QuRE: The Quantum Resource Estimator Toolbox

Martin Suchara (IBM Research)

In collaboration with: **Arvin Faruque, Ching-Yi Lai, Gerardo Paz, Fred Chong, and John Kubiatowicz**

IBM®

USC UNIVERSITY OF SOUTHERN CALIFORNIA

UCSB

Berkeley UNIVERSITY OF CALIFORNIA

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Why Quantum Computer Resource Estimator?

- Building a practical quantum computer is very difficult
- Goal: investigate impact of design choices on the performance of the computer without building one
  - Hardware: speed vs. reliability tradeoff
  - Error correction: choosing good strategies
  - Algorithms: which are efficient?
- This work: flexible configurable estimation tool

Inputs and Outputs of the QuRE Toolbox

Algorithm Specs

- # of logical qubits
- # of logical gates
- Circuit parallelism

Analysis of Error Correction

- Estimate cost of each logical operation as a function of error correction "strength"

Technology Specs

- Gate times and fidelities
- Memory error rates

Automated Resource Estimate

- Find out how strong error correction guarantees target success probability
- Estimate number of physical qubits, running time, physical gate and instruction count, etc.

QuRE Analyzes a Variety of Realistic Scenarios

- 7 quantum algorithms
- 4 quantum error correcting codes
- 12 physical technologies
- This talk
  - Overview of resource estimation methodology and highlights of our results

Overview

I. Properties of quantum technologies and algorithms

II. Estimation methodology – overhead of concatenated error correction codes

III. Estimation methodology – overhead of topological error correction codes

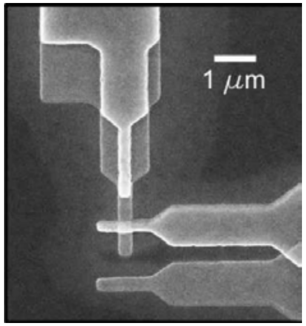
IV. Examples of estimates obtained with QuRE

How Quantum Computers Work

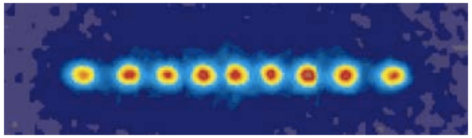
- Quantum instead of binary information
  - Quantum state $|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$, not just 0 or 1
- Operations and memory storage must be reliable
- Quantum computers must be able to initialize, store, manipulate and measure quantum states

A Number of Competing Candidate Technologies

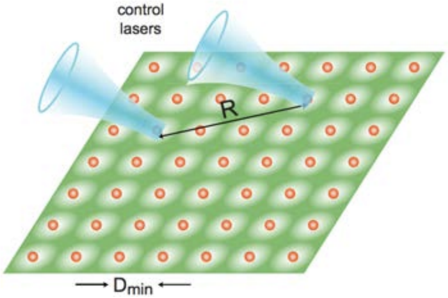
- Superconducting qubits
  - Josephson Junctions between superconducting electrodes



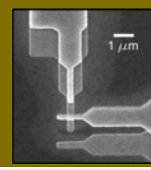
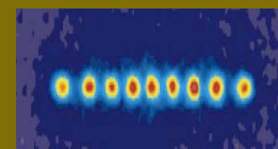
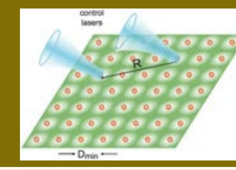
- Ion traps



  - Ions trapped in electromagnetic field, gates performed by applying lasers
- Neutral atoms
  - Ultracold atoms trapped by light waves in an optical lattice



Properties of Quantum Technologies: Gate Times and Errors

	Supercond. Qubits 	Ion Traps 	Neutral Atoms 
Average Gate Time (ns)	25	32,000	19,000
Worst Gate Error	1.00×10^{-5}	3.19×10^{-9}	1.47×10^{-3}
Memory Error	1.00×10^{-5}	2.52×10^{-12}	not available

- Ion traps slower but more reliable than superconductors
- Neutral atoms slower and error prone

The Best Known Quantum Algorithm

- Shor's factoring algorithm
  - Find prime factors of integer N
  - Quantum algorithm runs in polynomial time
  - Can be used to break public-key cryptography (RSA)
  - Algorithm uses quantum Fourier transform and modular exponentiation

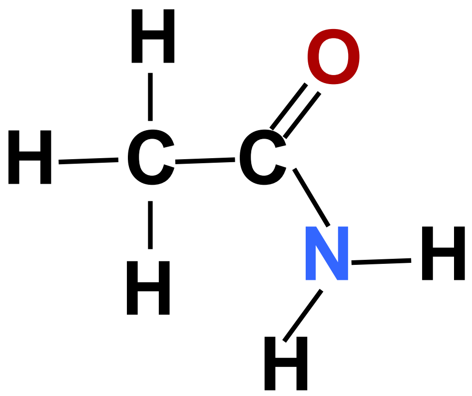
Shor's Factoring Algorithm – Logical Gate Count

- Factor a 1024-bit number
- Algorithm needs approximately 1.68×10^8 Toffoli gates and 6,144 logical qubits (*Jones et al., 2012*)

