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Winning by Losing: Incentive Incompatibility in Multiple Qualifiers

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Abstract

In sport tournaments, the rules are presumably structured in a way that any participant cannot benefit by losing instead of winning. We show that tournament systems, consisting of multiple round-robin and knockout tournaments with noncumulative prizes, which are ubiquitous around the world, are generically incentive incompatible. We use our model to discuss potential remedies and applications.

Keywords

tournaments, design, rules, incentives, football

Introduction

In any sport tournament, the rules define a strategic interaction between participants. Ideally, these rules should be structured, so that a team cannot advance by losing instead of winning a game. In practice, the rules are complex and sometimes create perverse incentives for participating teams. Optimal organization of tournaments has long been an area of study for economic theorists (e.g., Rubinstein, 1980; Slutzki & Volij, 2005).

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There are numerous situations in which a team might prefer losing a game to winning it.[1] First, players may be bribed. Second, the teams that performed worse may have advantages the next season.[2] Third, being ranked second in the qualifications might result in facing a preferred competitor in the knockout stage.[3] However, in the first situation, perverse incentives are not generated by tournament rules. In the second situation, prize distribution rules were deliberately designed to reward less fortunate teams. In the third case, an advantage is gained in expected terms (any team has a lower probability of winning against *Barcelona* or *Chelsea* than against a weaker team). Also, Chen, Deng, and Liu (2011), Faliszewski (2008), and Russell and Walsh (2009) consider possibility of a collusion between several teams. In this article, our focus is on the possibility that a team is strictly better off by losing.

In practice, tournament organizers exploit many different ranking principles. One strongly desired property of ranking methods is incentive compatibility. If only one tournament is played, under every reasonable ranking rule, a team cannot be better off by losing instead of winning. For example, in round-robin tournaments, all orderings by points are monotonic—more points equates to a higher place.

Our article demonstrates that incentive incompatibility is a generic feature in situations when results are aggregated across several independent tournaments with the same participants and prizes are noncumulative, that is, a team cannot win more than one unit of the prize.[4] In such situations, the monotonicity of ranking methods for each separate tournament is not enough to guarantee incentive compatibility for the whole qualification system. We show that the whole class of such allocation rules is inherently flawed and provide general results about misaligned incentives. The incentive incompatibility necessarily arises if there is more than one round-robin tournament with at least one prize or if there is one round-robin tournament and at least one knock-out tournament with at least one prize, and the allocation rule does not always favor the round-robin competition.

The following very simple example illustrates the basic logic of our general argument.

Example 1

Let there be two domestic round-robin tournaments and four teams, namely, A, B, C, and D. These teams participate in each of the two tournaments, which we will call "Tournament 1" and "Tournament 2." The best team in each tournament wins a noncumulative prize (e.g., Champions League qualification). There are two prizes that must be allocated to different teams under any circumstances. It could happen that one team wins both tournaments. In this case, one prize is vacant. Consider the following allocation rule: If one team wins both tournaments, the vacant place is allocated to the team that finished second in Tournament 1. Now, we construct a situation in which Team B is better off by losing the game against Team A (see Figure 1).

Tournament 1				
	A	B	C	D
A		Win	Win	Win
B	Loss		Win	Win
C	Loss	Loss		Win
D	Loss	Loss	Loss	

Tournament 2				
	A	B	C	D
A		?	Draw	Win
B	?		Draw	Loss
C	Draw	Draw		Win
D	Loss	Win	Loss	

Figure 1. An incentive incompatibility example.

> Under any "reasonable" ranking method in Tournament 1, Team A will be ranked first and Team B will be second. As for Tournament 2, Teams A and C are competing for first place. If Team B loses to A in the last game of the tournament, Team A wins both tournaments. In this case, according to the allocation rule, Team B wins the prize as the second team in Tournament 1. At the same time, if Team B defeats A, Team C is first in Tournament 2 (instead of A), and B does not win the prize. Consequently, Team B has to lose the game against A to win the prize. The same logic can be applied to a general case in which there are more than three teams, more winners and any reasonable ranking method.

In economics, the problem of the aggregation of results in sport tournaments is similar to the classic problem of voters' preferences aggregation (Harary & Moser, 1966). In a seminal contribution, Arrow (1963) formulated several highly desirable properties of aggregation rules for voter preferences and proved that there is only one aggregation rule (namely, a dictatorship) that satisfies these properties. Rubinstein (1980) used a similar approach to the problem of ranking participants in a round-robin tournament, defining properties of anonymity, positive responsiveness, and the independence of irrelevant alternatives, and proved that the only ranking rule that satisfies all three properties is a ranking with respect to the number of wins. Several authors defined other desirable sets of properties and found all ranking rules that satisfy these properties (see, e.g., Bouyssou, 2004; Herings, van der Laan, & Talman, 2005; Slutzki & Volij, 2005, 2006; van den Brink & Gilles, 2000).

In a political science context, the Gibbard–Satterthwaite (Gibbard, 1973; Satterthwaite, 1975) and Duggan–Schwartz (2000) theorems state that in the presence of "good enough" aggregation rules, there always exists a voter who can profitably deviate from his or her real preferences and vote strategically. A similar question arises in connection with tournament results aggregation: Under a given ranking rule, is there a team that has a positive incentive to deliberately lose a game for strategic reasons? Wright (2014) surveyed operations-research papers that address incentive incompatibility. Baumann, Matheson, and Howe (2010) demonstrated that in the playoff stage of the National Collegiate Athletic Association (NCAA) basketball "March Madness" tournament, the 10th and 11th seeded teams statistically advance further than the 8th and 9th seeded teams. This violation of the monotonicity of winning probabilities with respect to seeding number generates perverse incentives for participating teams during the regular season. Taylor and Trogdon (2002) confirmed that teams that lost the chance to advance to the playoff stage began to react to negative incentives by losing more frequently.

Pauly (2014) considers complex tournament systems consisting of two qualifying tournaments with disjoint sets of participants and a subsequent final tournament of four teams. In such situations, the properties of symmetry, nonimposition, anonymity, independence of irrelevant alternatives, and nonmanipulability defined for these tournaments are incompatible. Unlike Pauly (2014), we consider multiple tournaments with the same set of participants and focus on the consequences of a nonempty intersection of sets of winners of noncumulative prizes. Also, we allow for draws. Finally, Pauly (2014) requires *ex ante* fixing specific outcomes of the games in the final tournament to demonstrate incentive incompatibility at the qualifying stage. Instead, the noncooperative part of our analysis is "subgame perfect."

The rest of the article is organized as follows. The second section contains a real-world illustration of the phenomena that we study. The main model is presented and analyzed in the third section. In the fourth section, we develop a strategic noncooperative extensive-form game that takes into account the consecutive structure of the tournaments. The fifth section discusses the implications of our formal results for European football competitions, and the sixth section concludes.

A Real-World Example

The following real-world example[5] is more complicated than the story described in the theoretical model, as teams strive to qualify for two international tournaments instead of one. However, this does not affect the logic of our argument.

