Costly Information Acquisition, Stock Prices and Investment in a Neoclassical Growth Model*

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Abstract

Is the stock market another Las Vegas, where a lot of people just gamble? Or do the activities of informed stock traders contribute in a useful way to the optimal allocation of resources by providing guidance for physical investment?

This paper adresses this old question by incorporating the endogeneously costly acquisition of information about future returns and the partial transmission of that information to the stock market via prices into an infinite horizon rational expectations neoclassical growth model. The tradeoff between the social costs and benefits of information acquistion on the stock market are analyzed in numerical simulations.

We find that output may increase, but average welfare decrease due to informed speculation. If so, agents of medium wealth loose relatively more than poor agents. We find that a market portfolio mutual fund rules out costly information acquisition.

1 Introduction

Is the stock market another Las Vegas, where a lot of people just gamble? Or do the activities of informed stock traders contribute in a useful way to the optimal allocation of resources by providing guidance for physical investment? Asked differently: given the arrangement of a stock market[1], is costly information acquistion welfare improving? It is impossible to think about and judge policy regarding the stock market without taking a stand on this key question.

This paper provides a new model to adress this old question[2]. The aim is to tell the following simple story within a general equilibrium model: there are people trading on the stock market, who, through their efforts, are better in predicting future successes and failures of certain industries (firms, technologies, innovations, ...) and therefore earn a handsome profit from speculation. The demand of these informed traders has some influence on stock prices. Even though not all of their information will be revealed, stock prices tend to be the higher, the more successful some industry will be in the future. Thus, noninformed investors and society as a whole can use that information in stock prices to funnel investment resources into plant and equipment of that industry. Output will be higher than it would have been without the help of that information. The aim here is to analyze the trade-off between the resources spent by agents on information acquisiton and the output and welfare gained via the improved allocation of investment.

As it quickly turns out, implementing the story told above is not a simple

[1]We do not discuss in this paper, whether or how a stock market may emerge as an optimal arrangement, given asymmetric information. While this is an interesting problem and progress has been made in endogenizing the financial structure in macroeconomic models, see e.g. Farmer [11], [12], Bernanke and Gertler [2] and the survey in Gertler [14], this problem is beyond the scope of this paper. For the same reason, we do not analyze possible arrangements to share information between agents or intermediary arrangements to endogenize the externalities of information acquisition.

[2]Compare also Hayek [19], Hirshleifer [21], Grossman [15] and Leland [23].

matter. The key difficulty lies in keeping a signal extraction problem alive in modelling the stock market, i.e. keeping prices from fully revealing all information and keeping trade alive at the same time (see e.g. Radner [28], Grossman and Stiglitz [16], Milgrom and Stokey [27] and Tirole [34]). A "solution" often used in the literature is to introduce noise traders (see e.g. Grossman and Stiglitz [16], Hellwig [20], Verrecchia [37], Admati and Pfleiderer [1] or Wang [38]). We follow the idea of that literature in allowing agents only to either buy a particular stock or a mutual fund which holds a noisy version of the market portfolio. Since other parts of the model prevent us from using the popular normal-distribution setup, we develop a new way of creating noise which just takes finitely many values. The noise distribution has to be chosen "just right" for everything to work (see the "consistency condition", Proposition 4).

It is interesting to note that "noise" must be biased for this model (or any of the models listed above) to work: to achieve market clearing, noise traders must be more likely to sell, if the stock is about to outperform the market. The usual argument, that noise traders are willing to bear these costs because they trade for some pressing liquidity reason raises hard questions: how can these needs affect individual stocks rather than mainly the market portfolio? Why do noise traders end up on the wrong side of the market systematically? What choice do they have in submitting or withdrawing market orders, given the price of a particular stock? Indeed, if agents have access to a market portfolio mutual fund in the model, costly information acquisition is impossible (see proposition 5). The result follows in part, because there is no aggregate uncertainty, implying portfolio separation. Even though portfolio separation is in general hard to come by in models with aggregate uncertainty (see Cass and Stiglitz [4], Brennan and Kraus [3], Ross [29] and Dybvig and Ross [10]) it is hard to believe, that the day-to-day noisy demand for IBM stocks, say, comes about because uninformed traders need the special payoff structure of IBM stocks rather than that of the market portfolio to solve their individual portfolio problem. Why don't uninformed agents trade in a small set of portfolios, minimizing their exposure to traders with superior information? Why should we think of informed agents as aquiring information about individual stock returns rather than information about aggregate liquidity needs? Along with the literature, we sidestep these issues and assume that the noise is simply there without deeply explaining, why.

The main accomplishment of this paper is to actually incorporate the endogeneously costly acquisition of information about future returns and the partial transmission of that information to the market via prices into an infinite horizon rational expectations neoclassical growth model and to develop a computationally tractable model, in which the tradeoff between the social costs and benefits of information acquistion on the stock market can be analyzed. Aside from the theoretical insights mentioned above, results from numerical solutions are obtained. They show the intuitively appealing result, that the return to the noisy mutual fund is the lower and the expected return to the portfolio of the informed traders the higher, the better the quality of the signals privately received by the informed traders and the less information is revealed by prices, see figures 2.2 and 2.3. The calculations also suggest, that the presence of informed traders can decrease the welfare of agents with medium wealth, but at the same time increase the welfare of the very rich as well as the very poor agents in the economy.

The structure of this paper is as follows. In section 2 we describe the environment. In section 3 we describe the market setup and the equilibrium. Since the model turns out to be quite complex, section 4 motivates some of the details of the model. Section 5 shows the consistency condition and the result that a market mutual fund rules out costly information acquisition. It further describes the equilibrium and some additional theoretical insights. Section 6 describes the algorithm for computing equilibria numerically and describes the numerical results. Section 7 concludes.

2 The Environment

The model is an extension of the neoclassical growth model without aggregate uncertainty. Features are added to achieve endogeneously costly acquisition of information and partial revelation of that information on a stock market.

2.1 Technologies

There are infinitely many periods $t = 0, 1, 2, \ldots$. There is a continuum of output producing technologies $\tau \in [0; 1]$ to produce output according to

$$y_{t\tau} = \gamma_{t\tau} k_{t-1,\tau}^{\rho} n_{t\tau}^{1-\rho}, \tag{1}$$

where for period t and technology τ, $\gamma_{t\tau}$ is the productivity, $n_{t\tau}$ is labor and $k_{t-1,\tau}$ is the capital stock available for production in period t but "planted" in period $t-1$. Initial capital $k_{-1,\tau}$ is assumed to equal one in each technology. New capital is provided with the technology

$$k_{t\tau} = f(k_{t-1,\tau}, x_{t\tau}) \;\; x_{t\tau} \geq 0, \tag{2}$$

where f is a linear homogeneous function which is increasing in its second argument and where $x_{t\tau} \geq 0$ is irreversible investment. For the numerical calculations in section 6, we use

$$f(k, x) = (\kappa_1 k^{\alpha} + \kappa_2 x^{\alpha})^{1/\alpha} \tag{3}$$

with $\alpha = .5$, $\kappa_1 = .94$, $\kappa_2 = 1$. A production technology of this form for new capital has been used before in Hayashi [18]. Note, that $\kappa_2 = 0$ corresponds to "Lucas trees" as in Lucas [24] and that $\alpha = 1$, $\kappa_1 < 1$, $\kappa_2 = 1$ corresponds to the usual linear investment technology.

Each period, productivity grows randomly according to one of two factors, Γ_0 or Γ_1, where $\Gamma_0 > \Gamma_1$. In other words, the productivity $\gamma_{t\tau}$ evolves according to

$$\gamma_{t+1,\tau} = \Gamma_{g_{t,\tau}} \gamma_{t\tau}, \tag{4}$$

where $g_{t,\tau}$ is randomly drawn from the set $\{0;1\}$. Define

$$\bar{\gamma}_t \equiv \sup\{\gamma_{t\tau}\}.$$

We assume that $\bar{\gamma}_t < \infty$ and that any $\gamma_{0\tau}$ and thus any $\gamma_{t\tau}$ can be written as

$$\gamma_{t\tau} = \left(\frac{\Gamma_1}{\Gamma_0}\right)^l \bar{\gamma}_t$$

for some $l \in \{0, 1, 2, \ldots\}$. We call $l_{t\tau} = l$ the *level* of technology τ in period t. Since we will assume that all $g_{t\tau}$ are independent, it follows that

$$\bar{\gamma}_{t+1} = \Gamma_0 \bar{\gamma}_t. \tag{5}$$

Note that

$$l_{t+1,\tau} = l_{t\tau} + g_{t\tau}.$$

Since the $g_{t\tau}$ will be assumed to be independent, this means that the fraction of technologies which are on the top level $l = 0$ becomes ever smaller as time t passes. However, in equilibrium, these technologies will receive the majority of the investment into their capital stock so that as a fraction of total capital each period, their share can be constant: this is the case in steady state defined in section 3, see also proposition 3.