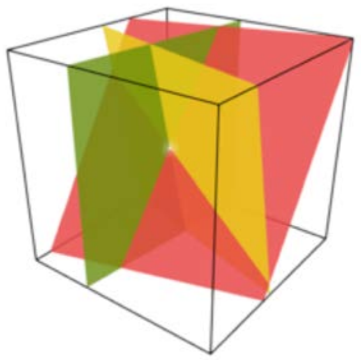
Gate	Occurrences	Parallelization Factor
CNOT	1.18×10^9	1
Hadamard	3.36×10^8	1
T or T†	1.18×10^9	2.33
Other gates	negligible	

More Examples of Studied Quantum Algorithms

- Ground state estimation algorithm
  - Find ground state energy of glycine molecule
  - Quantum simulation and phase estimation

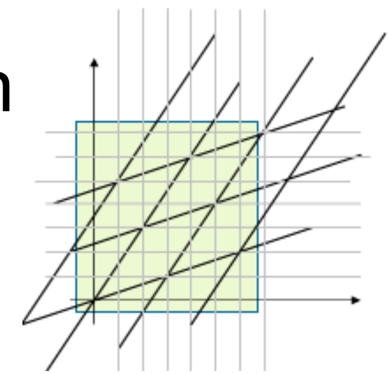


- Quantum linear systems algorithm
  - Find x in the linear system Ax = b
  - QFT, amplitude amplification, phase estimation, quantum walk

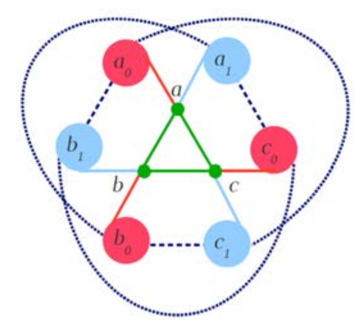


More Examples of Studied Quantum Algorithms

- Shortest vector problem algorithm
  - Find unique shortest vector in an integer lattice
  - QFT and sieving



- Triangle finding problem
  - Find the nodes forming a triangle in a dense graph
  - Quantum random walk and amplitude amplification



Example: Ground State Estimation Algorithm – Logical Gate Count

Gate	Occurrences	Parallelization Factor
CNOT	7.64×10^{10}	1.5
Hadamard	3.64×10^{10}	6
Prepare |0>	55	55
Measure Z	5	1
Z	1.21×10^{10}	3
S	1.21×10^{10}	3
Rotations	6.46×10^{9}	1.5

- Rotations decomposed into more elementary gates (*Bocharov et al., 2012*)

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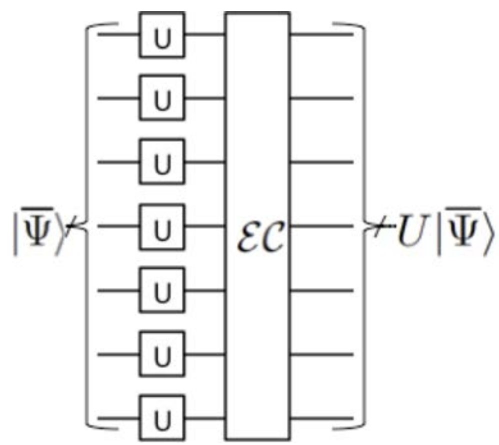
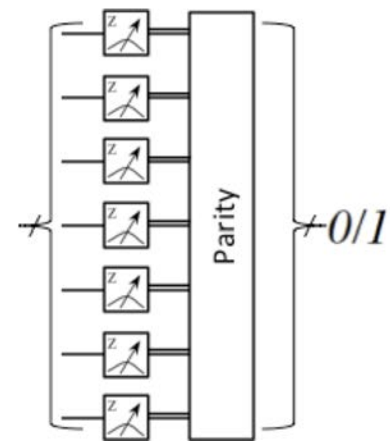
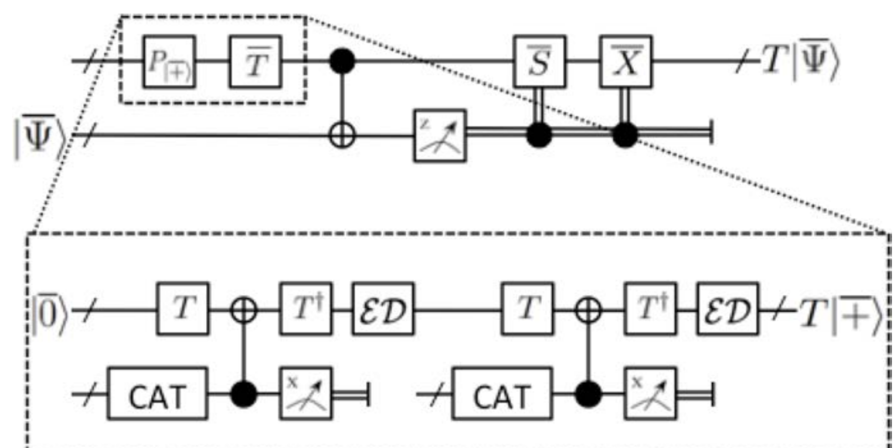
Steane [[7,1,3]] Concatenated Error Correction Code

- 7 data qubits encode a single logical qubit
- Most operations transversal:



