Central Bank Digital Currency: When Price and Bank Stability Collide

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Abstract

When a central bank introduces a central bank digital currency, it may lead to an erosion of deposits as the stable funding base of banks and result in challenges regarding the asset side of the central bank. We study the resulting economy and tradeoffs in a stylized nominal version of the Diamond-Dybvig (1983) model. The central bank is now involved in maturity transformation, if it so chooses. We posit that the central bank pursues three objectives: price stability, economic efficiency and financial stability. We demonstrate that a CBDC Trilemma arises: out of these three objectives, the central bank can achieve at most two. In particular, a commitment to price stability can cause a run on the central bank. mplementation of the socially optimal allocation requires a commitment to inflation.

Keywords: CBDC, currency crises, monetary policy, bank runs, financial intermediation, central bank digital currency, inflation targeting
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1 Introduction

Many central banks and policymaking institutions are openly debating the implementation of a central bank digital currency or CBDC.[1] With a CBDC, households will have access to an electronic means of payment and, thus, an attractive alternative to traditional deposit accounts.[2] One may fear that deposits as a stable funding base for the private banking system will erode. Some have proposed to make CBDC accounts sufficiently unattractive and limited in scope or to have private banks become more competitive in order to solve this threat of disintermediation.[3] In this paper, instead, we think through the consequences of disintermediation to its end.

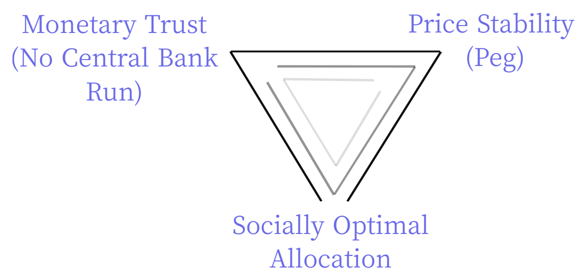


Figure 1: *CBDC Trilemma: For the consolidated central bank, it is impossible to attain all three objectives at a time. When one objective is fixed, at least one other objective has to be sacrificed.*

We study the resulting economy and tradeoffs in a stylized nominal version of the Diamond-Dybvig (1983) model. The central bank is involved in maturity transformation. We posit that the central bank pursues three objectives: assure price stability, support economic efficiency, and secure financial stability. As the main contribution of the paper, we demonstrate that a CBDC Trilemma arises, see figure 1: out of these three objectives, the central bank can achieve at most two. We show this impossibility result by fixing one central bank objective and demonstrating that at least one other objective is violated. We formally flesh out the following argument. Central banks have traditionally operated as "narrow banks," mainly holding highly liquid government debt on its asset side. When a CBDC becomes a wide-spread substitute for traditional bank deposit accounts, such a narrow balance sheet may maintain price stability and financial stability but will result in a loss of allocative efficiency as private sector funding dries up, hurting the second objective, see section 5.2. One might thus alternatively contemplate avenues for the central bank to refund the private financial sector[4] by either purchasing privately issued and appropriately regulated bank

[1] See Barrdear and Kumhof, 2016; Bech and Garratt, 2017; Chapman et al., 2017; Lagarde, 2018; Ingves, 2018; Kahn et al., 2019; Davoodalhosseini et al., 2020; Auer and Böhme, 2020; Auer et al., 2020, Group of Thirty, 2020 and Board of Governors of the Federal Reserve System, 2022).

[2] As Fernández-Villaverde et al. (2020) show that a CBDC offered by the central bank may be such an attractive alternative to private bank deposits that the central bank becomes a deposit monopolist, further consolidating its role as a financial intermediator.

[3] This argument has been made in particular by Darrell Duffie, see for instance Duffie et al. (2021). For a more formal analysis, see e.g. Monnet et al. (2021).

[4] This is a version of the equivalence argument by Brunnermeier and Niepelt, 2019, that a central bank can restore

bonds[5], see section 9, or by financing the real economy directly. This choice, in turn, implies that the central bank enters the business of financial intermediation and maturity transformation, traditionally reserved for the private sector. Maturity transformation enhances efficiency. However, as Diamond and Dybvig (1983) have shown in their banking model with real demand-deposits, maturity transformation gives rise to the possibility of bank runs and financial instability. We show in our nominal demand-deposit model here how financial instability in the form of a spending run[6] on the goods supply can result when the central bank is committed to keeping prices stable, see section 5.3. We call this a "run on the central bank": while the central bank can obviously always honor its nominal obligations, agents may fear the erosion of real resources available for purchase against their nominal CBDC balances. Therefore, a central bank run manifests itself as a collective spending spree where agents who have no instantaneous consumption needs nevertheless spend their CBDC balances on goods because they expect the real value of currency to decline. In that case, CBDC forfeits its purpose as the store of value. In such a run, issuing additional amounts of CBDC not only does not help but it makes inflation worse. The aggregate spending behavior at a given goods supply impacts the price level via market clearing. Therefore, a central bank run will manifest itself as a run on the price level. Likewise, nominal interest rates on CBDC only change price levels in the future when the interest is paid out rather than stopping the run now. The ability of a central bank to issue as much CBDC as desired is a useless tool against such runs, see section 6. Efficient maturity transformation in combination with a strong price stability objective will thus violate the third objective, financial stability. Financial stability can be achieved by threatening price instability in case of a run. Indeed and as our first result, we show that the central bank can implement the efficient allocation in dominant strategies and deter central bank runs ex ante but only via an inflation threat, see section 4. The inflation threat violates the first of the three objectives, should a run occur. We argue in section 5.1, that this inflation threat is neither a credible time-consistent nor a subgame-perfect commitment by a central bank that cares about price stability.

The issue of a central bank possibly engaging in maturity transformation arises specifically with CBDC and would typically not arise in the current cash-based system because electronic substitutes for cash are the prerogative of the private sector and traditional bank deposits fund financial intermediation. Moreover, the speed at which spending decisions will be feasible with a CBDC makes the threat to financial stability more dramatic than would be possible in a world based on cash

the original no-CBDC equilibrium by channeling funds to the same recipients.

[5] Central banks have engaged in large-scale, long-term lending to the economy ("quantitative easing") since the financial crises in 2008-2009. Refunding the banking sector with the introduction of a wide-scale CBDC will considerably enlarge these activities.

