Topological Subsystem Codes with Local Gauge Group Generators

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Quantum Error Correction (QEC)

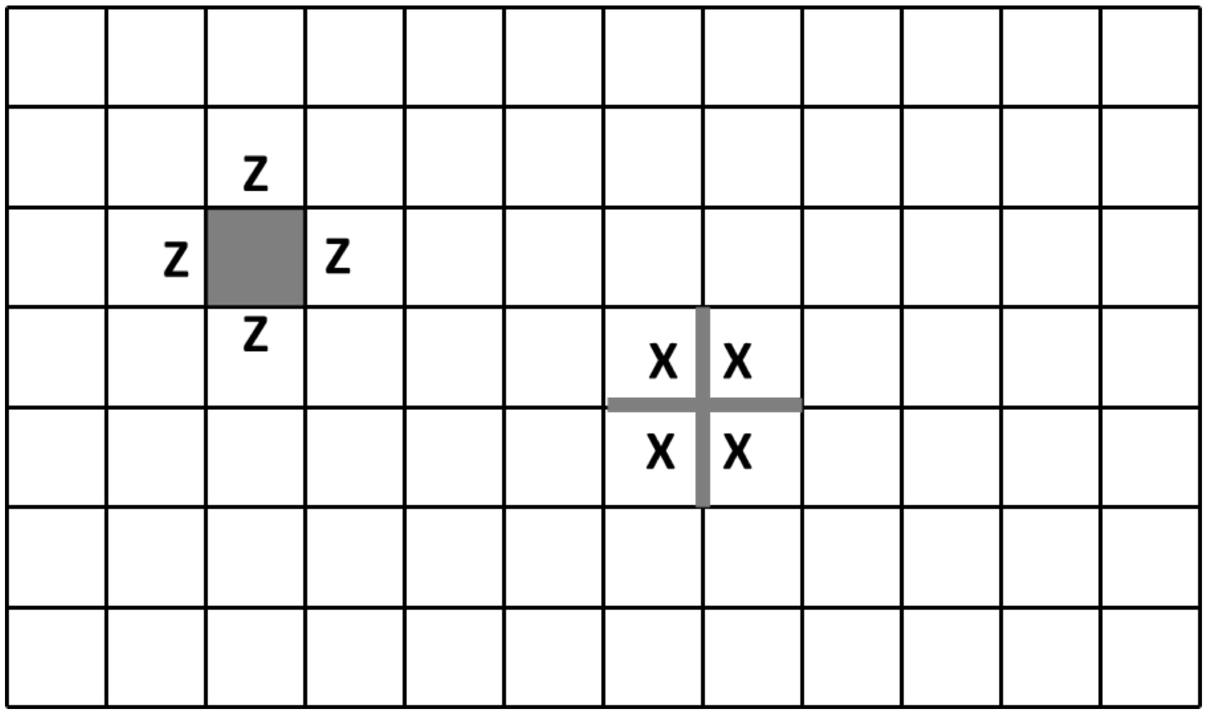
- Need error correction to build a practical quantum computer
- Reliable quantum information storage and computation with unreliable components
- Much more challenging than classically
  - Analog nature of quantum operations
  - New kinds of errors: partial bit flips, phase flips, small shifts

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Some Milestones in QEC Research

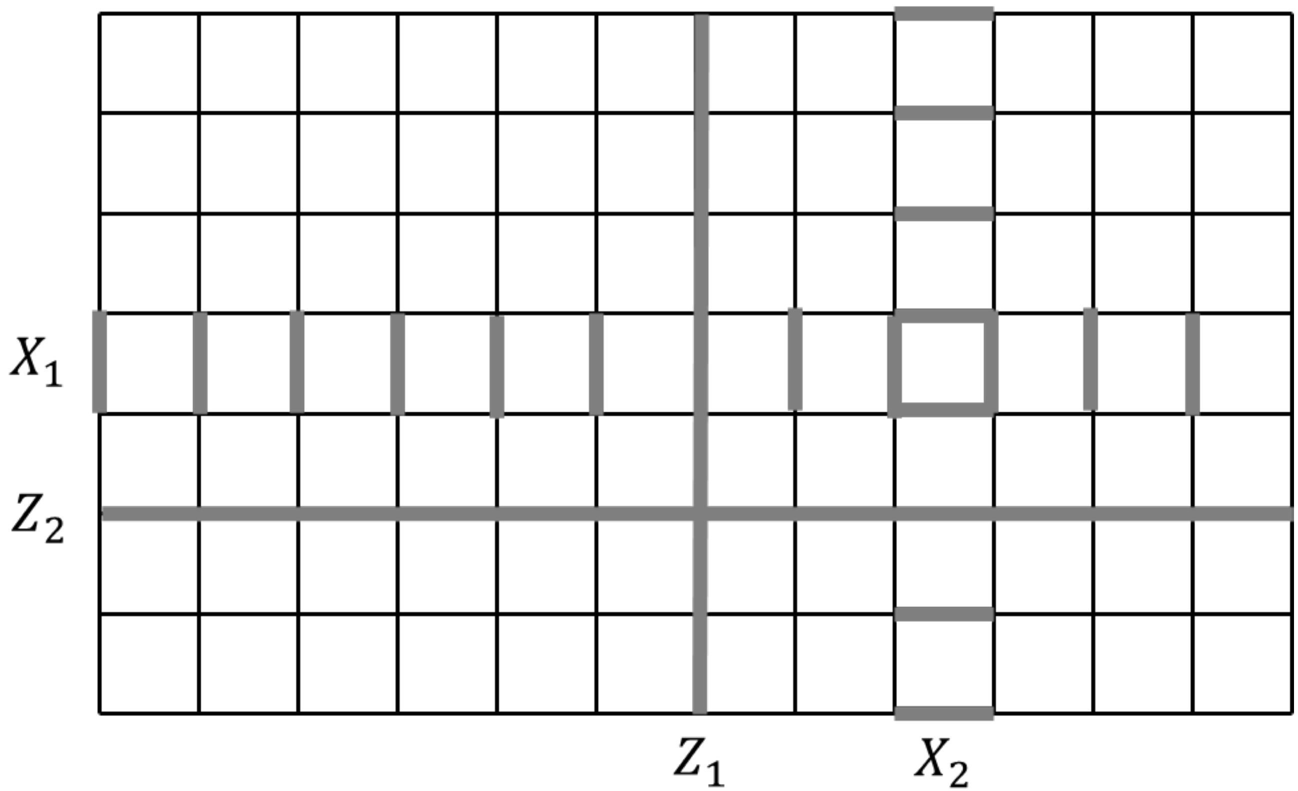
- Quantum block error correcting codes
  (Shor, 1995)
- Threshold theorem
  (Knill et al., 1998)
- Topological quantum codes
  (Kitaev, 1997)
- Subsystem codes
  (Poulin, 2005 and Bacon, 2006)

Example – Topological Quantum Memory



- Qubits on links (or sites) in the lattice
- Measuring these "check" operators yields error syndromes

Example – Topological Quantum Memory



- Logical operators

Advantages of Topological Stabilizer Codes

- Qubits laid on two-dimensional grid
- Local syndrome measurements
- Can increase lattice size rather than concatenate smaller blocks
- High threshold
- Permit encoding of multiple qubits and implementation of some gates by code deformation

Error Correction with 2-Body Measurements (*Bombin 2010*)

- Earlier codes: measurements of at least four neighboring qubits
- Our code: only local two-qubit measurements
- Simplifies physical implementation
- Main questions
  - How should the lattice look like?
  - How should the decoding algorithm work?
  - Numerical value of the threshold?

Overview

I. Construction of topological subsystem codes

II. The five-squares code

III. Experimental evaluation of the threshold

IV. Conclusion

Error Correcting Codes

- Codespace is a subspace of a larger Hilbert space that encodes logical information
- Syndrome measurements diagnose errors
- Decoding algorithm returns system to the original logical state
  - Threshold: below some noise level error correction succeeds w. h. p.