In the Russian Premier League, a win is awarded three points and a draw is worth one point. By May 8, 2012, each team in the league had one more game to play. The final of the national cup, the second major tournament, was to be held on May 9. Below, we show that conditional on results of other games, Lokomotiv would have been better off by losing its game against Spartak. This would let Spartak qualify for the Union of European Football Associations (UEFA) Champions League, let Dynamo (if it won over Rubin in the cup final) qualify for the Europa League, leaving Rubin out of the international competitions, and give Lokomotiv a place in the Europa League. If, instead, Lokomotiv beat Spartak, all other results being the

Table 1. Final Standings Without the Game Between Zenit and Anzhi.

Lokomotiv Wins			Lokomotiv Draws			Lokomotiv Loses		
Place	Team	Points	Place	Team	Points	Place	Team	Points
1	*Zenit*	85	1	*Zenit*	85	1	*Zenit*	85
2	*Dynamo*	74	2	*Dynamo*	74	2	*Spartak*	75
3	**CSKA**	74	3	**CSKA**	74	3	**Dynamo**	74
4	**Spartak**	72	4	**Spartak**	73	4	**CSKA**	74
5	**Anzhi**	70	5	**Anzhi**	70	5	**Anzhi**	70
6	Lokomotiv	69	6	Lokomotiv	67	6	**Lokomotiv**	66
7	**Rubin**	66	7	**Rubin**	66	7	Rubin	66

Note: There are three types of the teams. Some of them go to the Champions League (italicized), some of them go to Europa League (in bold) and some of them failed to qualify (roman).

same, Dynamo would qualify for the Champions League, thus making Rubin, the cup's runner-up, qualified for the Europa League and leaving Lokomotiv out of the international competitions.

At the last match day, the games were Kuban'–Dynamo, Lokomotiv–Spartak, Rubin–CSKA, and Anzhi–Zenit. The cup final on May 9 matched up Dynamo and Rubin.

For 2012-2013, Russia was awarded two slots in the Champions League and four Europa League places to be distributed according to the following rules.

1. Teams that are ranked first and second in the championship qualify for the Champions League.
2. Teams that are ranked third to fifth qualify for the Europa League.
3. If the cup winner is ranked first or second, it plays in the Champions League, and the cup runner-up qualifies for the Europa League.
4. If the cup winner is ranked third, fourth or fifth, the sixth team also qualifies for the Europa League.
5. Finally, if the cup winner is ranked below fifth place, it qualifies for the Europa League.

Now, consider the following scenario. First, suppose that Dynamo wins the Russian Cup and wins its last game in the championship. Second, suppose that Rubin vs. CSKA is a draw. As with an equal number of points, the ultimate relative standings are determined by the number of wins, Dynamo ranks above CSKA, and Lokomotiv ranks above Rubin. The outcome of the Anzhi–Zenit game is irrelevant for further consideration as Zenit has clinched the championship and Anzhi has already earned a place in the Europa League (regardless of the result of the last game, it could not rank lower than fifth or higher than fourth).

Thus, the only relevant game left is Lokomotiv–Spartak. There are three possible outcomes: Lokomotiv wins, draws, or loses. Table 1 shows the final standing of the teams in each of these cases.

In the scenario considered above, Lokomotiv has every incentive to lose the final game of the national championship. Although the team would finish sixth in each case, losing would result in qualification for the European tournament. Although this scenario did not come to pass as Rubin won the Russian Cup, beating Dynamo, it demonstrates that perverse incentives can easily arise in real circumstances.

Theory

In this section, we formalize the problem of results aggregation in round-robin and knock-out tournaments. We demonstrate when incentive incompatibility arises in qualification systems that consist of multiple round-robin and/or knock-out qualifiers.

Definition 1: A *round-robin tournament* is a pair (X, v), where X is a nonempty finite set of tournament participants (teams) and $v : (\mathsf{X} \times \mathsf{X}) \setminus \{(x, y) | x = y\} \to \{-1, 0, 1\}$ is an antisymmetric[6] function that is called the *characteristic function* of the tournament (X, v).

Definition 2: A *ranking method of a round-robin tournament* with a set of participants X is a function S for which the domain is the set of all characteristic functions of the round-robin tournaments with the set of participants X and that maps any characteristic function v into a partially ordered set $S(v)$ of elements of the set X.

Let $x_0, y_0 \in \mathsf{X}, x_0 \neq y_0$. We say that team x_0 *wins* over team y_0 if $v(x_0, y_0) = 1$, team x_0 *loses* to team y_0 if $v(x_0, y_0) = -1$, and teams x_0 and y_0 *tie* if $v(x_0, y_0) = 0$. Two teams play each other once, and function v describes the results of the games. For a round-robin tournament with the characteristic function v, let $N_v^1(x)$, $N_v^0(x)$, and $N_v^{-1}(x)$ denote the number of wins, draws, and losses of team x, respectively. A ranking method, then, is simply a rule that orders participating teams according to all results.

Example 2

Consider a round-robin tournament $T = (\mathsf{X}, v_0)$, where $\mathsf{X} = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$, and let S be the following ranking method:

1) a team earns three points for each victory, one point for each draw, and zero points for each loss,
2) if one team has more points than another, the former team is ranked higher than the latter,

Table 2. Characteristic Function v_0.

Team	A	B	C	D
A	—	1	−1	−1
B	−1	—	1	0
C	1	−1	—	−1
D	1	0	1	—

3) if two or more teams earn the same number of points, the team with more points in the games between them is ranked higher, and
4) if several teams receive an equal number of points and if these teams have an equal number of points in the games between them, these teams are ordered according to the following a priori seeding: A≻B≻C≻D.

Note that for any characteristic function v, the ranking method S defines a totally ordered set $S(v)$ of the teams from X.

Suppose that the characteristic function v_0 is given in the Table 2, where the value $v(x_0, y_0)$ is written in the intersection of row x_0 and column y_0.

Applying the ranking method S to the characteristic function v_0, we obtain $S(v_0) = D \succ B \succ C \succ A$, that is, D gets first place, B gets second, C gets third, and A gets fourth.

Next, we introduce a knock-out tournament.

Definition 3: A *knock-out tournament* T_{2^m} with 2^m participants $x_1, ..., x_{2^m}$, $m \geq 1$, is a full binary tree (each vertex[7] is either a leaf or has two child vertices) of height m with 2^m leaves. Each leaf is labeled with one of the teams $x_1, ..., x_{2^m}$. Two teams that are assigned to vertices with the same parent vertex play each other. The parent vertex is labeled with the winner of this game.

There exists a one-to-one correspondence between the trees that represent the knock-out tournaments and the functions, partially defined on the set X × X, which contain all the results of the tournament (similar to the characteristic functions of a round-robin tournament). For unification reasons, we will assume below that a knock-out tournament is defined by such a function, and we will refer to it as to the characteristic function of a knock-out tournament. However, unlike the characteristic function of a round-robin tournament, the characteristic function of a knock-out tournament cannot assume all possible results of individual games (e.g., to have a result from the game against Team B in the second round, Team A needs to defeat its opponent in the first round).

Definition 4: A *ranking method of a knock-out tournament* with a set of participants X is a function $\hat{S}$ for which the domain is the set of all characteristic functions of a knock-out tournament with a set of participants X and that maps any characteristic function $\hat{v}$ into a partially ordered set $\hat{S}(\hat{v})$ of elements from the set X.