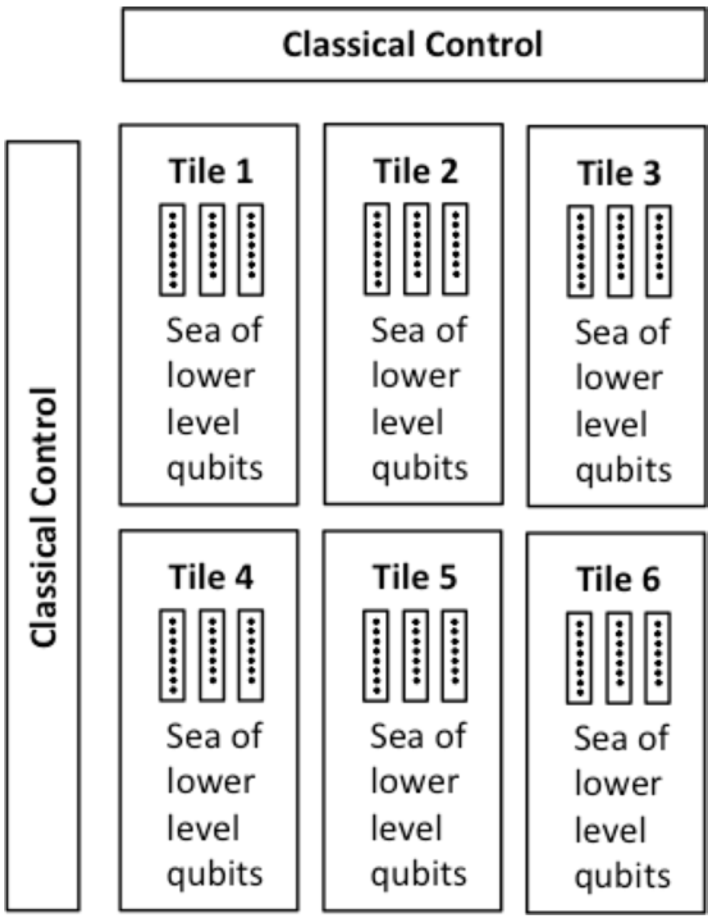


- Non-transversal T gate:



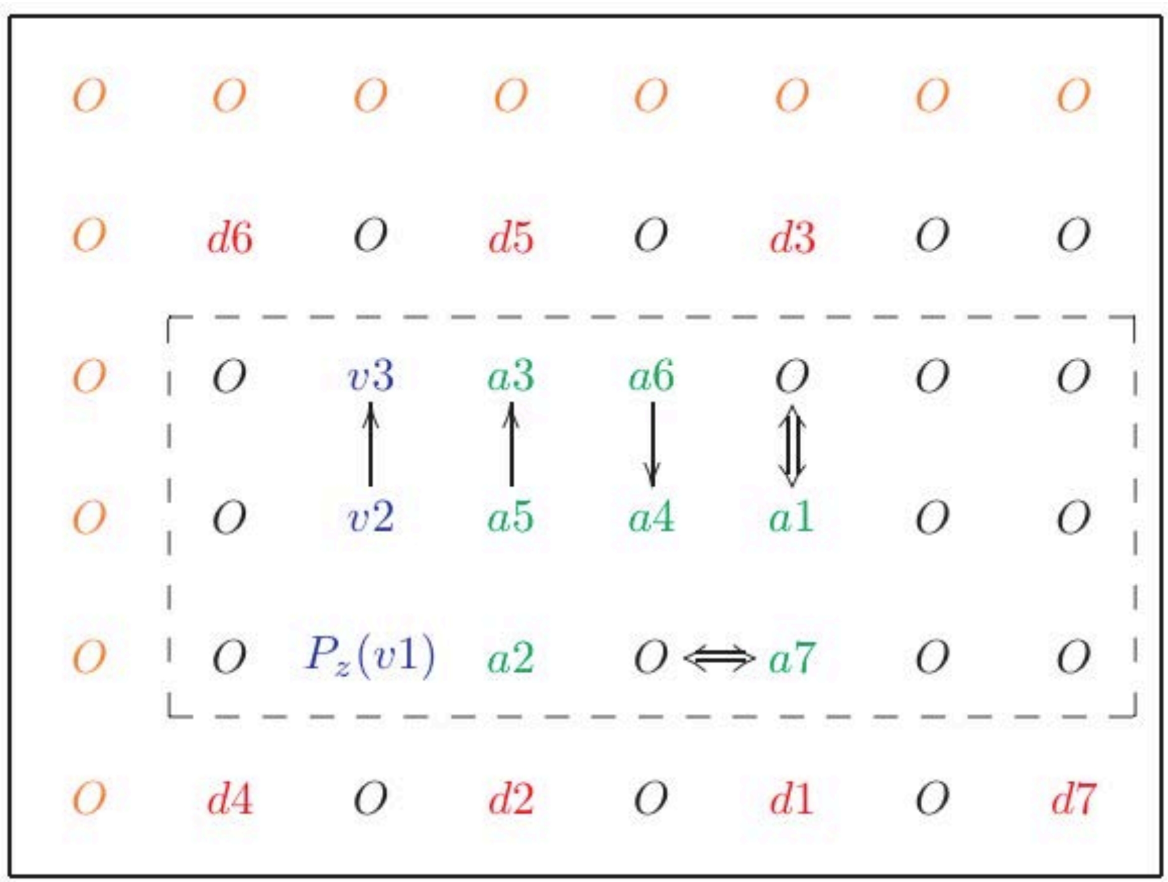
Tiled Qubit Layout for Concatenated Codes

- Each logical qubit is stored in a separate tile



- Tiles arranged in 2-D
- Supported operations:
  - Error correct a tile
  - Apply fault-tolerant operation
- Tiles must contain enough data and ancilla qubits

Optimized Layout in Each Tile
(*Svore et al., 2006*)