Stabilizer Subsystem Codes

- Stabilizer codes characterized by the stabilizer group generated by $S_1, S_2, \ldots S_{n-k}$
  - Have 2^k dimensional codespace: $S_j|\psi\rangle = |\psi\rangle$
  - $2k$ logical operators
- In subsystem codes some “logical” qubits do not encode any information
  - Can simplify decoding
  - Characterized by gauge group: stabilizers + logicals acting on gauge qubits

Desired Code Properties

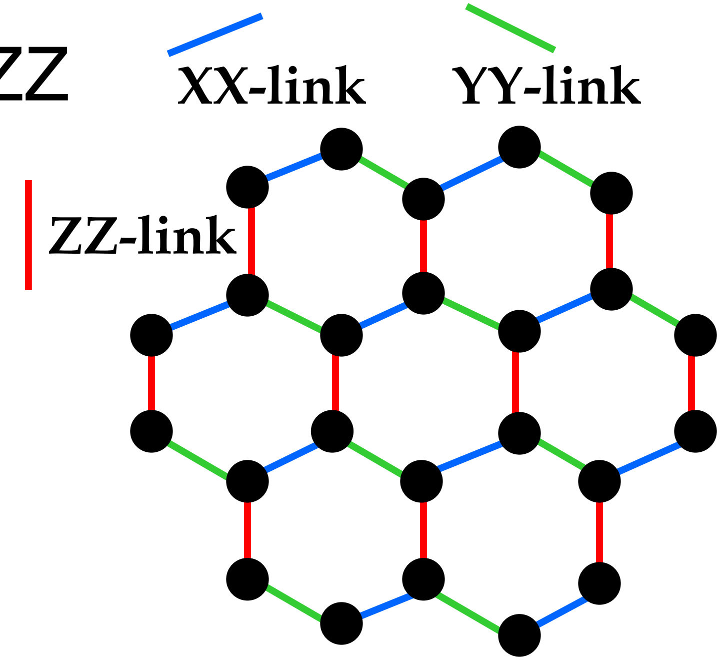
1. Syndromes can be extracted by measuring two-qubit gauge operators

2. Topological properties of the code: stabilizer group has spatially local generators

3. At least one logical qubit encoded

Kitaev's Honeycomb Model on a Torus

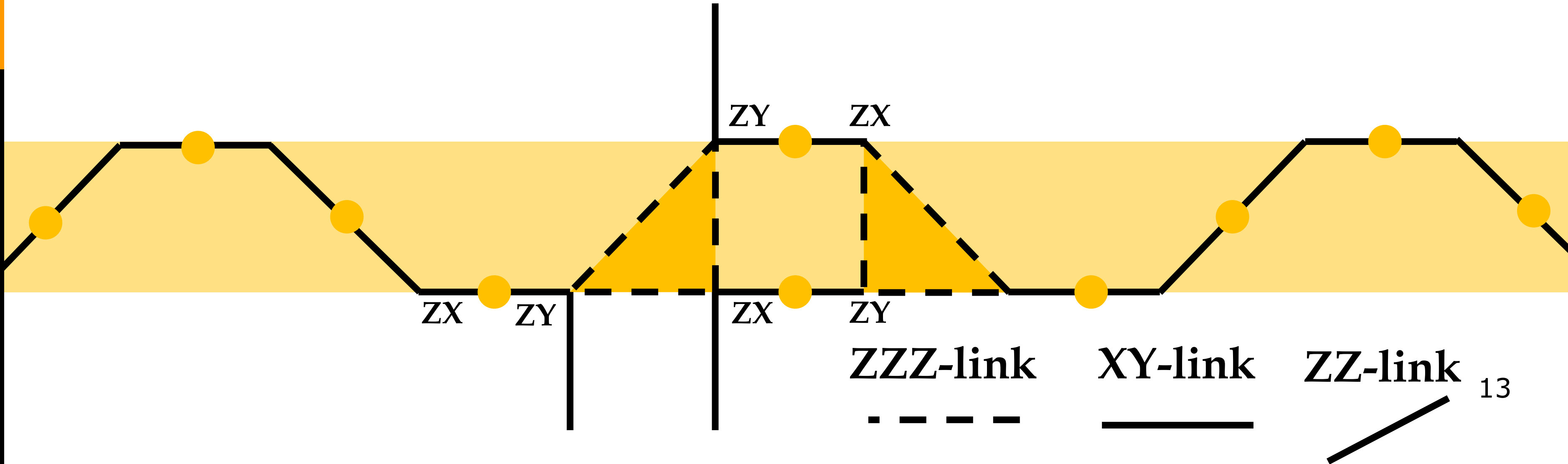
- Link operators XX, YY, and ZZ
  - Anticommute iff share one endpoint



- Has spatially local stabilizer generators (loop operators)
- Two-qubit measurements determine syndromes
- All loop operators commute
  - Does not encode any logical qubits!

Generalizing the 3-Valent Lattice

- Links connecting three sites – “triangles”
  - Loop has an even number of incident links at each site
  - Loop operators anticommute iff they share odd number of triangles



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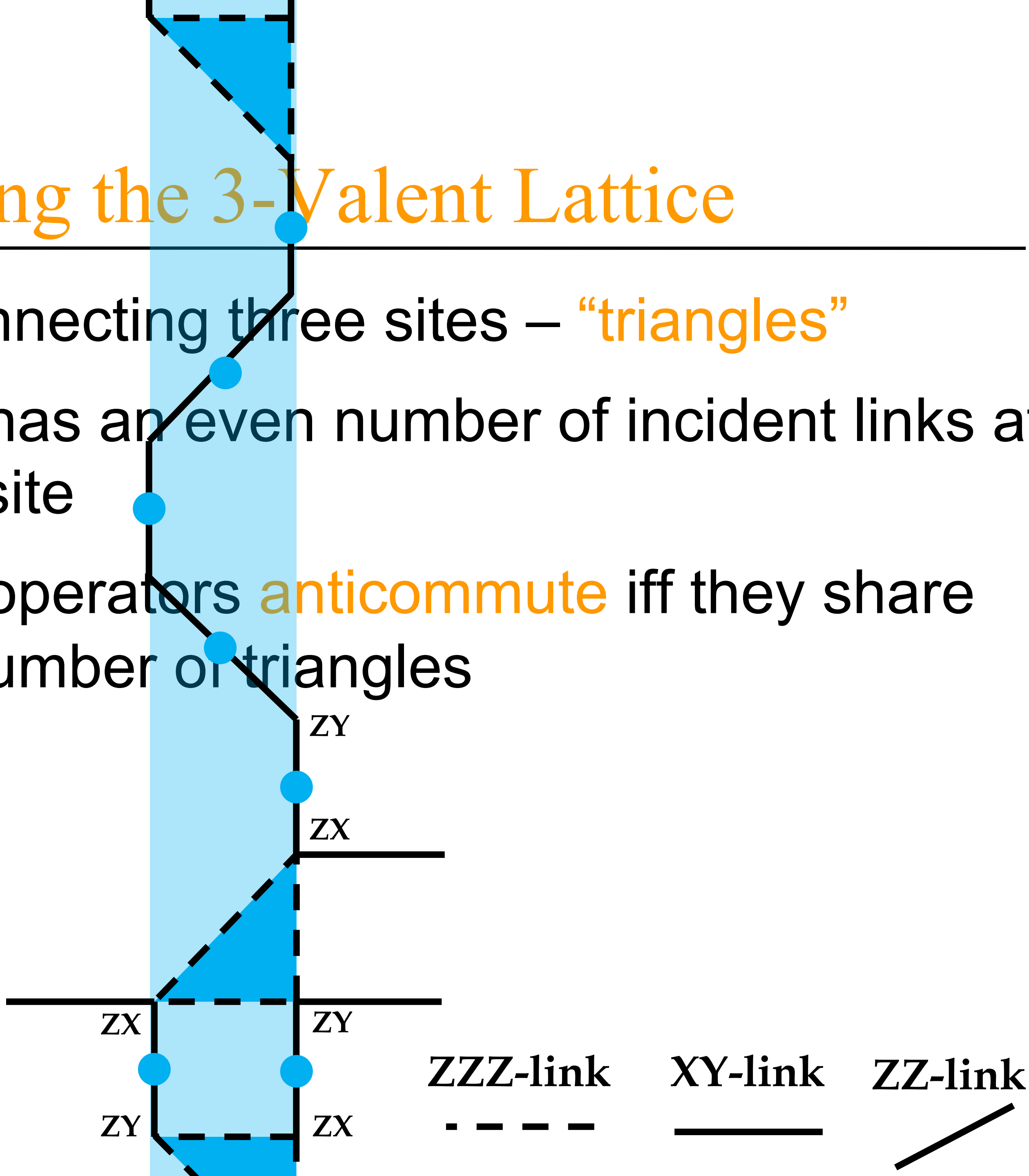