[6] As a parallel, one may think about the "electronic dollars" that many universities issue to faculty and students in their ID cards for purchases on campus. One can spend "electronic dollars" in different campus locations, such as vending machines and food courts, but one cannot "withdraw" the "electronic dollars" or transform them into other assets.

and deposits. Together, this implies that the arguments in this paper arise in particular for the introduction of a CBDC on a wide scale, and not otherwise.

We investigate the robustness of our analysis by examining solutions for avoiding runs on private banks. Suspension of convertibility there turns into suspension of spendability here because CBDC is already the most liquid means of payment. We argue that such a suspension can work within the confines of our model but is likely to undermine the trust in the monetary system, see section 6. In section 7, we examine the proposal by Jacklin (1987) to prevent runs by trade in equity shares. We extend Jacklin (1987) to a nominal version and show how trade in CBDC-denominated shares can result in the achievement of all three objectives when carefully designing the asset and liquidation policy by the central bank. Central bank equity shares would be an unusual implementation of a means of payment, but worthy of consideration as CBDCs get introduced. In section 8, we examine a fuller version of our model where the central bank competes with private banks. We show that the competitiveness of the private banking system requires the interest on CBDC to undercut the interest on reserves by private banks when implementing the efficient outcome, see appendix 13. We discuss extensions to a token-based CBDC, synthetic CBDC and retail banking in section 10.

The preservation of their currency's purchasing power is a main reason why central banks exist. This price stability objective is written explicitly in many central bank's statutes, such as the Federal Reserve's 1977 "dual mandate" in the U.S. and Article 127 of the "Treaty on the Functioning of the European Union" regulating the ECB. Money as a store of value, that is, as a way to shift consumption into the future, is widely accepted today, even though the intrinsic value of common paper money is zero. Therefore, such monetary trust is fragile and should not be taken for granted.

The conflict of interest between a central bank's traditional role as guardian of the price level and its new role in financial intermediation is the focus of this paper. To focus on this tension, the starting point of our analysis is the foundational tenet of economics, that people accept currency only because it enables future consumption and that people do not care for money per se. Therefore, we emphasize the intertemporal consumption problem of an agent when evaluating the acceptance of money as a store of value. The wedge between the agents' money balances versus expectations of the real value of currency is crucial for realizing optimal intertemporal consumption bundles and maintenance of monetary trust. Financial intermediators play a large role in enabling optimal consumption patterns of agents via risk-sharing across time. This paper, therefore, studies a highly stylized model, namely a nominal version of Diamond and Dybvig (1983). Agents interact with the central bank directly, abstracting from firms and other financial intermediators in the analysis. On the contrary, the central bank here also comprises the functionality of firm and government investment in the real economy and the bank's role as a financial intermediator, in addition to the functionality of a traditional central bank. In that way, we follow Velasco (1996), Calvo (1988) and Obstfeld (1996) who also consider a consolidated central bank, however without the financial intermediation role, as modeled here. We discuss allowing for a competing private banking system in section 8 and the role of the financial system more generally in section 9, and demonstrate that the lessons from the analysis so far continue to hold.

Related literature

This paper contributes to three literature strands: the literature on financial intermediation via nominal contracts, the literature on central bank digital currency, and the macro literature on currency runs:

Building on the seminal Diamond and Dybvig (1983) model, we contribute to the literature on financial intermediation and bank fragility by stressing the central bank's role in liquidity transformation when issuing a CBDC that allows depositors to share idiosyncratic liquidity risk. Similar to Diamond and Dybvig (1983) and Ennis and Keister (2009), we study the micro incentives of depositors to spend from the bank. But unlike them, we employ nominal instead of real demand-deposit contracts, giving "the bank" an additional tool –the price level– to prevent runs.

Nominal demand-deposit contracts and their proneness to runs have previously been analyzed in the literature. In a banking model with money, Diamond and Rajan (2006) show that nominal deposit contracts and a flexible price level response protect banks from failure due to liquidity shocks. Via a similar mechanism, Skeie (2008) shows that runs on nominal bank deposits do not occur in the unique equilibrium because price levels adjust flexibly and prompt competitive firms to adjust the goods supply. Unlike Diamond and Rajan (2006), we share with Skeie (2008) the possibility of miscoordation and panic runs by depositors. Here, withdrawals are strategic, and the run-deterrence mechanism works via the flexible adjustment of goods prices to the demand by depositors. Unlike both Diamond and Rajan (2006) and Skeie (2008), we show how nominal contracts and flexible prices help with implementing socially optimal allocations. In a different type of model, Allen, Carletti, and Gale (2014) show that the socially optimal allocation can be implemented as a competitive equilibrium via firms and banks. We instead characterize all asset liquidation policies that implement the socially optimal allocation in dominant strategies. As the main modeling difference to Allen, Carletti, and Gale (2014), 'runs' are non-strategic there, and deposit withdrawals are caused by exogenous liquidity shocks. By contrast, spending behavior by agents is strategic here because patient types can decide to consume early if they anticipate that their nominal balances buy them more goods today rather than tomorrow. Strategic spending allows for the occurrence of self-fulfilling spending panics and a discussion about how to prevent them by designing central bank policy. Unlike in Skeie (2008), Diamond and Rajan (2006) and Allen, Carletti, and Gale (2014), we characterize the set of run-deterring liquidation policies but abstract from modeling competitive firm and bank behavior: The reason for doing so is to model a central bank that issues a CBDC to citizens and conducts maturity transformation directly. We, therefore, envision a strategic, consolidated central bank that assumes the powers of a private bank, the central bank, and firms. The strategic central bank interacts with strategic CBDC depositors directly and invests in the real economy. Real asset liquidation acts as a central bank control variable which allows her to steer the agent's incentives. Most importantly, we differ from Allen and Gale (1998), Diamond and Rajan (2006), Skeie (2008), Allen, Carletti, and Gale (2014) by discussing the tradeoff between the three central bank objectives and the resulting trilemma, see figure 1. One can think of Skeie (2008) as covering one of the three corners when the price stability objective is of no concern. By contrast, much of our analysis concerns the impact of that price stability objective on efficiency and financial stability. We consecutively fix one out of the three central bank objectives and show that at least one other central bank objective is violated. Jointly all three results form the main contribution of the paper, the CBDC trilemma. Barlevy, Bird, Fershtman, and Weiss (2022) modify and expand our analysis here. They show that lending of last resort is possible without creating inflation. In a real Diamond-Dybvig model, Ennis and Keister (2009) study the planner's time-inconsistency problem when a bank run is happening. They show that asset liquidation beyond the ex ante optimal amount can be ex post efficient. We study time-consistent liquidation in section 5.1. Apart from the fact that we employ a nominal demand-deposit framework, an additional important modeling difference is the absence of a sequential service constraint in our model.

