Central Bank Digital Currency: When Price and Bank Stability Collide

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Central Bank Digital Currency or CBDC

- **In this paper:** A **CBDC** is an (interest bearing) account held by households at the central bank or **CB** (Barrdear-Kumhof, 2016)
- Likely to be introduced by many central banks within a few years.
- **Disintermediation Threat:** if HH hold CBDC rather than deposits, banks cannot fund firms ...
  - 1. ... unless CBDC is limited in scope and attractiveness (Fed) or
  - 2. ... unless HH re-invest CBDC at banks (Duffie) or ...
  - 3. ... Central Bank re-funds banks or projects (Brunnermeier-Niepelt).

This issue **arises due to introducing a CBDC.**

- **Here:** third option / Consolidate CB+firm+banks+gov into CB
The CBDC Trilemma

In our model: Only HH, CB, projects. CB is financial intermediary.

Key Mechanism
- Nominal Diamond-Dybvig (1983) model for a CB and its CBDC.
- Central bank can always deliver on its nominal obligations.
- But: CB runs can happen: “spending run” on available goods.

Three competing objectives:
1. Traditional CB objective: commitment to **Price Stability**
2. Social optimum, optimal risk sharing: **Efficiency**
3. Absence of runs, financial stability: **Monetary Trust**

Key Result:

Of the three objectives, the central bank can only achieve two.
Literature

**Academic:**
- Skeie (2008): nominal banking, keeping real resources fixed.
- Keister and Sanches (2019): Should central banks issue CBDC?
- Andolfatto (2021): CBDC competing with deposits.

**Policy:**
- Barrdear and Kumhof (2016): The macroeconomics of CBDCs.
The model: the real portion is Diamond-Dybvig, 1983

- **time** $t = 0, 1, 2$.

- **Continuum** $[0, 1]$ of agents:
  - $t = 0$: symmetric, endowed with one unit of a real good
  - $t = 1$: types reveal: “impatient” $\lambda$, “patient” $1 - \lambda$.
    Impatient agents: have to consume in $t = 1$.
  - $u(\cdot)$ strictly increasing, concave, RRA greater than one,
    $-x \cdot u''(x)/u'(x) > 1$.

- **Real Technology**:
  - long term: $1 \rightarrow 1 \rightarrow R$
  - storage $t = 1 \rightarrow t = 2$, available to all: $1 \rightarrow 1$

- **Optimal solution**:
  \[
  \max \lambda u(x_1) + (1 - \lambda) u(x_2) \quad \text{s.t.} \quad \lambda x_1 + (1 - \lambda) \frac{x_2}{R} = 1
  \]
  Unique solution, where $u'(x_1^*) = Ru'(x_2^*)$
  With that: $x_1^* > 1$. (Diamond and Dybvig, 1983)
The model: the nominal portion introduces CBDC.

**Definition**

A **central bank policy** is a triple \((M, y(\cdot), i(\cdot))\), where \(M\) is units of CBDC money per agent, \(y : [0, 1] \rightarrow [0, 1]\) is the central bank’s liquidation policy for every observed fraction \(n\) of spending agents, and \(i : [0, 1] \rightarrow [-1, \infty)\) is the nominal interest rate policy.

### Time Steps

\(t = 0\):
- Agents sell goods to CB for \(M\) CBDC units in \(t = 1\).
- CB: invests all received real goods in projects.

\(t = 1\):
- Agents learn type. Impatient agents **spend** \(M\). Patient agents may. Total fraction: \(\lambda \leq n \leq 1\).
- CB **observes agg. spending fraction** \(n\).
- CB liquidates fraction \(y = y(n) \in [0, 1]\) of projects.
- CB sells goods \(y\). Market clearing price \(P_1\).

\(t = 2\):
- Remaining agents **spend** \((1 + i(n))M\).
- CB sells remaining project payoffs \(R(1 - y)\).
- Market clearing price \(P_2\).

Note: “**spend**” not “**withdraw**” (into what? No physical cash).
A central bank policy is a triple \( (M, y(\cdot), i(\cdot)) \):

\[
y(n) = y^*(\lambda) = 0.4
\]

A liquidation policy

\[
i(n) = 0.1
\]

An interest rate policy

Set \( M \) so that \( P_1 = 1 \) clears the market, if \( n = \lambda \) agents spend in \( t = 1 \).
Equilibrium

Definition

Given a central bank policy \((M, y(\cdot), i(\cdot))\), an \textbf{equilibrium} \((n, P_1, P_2)\) is aggregate spending behavior \(n \in [0, 1]\), and price levels \(P_1\) and \(P_2\) such that:

1. The individual consumer’s spending decisions are optimal, given aggregate spending \(n\), the central bank’s policy \((M, y(\cdot), i(\cdot))\), the price level sequence \((P_1, P_2)\).