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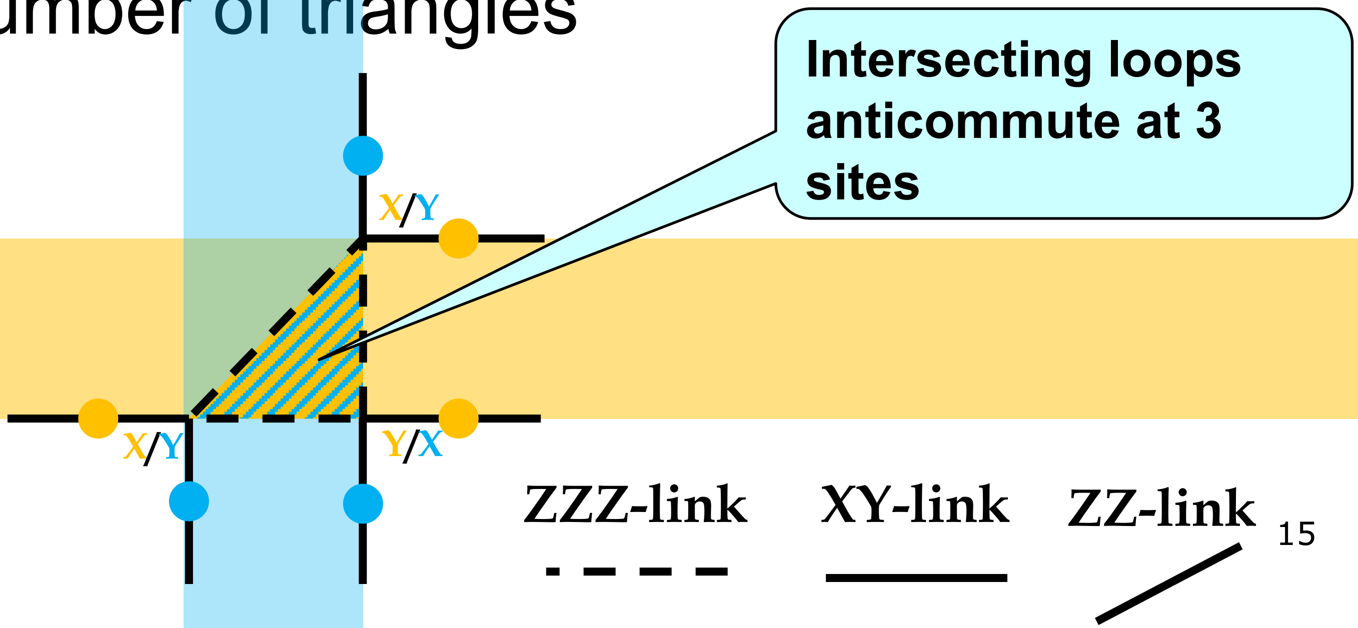
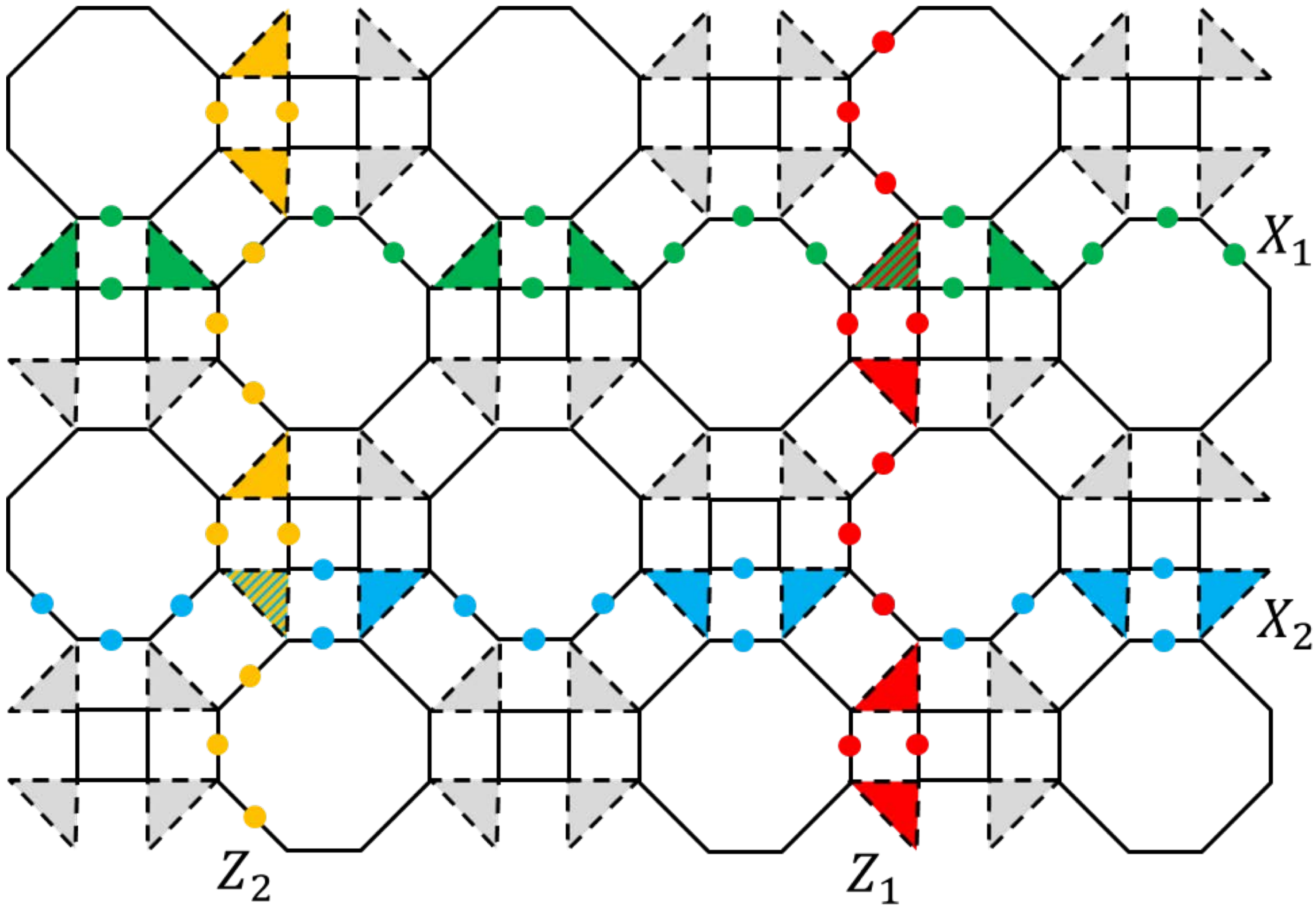
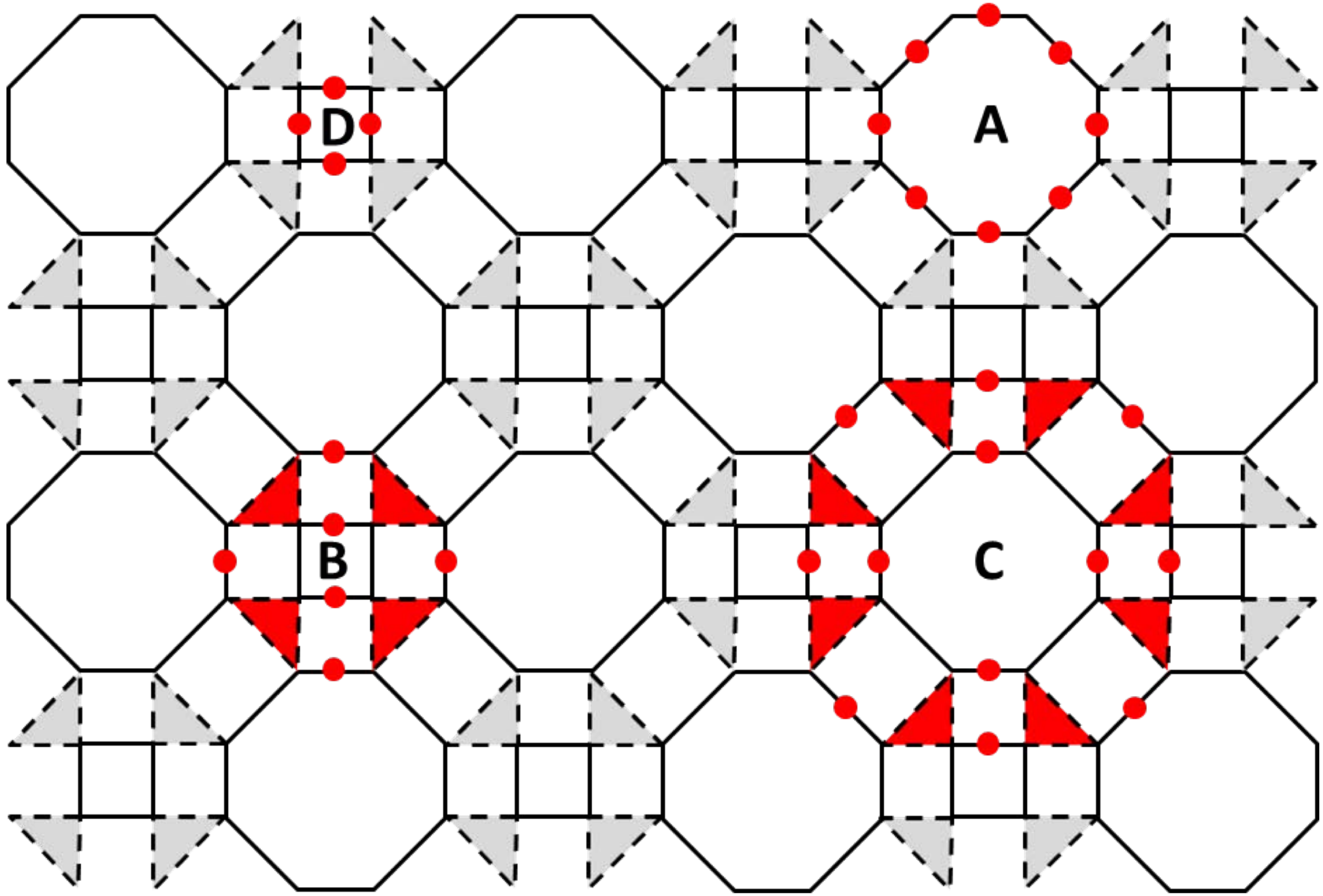


