome extraction circuit errors

CAN WE DO BETTER?

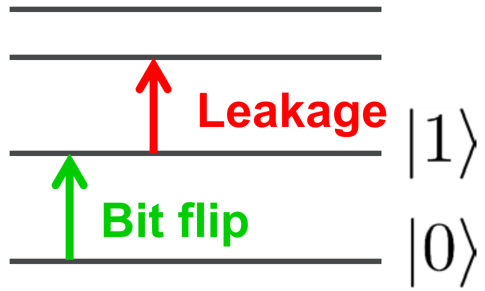
- Minimum Weight Matching ignores error degeneracy (multiple errors with same syndrome) and does not consider correlations of X and Z errors in the depolarizing noise model
- Approximate maximum likelihood decoding guesses the most likely error:
  - Thresholds improve slightly
  - Decoding error rate improves significantly

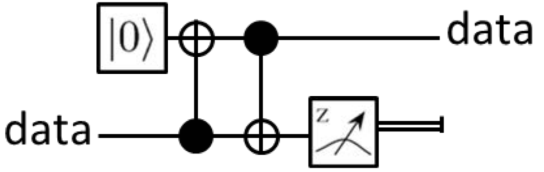


ADDITIONAL TYPES OF ERRORS – CORRECTING QUBIT LEAKAGE

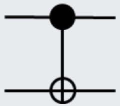
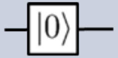
WHAT IS QUBIT LEAKAGE?

- Physical qubits are not ideal two-level systems and may leak out of the computational space
- With standard error correction techniques leaked qubits accumulate and spread errors
- Leakage reduction units based on quantum teleportation were suggested by Aliferis and Terhal.
- Threshold theorem for concatenated codes in the presence of leakage still holds



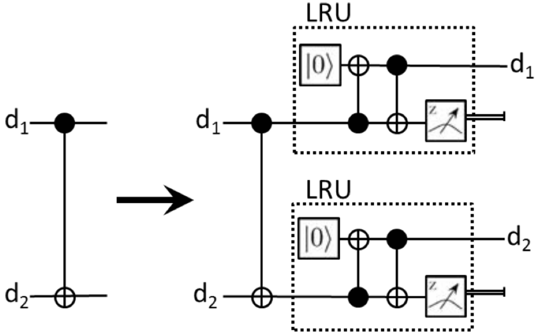


BEHAVIOR OF QUANTUM GATES IN THE PRESENCE OF LEAKAGE

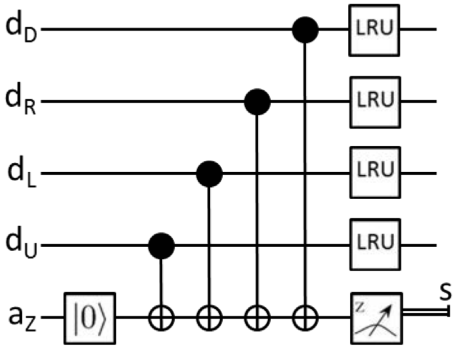
Gate	Possible Errors	Leakage Errors
Identity	X, Y, Z	if leaked relaxes w/ prob. pd, doesn't increase leakage
CNOT 	IX, XX, XZ, etc.	if leaked, applies random Pauli to the other qubit; leaks w/ prob. pu and relaxes w/ prob. pd
Preparation 	orthogonal state	leaks w/ prob. pu
Measurement 	Incorrect outcome	if leaked, always measures 1 (also consider possibility of leakage detection)

SEVERAL OPTIONS HOW TO DESIGN AN EFFECTIVE LEAKAGE REDUCTION CIRCUIT

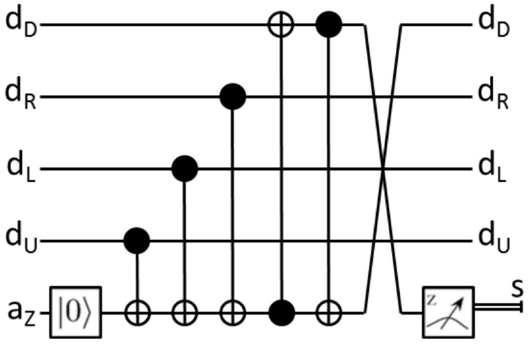
1. Full-LRU:
(resource heavy)