Next, we define several specific properties of ranking methods. When $S(v)$ is a totally ordered set, there exists a mapping from X to a K-tuple of team places $(s_1(v), ..., s_K(v))$, where $s_i(v)$ is the rank assigned to team number i by the ranking method S in the tournament with the characteristic function v, $i = 1, ..., K$. If for any two teams i and j, either $s_i(v) < s_j(v)$ or $s_j(v) < s_i(v)$ holds, $S(v)$ is a *strictly* totally ordered set. Usually, for tournament organizers, it is important that the ranking method makes it possible to order participating teams regardless of specific outcomes.

Definition 5: A ranking method of a round-robin tournament S (or a ranking method of a knock-out tournament $\hat{S}$) is *well-defined* if for any characteristic function v (or for any characteristic function $\hat{v}$), $S(v)$ (or $\hat{S}(\hat{v})$) is a strictly totally ordered set.

Our next step is to define a proper monotonicity concept for round-robin and knock-out tournaments.

Definition 6: A ranking method of a round-robin tournament S satisfies the *monotonicity property* (or, simply, is monotonic) if for any characteristic function v and for any two teams $x, y \in$ X such that $N_v^1(x) \geq N_v^1(y), N_v^{-1}(x) \leq N_v^{-1}(y)$, where at least one of these two inequalities is strict, inequality $s_x(v) < s_y(v)$ holds.

With a monotonic ranking method, if one of the two teams with different sets of results has not less wins and not more losses than the other, the former is ranked higher than the latter. This definition is compatible with the standard monotonicity concept for tournaments in which draws are not possible. Indeed, if for any team x, we put $N_v^0(x) = 0$, under a monotonic ranking method, a team that wins more games will be ranked higher.

For knock-out tournaments, the analogue of the monotonicity property is as follows.

Definition 7: A ranking method of a knock-out tournament $\hat{S}$ is *consistent* if $s_x < s_y$ implies that x was not eliminated earlier than y.

The next property underlines the equality of knock-out tournament participants. Such a property is often regarded as very desirable in a wide range of aggregation problems.

Definition 8: A ranking method of a knock-out tournament $\hat{S}$ is *anonymous* if for any characteristic function $\hat{v}$ and for any permutation of the teams π in the arguments of a characteristic function, $\hat{S}(\hat{v})$ turns into $\pi(\hat{S}(\hat{v}))$.

Example 3

Consider a knock-out tournament with participants $\mathsf{X} = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$, and let $\hat{S}$ be the following ranking method: The winner of the tournament is ranked first, the runner-up is ranked second, a semifinalist which loses to the winner of tournament is ranked third, and the remaining team is fourth. For each possible characteristic function $\hat{v}$, this rule assigns a strictly totally ordered set $\hat{S}(\hat{v})$. Thus, $\hat{S}$ is a ranking method. Furthermore, $\hat{S}$ is consistent: The runner-up is ranked below the champion, both semifinalists are ranked below both the champion and the runner-up. Finally, $\hat{S}$ is anonymous: The ranking does not depend on the name of the teams. On contrary, constant ranking method $\hat{S}$, such that $\hat{S}(\hat{v}) = \mathrm{A} \succ \mathrm{B} \succ \mathrm{C} \succ \mathrm{D}$ for each $\hat{v}$, is neither consistent (if D is the champion, it is ranked below the nonchampion A) nor anonymous (D cannot be ranked first).

Consider a qualification system consisting of $N > 1$ domestic tournaments and one international tournament, for which domestic teams want to qualify. A team can proceed to the international tournament only after a successful performance in one of the domestic competitions. Let the set of teams competing domestically be $\mathsf{X} = \{1, 2, ..., K\}$, $K \geq 1$, and let b_i be the number of slots for the international tournament contested in tournament i, $i = 1, ..., N$. We say that teams placed $1, \ldots, b_i$ in tournament i finish in the prize zone of tournament i.

It might happen that after all domestic tournaments are completed, one team earns more than one place in the international tournament, that is, the team finishes in the prize zone in several tournaments. In this case, there would be some vacant slots in the international tournament. For example, in the extreme case, when all teams are ranked the same in each tournament, there will be only $\max_i b_i$ slots filled instead of $\sum_i b_i$. Then, all vacant slots must be distributed among the remaining teams. It is easy to see that there cannot be more than $\sum_i b_i - \max_i b_i$ vacant slots.

Allocating the vacant slots to the remaining teams might be carried out in many different ways. It is natural to allow only such allocations of vacant slots that a team can win a slot only if all teams that finished above it in this tournament also qualified. Thus, it is sufficient to know the order in which vacant slots are allocated across tournaments.

For any finite sequence a_n and any $x \in \mathbb{R}$, let $Num(x, a_n)$ denote the cardinality of the set $\{i | a_i = x\}$.

Definition 9: *Allocation rule* R_n is a sequence of $(N-1)(\sum_i b_i - 1)$ elements, where $R_i \in \{1, ..., N\}$, for any $i = 1, ..., (N-1)(\sum_i b_i - 1)$, and $Num(x, R_n) \leq K - 1$, for any $x \in \{1, ..., N\}$. If there is any vacant slot, we try to allocate it to the tournament in order R_n, expanding the prize zone of this tournament by 1. If the corresponding team already qualified from other tournament(s), we look at the next element of the sequence R_n and expand the prize zone by 1 in the corresponding tournament.

The correctness of the definition follows from the fact that $(N-1)(\sum_i b_i - 1) \leq (N-1)(K-1) \leq N(K-1)$, and in the worst case when all tournaments provide the same ranking of teams, all $\sum_i b_i$ slots will be allocated after $N(\sum_i b_i - 1) + 1$ attempts, $\sum_i b_i$ of them having been performed before use of the rule; $N(\sum_i b_i - 1) + 1 - \sum_i b_i = (N-1)(\sum_i b_i - 1)$. For example, if $N = 2$, $b_1 = b_2 = 1$ and $R_1 = 2$, this means that if at the end there is a vacant slot, it would go to the team that finished second in the second tournament. After this, all slots are guaranteed to be allocated.

As a construction element for our proofs, we define a special tournament. Order the participating teams from 1 to K. A round-robin tournament is *transitive* if Team 1 beats all teams, Team 2 beats all teams but Team 1, . . ., team K loses all games. Straightforwardly, in a transitive tournament for any $i = 0, 1, ..., K-1$ exactly one team won i matches.

Consider the set of qualifying tournaments consisting of r round-robin tournaments with ranking methods $S_1, ..., S_r$ and k knock-out tournaments with ranking methods $\hat{S}_{r+1}, ..., \hat{S}_N$, $N = r + k$, and the allocation rule R_n. The following property singles out ranking methods that cannot generate perverse incentives.