The growth index $g_{t\tau}$ is part of a large random vector $Z_{t\tau}$ which is drawn at the beginning of period t for technology τ. This random vector

$$Z_{t\tau} \equiv \left(g_{t\tau}, i_{t\tau}, (m_{t\tau j})_{j\in[0;1]^2}\right)$$

consist out of

- $g = g_{t\tau} \in \{0;1\}$, the *growth index* for the productivity growth factor Γ_g,

- $i = i_{t\tau} \in \{0, \ldots, I\}$, the *information index.* Below we will use i together with g to describe the noisy demand for a stock as well as the information revealed on the stock market,

- $m = m_{t,\tau j} \in \{0, \ldots, M\}$, the *message* drawn for agent j.

So far, $Z_{t\tau}$ is just a vector of random integers with names such as "information index" or "message". The role of these integers will become apparent further below. The probability law for drawing $Z_{t\tau}$ will be described in subsection 2.3.

2.2 Preferences and Endowment

Each agent $j \in [0;1]^2$ cares about lifetime utility

$$U \equiv E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{tj}) \right], \text{ with } u(c) \equiv \frac{c^{1-\eta} - 1}{1 - \eta},$$

where $\eta > 0$, $\eta \neq 1$, $0 < \beta < 1$. Agents initially hold a market portfolio of capital with the size of that portfolio $\tilde{a}_{0j}$ drawn randomly across agents according to some distribution on $\mathbf{R}_{++}$. At each date t and for each agent j, a *stock pick* $\tau_{tj} \in [0;1]$, an *efficiency factor* $N_{tj} \in \mathbf{R}_{++}$ and a *random index* $r_{tj} \in [0;1]$ are drawn. The role of these ran***
τ_{tj} for the stock pick $\tau = \tau_{tj}$ only. Messages drawn for him for other technologies τ are discarded. If the agent decides to work, he learns N_{tj} and the unit of time is translated into N_{tj} units of labor.

2.3 Information and Probabilities

The complete probability structure is the list of all random variables

$$((\tilde{a}_{0j})_j, (\tau_{tj}, N_{tj}, r_{tj})_{tj}, (Z_{t\tau})_{t\tau})$$

on some underlying probability space $(\Omega, \mathcal{S}, P)$. Some assumptions regarding the distribution of these random variables and the measurability of endogeneous variables as well as some additional notation are now introduced.

It is assumed that all random variables except τ_{tj} are independent. We assume that $k_{t\tau}$ and $x_{t\tau}$ are functions only of the histories $(i_{s\tau}, g_{s\tau})_{s\leq t}$. Define

$b_{tj} = m_{t,\tau_{tj},j}$, if $d_{tj} = 0$ and $b_{tj} = N_{tj}$, if $d_{tj} = 1$. We assume that d_{tj} is a function only of $\tilde{a}_{0j}$ and $(r_{sj}, b_{sj}, i_{s,\tau_{sj}}, g_{s,\tau_{sj}})_{s<t}$ and that c_{tj} is a function only of the *history*

$$h_{tj} \equiv (\tilde{a}_{0j}, (r_{sj}, b_{sj}, i_{s,\tau_{sj}})_{s\leq t}, (g_{s,\tau_{sj}})_{s<t}).$$

The idea here is to capture formally what the agent knows and is able to use when making his decisions, see also section 3.

For a technology τ in period t define the *category* $\delta_{t\tau} = (l, i, g)$ to be the list of the three integers $l = l_{t\tau}$, $i = i_{t\tau}$ and $g = g_{t\tau}$. Let

$$\Delta_t(l, i, g) = \{\tau \mid \delta_{t\tau} = (l, i, g)\}$$

be the set of all technologies τ of category (l, i, g) at date t.

Conditional on the category (l, i, g), it is assumed that the probability for picking a particular technology $\tau_{tj} = \tau \in \Delta_t(l, i, g)$ for agent j at date t is proportional to the capital stock $k_{t-1,\tau}$. Conditional on $(i_{t,\tau}, g_{t,\tau}, l_{t,\tau}, k_{t-1,\tau})$, τ_{tj} is independent of all random variables in all periods.

Denote with $\pi(l, i, g)$ the probability that τ_{tj} is of category (l, i, g),

$$\pi(l, i, g) = P(\delta_{t,\tau_{tj}} = (l, i, g)).$$

We do not assume that $\pi(l, i, g) = P(\delta_{t,\tau} = (l, i, g))$ for any given $\tau \in [0; 1]$. Thus, τ_{tj} can depend on the random categories $(\delta_{t,\tau})_{\tau\in[0;1]}$. We assume that π does not depend on t and that it can be further decomposed into three factors,

$$\pi(l, i, g) = \pi(l)\pi(i)\pi(g \mid i), \tag{6}$$

where $\pi(i)$ and $\pi(g \mid i)$ are probability distributions on $\{0; \ldots; I\}$ and on $\{0; 1\}$, do not depend on l and are nonzero for all i, g.

Given a category (l, i, g) at date t and a capital stock size k, messages $m = m_{t\tau j}$ are drawn independently across j according to the conditional probabilities

$$\pi(m \mid g) \equiv P(m_{t\tau j} = m \mid g_{t\tau} = g, i_{t\tau} = i, l_{t\tau} = l, k_{t-1\tau} = k), \tag{7}$$

independent of i, l and k. It follows that

$$\pi(m \mid g) = P(m_{t\tau j} = m \mid g_{t\tau} = g, i_{t\tau} = i, l_{t\tau} = l, k_{t-1\tau} = k, \tau = \tau_{tj}). \quad (8)$$

We assume that $\pi(m \mid g = 0) \neq \pi(m \mid g = 1)$ for all m, i.e. messages are informative about g. We assume that $E[N_{tj}] = 1$ and that r_{tj} is uniformly distributed on $[0; 1]$.

The following proposition is helpful for the decision problem facing an agent who decided to become informed. The proposition gives a formula for combining two "priors" $\pi(g \mid i)$ and $\pi(g \mid m)$ into the "posterior" $\pi(g \mid i, m)$.

Proposition 1
Assume that $\pi(g) \neq 0$ for all $g \in \{0; 1\}$, where

$$\pi(g) = \sum_{i=0}^{I} \pi(g \mid i)\pi(i).$$

Assume that $\pi(m \mid g) \neq 0$ for at least one $g \in \{0; 1\}$. Let $g \in \{0; 1\}$. The probability $\pi(g \mid m)$ for $g_{t\tau_{tj}} = g$, given $m = m_{t\tau_{tj}}$ is given by

$$\pi(g \mid m) = \frac{\pi(m \mid g)\pi(g)}{\sum_{\tilde{g}=0}^{1} \pi(m \mid \tilde{g})\pi(\tilde{g})}. \quad (9)$$

The probability $\pi(g \mid i, m)$ for $g_{t\tau_{tj}} = g$, given $m = m_{t\tau_{tj}}$ and $i = i_{t\tau_{tj}}$ is

$$\pi(g \mid i, m) = \frac{\dfrac{\pi(g \mid i)\pi(g \mid m)}{\pi(g)}}{\displaystyle\sum_{\breve{g}=0}^{1} \frac{\pi(\breve{g} \mid i)\pi(\breve{g} \mid m)}{\pi(\breve{g})}} \quad (10)$$

and $\pi(g \mid i, m) \neq \pi(g \mid i)$ for all i, g and m.