Our paper contributes to a recent and fast-expanding literature on central bank digital currencies, such as Berentsen (1998); Böser and Gersbach (2019a,b); Brunnermeier and Niepelt (2019); Chiu, Davoodalhosseini, Hua Jiang, and Zhu (2019); Fernández-Villaverde, Sanches, Schilling, and Uhlig (2020); Ferrari, Mehl, and Stracca (2020); Keister and Sanches (2019); Niepelt (2020); Skeie (2019); Williamson (2019). Dirk Niepelt, ed. (2021) provides chapters by various authors, summarizing some of the frontier research. Auer, Banka, Boakye-Adjei, Faragallah, Frost, Narajan, and Prenio (2022) investigate the scope of CBDCs to enhance financial inclusion. Allen, Gu, and Jagtiani (2022) investigate the rise of e-CNY, the Chinese CBDC. Andolfatto (2021), Piazzesi and Schneider (2020), Gross and Shiller (2021) as well as Burlon, Montes-Galdon, Munoz, and Smets (2022) study the deposit base erosion in a dynamic model, when a CBDC is introduced. On the flip side, Monnet, Petursdottir, and Rojas-Breu (2021) argue that the introduction of a CBDC will force the banking system to become more competitive. We differ from this literature by paying particular attention to the asset side of the central bank and its role as financial intermediator when the deposit base at private banks has eroded and by pointing out the central bank's trade-off between efficiency, financial stability, and price stability.

This article is related to the first- and second-generation literature on self-fulfilling currency crises. Similar to Krugman (1979), a currency crisis is caused due to expectations of rationally behaving agents. Similar to Obstfeld (1984, 1988, 1996), multiple equilibria can arise due to self-fulfilling expectations. In Obstfeld (1996), a government holds foreign reserves to defend an exchange rate peg. The amount of foreign reserves and domestic currency holdings by the agents determine how resilient the government is against speculative currency attacks. High reserves can deter attacks completely whereas lower reserve holdings give rise to self-fulfilling currency attacks. In a different section of that paper, the government targets output and exchange rate stability subject to exogenous output shocks. The government can respond to shocks and maintain output high by devaluing its currency, that is, giving up the peg. Similarly, Obstfeld (1984) features exogenous shocks to domestic credit. Here instead, there is no exogenous randomness with output. Instead, output is endogenously set by the central bank by liquidating real assets following the endogenous spending decision by the agents (run on currency). Moreover, here, the central bank can deter the run on currency by credibly committing to abandon the peg whenever output is threatened in the short-run, see also Velasco (1996). In Calvo (1988), the government cannot commit to the real value of public debt and can repudiate via either taxation or inflating debt. The agents anticipate the government's repudiation, which may cause a self-fulfilling debt crisis. Unlike there, here, it is not the government but the spending agents who cannot commit. The central bank takes action, using repudiation as a threat to deter patient agents from running on the central bank, which requires currency to lose value in the short run. As the main difference to Calvo (1988), Obstfeld (1984, 1996), and Velasco (1996) our model emphasizes the maturity transforming role of the central bank for enabling optimal allocations via CBDC contracts , similar to Diamond and Dybvig (1983). Due to a liquidation externality, output is an endogenous function of both the agent's actions and the central bank's commitment to either price stability or the implementation of socially optimal allocations. Price stabilization via liquidation is costly because premature liquidation increases output at the expense of reducing output in the long-run. Due to this liquidation externality, short-term inflation can be socially optimal since it acts as an off-equilibrium path threat to deter speculation against the real value of currency.

2 The basic framework

Our framework builds on the classic Diamond and Dybvig (1983) model of banking. Time is discrete with three points in time $t = 0, 1, 2$, and no discounting. There is a $[0, 1]$-continuum of agents, each endowed with 1 unit of a real consumption good in period $t = 0$. Agents are symmetric in the initial period, but can be of two types in period 1: patient and impatient. An agent is impatient with probability $\lambda \in (0, 1)$ and otherwise is patient. The agent's type is randomly drawn at the beginning of period 1 and independently across agents. Types are private information. Since we have a continuum of agents, there is no aggregate uncertainty about the measure of patient and impatient types in the economy. Thus, λ also denotes the share of impatient agents. Impatient agents value consumption only in period 1. In contrast, patient agents value consumption in period $t = 2$. To make this precise, consider some agent $j \in [0, 1]$ and let c_t represent goods consumed by an agent j at period t. Preferences for agent j are then given by

$$U(c_1, c_2) = \begin{cases} u(c_1), & \text{if } j \text{ is impatient} \\ u(c_2), & \text{if } j \text{ is patient} \end{cases}$$

where $u(\cdot) \in \mathbb{R}$ is a strictly increasing, strictly concave, and continuously differentiable utility function over consumption $c \in \mathbb{R}_+$. We further assume a relative risk aversion, $-x \cdot u''(x)/u'(x) > 1$, for all consumption levels $x > 1$.

There exists a long-term production technology in the economy. For each unit of the good invested in $t = 0$, the technology yields either 1 unit at $t = 1$ or $R > 1$ units at $t = 2$. Additionally, there is a goods storage technology between periods 1 and 2, yielding 1 unit of the good in $t = 2$ for each unit invested in $t = 1$. Let $x_1 \geq 0$ denote the agent's real consumption when deciding to spend at $t = 1$, and let $x_2 \geq 0$ denote the agent's consumption when spending at $t = 2$.

2.1 Optimal risk sharing

Following Diamond and Dybvig (1983), we derive, first, the optimal allocation, when price stability considerations are absent. The social planner collects and invests the aggregate endowment in the long technology. Given that all agents behave according to their type, the social planner maximizes *ex-ante* expected utility from consumption

$$W = \lambda u(x_1) + (1 - \lambda)u(x_2) \tag{1}$$

by choosing (x_1, x_2), subject to the feasibility constraint $\lambda x_1 \leq 1$, and the resource constraint $(1 - \lambda)x_2 \leq R(1 - \lambda x_1)$. We call W the **allocative welfare** to distinguish it from the broader objective in equation (19), when price stability considerations are included. The interior first-order condition for this problem implies that the optimal allocation (x_1^*, x_2^*) satisfies:

$$u'(x_1^*) = Ru'(x_2^*). \tag{2}$$

Given our assumptions, the resource constraint binds in the optimum

$$R(1 - \lambda x_1^*) = (1 - \lambda)x_2^*. \tag{3}$$

This condition, together with equation (2), uniquely pins down (x_1^*, x_2^*) and delivers the familiar optimal deposit contract in Diamond and Dybvig (1983). Together with $R > 1$ and the concavity of $u(\cdot)$, equation (2) implies that the optimal consumption of patient agents is higher than the consumption of impatient ones: $x_1^* < x_2^*$.