The Square-Octagonal Lattice
The Logical Operators

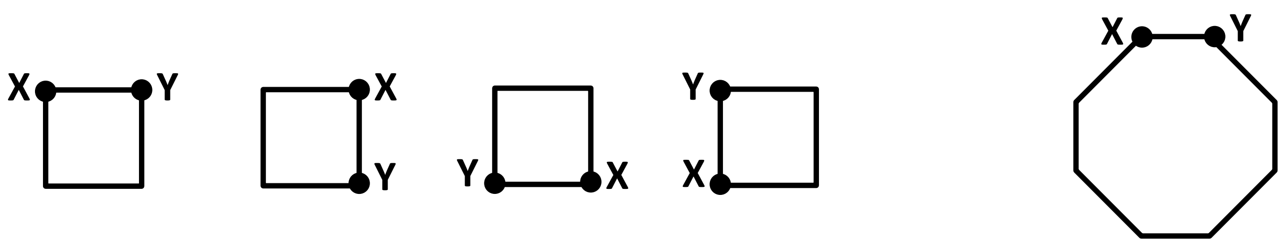


The Square-Octagonal Lattice
The Stabilizers

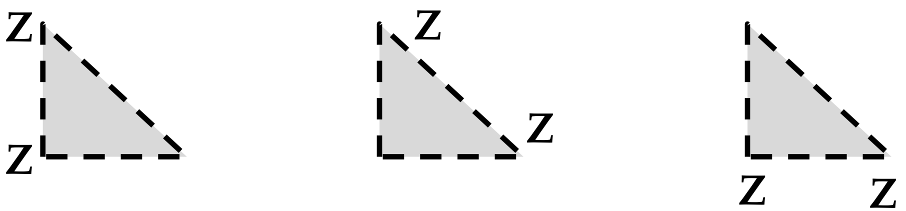


The Gauge Group Generators

- Link operators of weight two:



- Each triangle gives rise to a triple of generators:



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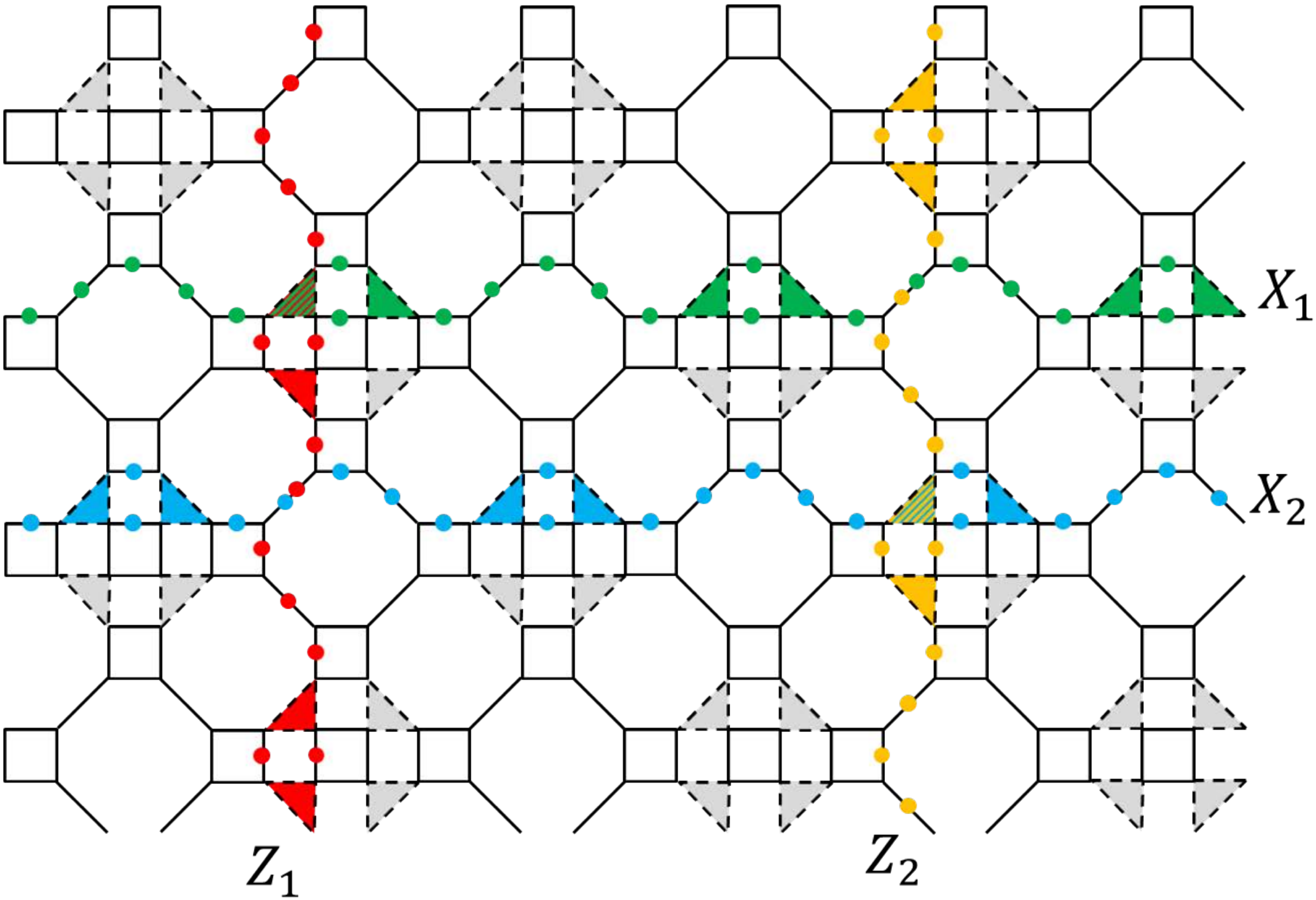
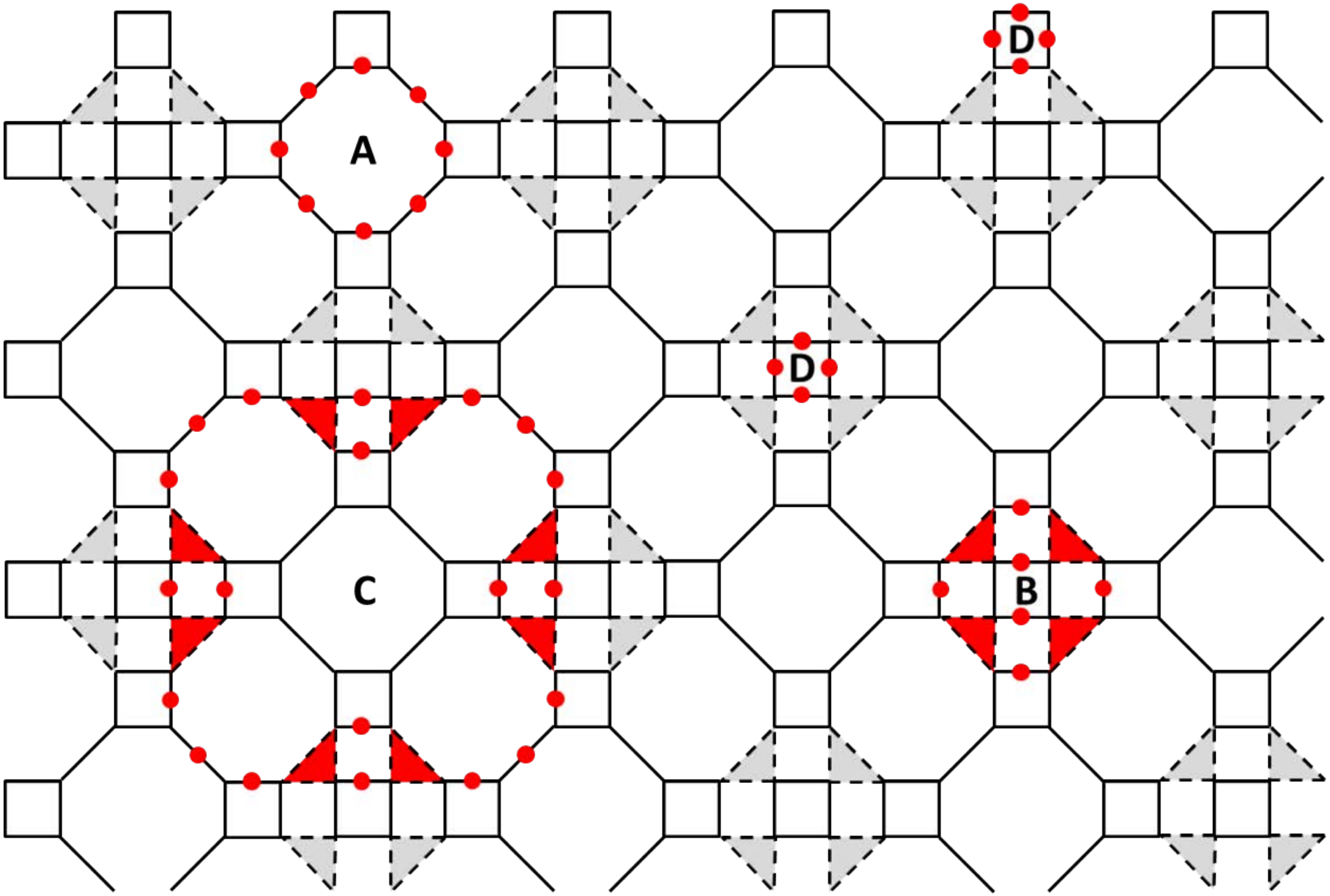
The Five-Squares Lattice
The Logical Operators