The proof is in appendix A. To introduce some further useful notation, let

$$\bar{k}_t(l, i, g) \equiv \int 1_{\Delta_t(l,i,g)}(\tau)\, k_{t,\tau}\, d\lambda(\tau)$$

be the aggregate capital stock available for production in period t+1 of all technologies in category (l, i, g) at date t. Here and everywhere below, λ is the Lebesgue measure and "$\int$" is the Pettis integral[3], see Uhlig [35]. Furthermore, let

$$
\begin{aligned}
\bar{k}_t(l) &\equiv \int 1_l(l_{t+1,\tau})\, k_{t,\tau}\, d\lambda(\tau) \\
&= \begin{cases} \sum_i \bar{k}_t(0,i,0) & \text{for } l = 0 \\ \sum_i \bar{k}_t(l,i,0) + \sum_i \bar{k}_t(l-1,i,1) & \text{for } l > 0 \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\bar{k}_t &\equiv \int k_{t,\tau}\, d\lambda(\tau), \\
&= \sum_{l=0}^{\infty} \bar{k}_t(l) \\
\bar{y}_t &\equiv \int y_{t\tau}\, d\lambda(\tau), \\
\bar{x}_t &\equiv \int x_{t\tau}\, d\lambda(\tau),
\end{aligned}
$$

[3]Technically, this achieves the solution of Uhlig [36] to the "law of large numbers" problem, raised by Judd [22]. The issue is to compute the integral of some function f from the unit interval into a space of random variables, where two different values of that function are independently distributed. Uhlig [36] has shown that the Pettis integral (see Diestel and Uhl [8] for a definition) delivers the law of large numbers $\int_{[0,1]} f(x)\,\lambda(dx) = \int_{[0,1]} E[f(x)]\,\lambda(dx)$ *a.e.* without additional assumptions on the underlying measure space, avoiding the measurability problems raised in Judd [22]. For simplicity, consider defining the random variable $I = \int_{[0,1]} f(x)\,\lambda(dx)$ via limits of Riemann sums. The problem is to choose a suitable concept for convergence. The Pettis integral corresponds to using the metric of mean squared difference and it is easy to see that the law of large numbers comes about as long as the variances $\mathrm{Var}[f(x)]$ are bounded. This idea is similar to the idea underlying the Ito integral which has found wide applications in finance (see Duffie [9]). The Pettis integral is one of the two standard generalizations of the Lebesgue integral to vector-valued functions. Judd's [22] approach corresponds to taking the limit pointwise almost everywhere, resulting in well-known measurability problems, and does not correspond to some standard approach for vector-valued integration. We will therefore use the law of large numbers below without further comment.

$$
\begin{aligned}
\bar{n}_t &\equiv \int n_{t\tau}\, d\lambda(\tau), \\
\bar{c}_t &\equiv \int c_{tj}\, d\lambda^2(j), \\
\bar{N}_t &\equiv \int d_{tj} N_{tj}\, d\lambda^2(j).
\end{aligned}
$$

Define

$$F_{k,t}(l,i,g) \equiv \bar{k}_t(l,i,g)/\bar{k}_t$$

and

$$
\begin{aligned}
F_{k,t}(l) &\equiv \bar{k}_t(l)/\bar{k}_t \qquad (11)\\
&= \begin{cases} \sum_i F_{k,t}(0,i,0) & \text{for } l = 0 \\ \sum_i F_{k,t}(l,i,0) + \sum_i F_{k,t}(l-1,i,1) & \text{for } l > 0 \end{cases}
\end{aligned}
$$

and call it the aggregate capital distribution. Aggregate feasibility requires that

$$\bar{x}_t + \bar{c}_t = \bar{y}_t$$

and

$$\bar{n}_t = \bar{N}_t.$$

Because of the law of large numbers, $\bar{y}_t$, $\bar{c}_t$ etc. are nonrandom, i.e. there is no aggregate uncertainty.

Let (l,i,g) be some category in period t and τ a technology of that category. We assume that there is a continuum of agents for which $\tau_{tj} = \tau$ whose distribution over histories and population mass per unit of capital equals the corresponding distribution and relative mass over all agents with τ_{tj} of category (l,i,g). While this assumption is consistent with the assumptions above, it may be hard to make it precise mathematically and requires tools beyond, say, Uhlig [35]. This assumption is convenient however since it guarantees for section 5 that the market for each technology τ of category (l,i,g) individually "looks like" the aggregate market for all technologies of category (l,i,g) together.

In the next section, the random variables τ_{tj} will be interpreted as *noise* or as the *stock picking technology* available to agents and to a mutual fund. The probabilities $\pi(l, i, g)$ will be the device to introduce a bias into this "noise" and l and i will be publicly available information, revealed by prices. To have τ_{tj} pick technologies τ according to their weight in the market portfolio of capital, given public information. the conditional probability for drawing good stocks $\pi(g = 0 \mid i)$ needs to equal the corresponding relative market weight

$$F_{k,t}(g = 0 \mid l, i) = \frac{F_{k,t}(l, i, g = 0)}{F_{k,t}(l, i, g = 0) + F_{k,t}(l, i, g = 1)}.$$

In that case, the stock picking technology τ_{tj} uses the correct market portfolio as its "dartboard", conditional on l and i. However, if

$$\pi(g = 0 \mid i) < F_{k,t}(g = 0 \mid l, i)$$

and thus

$$\pi(g = 1 \mid i) > F_{k,t}(g = 1 \mid i, l),$$

the stock picking technology τ_{tj} picks too many bad stocks and too few good stocks on average, given public information: the "dartboard" is biased and costly information acquisition can be profitable.

3 Equilibrium

In subsection 3.1, we describe the economic institutions assumed in this model, i.e. financial arrangements, trading constraints, firms and markets. The sequence of events within a period is explained in subsection 3.2. Equilibrium and steady state are defined in subsection 3.3.

3.1 Economic Institutions

To avoid timing issues with respect to payments and receipts within a period, it is assumed that all purchases and sales on all markets by all participants within a period are undertaken via *double-entry bookkeeping* in terms of the period-t consumption good. All accounts need to be settled in the last part of a period.

A (t, τ)-stock or (t, τ)-share is a claim to a unit of capital $k_{t\tau}$ for technology τ. Each period is divided into six parts, see figure 1 for an overview. There are two stock markets, i.e. markets for capital, within a period: *stock market 1* in part I of the period, where $(t-1, \tau)$- shares are traded, and *stock market 2* in part V, where $(t-1, \tau)$-shares as well as (t, τ)-shares are traded. Let the period-t consumption good be the numeraire for all prices in period t. Denote the price for a $(t-1, \tau)$-share on stock market 1 by $q_{1,t\tau}$, the price for $(t-1, \tau)$-share on stock market 2 by $q_{2,t\tau}$ and the price for a (t, τ)-share on stock market 2 by $q_{3,t\tau}$. Stock market 2 is the key market in this model, where the story told in the introduction is to take place: the pieces aid in making it happen (see section 4). Before describing the structure of a period in greater detail, some institutions need to be introduced.

There is one *mutual fund*. The mutual fund holds a diversified portfolio of shares. On stock market 1, the mutual fund sells and buys $(t-1, \tau)$-shares directly. We require the mutual fund to sell all $(t-1, \tau)$-shares on stock market 2, returning the proceeds to the owners of the mutual fund. It then buys (t, τ)-shares. In other words, the mutual fund briefly closes and then reopens again. As for purchasing (t, τ)-shares on stock market 2, we assume that the mutual fund is somehow prevented from acquiring shares in proportion to their market weights directly. Instead its technology for purchasing shares requires to use all agents $j \in [0;1]^2$ as stock pickers in the following way. Given that agent j observes the level $l = l_{t,\tau_{tj}}$ and the information index $i = i_{t,\tau_{tj}}$ for the stock τ_{tj} picked for agent j, the agent is instructed to buy $\bar{k}_t \phi(l, i)$ shares of the new capital in that technology (i.e. (t, τ_{tj})- shares) on behalf of the mutual fund. If the agent only observes the level l, he is instructed to buy $\bar{k}_t \tilde{\phi}(l)$ shares. Here $\phi(l, i) \in \mathbf{R}$, $\tilde{\phi}(l) \in \mathbf{R}$ are allowed to be negative: this corresponds to short sales by the mutual fund.

Note that with $\pi(g = 0 \mid i) < F_{k,t}(g = 0 \mid l, i)$, the mutual fund buys "too little" of the good stocks with $g = 0$ and "too much" of the bad stocks with $g = 1$. This happens, because for a given technology τ, there is a relatively greater chance that an agent bought this stock on behalf of the mutual fund if $g_{t\tau} = 1$ than if $g_{t\tau} = 0$. If the mutual fund kept proper records of its portfolio, it would realize that it has purchased relatively more of some stocks than of others with the same characteristics: the mutual fund could then identify the "bad stocks". The model essentially assumes, that the mutual fund does not keep such records or that it is too late then for the mutual fund to retrade. This is the "monkey wrench" needed in order to have the mutual fund trading on the "wrong side" of the market systematically and to make information acquisition profitable.