Moreover, the depositors' relative risk-aversion exceeding unity and the resource constraint yield

$x_1^* > 1$ and $x_2^* < R$.[7]

Diamond and Dybvig (1983) show that a demand-deposit contract can implement the efficient allocation. A key feature of their analysis is the use of a "real" demand deposit contract (i.e., a contract that promises to pay out goods in future periods). Due to a maturity mismatch between real long-term investment and real deposit liabilities, the Diamond and Dybvig (1983) environment, however, also features a bank run equilibrium under which the social optimum is not implemented. Our main contribution is to show that a nominal contract can lead to the implementation of the efficient allocation in dominant strategies. In other words, runs do not occur along the equilibrium path. The key mechanism is that the central bank can set the price level, thereby controlling the wedge between real long-term investment and nominal deposit liabilities. The implementation in dominant strategies comes at a price, requiring flexibility of the price level.

3 A nominal economy

Consider now an economy with a social planner that uses nominal contracts to implement the efficient allocation.

Nominal contracts. The planner offers contracts in a unit of account for which it is the sole issuer. Because central banks have a monopoly on currency, the planner in our analysis can be equated with the central bank or any other monetary authority with the ability to issue currency. In this paper, we refer to the unit of account as a central bank digital currency (CBDC) or digital euros. For our analysis, we abstract from the existence of competing national or digital currencies and assume full functionality of the CBDC account and ledger system.

Agents who sign a contract with the central bank hand over their real goods endowment and receive CBDC balances in return. The most straightforward interpretation of our environment is to think of a CBDC as an account-based electronic currency in the sense of Barrdear and Kumhof (2016) and Bordo and Levin (2017), i.e., to think of a CBDC as being akin to a deposit account at the central bank. In Section 10, we show that the results of our paper largely carry over to a token-based system or hybrid systems. Agents can spend their CBDC balances by redeeming them at the central bank in exchange for goods. Spending therefore reduces the CBDC supply. As with physical euros, we impose the constraint that agents cannot hold negative amounts of a CBDC.

Timing. At $t = 0$, the central bank creates an empty account, i.e., a zero-balance CBDC account, for each agent in the economy. In the benchmark model, we assume that in $t = 0$, all agents sell their unit endowment of the good to the central bank in exchange for $M > 0$ units

[7] Following the proof in Diamond and Dybvig (1983),

$$Ru'(R) = u'(1) + \int_1^R \frac{\partial}{\partial x}(x \cdot u'(x))\, dx = u'(1) + \int_1^R (x \cdot u''(x) + u'(x))\, dx < u'(1) \tag{4}$$

by $-x \cdot u''(x)/u'(x) > 1$ for all x.

of digital euros, credited to that agent's account. The central bank then invests all goods in the long-term technology. We consider voluntary participation of the agents in central bank contracts in section 8.

In $t = 1$, agents learn their type and decide whether to spend their CBDC balances, M, or to 'roll them over'. In $t = 1$, agents also have access to the goods storage technology between $t = 1$ and $t = 2$.[8] The central bank contract imposes the constraint that an agent either spends all of her balances or none at all. Because types are unobservable, the central bank cannot discriminate between patient and impatient agents to deny a patient agent access to her balances. Let $n \in [0, 1]$ denote the share and measure of agents who decide to spend in $t = 1$. The central bank observes n and then decides on the fraction $y = y(n)$ of the technology to liquidate, supplying that according quantity in the goods market at the market-clearing unit price P_1. Notice that through the resource constraint, early liquidation of the technology reduces the remaining investment and, hence, the supply of goods in $t = 2$. That is, there is a real payoff externality, and the central bank's liquidation choice in $t = 1$ determines the real supply of goods for both of the periods $t = 1$ and $t = 2$. There is no free disposal, thus, all returns that accrue to the technology in $t = 2$ are offered in the goods market for purchase against CBDC. Given n, the central bank also chooses a nominal interest rate $i = i(n)$ to be paid in period 2 on the remaining CBDC balances. Each digital euro held at the end of $t = 1$ turns into $1 + i(n)$ digital euros at the beginning of $t = 2$. Notice that $i(n) \geq -1$, given that agents cannot hold negative amounts of digital euros.

In $t = 2$, the remaining investment in the technology matures so that the central bank supplies $R\,(1 - y\,(n))$ units of goods in exchange for the remaining money balances. The measure of depositors $1 - n$ who rolled over each have $(1 + i)M$ digital euros to spend on goods at a market-clearing price P_2. Figure 2 summarizes this timing.

Definition 1. *A central bank policy is a triple* $(M, y(\cdot), i(\cdot))$*, where* $y : [0, 1] \to (0, 1]$ *is the central bank's liquidation policy and* $i : [0, 1] \to [-1, \infty)$ *is the interest rate policy for every possible spending level* $n \in [0, 1]$*.*

Notice that M is not state-contingent. The logic here is that, traditionally, 1 dollar today is 1 dollar tomorrow. In Section 6, we discuss an extension where we allow M to be state-contingent. We restrict attention to strictly positive liquidation policies $y(\cdot) > 0$ to rule out equilibria where impatient agents do not spend CBDC early since no goods are supplied in the economy.

Market clearing. In periods 1 and 2, agents spend their money balances for goods in a

[8] Our model is equivalent to Diamond and Dybvig (1983), where storage between $t = 1$ and $t = 2$ does not exist, but where patient agents can also consume in $t = 1$.