Definition 10: We say that qualification system $(S_1, ..., S_r, \hat{S}_{r+1}, ..., \hat{S}_N, R_n)$ is *incentive compatible* if there does not exist $\eta^+, \eta^- \in \{-1, 0, 1\}$, $\eta^+ > \eta^-$, characteristic functions $v_1, ..., v_r, w$, $\hat{v}_{r+1}, ..., \hat{v}_N$, and i, $1 \leq i \leq r$, such that the following four conditions simultaneously hold:

(1) there exists a pair (x_0, y_0) such that $v_i(x_0, y_0) = \eta^+$ and $w(x_0, y_0) = \eta^-$,
(2) for any pair (x, y), different from (x_0, y_0) and (y_0, x_0), the equality $w(x, y) = v_i(x, y)$ holds,
(3) according to standings $S_1(v_1), ..., S_{i-1}(v_{i-1}), S_i(v_i), S_{i+1}(v_{i+1}), ..., S_r(v_r), \hat{S}_{r+1}(\hat{v}_{r+1}), ..., \hat{S}_N(\hat{v}_N)$, team x_0 does not qualify for the international tournament, and
(4) according to standings $S_1(v_1), ..., S_{i-1}(v_{i-1}), S_i(w), S_{i+1}(v_{i+1}), ..., S_r(v_r), \hat{S}_{r+1}(\hat{v}_{r+1}), ..., \hat{S}_N(\hat{v}_N)$,team x_0 does qualify for the international tournament.

Informally, the above definition states that a ranking method is incentive compatible if and only if it does not allow for situations when, given a completed tournament, a team would prefer to have lost some game rather than to have won or drawn or one team would prefer to have drawn some game rather than to have

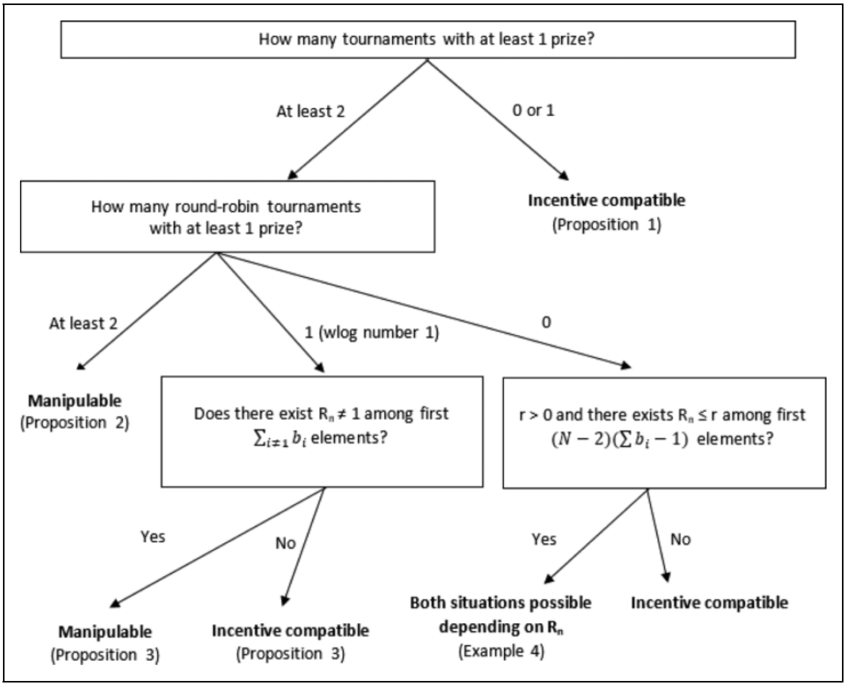

Figure 2. Summary of results.

won it. Qualification systems that are not incentive compatible are called incentive incompatible or manipulable.

Definition 10 stands for the profitability of deviating only in round-robin tournaments. We do not consider deviations in knock-out tournaments due to their specific properties. Indeed, one thinks of a tournament as a completed table in the case of a round-robin tournament or a completed tree in the case of a knock-out tournament. Consider the example of a completed knock-out tournament in which team X consequently won over teams $a_1, a_{2,}..., a_{t-1}$ and lost to team a_t. Unlike in a round-robin tournament, deviation from winning to losing in a knock-out tournament results in a team being eliminated from the competition and—this is critical—introduces new, unplayed games with unknown outcomes. If team X would ex post regret winning over team a_1, it could not know what would have happened in the case of losing to a_1, because the outcomes of the game between a_1 and a_2 and further games are undefined. So, there is no way to compare these two alternatives for team X.[8]

The remainder of the section is organized as follows. We consider general tournament systems consisting of r round-robin tournaments and k knock-out tournaments, $r, k \geq 0$, and allocation rule R_n. We investigate whether the system is incentive compatible. A short summary of results is provided in Figure 2. We will show that incentive incompatibility necessarily arises in the following cases:

(1) either there are multiple (more than one) round-robin tournaments with at least 1 prize
(2) or there is one round-robin and one knock-out tournament with at least one prize, and the allocation rule does not always favor round-robin tournament.

In both of the abovementioned cases, we will construct a situation that allows for profitable deviation using the idea from Example 1.

If there are some round-robin tournaments, but none of them provide any prizes, manipulability of the system depends on allocation rule. Since this case is rather degenerate and the more detailed analysis is cumbersome due to rich combinatorics of allocation rules, we confine our analysis to providing examples of manipulable and incentive compatible qualification systems in this case.

Now, we turn to the formal analysis. If all prizes are contested in one of the tournaments i, all slots in the international tournament are guaranteed to be distributed before the use of allocation rule based on the characteristic function of tournament i only. Then, if tournament i is round-robin, there are no incentives to deviate due to monotonicity of the ranking method S_i. Thus, the next proposition is straightforward.

Proposition 1:
1. Suppose that for some $i \leq r$ inequality $b_i \geq 1$ holds, and $b_j = 0$ for $j \neq i$. Then, for any monotonic ranking method S_i, for any ranking methods of other tournaments, and for any allocation rule R_n, the qualification system is incentive compatible.
2. Suppose that for some $i > r$ inequality $b_i \geq 1$ holds, and $b_j = 0$ for $j \neq i$. Then, for any ranking methods and for any allocation rule R_n, the qualification system is incentive compatible. The next proposition says that if there are at least two round-robin tournaments, each providing at least one winner with a prize, more than three teams, and more teams than prizes, then any monotonic ranking method and any allocation rule allow for a situation in which a team is better off by losing rather than winning.

Proposition 2: Consider a qualification system consisting of r round-robin tournaments (tournaments $1, \ldots, r$) and k knock-out tournaments (tournaments $r+1, \ldots, N$), $N = r + k$. Suppose that $r \geq 2$, $b_i \geq 1$ for $i = 1, 2$, and the number K of participating teams is large enough: $K > \max(\sum_i b_i, 3)$.

Then, for any well-defined monotonic ranking methods $S_1, ..., S_r$, for any well-defined consistent anonymous ranking methods $\hat{S}_{r+1}, ..., \hat{S}_N$, and for any allocation rule R_n, the qualification system is incentive incompatible.

Proof: Since there are at least two round-robin tournaments with at least one prize, at least one of these tournaments does not coincide with R_1. Suppose that $R_1 = 1$ and $b_2 > 0$ (see case $R_1 > r$ below).

Fix three arbitrary teams and call them X, Y, and Z (observe that our assumptions imply that $K \geq 4$).