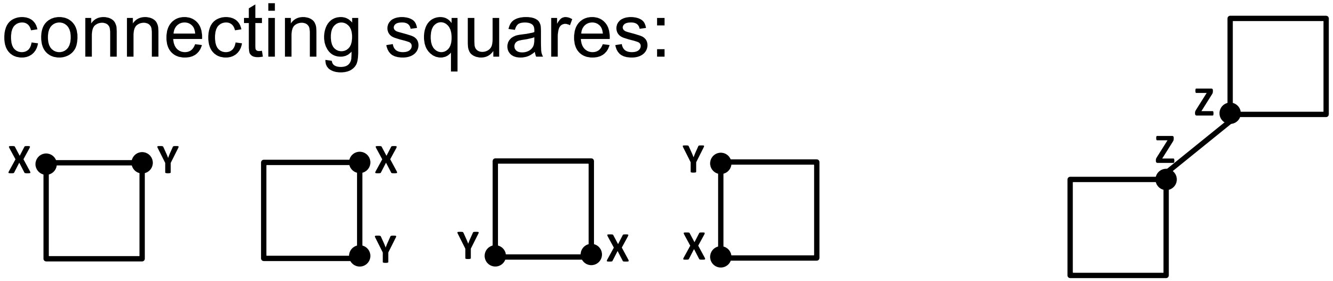


Examples of Lattices (with Stabilizers)
Five-Squares Lattice

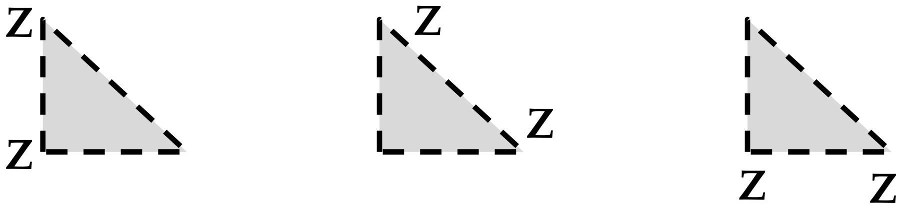


The Gauge Group Generators

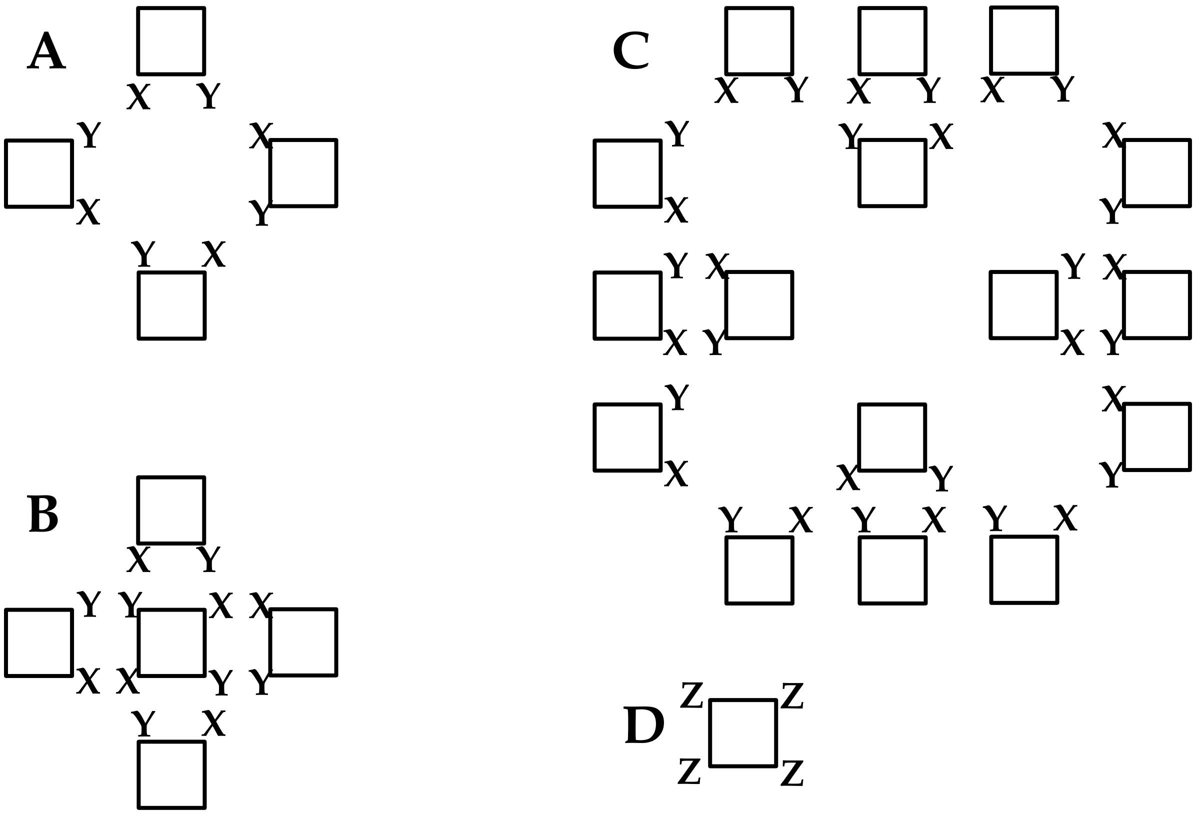
- Solid XY-links in squares, and ZZ-links connecting squares:



- Triangles:



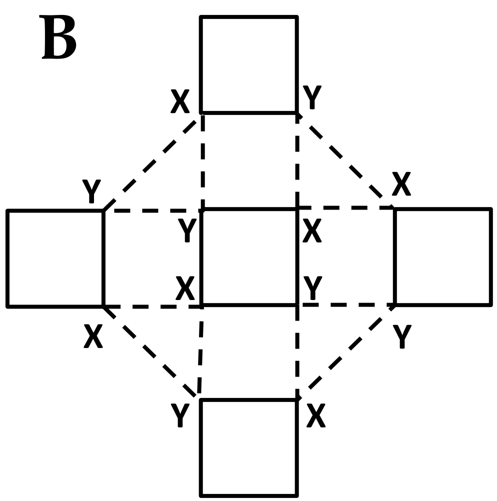
The Stabilizers

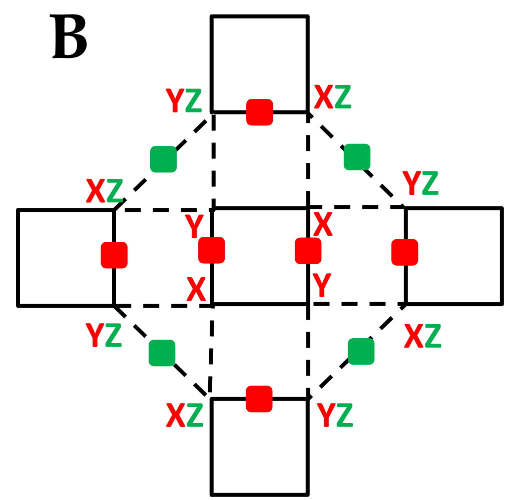


- How to measure them?