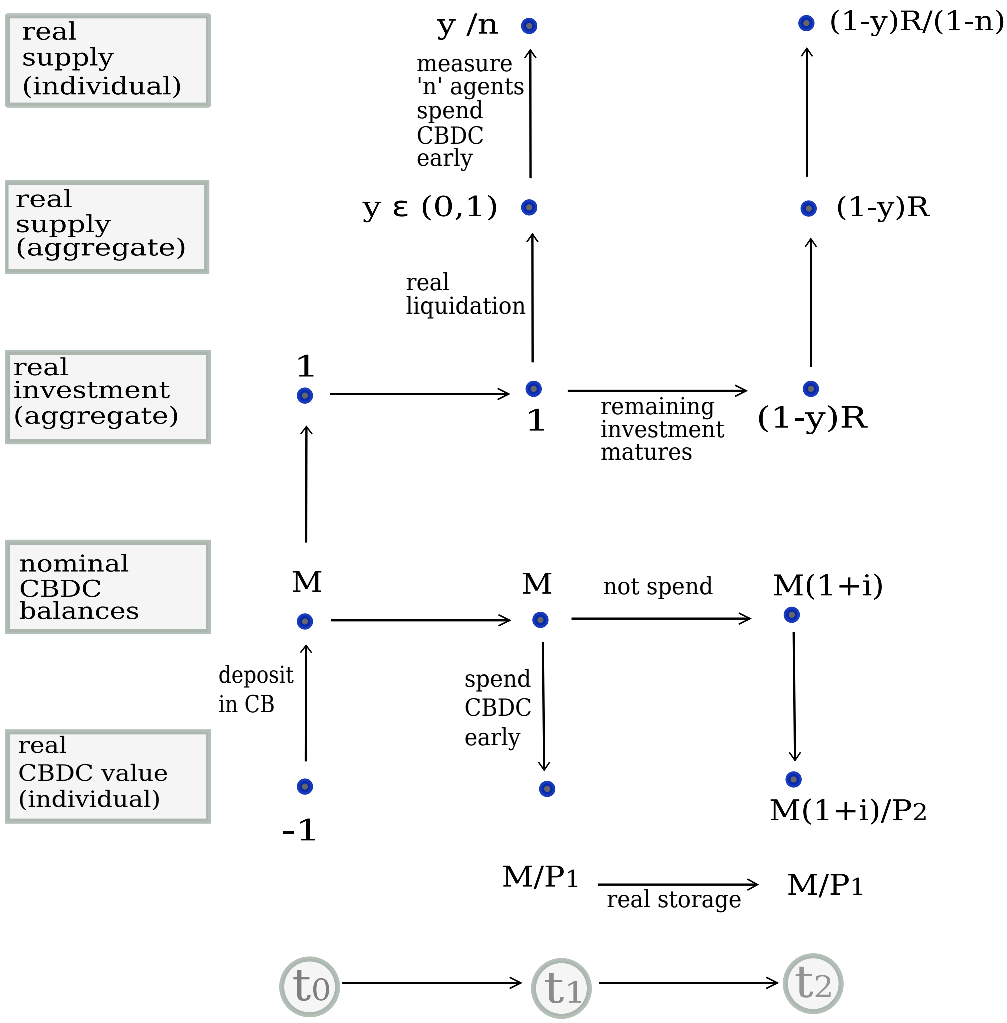


Figure 2: Nominal and real investment and contracts

Walrasian market. The market-clearing conditions are:

$$\underbrace{nM}_{\substack{\text{nominal CBDC}\\ \text{supply in } t_1}} = P_1 \cdot \underbrace{y(n)}_{\substack{\text{real goods}\\ \text{supply in } t_1}} \tag{5}$$

$$\underbrace{(1-n)(1+i(n))M}_{\substack{\text{nominal CBDC}\\ \text{supply in } t_2}} = P_2 \underbrace{R(1-y(n))}_{\substack{\text{real goods}\\ \text{supply in } t_2}}, \tag{6}$$

which take the form of the quantity theory equation in each period. As these equations reveal, a higher interest rate $i(n)$ results only in a higher price level P_2, when n and $y(n)$ remain unchanged. This is the standard Fisher relationship between nominal interest rates and inflation. Quantity theory then implies a higher nominal CBDC supply in t_2. Given aggregate spending n in $t = 1$, and the central bank's policy, these conditions determine the price level, $P_1 = P_1(n)$ and $P_2 = P_2(n)$, in each period:

$$P_1(n) = \frac{nM}{y(n)} \tag{7}$$

$$P_2(n) = \begin{cases} \frac{(1-n)(1+i(n))M}{R(1-y(n))}, & y(n) < 1 \\ \infty, & y(n) = 1,\ n < 1 \\ \in [0, \infty], & y(n) = 1,\ n = 1 \end{cases} \tag{8}$$

The special case $y(n) = 1$, $n < 1$ denotes the incidence where the goods supply in $t = 2$ equals zero while a demand for goods exists. The special case $y(n) = 1$, $n = 1$ denotes the incidence where both the goods supply and the goods demand in $t = 2$ equal zero. So far, we have not imposed price stability. Instead, the price levels flexibly adjust in aggregate spending and the central bank's liquidation policy. The central bank chooses the initial money supply before learning the measure of spending in the intermediate period. The central bank, however, controls the supply of goods, which is chosen conditional on the measure of spending. As a result, the central bank simultaneously, and interdependently controls the price level in period 1 and the real value of CBDC at time one versus time two.[9] The nominal interest rate allows the central bank to control the price level in period 2 independently of the price level in period 1. Because investment is real and since the intermediary is the central bank with a monopoly on the unit of account in which contracts are denominated, the liquidation policy is flexible. An additional CBDC euro spent does not necessarily translate into a specific, proportional raise in asset liquidation. Rather, liquidation is strategically directed to serve as a monetary policy tool.