Define the characteristic functions $v_3, ..., v_r, \hat{v}_{r+1}, ..., \hat{v}_N$ in such a way that they jointly satisfy the following two conditions:

(1) none of the teams wins more than one slot through tournaments $3, ..., N$;

(2) in the tournament i, each of the teams X, Y, and Z finishes below place b_i, $i = 3, ..., N$.

For round-robin tournaments, we can take the characteristic functions of several transitive tournaments whose sets of winners do not intersect. To ensure compatibility, it is sufficient to arbitrarily replace the teams in the prize zone of tournaments, leaving teams X, Y, and Z with 0, 1, and 2 victories, respectively, in each tournament. From the monotonicity property of ranking methods S_i, $i = 3, ..., r$, it follows that teams X, Y, and Z would be the three lowest placed in domestic tournaments $3, ..., r$. For knock-out tournaments, the existence of characteristic functions with required properties follows from anonymity (i.e., we permute the teams, generating new winners each time and leaving X, Y, and Z in the last three places each time). Neither team X, Y, or Z can win a slot from any of the tournaments $3, ..., N$ without using the allocation rule because there are no more than $K - 3$ slots in the total tournament prize pool of tournaments $3, \ldots, N$:

$$\sum_{i=3}^{N} b_i \leq \sum_{i=3}^{N} b_i + (b_1 - 1) + (b_2 - 1) = \sum_{i=1}^{N} b_i - 2 \leq K - 3,$$

where the first inequality follows from $b_i \geq 1$ for $i = 1, 2$, and the second inequality follows from the assumption that $K > \max(\sum_i b_i, 3)$.

Let v_1 be a characteristic function of the transitive tournament, such that the ranks of the teams Y, X, and Z are b_1, $b_1 + 1$, and K, respectively, and such that each team ranked from first to $(b_1 - 1)$ th place finishes below place b_i in the tournament i, $i = 3, ..., N$.

The next step is to construct characteristic functions for the second tournament, v_2 and w. These functions have the same values except for two entries. First, consider a transitive tournament such that:

(1) the ranks of the teams Y, Z, and X are b_2, $b_2 + 1$, and K, respectively,

(2) each team ranked from first to $(b_2 - 1)$ th place finishes below place b_i in the tournament i, $i=3, \ldots, N$, and

(3) none of the teams ranked from first to $(b_2 - 1)$ th place finished among the top b_1 teams in the first tournament.

Denote the characteristic function of this tournament $\hat{v}_2$.

Now, let us construct a function v_2. Let $v_2(X, Z) = v_2(Y, Z) = 0$ (i.e., Z drew the matches vs. X and Y) and $v_2(\hat{\alpha}, \hat{\beta}) = \hat{v}_2(\hat{\alpha}, \hat{\beta})$ for any pair $(\hat{\alpha}, \hat{\beta}) \in (\mathsf{X} \times \mathsf{X}) \setminus \{(X, Z), (Y, Z), (Z, X), (Z, Y)\}$. Now, consider team Y

and its place $s_Y(v_2)$. Team Y's record is worse than that of $b_2 - 1$ teams, which won at least $K - b_2 + 1$ matches, so $s_Y(v_2) \geq s_Y(\hat{v}_2) = b_2$. At the same time, team Y's record is better than that of all other teams, including teams X and Z (we again use the monotonicity property), so $s_Y(v_2) \leq b_2$. Thus, $s_Y(v_2) = b_2$. From tournament results $S_1(v_1), S_2(v_2), S_3(v_3), ..., S_N(v_N)$, it follows that team Y won a spot in the international tournament as a result of both the first and second domestic tournaments. According to the allocation rule R_n, and our definition of the first tournament, in this case, the team that finished in $(b_1 + 1)$ th place in the first tournament gets the vacant slot. Because $s_X(v_1) = b_1 + 1$, it is team X that qualifies.

Finally, we define the characteristic function w. Let $w(X, Y) = 1$ and $w(\hat{\alpha}, \hat{\beta}) = v_2(\hat{\alpha}, \hat{\beta})$ for any pair $(\hat{\alpha}, \hat{\beta}) \in (\mathsf{X} \times \mathsf{X}) \backslash \{(X, Y), (Y, X)\}$. Because of the monotonicity of the ranking method S_2 and inequality $K \geq 4$, the following relations hold: $s_X(w) > b_2$, $s_Y(w) > b_2$, and $s_Z(w) = b_2$. In this case, team X does not obtain a slot in the international tournament if results $S_1(v_1), S_2(w), S_3(v_3), ..., S_N(v_N)$ are realized. Hence, team X has an incentive to lose to team Y in Tournament 2.

Note that we assumed $R_1 = 1$ in the beginning. Another option is to give the slot to a knock-out tournament (if there are any). Without loss of generality, let $R_1 = r + 1$. All constructions from the previous case may be repeated here with just one correction: Tournament $r + 1$ now takes on the role of Tournament 1. It is possible to rank the teams in a tournament $r + 1$ in any order because ranking methods are anonymous and well-defined. Hence, it is possible to rank the teams X, Y, and Z at $(b_{r+1} + 1)$ th, b_r th, and Kth places, respectively. From this point on, the proof repeats the logic of the previous case. ■

Now suppose that there exists only one round-robin tournament i, such that $b_i \geq 1$, and at least one knock-out tournament l, such that $b_l \geq 1$. We will prove that the qualification system is incentive incompatible if and only if there exists j, $1 \leq j \leq \sum_{t \neq i} b_t$, such that $R_j \neq i$.

Proposition 3: Suppose that $b_1 > 0$, $b_2 = \ldots = b_r = 0$, and there exists at least one knock-out tournament l, such that $b_l \geq 1$, and $K \geq \sum_{i=1}^{N} b_i + 2$. Then, for any well-defined monotonic ranking methods $S_1, ..., S_r$, for any well-defined consistent anonymous ranking methods $\hat{S}_{r+1}, ..., \hat{S}_N$, and for any allocation rule R_n, the qualification system is incentive compatible if and only if $R_i = 1$ for any $i = 1, ..., \sum_{i=2}^{N} b_i$.

Proof: First, suppose that $R_i = 1$, for any $i = 1, ..., \sum_{i=2}^{N} b_i$. After the end of domestic tournaments, some slots in the international tournament may be

Tournament 1				
	Z	Y	Q	X
Z		Loss	Win	Win
Y	Win		Draw	Loss
Q	Loss	Draw		Win
X	Loss	Win	Loss	

Figure 3. Four teams in Tournament 1.

still vacant. Then, following the allocation rule R_n, the teams from the first tournament begin to obtain slots one by one. Note that no more than $\sum_{i=2}^{N} b_i$ attempts are needed because after $\sum_{i=2}^{N} b_i$ attempts, $\sum_{i=1}^{N} b_i$ different teams from Tournament 1 will qualify for the international tournament. In this case, a team has no incentive to lose in a round-robin tournament due to the monotonicity of the ranking method S_1.

Second, suppose that there exists i, $1 \leq i \leq \sum_{i=2}^{N} b_i$, such that $R_i \neq 1$. We will prove that the qualification system is incentive incompatible. Let $n_0 = \min\{i | R_i \neq 1\}$. Let $R_{n_0} = 2$.