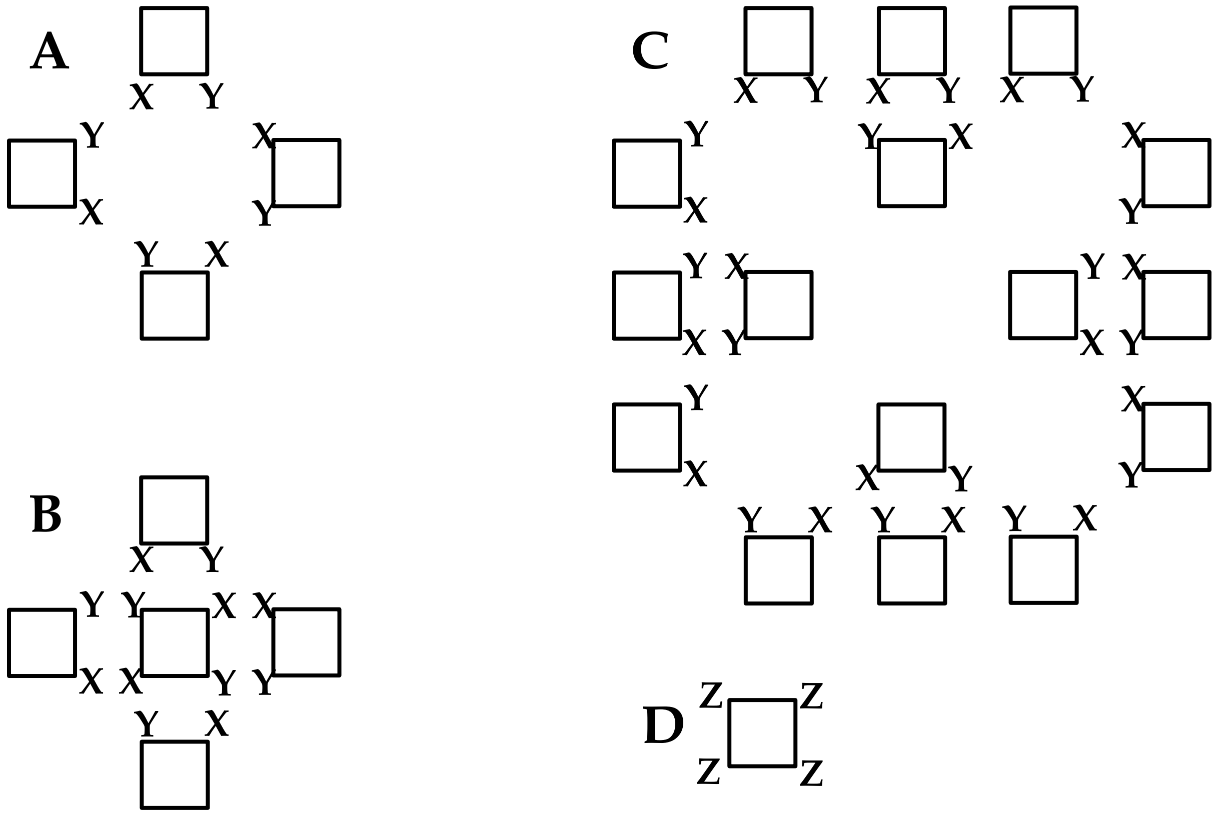
Measuring the Stabilizers

- To measure syndromes A, B, C, D need to take 8, 10, 40, and 4 two-qubit measurements





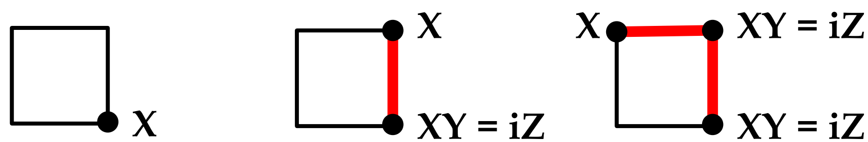
The Stabilizers

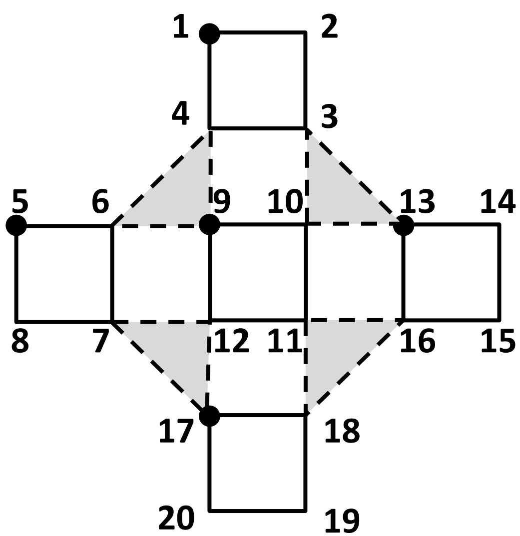


- How to use them to correct errors? Correct X and Z errors separately.

Correcting X Errors

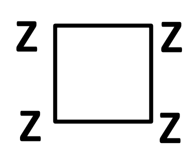
Consider an arbitrary X error:



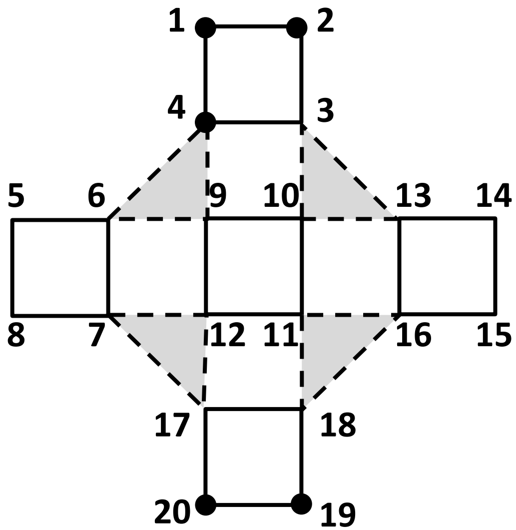


Only need to correct errors X_1, X_5, X_9, X_{13}, and X_{17}.

Easy to do with syndrome

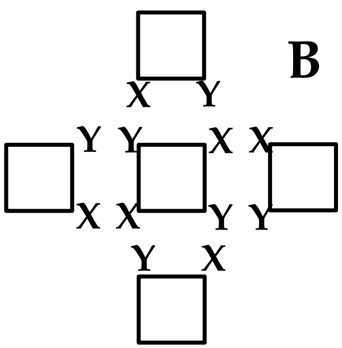


Correcting Z Errors



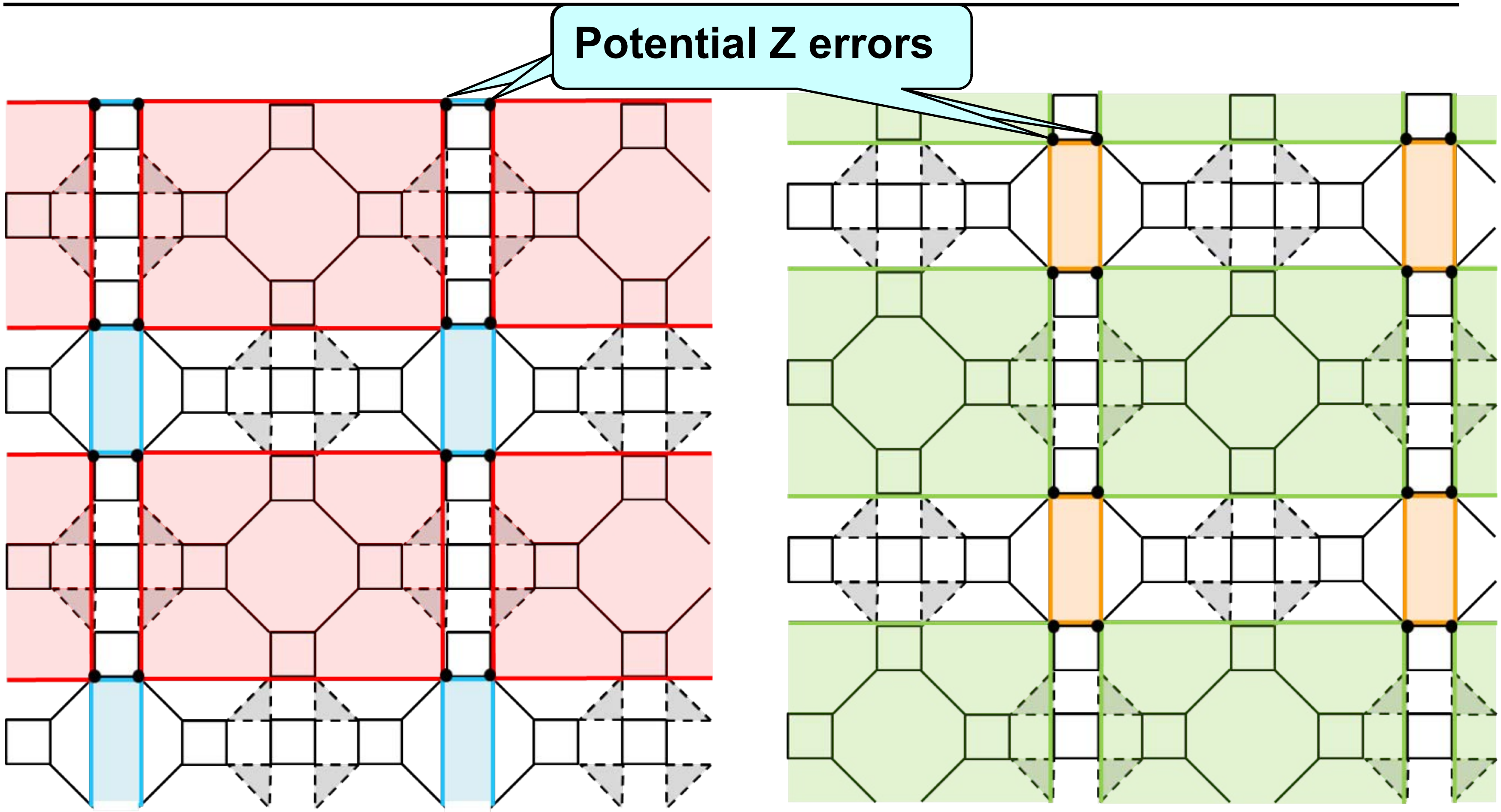
Similarly, only need to consider Z_1, Z_2, Z_4, Z_{19}, and Z_{20}.