Implied real contract. Patient agents have no instantaneous consumption needs in $t = 1$. Because storage of consumption goods is possible between $t = 1$ and $t = 2$, patient agents strategically spend their CBDC early or late. The individual real allocation that a patient agent can afford with her CBDC balances when spending early versus late is all that matters. The real value of the CBDC balances in $t = 1$ equals

$$x_1 = \frac{M}{P_1}, \tag{9}$$

[9] A private bank, in contrast, would need to take P_1, P_2 as given, which together with the observation n implies a unique liquidation $y(n, P_1)$. In a more detailed model, the central bank could determine the supply of goods by different instruments, such as calling loans to private banks or by moving the policy interest rate (as in New Keynesian models). The details of how that happens are not central to our argument.

while the real value of CBDC balances in $t = 2$ equals

$$x_2 = \begin{cases} \frac{(1+i(n))M}{P_2}, & P_2 < \infty \\ 0, & P_2 = \infty \end{cases} \tag{10}$$

Aggregate spending n and the liquidation policy $y(n)$ jointly determine the allocation of goods via the market-clearing conditions. The real allocations when spending in $t = 1$ versus $t = 2$ can therefore be rewritten via (7) and (8) as

$$x_1(n) = \begin{cases} \frac{y(n)}{n}, & n > 0 \\ \infty, & n = 0 \end{cases} \tag{11}$$

$$x_2(n) = \begin{cases} \frac{1-y(n)}{1-n}R, & n < 1 \\ 0, & n = 1, y(n) = 1 \\ \infty, & n = 1, y(n) < 1 \end{cases} \tag{12}$$

That is, for given aggregate spending, via her liquidation policy, the central bank directly sets the real value of CBDC in $t = 1$ *and* $t = 2$. Because all agents that spend CBDC in the same period have the same nominal expenses, and since the goods market is centralized, the real goods supply $y(n)$ is equally distributed across all spending agents in period 1, and the supply $R(1 - y(n))$ is equally allocated to all spending agents in period 2.[10]

Given an aggregate spending level $n \in [0, 1]$, for a patient agent $j \in [0, 1]$ it is optimal to 'spend' CBDC money balances M in $t = 1$ if $x_1(n) \geq x_2(n)$ while it is optimal to 'not spend' if $x_1(n) \leq x_2(n)$. Since $y(n) > 0$ for all $n \in [0, 1]$, and thus $x_1(n) > 0$ for all $n \in [0, 1]$ 'spend' is always optimal for an impatient agent. We restrict attention to pure strategy Nash equilibria with regard to the depositors' coordination game. Therefore, in the case $x_1(n) = x_2(n)$ and $\lambda < n < 1$, a mass $n - \lambda$ of patient agents spends their CBDC money balances in $t = 1$ and the remaining mass of agents $1 - n$ does not. This is consistent with optimal behavior. Our analysis can be extended to allow mixed strategy equilibria via the law of large numbers applied to the continuum of agents, see (Uhlig, 1996).

To summarize: in $t = 0$, the central bank announces and commits to a policy $(M, y(\cdot), i(\cdot))$, pinning down a spending-contingent real goods supply and an offer of a nominal contract $(M, M(1+i(\cdot)))$ in exchange for 1 unit of the good. All consumers accept the contract and the policy, meaning they have the option to spend either M digital euros in period 1 or $M(1 + i(n))$ digital euros in period 2, for every possible level of aggregate spending $n \in [0, 1]$. We discuss voluntary participation in contracts in Section 8.

In $t = 1$, the aggregate spending level n is realized. Finally, the central bank's policy, to-

[10] These equations remain intuitive even if $y(n) = 0$ or $y(n) = 1$. Therefore, we assume that they continue to hold, despite one of the price levels being potentially ill-defined or infinite.

gether with the market-clearing conditions, result in the real consumption amounts $(x_1(n), x_2(n)) = (\frac{M}{P_1}, \frac{M(1+i(n))}{P_2}) = \left(\frac{y(n)}{n}, \frac{1-y(n)}{1-n}R\right)$. Notice that the central bank is fully committed to carry through with its policy (M, y, i), regardless of which n obtains and independently of the implications for the price levels (P_1, P_2). We, therefore, define

Definition 2. *An* **equilibrium** *consists of a central bank policy $(M, y(\cdot), i(\cdot))$, aggregate spending behavior $n \in [0, 1]$ and price levels (P_1, P_2) such that:*

(i) The spending decision of each individual consumer is optimal given aggregate spending decisions n, the announced policy $(M, y(\cdot), i(\cdot))$, and price levels (P_1, P_2).

(ii) Given aggregate spending n, the central bank provides $y(n)$ goods and sets the nominal interest rate $i(n)$.

(iii) Given $(n, y(n), M)$, the price level P_1 clears the market in $t = 1$.
Given $(n, y(n), i(n), M)$, the price level P_2 clears the market in $t = 2$.

As a particular consequence of this equilibrium concept, the price levels (P_1, P_2) flexibly adjust to the aggregate spending realization and the announced central bank policy.

4 Implementation of socially optimal allocation

Given the preferences and technology that we postulated above, only the real allocation of goods matters to the two types of agents. If the central bank acts to enable optimal financial intermediation as in (Diamond and Dybvig, 1983), the implementation of the optimal risk-sharing arrangement (x_1^*, x_2^*) is the central bank's key objective when determining her policy. There is, consequently, no additional motive for the monetary authority to keep prices stable.

However, focusing only on real allocations is a narrow perspective. There is a vast literature arguing in favor of central banks keeping prices stable or setting a goal of low and stable inflation for reasons that are absent from our model.[11] Having to hold cash to accomplish transactions, such as in cash-in-advance or money-in-utility models, creates a whole range of distortions that can be minimized by deft management of the price level (think about the logic behind the Friedman rule). Rather than extending the model to include these considerations, for simplicity, we shall proceed by discussing the tradeoffs between achieving the optimal real allocation of consumption and the implications of such an effort for the stability of prices. We return to the price stability objective in section 5.

Runs on the central bank. A nominal contract, *per se*, does not rule out the possibility of a run on the central bank. Since impatient agents only care for consumption in $t = 1$, every

[11] For instance, stable prices minimize the misallocations created by nominal rigidities as in Woodford (2003).

equilibrium will exhibit aggregate spending behavior of at least λ, implying $n \geq \lambda$.[12] Patient agents, on the other hand, spend their CBDC balances strategically in $t = 1$ or $t = 2$. They spend in $t = 1$ if they believe that the central bank's policy implies a higher real value of CBDC balances in $t = 1$ rather than $t = 2$, $x_1 > x_2$. In that case, patient agents will use the storage technology to consume x_1 in period 2. Otherwise, patient agents will find it optimal to wait until the final period. We say,

Definition 3 (Central Bank Run)**.** *A run on the central bank occurs if not only impatient but also patient agents spend in $t = 1$, $n > \lambda$.*

In a bank run, the central bank is not running out of the item that it can produce freely (i.e., it is not running out of digital money). This feature distinguishes the run equilibrium here from the bank run equilibrium in Diamond and Dybvig (1983), in which a commercial bank prematurely liquidates all of its assets to satisfy the demand for withdrawals in period 1, therefore, ultimately running out of resources. Yet, the real consequences of a run on the central bank with nominal contracts can be similar to its counterpart in the model with real contracts. Importantly, by equations (11) and (12), a patient agent's optimal decision whether to run on the central bank, to spend or not, depends on the central bank's policy choices only through the liquidation policy $y(\cdot)$ and not via the nominal elements M and $i(n)$. By our equilibrium definition, the aggregate spending behavior n has to be consistent with optimal individual choices. These considerations imply the following lemma.