"empty" qubit

data qubit

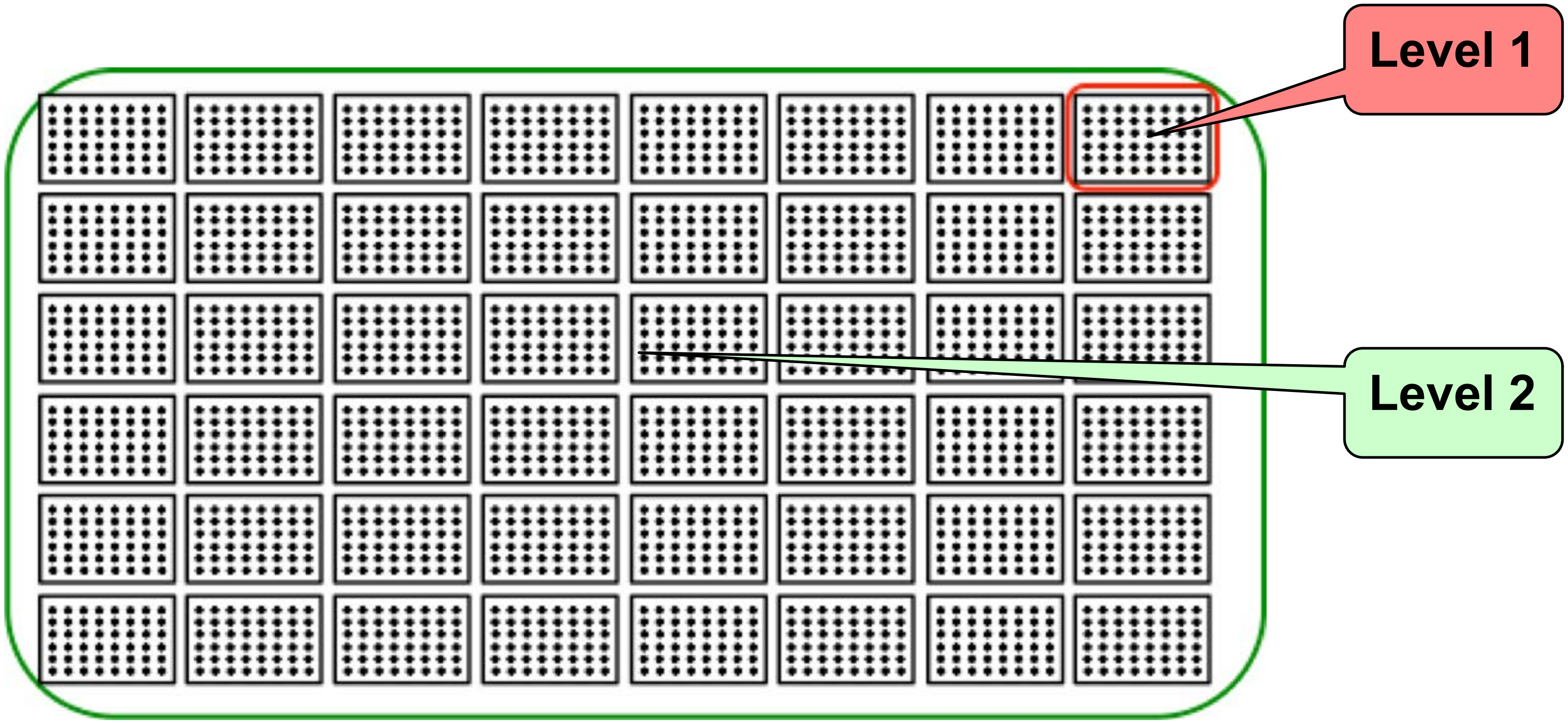
verification qubit

ancilla qubit

⟺ SWAP

↑ CNOT

Tiles Have a Hierarchical Structure that Allows Code Concatenation



- Sufficient number of concatenations to achieve constant probability of success of computation

Counting the Gates and Computation Time

- For each logical operation (CNOT, error correction, Paulis, S, T, measurement, etc.)
  - Count number of elementary gates
  - Count time taking parallelism into account
- Methodology: recursive equations that follow the concatenated structure

$$ops(X_{(m)}) = 7X_{(m-1)} + \mathcal{EC}_{(m)}$$

$$time(X_{(m)}) = X_{(m-1)} + \mathcal{EC}_{(m)}$$

$$\begin{aligned} ops(\mathcal{EC}_{(m)}) = {} & 14hCNOT_{(m-1)} + 28vCNOT_{(m-1)} + 7H_{(m-1)} + 18hSWAP_{(m-1)} \\ & + 15vSWAP_{(m-1)} + 8P_{|+\rangle(m-1)} + 12P_{|0\rangle(m-1)} + 20\mathcal{M}_{X(m-1)} \end{aligned}$$