We define characteristic function v_1 in several steps. Consider a transitive tournament with K participating teams. Let Z, Y, Q, and X be the teams that finished from $(b_1 + n_0 - 1)$ st to $(b_1 + n_0 + 2)$ nd in descending order (this operation is correctly defined because $b_1 \geq 1$, $n_0 \geq 1$, and $K \geq \sum_{i=1}^{N} b_i + 2$). Redefine the outcomes of the games between these teams as shown in Figure 3, with all other results staying the same. Let v_1 be the corresponding characteristic function.

Because the ranking method S_1 is monotonic, teams Z and X will again be in the $(b_1 + n_0 - 1)$ st and $(b_1 + n_0 + 2)$ nd positions, respectively. Complete definition of v_1 and define characteristic functions $v_2, \ldots, v_r, \hat{v}_{r+1}, \ldots, \hat{v}_N$ arbitrarily in such a way that they jointly satisfy the following conditions:

(1) the pairwise intersections of the sets of winners of tournaments $2, \ldots, N$ are empty (i.e., none of the teams finished in the prize zone of tournaments $2, ..., N$ more than once),
(2) among top $b_1 + n_0 - 1$ teams in Tournament 1, there are exactly $n_0 - 1$ teams that won the slot twice and other b_1 teams that won the slot once,
(3) top $n_0 - 1$ teams of Tournament 1 are the only teams that have won the slot more than once,
(4) the ranks of the teams Y, X, and Z in Tournament 2 are $1, b_2 + 1$, and K, respectively, and
(5) team X did not finish in the prize zone in any of the tournaments.

These conditions can be fulfilled by exploiting transitive round-robin tournaments, due to the anonymity of ranking methods of knock-out tournaments, and because the total number of participants is large enough: $K \geq \sum_{i=1}^{N} b_i + 2$.

By construction, the last team that gets a vacant slot after all reallocations are made is Z. Thus, team X will not receive any slot(s). However, if X lost to Y in Tournament 1 instead of winning the game, X would go to the international tournament as the $(b_2 + 1)$ st-placed team from Tournament 2 (team Y would be pushed into the $(b_1 + n_0 - 1)$ st position in Tournament 1 at the expense of Z, Y would have two slots, and another reallocation would be needed; this reallocation would be in favor of Tournament 2 according to the allocation rule R_n). This proves the incentive incompatibility of the qualification system.

Note that if $R_{n_0} > r$, exactly the same construction works. ■

Now consider the remaining case: Neither of round-robin tournaments provides any prizes. Although such qualification system is rather theoretical, it nicely demonstrates the richness of the set of prizes distribution mechanisms.

Example 4

Consider qualification system consisting of one round-robin tournament (Tournament 1) and two knock-out tournaments (Tournaments 2 and 3). Let $b_1 = 0$, $b_2 = 2$, and $b_3 = 2$. We assume that all ranking methods satisfy the standard set of properties. If $R_1 = R_2 = R_3 = R_4 = 1$, this system is incentive compatible since manipulation is possible in round-robin tournaments only, and there is no sense in worsening the team's ranking in Tournament 1 (all additional allocations are made through Tournament 1). If $R_1 = 2$ and $R_2 = R_3 = R_4 = 1$, the qualification system is again incentive compatible. In contrast, if $R_1 = 1$ and $R_2 = 2$, the system is incentive incompatible. As an example of perverse incentives, suppose that Teams

1 and 2 are first and second in each of the Tournaments 1, 2, and 3. Team 3 is ranked third in Tournaments 2 and 3, and is ranked fourth in Tournament 1, having skipped Team 5 to third position with one extra draw compared to Team 3. Also, suppose that Team 4 is ranked fourth in Tournament 2 and is ranked last in Tournament 1 with only one win over Team 3. Then, Teams 1, 2, 3, and 5 qualify for international tournament. However, Team 4 would prefer to lose to Team 3 in Tournament 1. This loss would push Team 3 to third place, thus giving the fourth prize to Team 4 from Tournament 2. It appears that incentive incompatibility arises if there exists round-robin tournament i, such that allocation rule contains i in some place, and not all other elements of allocation rule equal i as long as there exists a theoretical chance to reach the element of allocation rule. The latter condition is not easy to formalize, though idea of the incentive incompatible example remains the same. We leave this suggestion without formal proof.

A Noncooperative Game

In this section, we propose a simple model for the championship, whereby in each game, the teams choose their effort level. Applying the statement of Proposition 2 to the model, we obtain the manipulability of any reasonable qualification system consisting of several round-robin tournaments. In fact, a noncooperative game theory approach is standard in the literature focusing on incentive compatibility. However, not all of the results discussed above may be restated within a noncooperative game framework. As discussed, the standard approach to study of tournaments does not allow to work with deviation in knock-out tournaments.

Consider N round-robin tournaments with K competing teams in each of them. There are $\frac{K(K-1)}{2}$ games played in a single tournament. Thus, there are $N\frac{K(K-1)}{2}$ games in total. In the real world, these games are occasionally ordered by rounds (as in national football championships in Europe). Sometimes, the schedule is more flexible, and one game starts after another finishes (as in the American Major League Soccer (MLS)). For the sake of simplicity, we will consider the latter case.

Definition 11: *A schedule of the round-robin tournament* involving K teams is an ordered set of all $\frac{K(K-1)}{2}$ tournament games. *A schedule of N round-robin tournaments* involving K competing teams is an ordered set of all $N\frac{K(K-1)}{2}$ games; that is, a schedule is a permutation π of numbers $1, \ldots, N\frac{K(K-1)}{2}$.

Denote by G_n, the schedule generated by permutation π. The schedule is an additional parameter for qualification systems compared with the model in the previous section. Denote by $(S_1, \ldots, S_N, R_n, G_\pi)$, the qualification system consisting

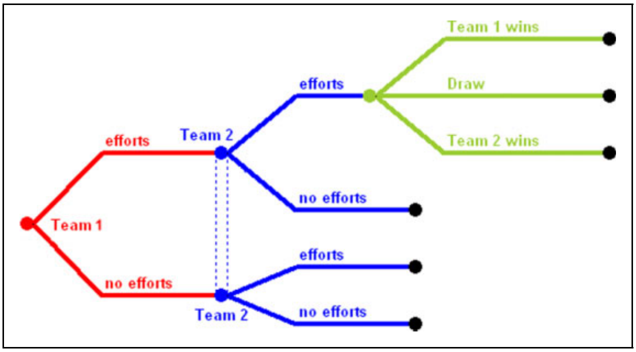

Figure 4. Block B_i represents a single match *i*.

of N round-robin tournaments with ranking methods $S_1, ..., S_N$, allocation rule R_n, and a schedule G_π.