The mutual fund is assumed to act as price-taker and to maximize expected return from one market to the next. Since the portfolio is diversified, the return is riskless. The return from stock market 2 at date t to stock market 1 at date $t + 1$ is denoted with R_{t+1}. The return from stock market 1 to stock market 2 is 1, since it is the return within a period. The mutual fund therefore is willing to buy any share with at least an actuarily fair price on stock market 1 and buys any share with an expected return of at least R_{t+1} on stock market 2, given its stock picking technology characterized by $(\tau_{tj})_{tj}$.

There is a competitive sector of *output-producing firms*, renting capital $k_{t-1,\tau}$ between stock market 1 and stock market 2 from the owners and paying dividends in return, and hiring labor on the labor market in part III of the period, paying a wage w_t per unit of labor. The output is sold in part V of the period.

There is a competitive sector of *capital-producing firms*, buying capital $k_{t-1,\tau}$ on stock market 2 as well as investment goods $x_{t\tau}$ on the goods market in part V of the period. The produced capital $k_{t\tau}$ is sold as (t, τ)-shares on stock market 2.

There is a *lottery fund* operating in part II of the period, accepting all actuarily fair gambles in units of the period-t consumption good by agents.

Below we will describe the gambles that agents enter into and they will turn out to be idiosynchratic. Thus the lottery operates at zero profit by the law of large numbers.

3.2 Sequence of Events.

We now describe the sequence of events by following an agent through a period, see figure 1. Recall, that all transactions within the period are performed via double-entry bookkeeping.

In *part I*, the agent enters period t, holding shares of the mutual fund as well as $(t-1, \tau_{t-1,j})$-shares. The agent may hold either share short. If so, he is required to settle his short-position on stock market 1. In other words, let a_{tj} be the value on stock market 1 of his portfolio in terms of the period-t consumption good. We require that $a_{tj} \geq 0$ almost surely and thereby impose a borrowing constraint on the agent. This type of constraint has been used before e.g. in Foley and Hellwig [13], Lucas [25], Scheinkman and Weiss [30] and Deaton [6].

On stock market 1, the agent can trade mutual fund shares as well as $(t-1, \tau_{t-1,j})$-shares going long or short (see also the description for stock market 2 below). Period 0 opens in the "middle" of the stock market with all agents just holding shares of the mutual fund and possibly trading them against some individual $(-1, \tau_{-1,j})$-shares.

In *part II* of the period, agents can enter actuarily fair lotteries in units of the consumption good with the outcome determined by the random variable r_{tj}. Thus, suppose agent j enters a lottery where he can either loose two units of the consumption good or win one unit o***
N_{tj}. If the agent decides to become informed instead, he does not work. Thus, the costs of acquiring information are the opportunity costs of not working and are endogeneously determined within the model. The agent receives dividends for any shares he ownes. Dividends paid to the mutual fund are paid to the owners of mutual fund shares in turn.

In *part IV* of the period, a new stock τ_{tj} is picked for each agent. If the agent decided to become informed, he also learns the message $m_{t,\tau_{tj},j}$ about that stock.

In *part V* of the period, we require the agent to sell all $(t-1,\tau_{t-1,j})$-shares (or to buy them back, if he held a short position). Note that the agent also receives the value of his mutual fund shares, since the mutual fund closes briefly. The agent then buys shares in the mutual fund (which in turn purchases a portfolio of (t,τ)-shares) and (t,τ_{tj})-shares. Shortselling in either shares is allowed subject to the borrowing constraint explained above. Note that the agent is not allowed to purchase individual shares other than (t,τ_{tj})-shares: this means in particular, that the mutual fund is the only means of diversification. The agent purchases (t,τ)-shares on behalf of the mutual fund in the manner described in subsection 3.1 above.

In part V of the period the agent also buys consumption contingent on the history h_{tj} described in subsection 2.3 above. We assume that the share purchases $s_{t,\tau_{tj},j}$ resp. $s_{t,\text{mut.fund},j}$ by agent j on stock market 2 are functions of h_{tj} as well.

In *part VI* of the period the agent needs to have a cleared account, i.e. total receipts have to exactly pay for total purchases. This simply means, that the period-t budget constraint needs to hold exactly.

3.3 Equilibrium and Steady State

An *equilibrium* is an allocation and a list of prices, such that

- Given prices and the constraints described in the previous two sections, agents maximize lifetime utility U.
- Given prices, the output-producing firms and the capital-producing firms maximize profits and the mutual fund maximizes expected return from one stock market to the next.
- The market for labor and the market for goods clears.

- For each technology τ, the markets for $(t-1,\tau)$-shares on stock market 1 (before dividends) and on stock market 2 (after dividends) as well as for (t,τ)-shares on stock market 2 clear.

- Prices $q_{2,t\tau}$ and $q_{3,t\tau}$ are independent of $i_{t\tau}$, if almost all agents decided, not to acquire information, i.e. if $d_{tj} = 1$ for λ-a.e. j.

- The market for mutual fund shares on stock market 1 as well as stock market 2 clears.

Let $a_{tj} \in \mathbf{R}_+$ be the relative wealth or relative asset position of agent j in period t, i.e. the value of the portfolio of agent j at the beginning of period t as evaluated by stock market 1, relative to the value of the entire market portfolio. Let $F_{a,t}$ be the resulting population distribution over a_{tj}, which we call the *relative wealth distribution.* Let $\hat{a}_{tj} \in \mathbf{R}$ be the sum of a_{tj} and the random gain due to the lottery in part II of the period relative to the total value of the entire market portfolio. Let $A_{t,j}(\hat{a};a)$ be the distribution of $\hat{a}_{tj}$ conditional on a_{tj}. Let $\check{a}_{tj} = \hat{a} + d_{tj}N_{tj}\check{w}_t$, where $\check{w}_t$ is the wage w_t divided by the total value of the entire market portfolio.

A *steady state equilibrium* is an equilibrium which additionally satisfies that

- The distributions $F_{a,t}$, $A_{t,j}$ and $F_{k,t}$ do not depend on t or on j.

- The *mutual fund return factor* R is independent of time t.

- The decisions of agents can be written as time-invariant decision rules,

$$d_{tj} = d(\hat{a}_{tj}) \tag{12}$$

as well as

$$s_{t,\tau_{tj},j} = \begin{cases} s_{\text{inf}}(\hat{a_{tj}}, l_{t,\tau_{tj}}, i_{t,\tau_{tj}}, m_{t,\tau_{tj},j}) & \text{if } d_{tj} = 0, \\ s_{\text{uninf}}(\check{a_{tj}}, l_{t,\tau_{tj}}, i_{t,\tau_{tj}}) & \text{if } d_{tj} = 1 \end{cases}$$

and likewise for $s_{t,\text{mut.fund},j}$ and c_{tj}.

- Share prices are given by time invariant functions,

$$\begin{aligned} q_{1,t\tau} &= q_1(l_{t\tau}) \\ q_{2,t\tau} &= q_2(l_{t\tau}, i_{t\tau}) \\ q_{3,t\tau} &= q_3(l_{t\tau}, i_{t\tau}). \end{aligned}$$

- The functions $q_2(l,i)$ and $q_3(l,i)$ are independent of i, if almost all agents decide, not to acquire information, i.e. if $d_{tj} = 1$ for λ-a.e. j.

- Aggregates grow at factor ζ, where

$$\zeta = \Gamma_0^{1/(1-\rho)}. \tag{13}$$

  In other words,

$$\bar{y}_t = \zeta^t \bar{y}_0,\ \bar{w}_t = \zeta^t \bar{w}_0,\ \bar{k}_t = \zeta^t \bar{k}_0,\ \bar{c}_t = \zeta^t \bar{c}_0,\ \bar{x}_t = \zeta^t \bar{x}_0.$$

It follows in steady state, that aggregate labor supply $\bar{N}_t$ is a constant, independent of time, and that the relative investment per unit of old capital $x_{t\tau}/k_{t-1,\tau}$ is a time-invariant function $x(l,i)$ of $(l,i) = (l_{t\tau}, i_{t\tau})$.