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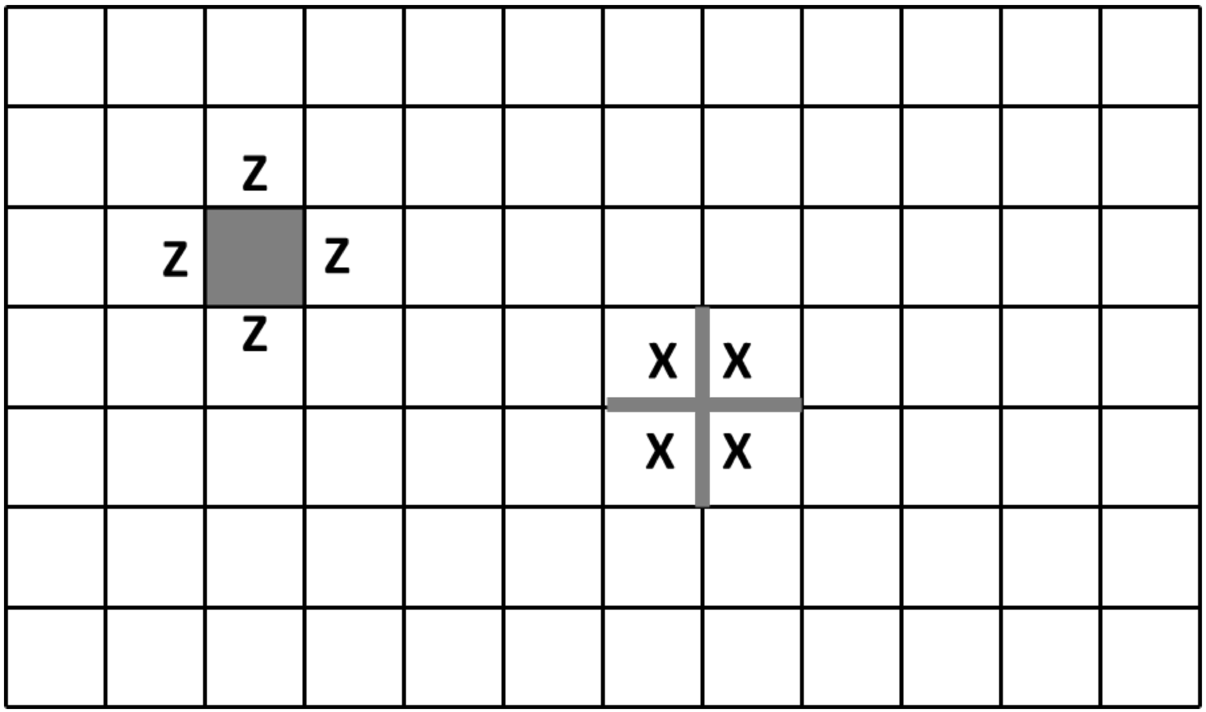
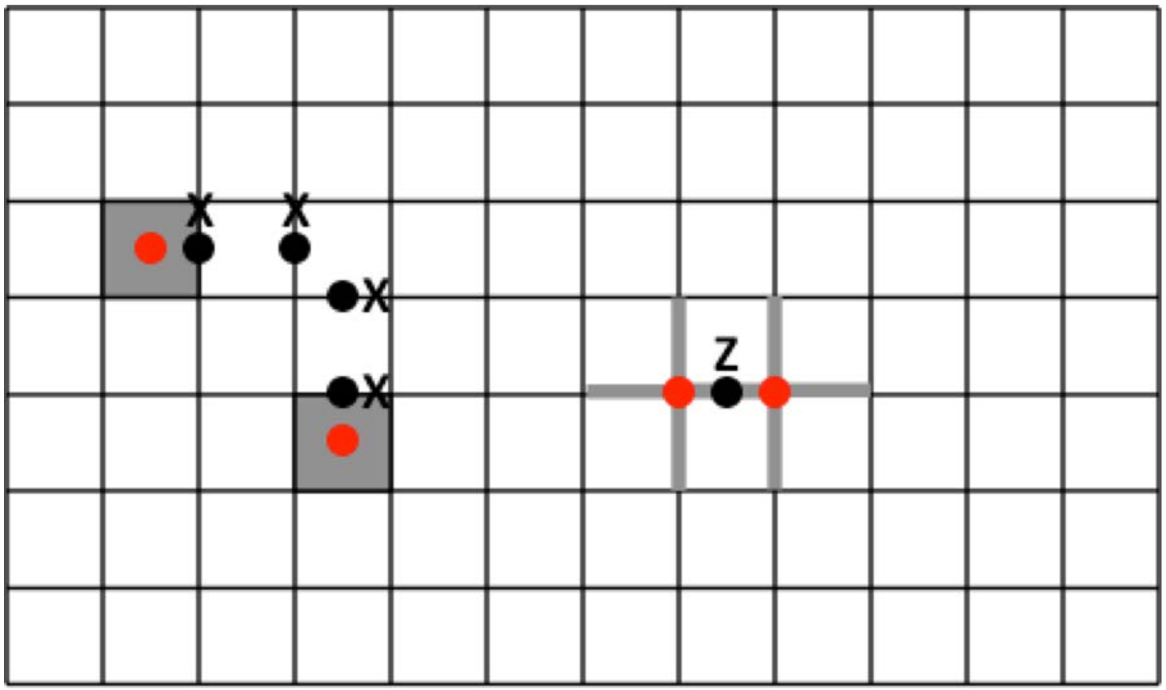
Topological Quantum Memory – The Surface Error Correction Code



- Physical qubits on links in the lattice
- Measuring the shown “check” operators yields error syndromes