Suppose that all assumptions of Proposition 2 hold. Consider an extensive form game consisting of $N\frac{K(K-1)}{2}$ consecutive blocks B_i, $i = 1, ..., N\frac{K(K-1)}{2}$. For each i block B_i describes the strategic interaction between two teams that compete within this particular game number i. There are two teams playing in game i, and both teams decide simultaneously whether to make the effort to win a game (see Figure 4); the efforts are costless. If both teams decide to make the effort, then nature decides the winner: Each of the two teams has a positive probability of winning, and there also exists a positive probability of a draw. If only one team decides to make an effort, this team wins with probability 1. Finally, if both teams decide to avoid playing for victory, both teams are disqualified and obtain a negative payoff $-c$, where $c > 0$. The idea behind the latter assumption is that no effort on the part of both competitors is observable by everyone including officials. It is extremely hard to lose to an opponent who is also trying to lose, and both sides engage in obvious irrational actions in their willingness to lose. In this case, the officials disqualify both opponents, as happened during the 2012 London Olympic Games with four badminton pairs (see footnote 3). In comparison, no effort on the part of only one opponent is not observable because it is impossible to distinguish an ordinary loss from a deliberate loss.

Independently of the result of game i, game $i+1$ starts, $i = 1, ..., N\frac{K(K-1)}{2} - 1$ (see Figure 5), and so on. After all the games have been played, the payoffs are realized. If a team was disqualified at least once, its payoff is $-c < 0$. If a team was not disqualified after the end of the tournament, its payoff is 1 in the case of qualification for the international tournament and 0 otherwise.

Note that there are six terminal vertices in each block. First game is described by a single block B_1 with six terminal vertices. Second game is added to the tree by adding block B_2 to each of six terminal vertices of block B_1. Adding another game number l requires that a block B_l is conjuncted to each of the 6^{l-1} existing terminal vertices.

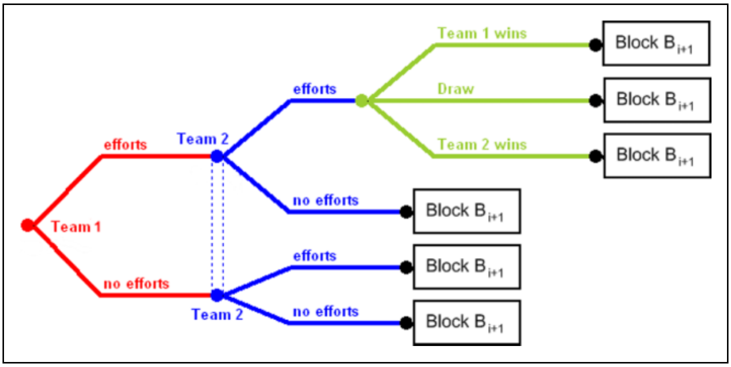

Figure 5. Induction step in tree construction.

Definition 12: A qualification system $(S_1, ..., S_N, R_n, G_\pi)$ is *incentive incompatible* (or *manipulable*), if the strategy profile whereby all teams always make an effort is not a subgame perfect Nash equilibrium.

Proposition 4: Suppose that $N \geq 2$, $b_i \geq 1$ for each $i = 1, ..., N$, and the number of participating teams satisfies the inequality $K > \max(\sum_i b_i, 3)$. Then, for any well-defined monotonic ranking methods $S_1, ..., S_N$, for any allocation rule R_n, and for any schedule π, the qualification system $(S_1, ..., S_N, R_n, G_\pi)$ is incentive incompatible.

Proof: Observe that qualification system $(S_1, ..., S_N, R_n, G_\pi)$ satisfies the assumptions of Proposition 2. It follows from Proposition 2 that there exists such characteristic function v and teams x and y that $v(x,y) = 1$, but it would be profitable to team x to lose the game against y instead of winning it (given all other results are the same as in v). Now, look at the tree of the game generated by the schedule G_π and find index i and all block B_i corresponding to the game between teams x and y. There will be 6^{i-1} blocks B_i in the tree. Choose block B_i, such that the subgame that starts at this block B_i lies on the path from the root to the leaf of the tree, which corresponds to the match outcomes of the characteristic function v, where all teams make an effort. There is a positive probability that team x wins over y and therefore does not qualify for the international tournament. However, if team x deviates and loses deliberately, then team x qualifies for the international tournament with probability 1 by construction. Hence, always making an effort is not subgame perfect. ■

Thus, one can say that under the assumptions of Proposition 2, it is impossible to design an allocation rule that will force all players to exert an effort in a subgame perfect Nash equilibrium.

For the fixed ranking methods $S_1, ..., S_N$ and an allocation rule R_n that satisfy the assumptions of Proposition 2, one may choose such a schedule that deviation will be profitable in the last game. Then, it may occur that some team has an ex ante incentive to deviate in the last game contingent on information available before the start of a game. The presence of ex ante perverse incentives is, in some sense, a stronger and more realistic result compared with *ex post* regret effect illustrated in the second section.

Extensions and Discussion

Most European football national championships are played in two rounds on a home-away basis; that is, each participating team plays each other twice. To formally describe this type of competition, the notion of a generalized round-robin tournament has been introduced (see, e.g., Slutzki & Volij, 2005). Namely, an l-rounds round-robin tournament is a tuple $(\mathsf{X}, v^1(x,y), ..., v^l(x,y))$, where X is the set of all participating teams, and v^i is a characteristic function of the round i, satisfying the same conditions as in the definition of a round-robin tournament, $i = 1, ..., l$, $l \geq 1$. It is easy to replicate all incentive incompatibility results for l-rounds tournaments. The following result is an analogue to Proposition 2.

Proposition 5: Consider the qualification system consisting of r l-rounds round-robin tournaments (tournaments $1, \ldots r$), and k knock-out tournaments (tournaments $r+1, \ldots, N$), $N = r + k$, $l \geq 1$. Suppose that $r \geq 2$, $b_i \geq 1$ for $i = 1, 2$, and the number K of participating teams satisfy $K > \max(\sum_i b_i, 3)$. Then, for any well-defined monotonic ranking methods $S_1, ..., S_r$, for any well-defined consistent anonymous ranking methods $\hat{S}_{r+1}, ..., \hat{S}_N$, and for any allocation rule R_n, the qualification system is incentive incompatible.

Proof: Let v_j^i denote a characteristic function of the i th round of the j th round-robin tournament, $i = 1, ..., l, j = 1, ..., r$, and $\hat{v}_k$ denote a characteristic function of k th knock-out tournament. Let v_j^1 and $\hat{v}_k$ be the characteristic function of the j th round-robin tournament and k th knock-out tournament, respectively, from the proof of Proposition 2. For any $i > 1$, any $j = 1, ..., r$, and any teams x and y, put $v_j^i(x,y) = 0$. In other words, we take the example from Proposition 2 and suppose that all other games are drawn. It is easy to check that team X has incentives to lose to team Y in Tournament 2. Therefore, this qualification system is incentive incompatible. ■

Proposition 3 may be replicated in the same way.

In most UEFA countries, qualification for the Champions League and the Europa League is determined via two tournaments: the national round-robin championship and the national cup, a knock-out competition. There are several exceptions: For example, Liechtenstein has a national cup only, whereas in England, it is possible to obtain a slot in an international tournament from three competitions: the Premier

Table 3. All Allocation Rules.

Intersection type	R_1	R_2	R_3	R_4
Type 1	Championship	Championship	Cup	Cup
Type 2	Championship	Cup	Championship	Cup

League, the Football Association Cup, and the League Cup. Proposition 3 leads to an important practical implication: *If one wants to make the ranking method incentive compatible, the allocation rule needs to be defined in such a way that all vacant* slot*s are awarded to the teams based on the results from the round-robin tournament.* Until the 2015/2016 season, in many European countries, if the cup winner qualified in the Champions League, the vacant Europa League slot would go to the cup runner-up. These rules left the door open for possible incentive misalignment. UEFA decided to abandon this rule starting from the 2015/2016 season (see http://www.uefa.com/uefaeuropaleague/news/newsid=2137611.html, retrieved July 28, 2016).