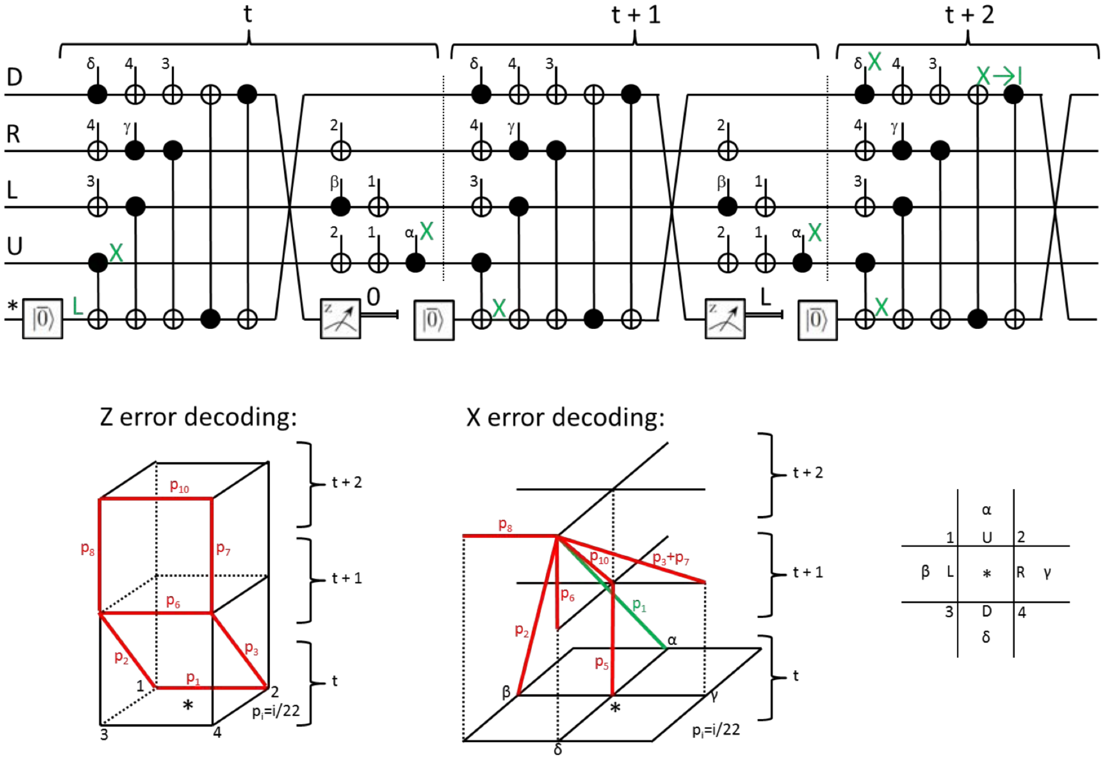
2. Partial-LRU:
(fewer gates)



3. 'Quick' circuit:
(swap data and ancilla)

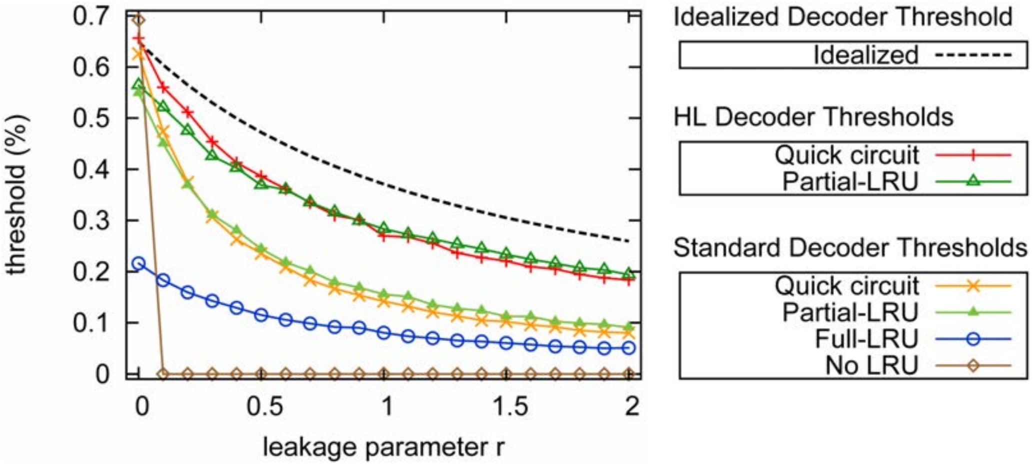


DESIGNING A DECODER THAT USES LEAKAGE DETECTION AND THE ‘QUICK’ CIRCUIT

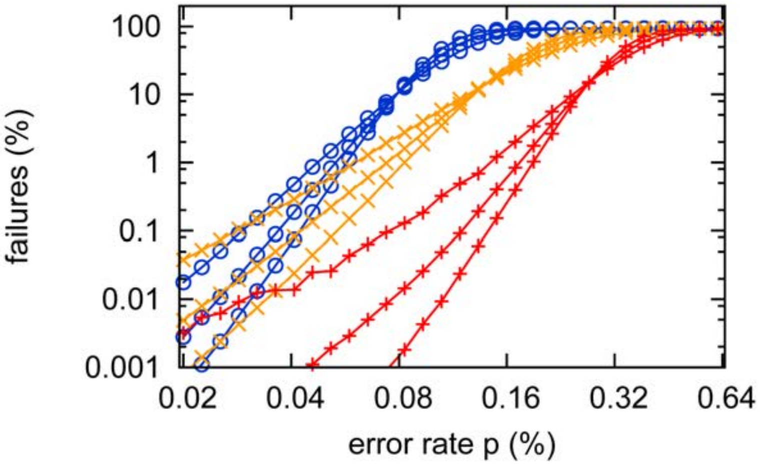


SEVERAL OPTIONS HOW TO DESIGN AN EFFECTIVE LEAKAGE REDUCTION CIRCUIT

Threshold comparison:



Decoding failure rates:



- More complicated circuits have lower threshold
- Measurement that detects leakage (HL) boosts performance
- Full-LRU performs well at low error rates

THANK YOU!