Syndromes Caused by Errors

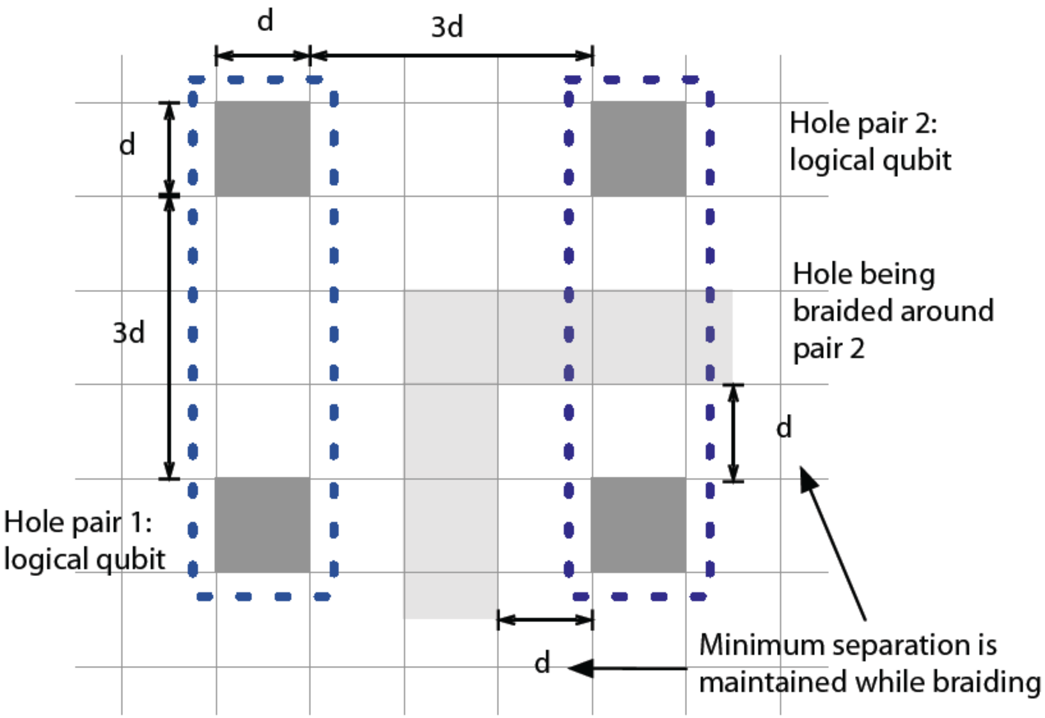


- Guess the most likely error consistent with observed syndromes
- Error correction performed continuously

Tiles Represent Logical Qubits

- Each logical qubit represented by a pair of holes
- CNOT gates performed by moving holes around each other

Code Distance Determines Fault Tolerance and Size of the Tiles



N: number of gates

p: physical error rate

C_1, C_2: constants

$P_{th} \approx 0.1$: error correction threshold

- Distance sufficient for high success probability:

$$\frac{0.5}{N} \geq C_1 \left(C_2 \frac{p}{p_{th}} \right)^{\lfloor \frac{d+1}{2} \rfloor}$$

(*Jones et al., 2012*)

Counting the Qubits and Gates

- Qubit count: multiply number of tiles and size of tile
- Gate count:
  - Calculate total running time T
  - Calculate number of gates required to error correct the entire surface during interval T
  - Estimate the small number of additional gates required by logical operations

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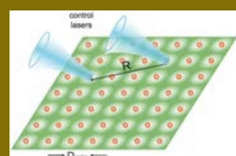
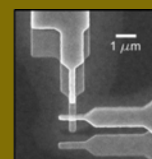
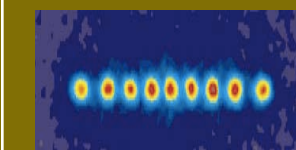
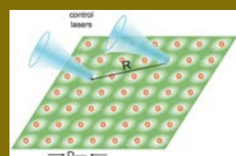
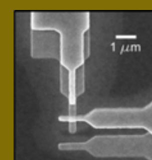
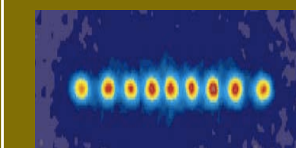
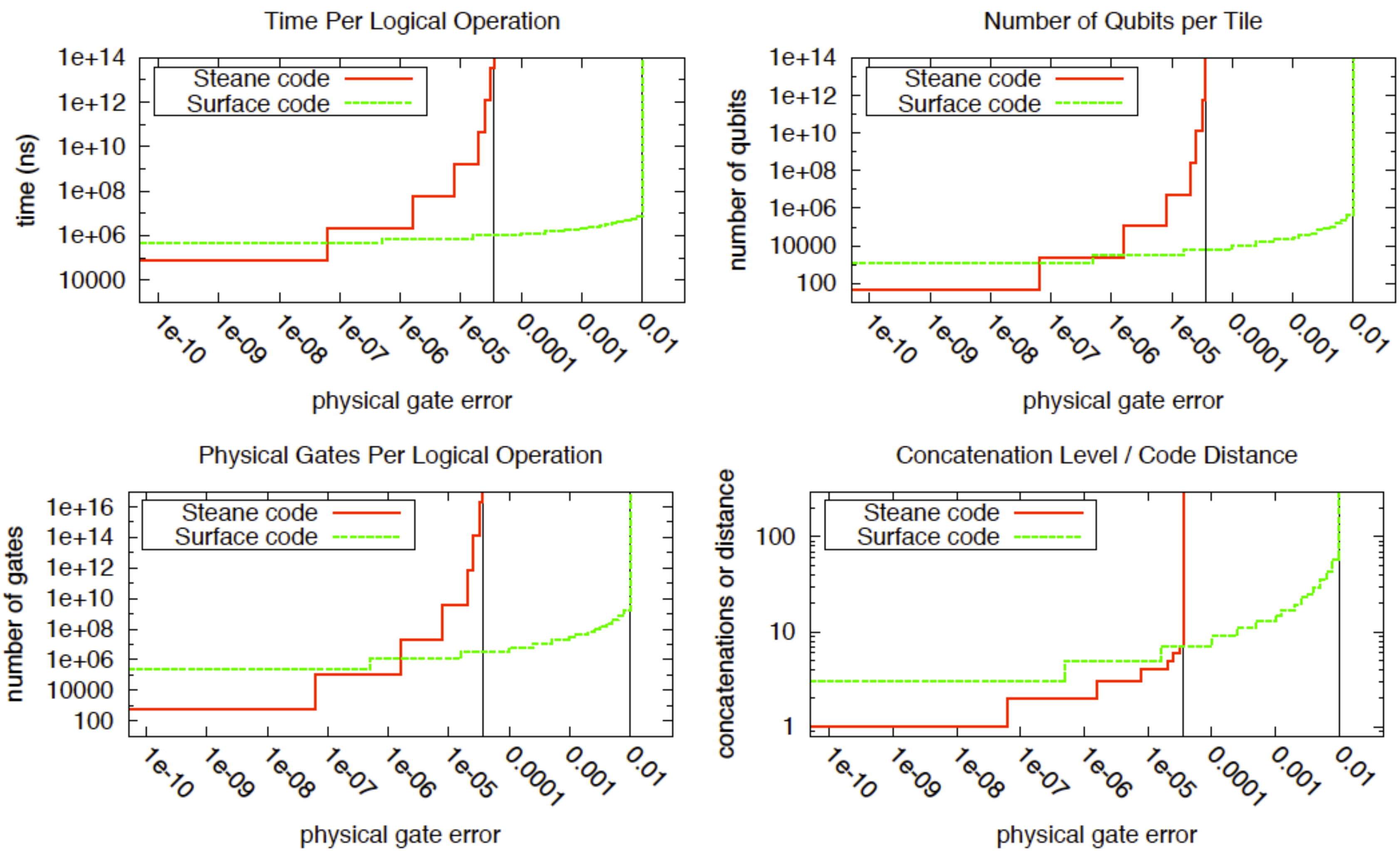
Numerical Results – Shor's Factoring Algorithm, Three Technologies