2. Given the aggregate spending realization \(n\), the central bank liquidates \(y(n)\) and sets the nominal interest rate \(i(n)\).

3. Given the realization \((n, y(n), i(n))\) and \(M\), the price levels \((P_1, P_2)\) clear the goods market in each period;
Market Clearing

\[ nM = P_1 y(n) \]
\[ (1 - n)(1 + i(n))M = P_2 R(1 - y(n)), \]

\[ \Rightarrow n, y(n), i(n) \text{ pin down the price levels } P_1, P_2. \]

\[ P_1(n) = \frac{nM}{y(n)} \quad \text{and} \quad P_2(n) = \frac{(1 - n)(1 + i(n))M}{R(1 - y(n))} \]

Note: \( P_2(n) \) can be “anything” per \( i(n) \), but \( i(n) \) does not affect \( P_1(n) \).

**Real allocation: only depends on \( n \) via \( y(n) \):**

\[ x_1(n) = \frac{M}{P_1} = \frac{y(n)}{n} \quad \text{and} \quad x_2(n) = \frac{(1 + i(n))M}{P_2} = \frac{1 - y(n)}{1 - n} R \]

**Given \( n \), patient agents spend in } t = 1, \text{ iff } x_1(n) \geq x_2(n). \]
Three competing objectives of the Central Bank

The central bank picks a policy \((M, y(\cdot), i(\cdot))\) to achieve three objectives:

1. **Price stability**: assure stable prices for all/most \(n\).
   - **Key focus**: Get \(P_1(n)\) to be close to some target \(\bar{P}_1\) for all/most \(n\).
   - **Interpretation**: Think of \(\bar{P}_1 = P_0 \times (1 + \pi^T)\) for some un-modelled price level \(P_0\) and some inflation target \(\pi^T\) between \(t = 0\) and \(t = 1\).
   - **Paper also**: Achieve some inflation target between period 1 and 2.

2. **Efficiency**: optimal risk-sharing. Achieve the socially optimal real allocation \(x_1^*\) arising from the Diamond-Dybvig portion.

3. **Monetary trust** and stability: no equilibria with \(n > \lambda\).
The social optimum \((x_1^*, x_2^*)\) is an equilibrium, if \(y(\lambda) = y^* = \lambda x_1^*\).
Objective 3: Monetary Trust

A Run on the Central Bank is a Spending Run:

**Definition**
A *run* occurs if \( n > \lambda \): patient agents also spend in \( t = 1 \).

**Definition**
**Monetary Trust**: a central bank policy, so that a run cannot occur in equilibrium.

In a run, money loses its ‘store of value’ function.
- Patient agents purchase goods now and hoard them.
- Trust in the monetary system evaporates, monetary instability.
- Compare to:
  - temporary pandemic stockouts.
  - hyperinflations.
  - currency crises.
- But: different from a systemic bank run, where HH turn deposits into cash without spending it.
No run

t=0

t=1

t=2
Run

t=0

t=1

<table>
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<th>Run</th>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
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14 / 24
Lemma

The central bank policy \((M, y(\cdot), i(\cdot))\) implements optimal risk sharing \((x_1^*, x_2^*)\) in dominant strategies if the central bank

1. sets \(y(\lambda) = y^*\) for any \(n = \lambda\).
2. sets a liquidation policy that implies \(x_1(n) < x_2(n)\) for all \(n > \lambda\).

This is achieved with a run-deterring policy

\[
y(n) < \bar{y}(n) = \frac{nR}{(1 + n(R - 1))}.
\]

Corollary (Trilemma 1: no Price Stability)

Run-deterring policies imply

\[
P_1(n) > \frac{M}{R} (1 + n(R - 1)), \quad \text{for all } n \in (\lambda, 1].
\]
For \( y(n) \equiv y^* \): \( P_1(n) = n \frac{M}{y^*} \).

For \( y(n) = \bar{y}(n) \): \( P_1(n) = \frac{M}{R} (1 + n(R - 1)) \).

These two policies violate the **Price Stability** objective for \( P_1(n) \).

This arises “off equilibrium”. “Threat” of high inflation for \( n > \lambda \).

Commitment / credibility vs sub-game perfection / time consistency: if \( n \neq \lambda \) arises, will a price-stability oriented central bank follow through on that threat?

What about \( P_2(n) \)? It depends on \( i(n) \). Nothing else does.
Objective 1: Price Stability

Why price stability? 50+ years of extensive literature. Pick your poison.

- Traditional CB objective.
- Dual mandate. Maastricht treaty.
- Efficiency loss with higher inflation.
- (Fully) sticky prices in $t = 1$, fixed in $t = 0$: a constraint on CB.