The *steady state growth factor* ζ of the economy follows from (5) and from aggregating (1) with the time-invariant distribution F_k.

4 Why is this model so complicated?

The ingredients to this model are a consequence of three basic choices about the model structure.

1. The use of a general equilibrium neoclassical growth framework rather than partial equilibrium as in e.g. Leland [23],

2. Keeping track of individual wealth accumulation rather than reuniting all agents at the end of each period into one family to restore the representative agent framework as in e.g. Lucas [26],

3. Avoiding "aggregate illusions" as in De Long et al [7].

Choice 1 leads to equations (1), (2) and (4) and to the interpretation of stocks as claims to a unit of capital. Choice 3 leads to the use of a continuum of technologies, each with their own "noise" rather than a single technology. Thus, there is no aggregate uncertainty and agents cannot draw inferences from aggregate variables about stock movements. This also simplifies the computation of the equilibrium. Using a continuum of technologies in turn requires the use of a bounded distribution for the technology growth factors $\Gamma_{t\tau}$, since the aggregate steady state growth factor of the economy is

$$\zeta = \left(\sup_{\tau} \Gamma_{t\tau} \right)^{1/(1-\rho)} .$$

A two-point distribution for $\Gamma_{t\tau}$ as used in this paper is the most simple, non-trivial choice. The popular normal-distribution structure for asymmetric information models cannot be used here.

The introduction of a mutual fund and the restriction of agents to trade in at most one risky stock simplifies the portfolio choice problem faced by agents while at the same time allowing for diversification. The economic motivation for this assumption lies in the fact that it is costly in practice to achieve diversification individually rather than through the purchase of mutual funds. In the model, the mutual fund is the natural candidate for noise trading i.e. for biased stock picking. By formalizing the bias via the probabilities $\pi(l, i, g)$, the inference problem on stock market 2 becomes easy to analyze (see the next section).

Consider figure 1 again. Stock market 1 assures that the informational advantage of traders only last one period: this simplifies the analysis of prices and the portfolio choice problem. It also allows the restriction of unlimited short sales or credit-financed long positions via the imposed borrowing constraint.

The lottery in part 2 is needed as a concavification device. Let $\hat{a}$ be the value of the portfolio of some agent after the lottery. Let $v_{\text{uninf}}(\hat{a})$ be the expected discounted value of all present and future utility, given that the agent chooses to stay uninformed and work. Similarly, let $v_{\text{inf}}(\hat{a})$ be that value, provided the agent becomes informed. Before that choice, the value is therefore

$$v(\hat{a}) = \max\{v_{\text{uninf}}(\hat{a}), v_{\text{inf}}(\hat{a})\}$$

and there is no reason to expect $v(\hat{a})$ to be concave, even if $v_{\text{uninf}}(\hat{a})$ and $v_{\text{inf}}(\hat{a})$ are. If $v(\hat{a})$ is not concave, agents have an incentive to engage in actuarily fair lotteries: the model allows for this and the resulting before-the-lottery value function is concave as a result.

The random labor productivity shock N_{tj} is needed to assure a constant resupply of wealthy individuals who will have an incentive to acquire information in the next period. There is a market for old capital and for new capital on stock market 2, since the key to the model is the allocative role of information in stock prices on investment.

Thus given the three choices above this model may well be the simplest model to formalize the idea explained in the introduction.

5 Theoretical Insights

Consider a candidate steady state equilibrium. Recall that the mutual fund achieves a safe rate of return on its portfolio, given by the mutual fund return factor R between periods. The mutual fund acts as risk-neutral with respect to each individual stock. To achieve market clearing, prices must prevent the mutual fund from wishing to take an arbitrarily long or short position in any particular stock, i.e. it needs to be the case that the mutual fund no-arbitrage condition

$$q_3(l, i) = \frac{\pi(g = 0 \mid i) q_1(l) + \pi(g = 1 \mid i) q_1(l + 1)}{R} \tag{14}$$

holds, if prices $q_3(l, i)$ reveal i at all. Note that the expected return in the numerator of (14) takes into account the bias inherent in the stock-picking technology τ_{tj}. Given prices $q_3(l,i)$ as well as Probabilities $P(i_{t\tau} = i, g_{t\tau} = g \mid l)$ and wages $\bar{w}_0$, the prices $q_2(l,i)$, $q_1(l)$ and the investment $x(l,i)$ per unit of capital can be calculated:

Proposition 2
In steady state equilibrium, dividends per unit of capital are given by

$$dividend(l) = \frac{\rho \bar{w}(0)}{1-\rho} \left((1-\rho) \left(\frac{\Gamma_1}{\Gamma_0} \right)^l \frac{\gamma_0}{\bar{w}_0} \right)^{1/\rho} . \tag{15}$$

Stock prices are given by

$$q_1(l) = dividend(l) + \sum_i P(i \mid l) q_2(l,i) \tag{16}$$

and

$$q_2(l,i) = \max_{x(l,i)\geq 0} q_3(l,i) f(1, x(l,i)) - x(l,i). \tag{17}$$

Investment per unit of capital $x(l,i)$ is given by the solution to the maximization problem in (17).

Proof: *Equations (15) follows from the first order condition for profit-maximization of the output-producing firms. Equation (16) gives the only price at which the mutual fund cannot arbitrage between parts 1 and part 5 of a period. Equation (17) is the zero-profit condition for the competitive sector of capital producing firms.* •

Hence, given Probabilities $P(i_{t\tau} = i, g_{t\tau} = g \mid l)$ and wages $\bar{w}_0$, equation (14) with q_1 substituted out via equations (15), (16) and (17) yields a fixed point problem in prices $q_3(l,i)$. Under suitable assumptions on f, one can show that a unique solution exists, that it can be computed via a contraction mapping and that $q_1(l)$ is strictly monotonically decreasing (see Uhlig [35]).

Equation (14) shows, that prices $q_3(l,i)$ are higher if it is more likely (indicated via i) that the stock is "good", i.e. that its productivity is growing at the high factor Γ_0 rather than at the low factor Γ_1. As a result, investment $x(l, i)$ per unit of capital is higher as well for the stocks indicated likely to be "good". This is the central mechanism by which information acquisition improves the productive allocation of information: given that the information acquisition leads to the partial revelation $i_{t\tau} = i$ of the information $g_{t\tau} = g$ through prices $q_2(l, i)$ and $q_3(l, i)$, investment will be channelled into those technologies which are more likely to show high productivity growth, everything else (i.e. the level l) being equal. In fact, with the usual choice of $\alpha = 1$, $\kappa_2 > 0$ in (3), proposition 2 implies that $x(l, i) = 0$ for all l, i except for $l = 0$ and that value of i, which maximizes $\pi(g = 0 \mid i)$: investment will only be undertaken in the very best technologies. For $0 < \alpha < 1$, $\kappa_2 > 0$, one obtains strictly interior solutions for all $x(l, i)$, which is why that choice of parameters was made for the numerical calculations.

So far it has been assumed that prices $q_3(l, i)$ will indeed reveal the index i but not $g_{t\tau} = g$. To that end, it needs to be checked that stock market clearing is achieved for each technology τ individually. The supply of (t, τ)-shares, if τ is of category (l, i, g), is given by

$$k_{t\tau} = f(1, x(l, i))\, k_{t-1,\tau}. \tag{18}$$

To find the aggregate distribution of capital across different categories, the following proposition is helpful.