	Neutral Atoms	Supercond. Qubits	Ion Traps	
	$e = 1 \times 10^{-3}$ t = 19,000 ns	$e = 1 \times 10^{-5}$ t = 25 ns	$e = 1 \times 10^{-9}$ t = 32,000 ns	
				
Surface Code	2.6 years	**10.8 hours**	2.2 years	**Time**
	5.3×10^{8}	$\mathbf{4.6 \times 10^{7}}$	1.4×10^{8}	**Qubits**
	1.0×10^{21}	$\mathbf{2.6 \times 10^{19}}$	5.1×10^{19}	**Gates**
Steane Code	-	5.1 years	**58 days**	**Time**
	-	2.7×10^{12}	$\mathbf{4.6 \times 10^{5}}$	**Qubits**
	-	1.2×10^{32}	$\mathbf{4.1 \times 10^{18}}$	**Gates**

Numerical Results – Ground State Estimation, Three Technologies

	Neutral Atoms	Supercond. Qubits	Ion Traps	
	$e = 1 \times 10^{-3}$, $t = 19{,}000$ ns	$e = 1 \times 10^{-5}$, $t = 25$ ns	$e = 1 \times 10^{-9}$, $t = 32{,}000$ ns	
				
Surface Code	6.2×10^{21}	**3.6×10^{18}**	6.0×10^{21}	Time (ns)
	4.2×10^{8}	**5.5×10^{7}**	2.5×10^{8}	Qubits
	6.1×10^{25}	**2.8×10^{24}**	7.5×10^{24}	Gates
Steane Code	-	1.5×10^{23}	**1.6×10^{22}**	Time (ns)
	-	1.4×10^{10}	**1.3×10^{5}**	Qubits
	-	1.0×10^{36}	**1.5×10^{25}**	Gates

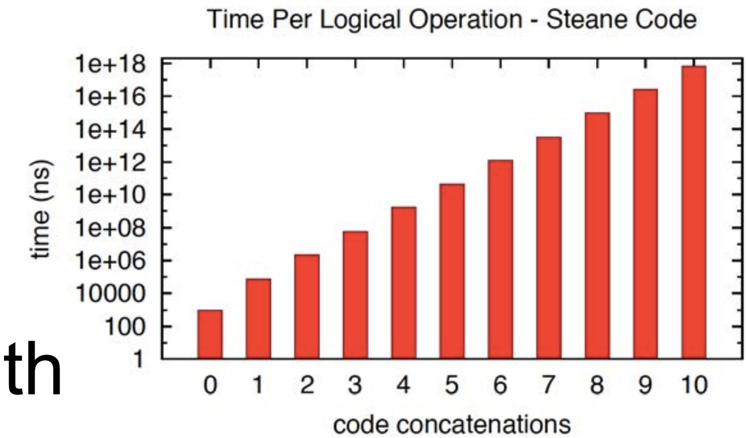
Abstract Technology (1 μs gates) with Varying Physical Error Rate