And Z_4 is corrected using stabilizer B



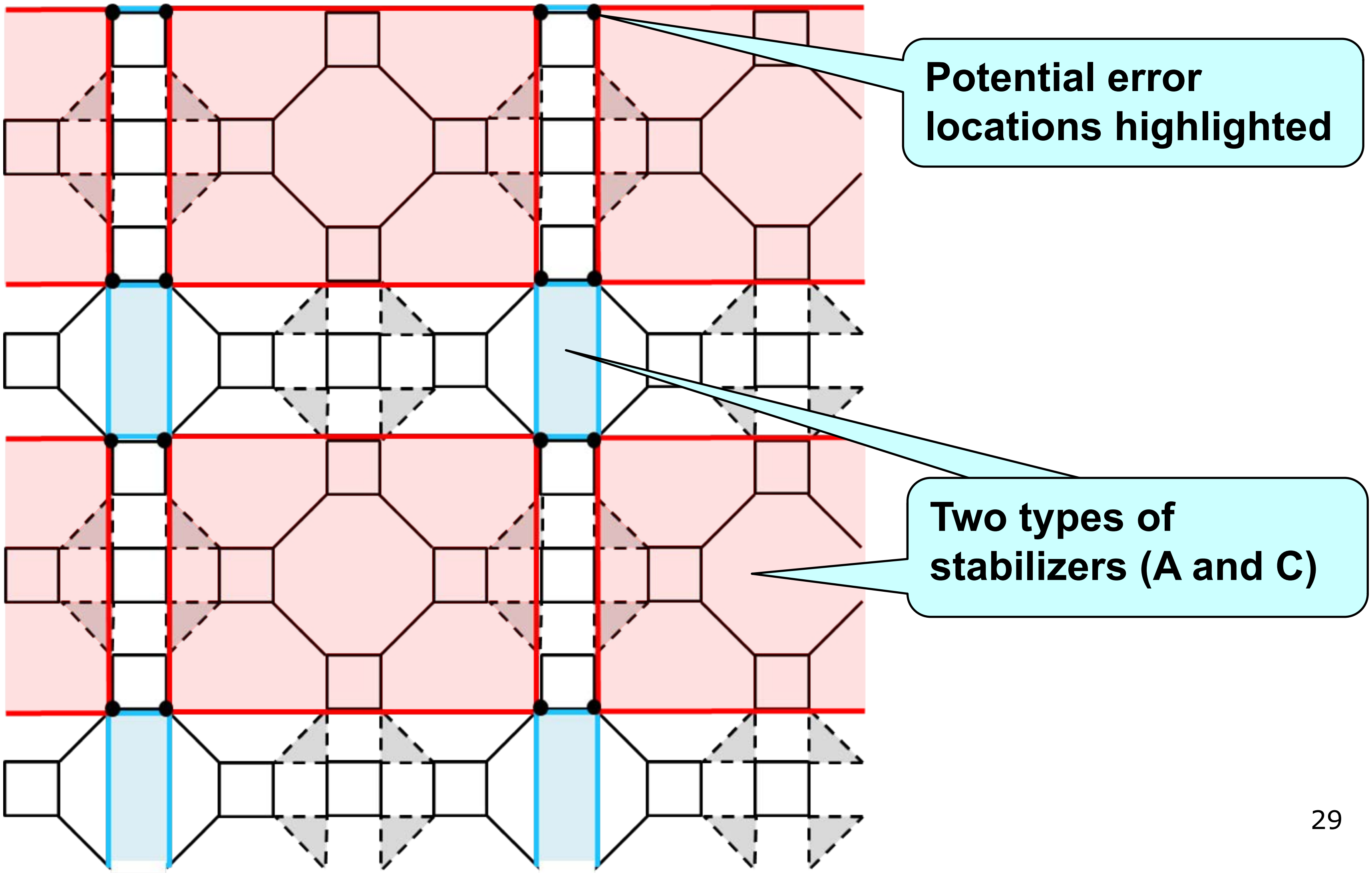
How do the remaining stabilizers act on the errors?

Correcting Z Errors



Two non-overlapping sublattices

Correcting Z Errors



Correcting Z Errors

- Find the smallest possible weight error
- Minimum weight matching algorithm
  - Match pairs of non-trivial syndromes
  - Minimize total weight of the matching
  - Correct Z errors on lines connecting matched syndromes
- Error correction fails if $E_{actual}E_{guessed}$ is a logical operator

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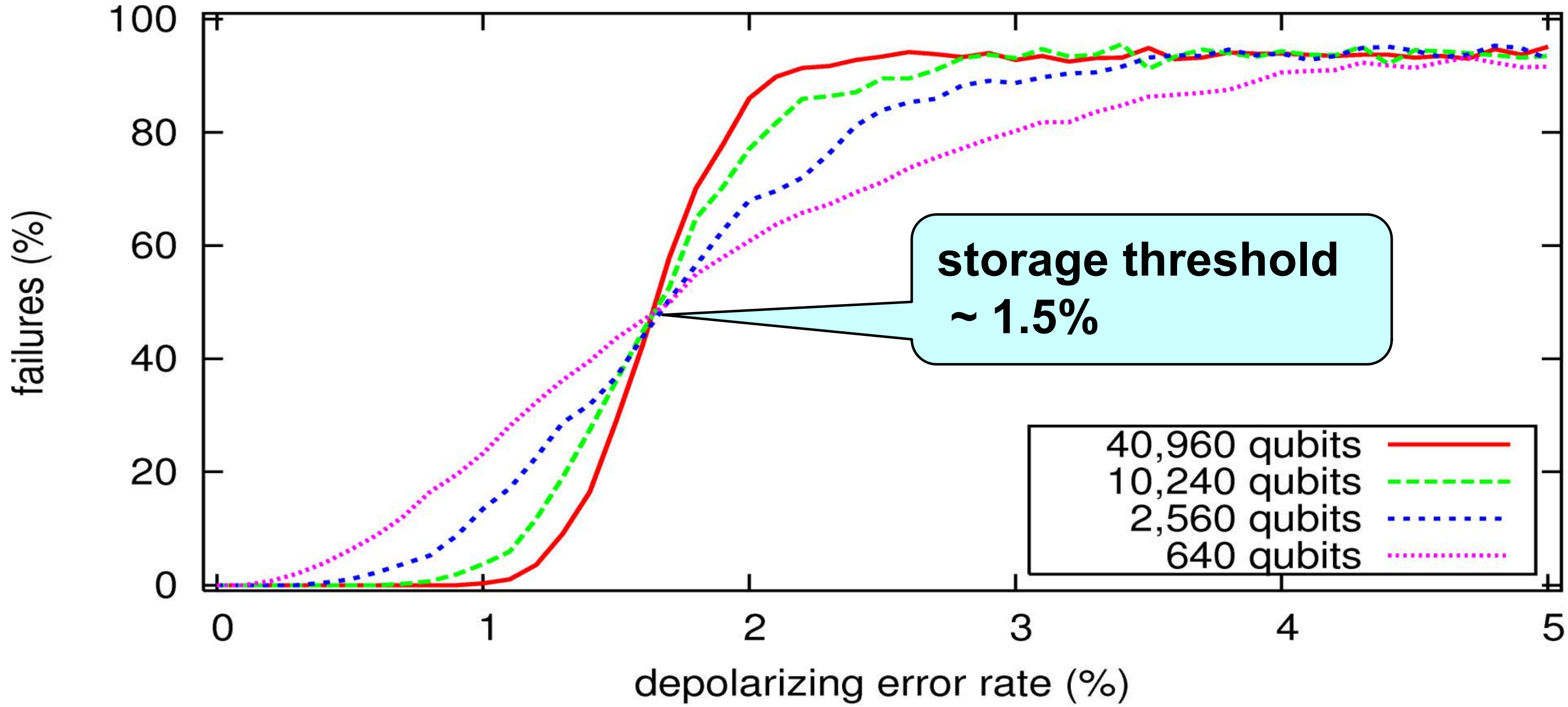
Assumptions and Experimental Setup