Proposition 3

Given investment $x(l, i)$ per unit of capital and probabilities $P(i, g \mid l)$, the steady state distribution $F_k(l, i, g)$ of capital across different categories is given by

$$F_k(l, i, g) = P(i, g \mid l)\, f(1, x(l, i))\, F_k(l)/\zeta, \tag{19}$$

and

$$F_k(l) = \tilde{F}_k(l) \,/\, \sum_{l'=0}^{\infty} \tilde{F}_k(l')$$

where

$$\begin{aligned}
\tilde{F}_k(0) &= 1 \\
\tilde{F}_k(l) &= \psi(l)\,\tilde{F}_k(l-1),\ l = 1, 2, \ldots \\
\psi(l) &= \frac{\sum_{i=0}^{I} P(g=1, i \mid l-1)\, f(1, x(l-1, i))}{\zeta - \sum_{i=0}^{I} P(g=1, i \mid l)\, f(1, x(l, i))}.
\end{aligned}$$

Proof: *Given* $(\bar{k}_{t-1}(l))_{l=0}^{\infty}$*, we have*

$$\bar{k}_t(l, i, g) = P(i, g \mid l)\, f(1, x(l, i))\, \bar{k}_{t-1}(l),$$

which delivers the first equation. Given $F_k(0)$*, it follows with (11) and (19) that*

$$F_k(l) = \psi(l)\, F_k(l-1),\ l = 1, 2, \ldots.$$

The constraint $\sum_{l=0}^{\infty} F_k(l) = 1$ *then allows the calculation of* $F_k(0)$*.* •

Since $F_k(0) = \sum_i F_k(0, i, 0)$, it needs to be true that

$$\sum_{i=0}^{I} P(i \mid 0)\, f(1, x(0, i)) = \zeta. \tag{20}$$

Since dividends and thus $x(l, i)$ is a function of the wage w_0, which so far has been treated as a parameter, equation (20) delivers the equilibrium wage (this assertion needs some assumptions and a proof, see Uhlig [35] for details).

By construction, the demand of the mutual fund for a (t, τ)-stock of category (l, i, g) is given by

$$\begin{aligned}
\text{Demand}_{\text{Mutual Fund}}(t, \tau) &= \frac{k_{t-1,\tau}}{P(i, g \mid l) F_k(l) \bar{k}_{t-1}} \bar{k}_t \phi(l, i) \pi(l, i, g) \\
&= \frac{k_{t,\tau}}{F_k(l, i, g)} \phi(l, i) \pi(l, i, g),
\end{aligned}$$

provided prices q_3 reveal i, but not g. Aggregating, the mutual fund thus buys

$$\text{Demand}_{\text{Mutual Fund}}(l, i, g) = \bar{k}_t \phi(l, i) \pi(l, i, g) \tag{21}$$

total units of new capital in technologies of category (l, i, g) on stock market 2, if the information index i is always observed.

To calculate the demand for stocks by individual agents, note that an uninformed agent receives the same expected return on the picked stock τ_{tj} as on the mutual fund by virtue of (14). Since his value function, i.e. his expected discounted sum of present and future utilities, can be shown to be concave (see the remarks about the lottery in the previous section), the agent will at least weakly prefer to hold zero shares of the picked stock and to save entirely by means of the mutual fund. To simplify matters, we assume that all uninformed agents do.

Informed agents however have an informational advantage, since messages are informative, i.e. since $\pi(g \mid i, m) \neq \pi(g \mid i)$ for all i, g and m, see proposition 1. Since $\pi(i)$ and $\pi(g \mid i)$ do not depend on the level l, the portfolio choice problem of an agent informed about some technology of level l is equivalent to the portfolio choice problem of an agent informed about some technology of level $l = 0$: any solution to the latter can be converted to a solution of the former and vica versa. As a result, the aggregate demand of the informed agents for stocks of category (l, i, g) is given by

$$\begin{aligned} &\text{Demand}_{\text{Informed}}(t, \tau) \\ &= \frac{k_{t-1,\tau}}{P(i, g \mid l) F_k(l) \bar{k}_{t-1}} \pi(l, i, g) \end{aligned}$$

Market clearing for each technology τ is therefore achieved if and only if the following stock market clearing condition is satisfied for all categories:

$$\pi(l, i, g) \left(\phi(l, i) + \frac{q_1(0) - q_1(1)}{q_1(l) - q_1(l+1)} D_{\text{inf}}(i, g) \right) = F_k(l, i, g). \tag{23}$$

Fix some (l, i). Since with (14), the mutual fund is indifferent between different choices for $\phi(l, i)$, "average" market clearing across the two categories $(l, i, g = 0)$ and $(l, i, g = 1)$ is achieved by adjusting $\phi(l, i)$. Since $\phi(l, i)$ however cannot be adjusted to achieve market clearing for each category individually, one obtains:

Proposition 4 (Consistency Condition)

Equation (23) can be satisfied for a suitable choice of $\phi(l,i)$, if and only if the following consistency condition for the probability structure is satisfied for all categories (l,i,g):

$$P(g \mid l,i) = (1 + \xi(l,i,g))\, \pi(g \mid i), \tag{24}$$

where

$$\xi(l,i,g) = \chi(l,i)\, \pi(1-g \mid i)\, \Big(D_{inf}(i,g) - D_{inf}(i,1-g)\Big)$$

and

$$\chi(l,i) = \frac{\pi(l)}{F_k(l)} \frac{\pi(i)}{P(i \mid l)\, f(1, x(l,i))} \frac{q_1(0) - q_1(1)}{q_1(l) - q_1(l+1)}.$$

Proof: *Given (l,i), solve (23) for $\phi(l,i)$ and compare the solutions for $g = 0$ and $g = 1$.* •

The consistency condition is a necessary condition for an equilibrium to exist. It is knife-edge in the sense that arbitrary choices of probabilities are unlikely to satisfy it. The consistency condition corresponds to the feature that the same price $q_3(l,i)$ clears the market for both categories $(l,i,g=0)$ and $(l,i,g=1)$: this is needed so that prices can be partially revealing.

One way to read the consistency condition is as follows: suppose, one has chosen all probabilities except for $P(g=0 \mid l,i)$. Equation (24) gives a formula for choosing these remaining probabilities so that a partially revealing steady state may exist. This is what is done in the numerical calculations. It amounts to "backsolving" for probabilities which are primitives of the model in the spirit of Sims [31], [32]. Performing these calculations is not quite so simple, however, since the right hand side of (24) depends on F_k which in turn depends on $P(g=0 \mid l,i)$, leading to a fixed point problem. One furthermore needs to check that $P(g=0 \mid l,i)$ is between 0 and 1.

Alternatively, one may use the consistency condition the other way around for calculating the probabilities $\pi(g = 0 \mid i)$, i.e. the probability that a share purchased by an uninformed agent or the mutual fund happens to be a "good stock". This is actually not practical numerically, since one runs into almost insurmountable difficulties to achieve the independence $\pi(g \mid i)$ from the level l: this independence is needed, however, to derive equation (**??**). Note that the mutual fund buys the entire supply of shares net of the demand by informed agents by virtue of market clearing. One could thus imagine some mechanism in which $\pi(g = 0 \mid i)$ arises endogeneously through this "residual demand" feature of the mutual fund. We have chosen $\pi(g = 0 \mid i)$ to be a primitive parameter of the model to keep the inference machinery and the analysis simple.

There is a deeper issue involved. Given a partially revealing steady state, suppose that on some stock market a price $\tilde{q}_3(l, i)$ is quoted, from which i can be inferred. Everybody, including the mutual fund, realizes that if market clearing occurs at this price, it will be due to one of two possibilities: either $g = 0$ with probability $P(g = 0 \mid l, i)$ and the mutual fund buys "too few" shares or $g = 1$ with probability $P(g = 1 \mid l, i)$ and the mutual fund buys "too many" shares. Given $\tilde{q}_3(l, i)$ and the bias inherent in the mutual fund stock picking technology, one can infer the probability $\pi(g = 0 \mid i)$ that a share purchased by the mutual fund is actually a "good stock" with $g = 0$. Given $\pi(g = 0 \mid i)$, the mutual fund can now calculate the price $q_3(l, i)$ at which it is exactly indifferent between going long or short for a stock with index i: this price is given by (14). The consistency condition states that probabilities have to be such that $\tilde{q}_3(l, i) = q_3(l, i)$.

Is the bias in the mutual fund stock picking technology necessary to induce some agents to become informed rather than to earn wages by working? The next proposition answers this question.

Proposition 5 *Access to a market portfolio mutual fund rules out costly information acquistion.*

Proof: *A market portfolio and the entire capital stock have the same safe return R_M, say. In order for an agent with a concave value function to acquire information at the opportunity cost of not working, he must be assured of an unconditionally expected portfolio return strictly higher than R_M, where that portfolio choice will in general depend on the acquired information. Likewise, any uninformed agent with a concave value function will choose a portfolio with an unconditionally expected portfolio return of R_M or higher. Since the return on the entire capital stock must equal the properly weighted average return on all agents' portfolios, the fraction of agents who choose to become informed must be zero.* •

Stated differently, the market portfolio return is split between the informed traders and the uninformed "noise" traders. In order for the informed traders to receive a higher expected return than the uninformed traders, the uninformed traders need to underperform the market.