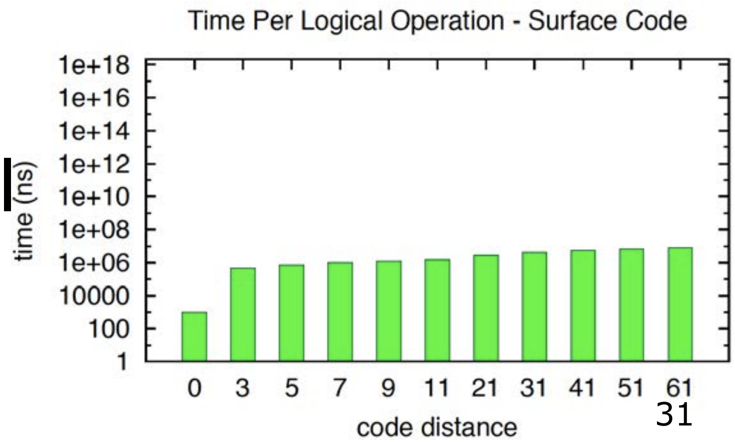
For low error rates concatenated codes outperform topological codes. Why?

The Topological and Concatenated Code Families are Very Different

- Concatenated codes
  - Lightweight with 1-2 levels of concatenation
  - Exponential overhead with additional concatenations

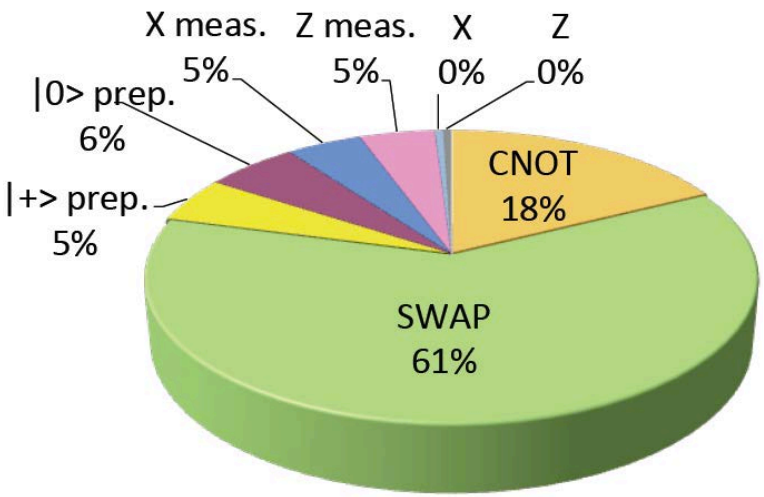


- Topological codes
  - Operations highly parallel
  - Moderate overhead with increasing code distance

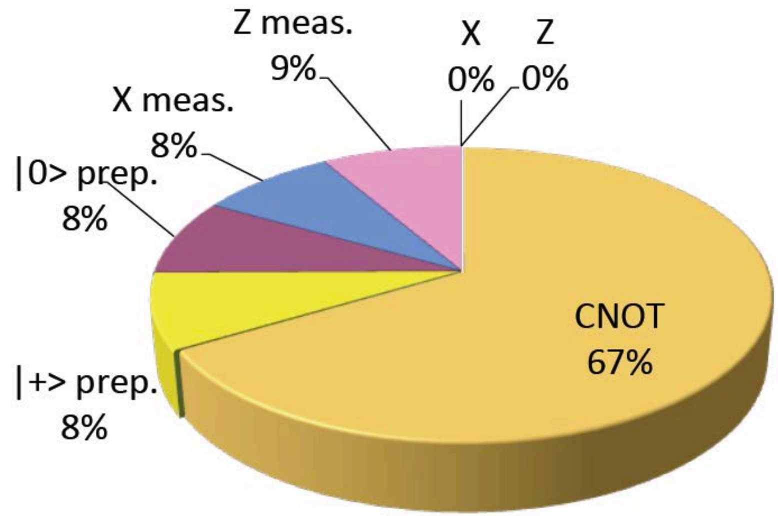


Qualitative Difference in Gate Composition

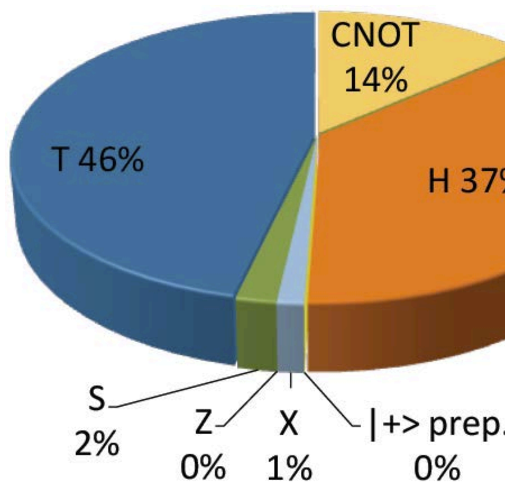
Steane code:



Surface code:



Logical circuit:



Resource Estimates Useful for Identifying Topics for Future Work

- Low parallelism of studied circuits
  - How to exploit parallelism and move some operations off the critical path?
- Decomposition of arbitrary rotations very costly
  - More efficient techniques?
- Costly T and CNOT gates dominate
  - Circuit transformations to avoid these gates?
  - More efficient offline implementation?

Conclusion

- QuRE is an automated tool that quickly estimates the properties of the future quantum computer
- Reports a number of quantities including gate count, execution time, and number of qubits
- Is easily extendable for new technologies and algorithms
- Allows to identify sources of high overhead and quickly asses the effect of suggested improvements

Thank You!