- Depolarizing error model
  - X, Y and Z errors with probability p/3
- Noiseless syndrome measurements
- Monte Carlo simulation in C++
  - 640 to 40,960 qubits
  - Generate and correct random error
  - Repeat 1,000 times

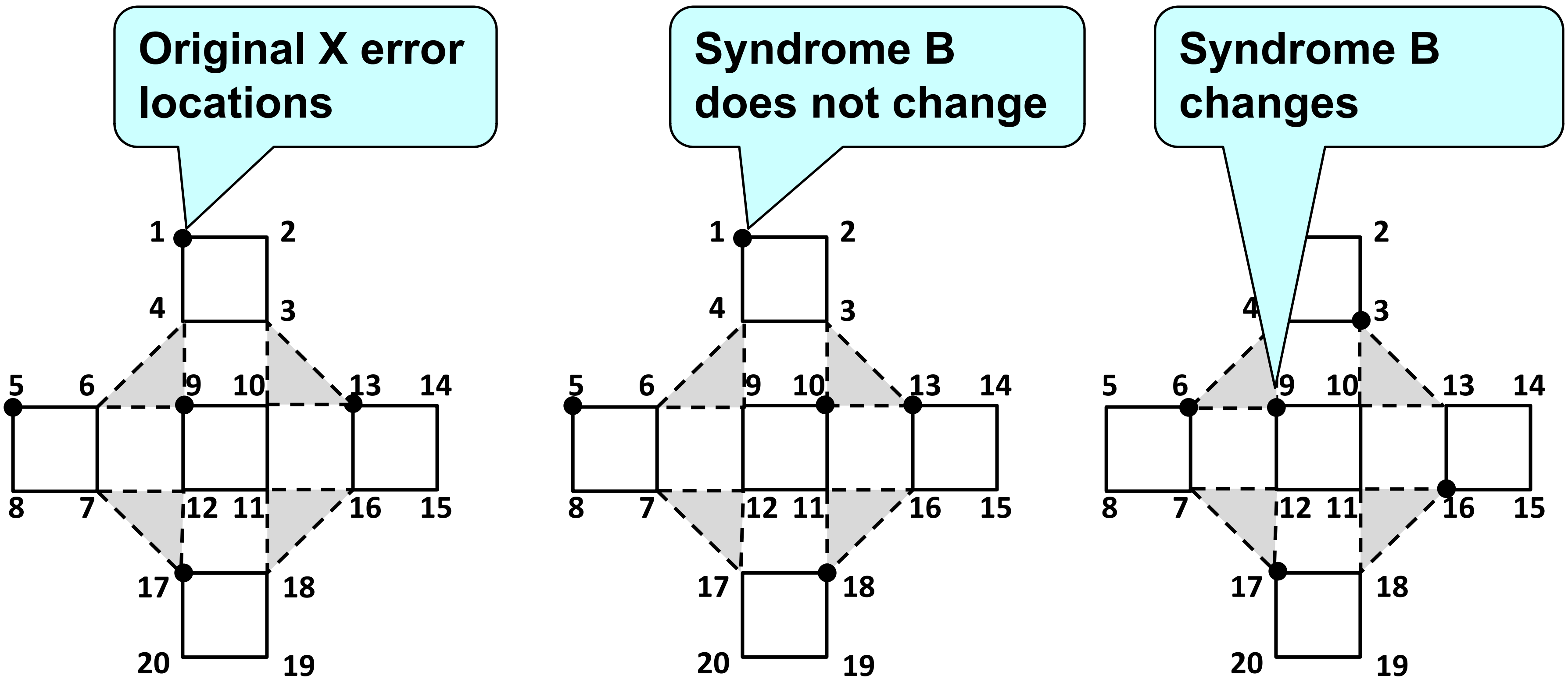
Threshold – Simple Decoding Algorithm



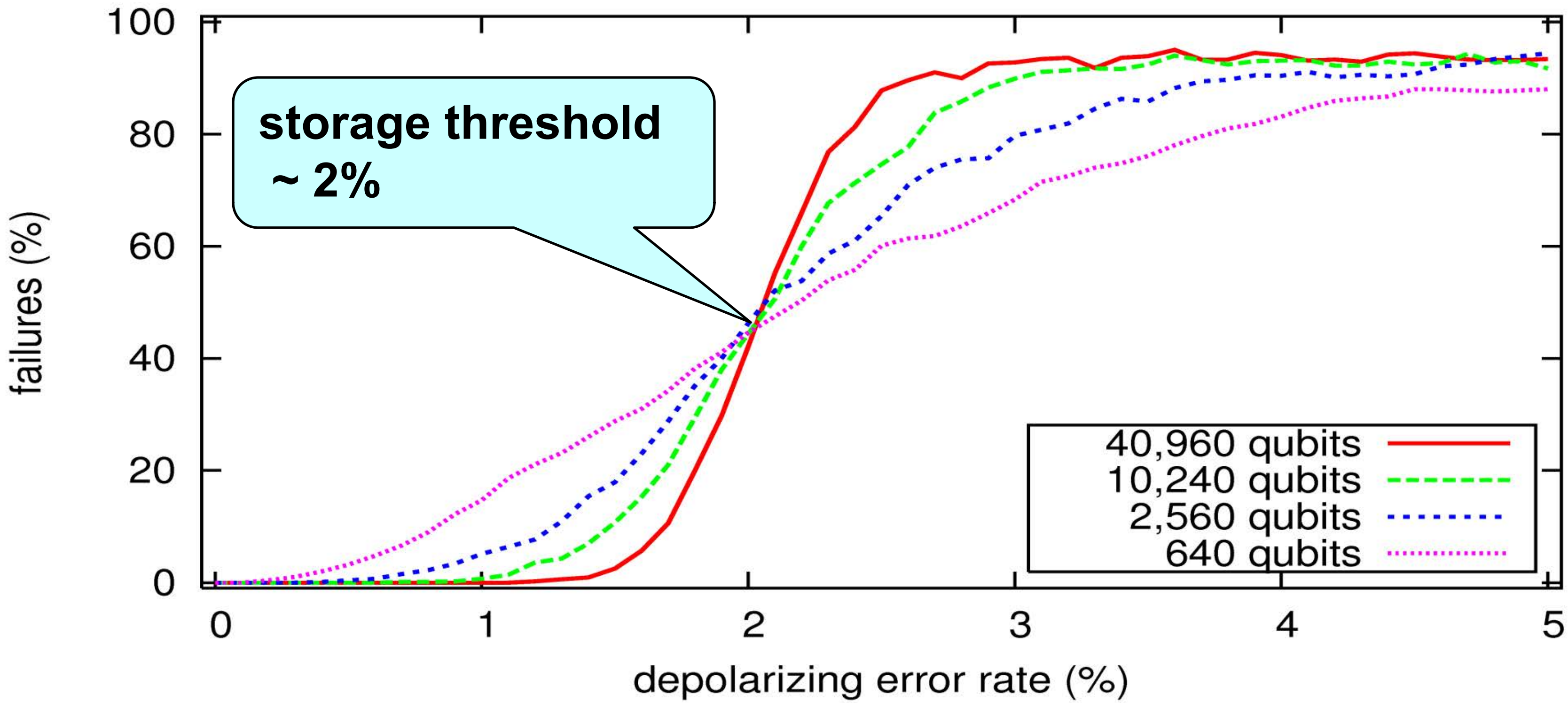
- Transition sharper for larger lattice size

Improving the Threshold

- In the first two correction steps guess the smallest weight error!



Threshold – Improved Decoding



- Noticeable improvement of threshold

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Conclusion

- Construction of new codes that only require two-qubit measurements for error correction
- Decoder for the five-squares code achieves 2% storage threshold
- Price to pay: threshold value and number of measurements needed for syndrome calculation
- Open question: how to encode multiple logical qubits by creating holes?

Thank You!