A characteristic of an equilibrium is that the richest agents (as measured by their beginning-of-period value of their portfolio net any gains or losses due to the lottery) choose to acquire information whereas the poorest agents decide to work. The reason lies in the borrowing constraint imposed at the beginning of period 1: the short or long position an informed agent can take the stock picked is limited by an increasing function of the stock market 2 value of his portfolio. In short, information is more valuable to rich agents because they are allowed to take larger bets, but the opportunity costs of not working are the same.

It could not be shown that there is a critical level of wealth, below which an agent chooses to work and above which an agent chooses to acquire information: since the value of information also depends on the curvature of the beginning-of-period value function and since the value function may include locally linear parts due to the lottery in part 2 of the period, agents of medium wealth may be locally risk neutral whereas very rich agents are not.

In the numerical calculations, however, a critical level of wealth could always be found and the value function turned out to be concave even without the lottery in part 2 of the period.

Completely proving the existence of an equilibrium subject to choosing probabilities $P(g = 0 \mid l, i)$ turns out to be quite hard and lengthy, since e.g. aggregate demand depends on the steady state distribution of wealth across agents which in turn depends on endogeneous parameters of the model. For a proof subject to several additional assumptions, arbitrarily small perturbations and one conjecture, see Uhlig [35].

6 Numerical Results

6.1 The Algorithm for Computing Equilibria

The key to computing equilibria is to find a "hierarchical decomposition" of the problem into smaller subpieces. In particular the interest factor R on the mutual fund turns out to be the central variable: conditional on R, everything becomes essentially static and the "decision problem side" and the "production side" of the economy can be solved for almost separately. Briefly, the program has the following structure:

1. Input and Initializations
2. R-Loop,
    (a) Decision Problem Module
        i. Value Function Loop to calculate value functions.
        ii. Asset Distribution Loop to calculate the asset distribution F_a.
        iii. Calculation of Aggregate Demands at the Normalized Wage $w_0 = 1$.
    (b) Probability Adjustment Loop

i. Wage Loop to calculate w_0.

A. Price Loop to calculate q_1, q_2, q_3 and x.

ii. Calculation of Capital Distribution, Aggregate Capital and Aggregate Demands

iii. "Backsolving" for the Probabilitites via the consistency condition.

(c) Calculation of Remaining Aggregates

3. Output

In part 2a(i), a grid method similar to Colemans' method (see Coleman [5]) is used for solving the dynamic programming problem by iterating on the Bellman equation. Note that it is necessary here to keep track of the value function itself rather than just Euler equations in order to determine the worker vs. informed trader decision.

In part 2b(iii), we choose probabilities "conveniently" to assure the consistency condition and the numerically appealing condition that $\chi(l,i) = \bar{\chi}$ for some $\bar{\chi}$. This is done by choosing $\pi(i)$ and $\pi(g \mid i)$ as parameters of the program. The program then calculates $P(g \mid i,l)$, $P(i \mid l)$ and $\pi(l)$. It follows from $x(l,i) \to 0$ for $l \to \infty$ and from $\chi(l,i) = \bar{\chi}$, that $P(i \mid l) \to \pi(i)$ for $l \to \infty$.

Most of the loops use contraction mapping principles (see Stokey, Lucas with Prescott [33]) to find solutions. The programming was done in the programming language C and resulted in approximately 4000 lines of rather sparse code. Computation of one experiment on a Sparc I took about 40 to 90 minutes of CPU time with informed agents and 6 to 8 minutes without informed agents. Additional details can be found in Uhlig [35, p. 244–273].

6.2 Some Experiments

A total of 66 numerical experiments were performed. The fixed parameters for all experiments are in table 1. We chose one of six different values for the

$I = M =$	1
$\pi(g = 0) = \pi(i = 0) =$	.7
$\Gamma_0 =$	1.05
$\Gamma_1 =$	1.02
$\beta =$	.95
$\eta =$	1.5
$\alpha =$	.5
$\kappa_1 =$	.94
$\kappa_2 =$	1.0
$\rho =$	.3
F_e:	exponential distribution, $\lambda = 1$

Table 1: Fixed Parameters

spread $\pi(g = 0 \mid i = 0) - \pi(g = 0 \mid i = 1)$, given in table 2, and one of eleven values for the message quality or signal quality $P(m = 0 \mid g = 0) = P(m = 1 \mid g = 1)$, given in table 3.

The spread $\pi(g = 0 \mid i = 0) - \pi(g = 0 \mid i = 1)$ is interpreted as the informativeness of prices: if the spread is zero, prices $q_3(l, i)$ as calculated with equation (14) would not reveal anything about g, whereas with a spread of 1, prices would completely reveal g. Given the spread and given $\pi(g = 0)$ and $\pi(i = 0)$, the conditional probabilities are given by

$$\begin{aligned} \pi(g = 0 \mid i = 0) &= \pi(g = 0) + \text{spread} * \pi(i = 1), \\ \pi(g = 0 \mid i = 1) &= \pi(g = 0) - \text{spread} * \pi(i = 1) \end{aligned}$$

For each set of parameters, we also computed the result for an economy without informed agents and without information in prices, but with the same unconditional probability $P(g = 0 \mid l)$ as computed in the with-informed-agents economy.

.1	.2	.3	.4	.5	.6

Table 2: Spreads (Information in Prices), $\pi(g = 0 \mid i = 0) - \pi(g = 0 \mid i = 1)$

.64	.66	.68	.685	.69	.695	.70	.705	.710	.715	.72

Table 3: Signal Qualities, $P(m = 0 \mid g = 0) = P(m = 1 \mid g = 1)$

To judge the social gains from information acquisiton we calculated the percentage gain (or loss) in total output of the with-informed-agents economy as compared to the without-informed-agents economy[4]. It is important to emphasize, that by construction, this gain cannot exceed the difference between the best-case scenario of always investing physical resources only in the good stocks $\Gamma_{g_{t\tau}} = \Gamma_0$ and the scenario of always investing physical resources only in the bad-growth stocks $\Gamma_{g_{t\tau}} = \Gamma_1$. The steady state growth factor of the economy is not affected and given by ζ according to equation (13). The maximal output gain due to information acquisition is therefore given by a permanent level increase by the factor $(\Gamma_0/\Gamma_1)^{1/(1-\rho)}$, i.e. is given by a permanent increase of 4.2 % with the parameters in table 1. The results below should be read relative to this maximum. Losses due to information acquistion can be quite large, however, since information acquiring agent do not participate in the labor market.

Likewise, individual and average welfare can be computed and compared. It can be shown that a gain in welfare for an agent (or a gain in aggregate welfare) from v_1 in some economy 1 to v_2 in some economy 2 is equivalent to a permanent level increase of all consumptions in economy 1 by the factor

$$\left(\frac{v_1 + \frac{1}{1-\beta}\frac{1}{1-\eta}}{v_2 + \frac{1}{1-\beta}\frac{1}{1-\eta}} \right)^{\frac{1}{1-\eta}}, \tag{25}$$

allowing intuitively meaningful comparisons.

[4]We also look at distributional issues, see below.

For the spread fixed at .4 and the message quality at .7, we compared the effect of information acquisition on prices q_3 in figures 1.4.1 and 1.4.2 and on investment in figures 1.5.1 and 1.5.2. The figures 1.4.1 and 1.5.1 are for the economy with information acquistion whereas figures 1.4.2 and 1.5.2 are for the corresponding economy without information acquisition. There are two curves in figures 1.4.1, the top curve for $i = 0$ and the bottom curve for $i = 1$. Agents can read i of prices since these two curves differ: this is the partial revelation of information through prices. Figure 1.5.1 shows the resulting effect of that partial revelation on investment: investment is a lot higher for $(i = 0)$-technologies (top curve) than for $(i = 1)$-technologies (bottom curve). Figures 1.4.1 and 1.5.1 thus demonstrate numerically the story told in the introduction. The gains to information acquisition in terms of total output is 1.2 % for these parameters, whereas average welfare actually decreases corresponding to a 4.3 % permanent level reduction of consumption.

Figures 2.1 through 4.7 summarize all experiments: figures 2.1 through 2.7 show the "raw results" whereas figures 3.1 through 4.7 are obtained by essentially interpolating the corresponding figures among 2.2 though 2.7 with the help of figure 2.1.