Lemma 4.1. *Given the central bank policy $(M, y(\cdot), i(\cdot))$,*

(i) The absence of a run, $n = \lambda$, is an equilibrium only if $x_1(\lambda) \leq x_2(\lambda)$.

(ii) A central bank run, $n = 1$, is an equilibrium if and only if $x_1(1) \geq x_2(1)$.

(iii) A partial run, $n \in (\lambda, 1)$, occurs in equilibrium if and only if patient agents are indifferent between either action, requiring $x_1(n) = x_2(n)$.

Given this equilibrium characterization for a given policy-implied real allocation, how can central bank policy attain the first-best allocation?

4.1 Implementation of optimal risk sharing via liquidation policy

By $(x_1^*, x_2^*) = \left(\frac{y^*}{\lambda}, \frac{R(1-y^*)}{1-\lambda}\right)$, the feasibility constraint $y \in [0, 1]$, and the optimality conditions in Section 2.1, the implementation of optimal risk sharing requires a liquidation policy to satisfy

$$y^*(\lambda) = x_1^*\lambda \in (\lambda, 1]. \tag{13}$$

[12] When $y(n) = 0$, impatient agents are indifferent between spending and not spending. To break ties, we assume that they spend their CBDC balances in $t = 1$.

That is, given that only impatient types spend, the central bank needs to liquidate enough of the technology to provide the optimal x_1^*. Similarly to Diamond and Dybvig (1983), the resource constraint $y \in [0,1]$ and $x_1^* > 1$ imply that optimal risk sharing is not feasible when all agents spend: If $n = 1$, then the goods provision would need to exceed one, $1 \cdot x_1^* > 1$ but the central bank cannot liquidate a share larger than one of the entire investment. Combining the previous derivation with Lemma 4.1, we arrive at the following lemma.

Lemma 4.2. *The central bank policy $(M, y(\cdot), i(\cdot))$ implements optimal risk sharing (x_1^*, x_2^*) in dominant strategies if the central bank*

(i) sets $y(\lambda) = y^*$ for any $n \leq \lambda$.

(ii) sets a liquidation policy that implies $x_1(n) < x_2(n)$ for all $n > \lambda$.

Given that only impatient agents are spending, $n = \lambda$, then a policy choice with $y(\lambda) = y^*$ implements the social optimum. That is, there is a multiplicity of monetary policies that implement the first-best since the pair $(M, i(\cdot))$ is not uniquely pinned down. While the pair $(M, i(\cdot))$ does not affect depositors' incentives, it has an impact on prices via equations (7) and (8). In the second part of Proposition 4.2, the central bank steers the incentives of the patient agents. Patient agents can but do not have to spend their CBDC balances at $t = 1$, and spend at $t = 2$ for sure only if for every possible spending level n the real allocation at $t = 2$ exceeds the allocation at $t = 1$. The central bank internalizes her depositors' decision making. It observes aggregate spending behavior n *before* it liquidates any asset. The central bank can, therefore, liquidate in a spending-contingent way, and is not committed to liquidating y^* if also patient agents are spending. Condition (ii) of this lemma corresponds to the classic incentive-compatibility constraint in the bank run literature: since the depositors' and the central bank's expectations are rational, and since the central bank policy is announced in $t = 0$, the depositors correctly anticipate the real value of their CBDC balances that would follow every aggregate spending behavior n. To deter patient agents from spending, the central bank can threaten to implement a liquidation policy $y(\cdot)$ that makes spending early sub-optimal *ex-post*, i.e., so that $x_1(n) < x_2(n)$ for every $n \in (\lambda, 1]$. If the monetary authority can credibly threaten patient agents by announcing such a liquidation policy, it deters them from spending ex-ante, and a central bank run does not occur in equilibrium. Therefore, in the unique equilibrium, only impatient agents spend, all patient agents roll over, and the social optimum is always attained.

The central bank implements "spending late" as the dominant equilibrium strategy for patient agents by fine-tuning the real goods supply via its liquidation policy, i.e., by making real asset liquidation spending-contingent.

Definition 4. *We call a central bank's liquidation policy $y(\cdot)$ "run-deterring" if it satisfies $y(n) <$*

$y^d(n)$ *for all* $n \in (\lambda, 1]$, *where the* **run-deterrence boundary** $y^d(n)$ *is defined by*

$$y^d(n) = \frac{nR}{1 + n(R-1)}, \quad \textit{for all } n \in (\lambda, 1]. \tag{14}$$

Such a liquidation policy implies that "roll over" is ex-post optimal $x_1(n) < x_2(n)$ *whenever patient agents are spending early* $n \in (\lambda, 1]$.

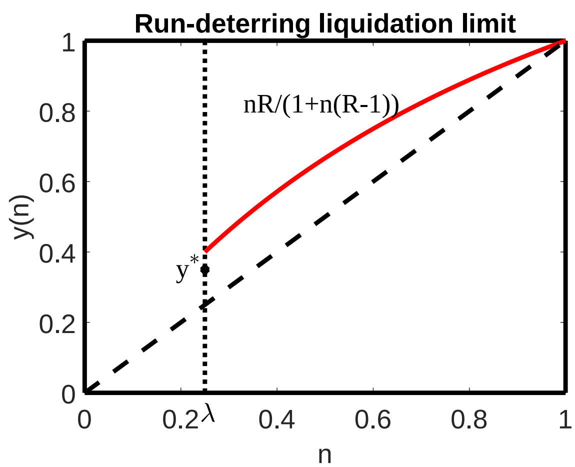


Figure 4: The upper bound of a "run-deterring" liquidation policy as a function of n is plotted in red. The bound starts at λ (for illustration purposes, here 0.25) because "impatient agents" will always spend. Note the social optimum, y^*, which is at λ in the n-axis and below the upper bound in the $y(n)$-axis and, to make interpretation easier, the 45-degree line in discontinuous segments.