Sometimes, teams compete for slots in several international tournaments. For example, national football federations from the UEFA zone send their teams to two international tournaments—the Champions League and the Europa League. A general formal analysis is cumbersome as the number of types of "joint wins" for domestic tournaments increases dramatically. Thus, it is harder to define general allocation rules. Below, we consider in detail one important special case, which is particularly relevant in the real world.

Consider two domestic tournaments, a round-robin championship and a cup, as well as two international tournaments, the Champions League and the Europa League. Let the best a of the championship teams obtain slots in the Champions League and the next b best teams will play in the Europa League along with the cup winner, $a, b \geq 1$. There are two possible types of intersections of winners' sets: First, the cup winner may finish among the top a teams in the championship; second, the cup winner may finish among the top $a + b$ teams in the championship but not among the top a teams. Both situations result in vacant slots. The allocation rule must describe what should happen in both cases. There are two options (allocate an additional slot to the championship or to the cup) for each of the two types of intersections. Thus, there are four possible allocation rules. Denote them $R_1, R_2, R_3, \text{and} R_4$ and define how they allocate the vacant slot in Table 3.

The following formal result holds.

Proposition 6: Suppose that $a, b \geq 1$ and $K \geq a + b + 3$. Then, for any well-defined monotonic ranking method of the round-robin tournament S, for any well-defined consistent anonymous ranking method of the knock-out tournament $\hat{S}$ and for allocation rule $R \in \{R_1, R_2, R_3, R_4\}$, the qualification system $(S, \hat{S}, R)$ is incentive compatible if and only if $R = R_1$.

Proof: Consider allocation rule R_1. All allocations favor the championship. Thus, if a team does not win the cup, it has no further chances to qualify for European cups from the national cup. Hence, the only objective function of a tournament participant is to maximize its achievements in both tournaments independently. Due to the monotonicity of ranking method S, a team cannot climb in the standings of the championship by worsening its results in any single game. There is no reason to lose in the championship, and the qualification system $(S, \hat{S}, R)$ is incentive compatible.

Now, consider allocation rule R_2. We introduce an auxiliary qualification system that satisfies the conditions of Proposition 3. Namely, let the ranking methods and the allocation rule be the same, but the difference between the Champions League and the Europa League disappears; that is, the only type of prize is a slot in a European cup, regardless of which one it is. In the earlier setup, this corresponds to the case $b_1 = a + b$, $b_2 = 1$. By Proposition 3, the latter qualification system is incentive incompatible.

Finally, to prove the statement of the proposition for allocation rules R_3 and R_4, we exploit the auxiliary qualification system with $b_1 = a$, $b_2 = 1$. ■

As mentioned above, until the 2015/2016 season, most UEFA national federations exploited allocation rule R_3. In 2013, UEFA advised national federations to switch to rule R_1 and eliminate the incentive incompatibility. Still, the problem of misaligned incentives is not restricted to national tournaments. For example, competition rules of the European qualification tournament for the 2014 FIFA World Cup in Brazil suffered from the same problem. And the "perverse incentives" situation was not merely a theoretical possibility. Two months before the end of the tournament, with 80% of games completed, there still was a scenario under which a team might need to achieve a draw instead of winning to go to Brazil.

Finally, we note that the assumption that number of teams in knock-out tournaments and, consequently, also in round-robin tournaments, must be equal to power of 2, is without loss of generality. Indeed, it is always possible to introduce dummy teams which by definition lose any game to any nondummy team. Then, if there are K teams in the country, with $2^n < K < 2^{n+1}$, we add $2^{n+1} - K$ dummy teams and reduce the model to the original one. Since we consider monotonic ranking methods, all dummy teams will be placed at the bottom of the tournaments and will not influence the standings of "real" teams.

Conclusions

Optimal design of aggregation rules for tournaments is an important theoretical problem. Neglecting the analysis of incentive compatibility, tournament organizers may suddenly face a situation in which one team (or even several) would prefer to lose a game. While this is a low-probability event, the potential costs of the rational misbehavior of the teams are too high. In this article, we demonstrated that recent regulations that determine who qualifies for major football tournaments allow for a situation in which a team would need to lose to qualify. We showed that the existence of incentive compatible ranking methods and allocation rules depends on the structure of qualifiers. In a single round-robin tournament, any monotonic ranking method prevents deliberate losses. If there are at least two round-robin qualifiers, it is impossible to implement an appropriate ranking method. In qualification systems with one round-robin and several knock-out tournaments, incentive compatibility may be achieved by allocating the vacant slots according to team performance in the round-robin tournament. Estimating the frequency of tournament situations which allow for perverse deviations could be an interesting direction for future research.

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Notes

1. An example of misaligned incentives provided by tournament rules is the 1994 Shell Caribbean Cup (see Gardiner, 2005). In the last game of the preliminary stage, Barbados had to win with a goal difference of 2 or more, while for its competitor, Grenada, a loss with a goal difference of -1 was enough to advance. In the case of a draw, the teams had to play an extra 30 min. If a goal was scored in extra time, the game ended, and most unusually, the goal added two points to the final score. Barbados was leading 2-0 when Grenada scored with 7 min left. Now, Barbados had the option to win by, first, scoring an own goal (making it 2-2) and then scoring in extra time to win 4-2. After Barbados scored an own goal, Grenada had to score one goal in either net! Barbados divided its players to successfully defend both goals, scored in extra time, and advanced to the next stage.

2. For example, in the National Basketball Association, the draft lottery favors less successful teams to level off teams' chances in the next season.
3. In the London Summer Olympics 2012, four badminton pairs were disqualified for doing this. The Badminton World Federation (BWF) charged them with "not using one's best efforts to win a match" and "conducting oneself in a manner that is clearly abusive or detrimental to the sport" (see the BWF website, checked April 29, 2017 http://bwfbadminton.com/2012/08/01/london-2012-koreans-appeal-rejected-indonesias-withdrawn/).
4. Noncumulative prizes are ubiquitous. For example, the distribution of entries to major European tournaments, the UEFA Champions League and the UEFA Europa League, based on the results of the domestic championship and domestic cup(s) is noncumulative: There is no possibility to win more than one slot in UEFA competition.
5. This case was initially described in a comment posted by Dr. Andrei Brichkin (nickname Quant) at http://www.eurocups.ru/guestbook (see message 170910).
6. The antisymmetry property requires that $v(x_0, y_0) = -v(y_0, x_0)$ for each $x_0, y_0 \in X$, $x_0 \neq y_0$.
7. Using another terminology, a vertex is a node, and a leaf is a terminal node.
8. In Pauly (2014), there is an attempt to solve this problem by defining hypothetical outcomes of unplayed matches. However, we think that this approach is unsatisfactory as it gives the researcher too much freedom: Basically, it allows the strategic incentives of one team to be affected by the outcome of a future game involving the same team.

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