Here: traditional CB objective of Price Stability. CB maximizes

$$V(y, n, \bar{P}) = \alpha W(y, n) - (1 - \alpha)(\bar{P} - P_1(n))^2$$  \hspace{1cm} (2)

where $0 \leq 1 - \alpha < 1$ is weight on price stability goal and $W(y, n)$ is expected consumer utility, given liquidation $y$ and spending fraction $n$,

$$W(y, n) = n u \left( \frac{y}{n} \right) + (1 - n) u \left( \frac{R(1 - y)}{1 - n} \right)$$ \hspace{1cm} (3)

- Consider time-consistent or subgame-perfect equilibria: CB sets optimal $y(n)$, given $n$.
- Let $P_1^* = M/x_1^*$ be the price level at efficient outcome, when $n = \lambda$. 


Price Stability and Efficiency: $\bar{P} = P_1^*$. 

Figure: subgame-perfect liquidation policies $y_\alpha(n)$ and pricing implications.

- At $n = \lambda$: All levels $\alpha$ reach $y^*$ (because $P_1(\alpha) = \bar{P}$)
- For $\alpha$ near 1: run-deterring (for $n < 1$).
- For $\alpha$ near 0: subgame-perf liquidation policies give rise to runs.
- **Trilemma 2**: no Monetary Trust.
Price Stability and Efficiency, $\alpha \to 0$.

- We call this “Partial Price Stability”.
- Also arises for: fully sticky prices, unless all is sold.
- **Trilemma 2**: no Monetary Trust.

Figure: subgame-perfect partially price stable policies.
Price Stability and Monetary Trust

For run-deterring policies: need to raise the price stability target $\bar{P}$.

- Given $\alpha$: Compute the smallest $\bar{P}(\alpha) \geq P_1^*$ so that the subgame-perfect liquidation policy is run-deterring following every subgame $n > \lambda$.

- **Trilemma 3**: no Efficiency.
We call this “Full Price Stability”.

Also arises for: fully sticky prices, unconditionally.

Or: CB only invests in “storage”, i.e. short-term govt bonds.

Vollgeld, Narrow Banking, Chicago Plan.

**Trilemma 3**: no Efficiency.

Figure: Subgame-perfect fully price stable policies.
Why not change the money supply in $t = 1$?

- **Also allow** for state-contingent $M(n)$ in $t = 1$.
- Suppose $y(n) \equiv y^*$. To maintain price stability at some $\bar{P}$:
  \[ nM(n) = \bar{P}y^* = \lambda M(\lambda) \]  
  \[ (4) \]

- Total money spent in $t = 1$ is constant. Money spent per agent decreases with $n$. CB commits itself to **reduce** quantity of money in response to demand shock and spending spree.

- Implementations:
  - State-contingent money supply.
  - Taxation of individual money holdings.
  - Suspension of spending: only some of the money can be used.
  - Change $i(n)$? Or OMOs, i.e. sell bonds? Won’t do the trick: $i(n)$ does not impact real allocation.

- “Suspension of convertibility” becomes “**Suspension Of Spendability**” or **SOS**.

- No runs, stable prices! Problem solved?

- Doubtful. **SOS will undermine trust in monetary system**.

- With general $y(n)$: run issues as before. Agents only care about real allocation. Money is neutral.
Jacklin-inspired solution

- Jacklin (1987) has proposed to issue “equity” instead of deposits.
- Here: in period $t = 0$, do not provide agents with cash. Rather, provide them
  1. with a short-maturity bond, paying $D_1 = \bar{P} y^*$ in $t = 1$.
  2. with a long-maturity bond, paying $D_2 = \bar{P} R (1 - y^*)$ in $t = 2$.
- Agents trade bonds in period $t = 1$.
- Now, the liquidation policy $y(n) \equiv y^*$ will be efficient, financially stable and price stable!
- Trinity resolved? Runs always avoided?
- Not quite. Whether a “run” occurs or not, depends entirely on the real liquidation policy, in contrast to Jacklin.
- Not subgame perfect: given $n$, $y(n) = y^*$ is not optimal.
- For that and to get $P_1 \equiv \bar{P}$, introduce a discount window at the central bank, to obtain cash in $t = 1$ for the long-maturity bond.
- Promising approach! Match CBDC bonds to asset side!
Conclusions

In a nominal banking model for a central bank and its CBDC.
- The central bank can always deliver on its nominal obligations.
- But: “spending runs” can still happen.

We show the **CBDC Trilemma**

- Implement social optimum, no runs, **but** threaten inflation.
- Keep prices always stable: no runs, **but** inefficient (“Vollgeld”).
- Keep prices mostly stable: efficiency, **but** runs may happen.