Figure 2.1 shows the demand by insiders (= informed agents) for stocks of categories with $i = g = 0$ in percent of the total capital in these stocks. The better the quality of signals or messages and the less information is revealed by prices, the lower the demand by informed agents. Figure 2.2 shows the shure return on the mutual fund, whereas figure 2.3 shows the expected return on the portfolio of informed agents. These two figures are more or less mirror images of each other, since together they show how the market portfolio return is split between informed and uninformed agents. Figure 2.4 shows the fraction of assets owned by insiders as evaluated on stock market 1. Comparing these results to figure 2.5, which shows the fraction of the population which acquires information, demonstrates, that the informed agents constitute a small fraction of the population but control a large fraction of the total capital stock.

Figure 2.6 shows the level increase of output in percent due to the presence of insiders. The increase is the higher, the better the information in prices: for $P(m = 0 \mid g = 0) = .7$ and a spread of .6, that increase is quite close to the maximum of 4.2 **
of the paper in the following way: the presence of informed agents improves average welfare, if these agents know relatively little and if most of the information gets revealed by prices, and worsens average welfare otherwise.

Consider figure 2.1 again. Since the mutual fund buys the supply net of the demands by informed agents, figure 2.1 can also be interpreted as measuring the biased noise inherent in the stock picking technology of the mutual fund. What does this measure of the noise imply? Figures 3.1 through 3.4 answer that question. For figure 3.1, we flipped the axis in figure 2.1, solving for the signal quality, given some level of noise (i.e. demand of the informed agents for $(i = g = 0)$-stocks in percent of supply) and the informativeness of prices (given by the spread variable). We can then use the results of figure 3.1 to present the results of say, figure 2.3 and 2.4, in terms of noise and price informativeness: this is done in figures 3.3 and 3.4. They show that the return to the portfolio of informed agents as well as the fraction of assets owned by informed agents is almost entirely a function of this measure of noise: the informativeness of prices matters little.

In interpreting the model and the figures so far, one may wish to regard noise as the fundamental parameter and the informativeness of prices as the endogeneous variable rather than the other way around. This perspective is used for figures 4.1 through 4.7. In figure 4.1, we solve for the informativeness of prices as a function of the signal quality and the noise: with the data from figure 2.1 and for each of the 11 levels of signal quality listed in table 3, a linear function is fitted via least squares to the six values of the percent insider demand for $(i = g = 0)$-stocks resulting from the six values for the spread given in table 2, and inverted to solve for the spread as a function of noise. The mapping from figure 4.1 is then used to redraw figures 2.2 through 2.7 as functions of signal quality and the noise in figures 4.2 through 4.7.

Figures 4.2 and 4.3 show the return to the mutual fund and the expected return to the informed agents portfolio: the higher the noise, the lower the return to the mutual fund and the higher the expected return to informed agents. Similarly figures 4.4 and 4.5 show that the fraction of assets owned by informed agents as well as the fraction of the population which acquires information is an increasing function of the noise and, somewhat surprisingly perhaps, a decreasing function of the signal quality. An explanation for why the signal quality decreases rather than increases the fraction of informed agents may be, that with better signals and the same level of noise, prices are more informative (see figure 4.1) and thus the potential gain to "betting right" smaller: this effect is apparently not offset sufficiently by the fact that informed traders can be surer about "betting right" with better signals.

Figures 4.6 and 4.7 correspond to figures 2.6 and 2.7: again, note how an output gain often corresponds to a loss in average welfare. The loss in average welfare due to the presence of informed agents is the smaller, the lower the noise and the better the signal of the informed agents. Thus holding noise fixed rather than the informativeness of prices as in figure 2.7, a higher signal quality leads to an improvement rather than a worsening of average welfare.

Average welfare gains or losses may be quite far from the welfare gain or loss of an individual agent, however, since this is not a representative agents model. Given a certain value a for the assets an agents owns at the beginning of a period, would this agent rather vote in favor of an economy with informed speculators or would he rather vote against it? Given the "before" and "after" value v_1 and v_2 for such an agent, he would vote in favor of allowing informed speculation, iff the welfare corresponding to the "with"-economy exceeds the value from the "without"-economy. The relative gain or loss can be computed by calculating the corresponding level improvement via equation (25): a factor below 1 indicates that the agent would vote against legalizing information acquistion and gives an intuitive measure of how much he stands to gain or to loose.

We performed agent-individual calculations for the main example, where the spread is .4 and the signal quality is .7. Figures 5.1 through 5.4.2 show the results. There are subtle differences in the answer, depending exactly on how the comparison is performed, since the numerical methods only allow the computation of steady states. To match an agent in a steady state of a without-economy to an agent in a steady state in a with-economy, one can match the value of the beginning-of-period-0 portfolio in terms of the consumption good (this is the solid line in figures 5.1, 5.2 and 5.3) or the size of the portfolio relative to the total capital stock (this is the dotted line in figures 5.1, 5.2 and 5.3). Figure 5.1 shows the level improvement calculated via equation (25) on a logarithmic scale of asset holdings: poor agents are worse off whereas very rich agents are better off by allowing informed speculation in this example. It is remarkable that agents of medium wealth (around 10^1 units in figure 5.1) actually fare worse than agents of very low wealth. It is probably possible to find parameter values for which the poorest and the richest agents experience an improvement due to informed speculation whereas the middle-class experiences a welfare loss. The reason is that very poor agents care a lot less about the depressed return on the mutual fund due to the presence of informed agents than medium wealthy agents, since very poor agents have very little savings. Since informed speculation leads to an improvement in aggregate output for several parameters (see figure 2.6), poor agents may end up being better off, since the corresponding improvement in wages may overcompensate the welfare loss resulting from lower savings returns.

In figure 5.2 we have rescaled the ordinate axis to show the proportion of the population with assets not more than some particular value rather than that particular value itself as in figure 5.1. We can interpret these proportions as percentages of voters in favor or against allowing informed speculation, depending on whether the calculated factor exceeds unity or is below 1. As one can see, practically everybody in the steady state of the "without" economy would suffer a welfare loss by moving to the steady state of the "with" economy. Matters are somewhat different if doing the same comparison but using the asset distribution from the "with" economy as done in figure 5.3: since there are more very rich agents in the "with" economy, the fraction of voters in favor of informed speculation is around 1 % rather than practically negligible as in figure 5.2.

In figure 5.4.1 and 5.4.2, agents are moved from the steady state in the "without" economy to the steady state in the "with" economy by maintaining their relative status within the society. Figure 5.4.1 shows all agents except for the richest 1% whereas figure 5.4.2 shows only these agents. As figure 5.4.1 shows again, almost nobody prefers the "with" economy. The richest 0.7 % however prefer it quite dramatically, as figure 5.4.2 shows.

7 Conclusion

We have constructed a model to analyze the tradeoffs between the costs of private information acquisition by speculators and the benefits of the improved allocation of resources due to a stock market. We have evaluated the tradeoff in numerical simulations. We have found, that output may increase, but average welfare decrease due to informed speculation. We also found that agents of medium wealth stand more to loose from informed speculation than poor agents, since the latter care mainly about wages, whereas the former care about the lowering of the expected return on portfolios of uninformed agents.

We found that a market portfolio mutual fund rules out information acquisition. We found a knife-edge consistency condition on the probability structure necessary for the existence of a steady state equilibrium with partial revelation of information.

Appendix

A The Proof for Proposition 1.

Proof: *Equation (9) is just Bayes' rule. Note that it can be rewritten as*

$$\pi(m \mid g) = \left(\sum_{\tilde{g}=0}^{1} \pi(m \mid \tilde{g}) \pi(\tilde{g}) \right) \frac{\pi(g \mid m)}{\pi(g)}. \tag{26}$$

For equation (10), observe that Bayes' rule implies that

$$\pi(g \mid i, m) = \frac{\pi(m \mid g)\pi(g \mid i)}{\pi(m \mid i)}, \tag{27}$$

where

$$\pi(m \mid i) = \sum_{\breve{g}=0}^{1} \pi(m \mid \breve{g}) \pi(\breve{g} \mid i).$$

Substituting (26) into (27) yields the result. Furthermore, since $\pi(m \mid g = 0) \neq \pi(m \mid g = 1)$ *for all* m*, we have* $\pi(g \mid m) \neq \pi(g)$*, all* g*,* m*, and thus* $\pi(g \mid i, m) \neq \pi(g \mid i)$ *for all* i*,* g *and* m*.* •

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