The implementation of a run-deterring policy is only possible because the contracts between the central bank and the agents are nominal. The liquidation of investments in the real technology is at the central bank's discretion, thereby controlling the real goods supply and, for a given spending level, the real allocation in either time period. A spending-contingent liquidation policy implies a spending-contingent price level. In the case of real contracts between a private bank and depositors such as in Diamond and Dybvig (1983), in contrast, the real claims of the agents are fixed already in $t = 0$, thus pinning down a liquidation policy for every measure of aggregate spending n. In the case of high spending, rationing must occur. Similarly, in the case of nominal contracts between a private bank and depositors, the private bank has to take the price level as given, which then again pins down the liquidation policy. Alternatively, the price level adjusts via market clearing to high aggregate nominal spending (Skeie, 2008), while here it can serve as a strategic control variable.

As the main result of this paper,

Corollary 5 (Trilemma part I - No price stability)**.** *Every central bank policy* $(M, y(\cdot), i(\cdot))$, $n \in [0, 1]$ *with*

$$y(\lambda) = y^* \textit{ and } y(n) < y^d(n), \quad \textit{for all } n \in (\lambda, 1], \tag{15}$$

deters central bank runs and implements the social optimum in dominant strategies. Such a deterence policy choice requires the interim price level $P_1(n)$ to exceed the spending-dependent bound:

$$P_1(n) > \frac{M}{R}(1 + n(R-1)), \quad \textit{for all } n \in (\lambda, 1]. \tag{16}$$

Under a credible liquidation policy (15) all agents have a dominant strategy to spend if and only if the agent is impatient; otherwise they wait. Thus, under rational behavior, runs do not occur, and by $y(\lambda) = y^*$ the social optimum always obtains. That is, a strategic real supply shock enforced by the central bank *causes* a demand shock to CBDC spending that deters runs. The implementation, however, comes at a price. Feasibility of a run-deterring policy $y(\cdot)$ requires sacrificing price stability. By condition (16), the more agents spend, the larger the required price level threat to deter runs. Intuitively, to deter high levels of early CBDC spending, a high CBDC supply must meet a low supply of goods, so that, via market clearing, each good must have an exorbitantly high price. The threat has to be credible to deter runs *ex-ante*. Agents have to believe that *ex-post* the central bank will give up price stability whenever realized spending behavior is excessive. Only then do runs and inflation not occur on the equilibrium path. In that case, inflation arises via (16) only off the equilibrium path. It is not possible to avoid inflation as in (16) by introducing a nominal interest rate between $t = 0$ and $t = 1$, unless the interest rate is spending-contingent and thus random in $t = 0$. A random nominal interst rate brings new challenges, see the discussion in section 6.

In Diamond and Dybvig (1983), we learned the dilemma that offering the optimal amount of risk sharing via demand-deposit contracts requires private banks to be prone to runs. Thus, a bad bank run equilibrium also exists. Our result brings this dilemma to the next level. If the bank is a central bank equipped with the power to set price levels and control the real goods supply, then optimal risk sharing can be implemented in dominant strategies such that a bank run never occurs, but only at the expense of price stability.

Observe that by the optimality conditions and the resource constraint, $y^* < y^d(\lambda)$ holds and that the run-deterrence boundary $y^d(n)$ is increasing in n. Therefore, the constant liquidation policy

$$y(n) \equiv y^*, \text{ for all } n \in [0, 1] \tag{17}$$

implements optimal risk sharing in dominant strategies. There, however, exist infinitely many other run-deterring liquidation policies, see Figure 4.

Besides its simplicity, policy (17) is particularly interesting, since it is equivalent to the run-proof dividend policy in Jacklin (1987), which implements the social allocation via interim trade in equity shares. Section 7 discusses the connection of this result to our model and argues that Jacklin (1987) features a special case of a run-deterring policy. The policy (17) also implements the same allocation as the classic suspension-of-convertibility option, which is known to exclude bank runs in the Diamond-Dybvig world.

There is a subtle but essential difference, though, between suspension and our liquidation policy. Suspension of convertibility requires the bank to stop paying customers who arrive after a fraction λ of agents have withdrawn. By contrast, in our environment, there is no restriction on agents to spend their digital euros in period 1, and there is no suspension of accounts. Instead, it is the supply of goods offered for trade against those digital euros and the resulting change in the price level that generate the incentives for patient agents to rather prefer 'rolling over'. This reasoning also implies that, in our model, (nominal) deposit insurance will not deter agents from running on the central bank. Only a true commitment to a run-deterring policy is a guarantee or insurance of a positive real return on CBDC.

More concretely, low liquidation and thus a low goods supply push the price level P_1 above an upper bound that is increasing in the aggregate spending.[13] The low liquidation policy, on the other hand, deters large spending *ex-ante*, such that the high price level (16) is a threat that is realized only off-equilibrium. But each time we have an off-equilibrium threat, we should worry about the possibility of time inconsistency. In comparison with the classic treatment of time inconsistency in Kydland and Prescott (1977), the concern here is not that the central bank will be tempted to inflate too much, but that it would be tempted to inflate too little. The central bank can avoid suboptimal allocations by committing to let inflation grow whenever necessary. A similar concern appears in models with a zero lower bound on nominal interest rates. Eggertsson and Woodford (2003) have shown that a central bank then wants to commit to keeping interest rates sufficiently low for sufficiently long, even after the economy is out of recession, to get the economy off the zero lower bound (see also Krugman, 1998, for an early version of this idea). But once the economy is away from the zero lower bound, there is an incentive to renege on the commitment to lower interest rates and avoid an increase in the price level.

In our model, we assume that the central bank fully commits such that the threat is credible. But what if the central bank is concerned with price stability and, therefore, refuses to induce a high price level?

5 The classic policy goal: Price level targeting

There are many possible reasons why central banks view the stabilization of price levels or low inflation rates as one of their prime objectives. This price stability objective is written explicitly in many central bank's statutes such as the Federal Reserve's 1977 "dual mandate" in the U.S., and Article 127 of the "Treaty on the Functioning of the European Union" regulating the ECB. It also is a key objective for central banks in a substantial part of the literature on monetary economics,

[13] Our result resembles Theorem 4 in Allen and Gale (1998) and has a similar intuition. In Allen and Gale (1998), a central bank lends to a representative bank an interest-free line of credit to dilute the claims of the early consumers so that they bear a share of the low returns to the risky asset. In their environment, private bank runs are required to achieve the first-best risk allocation.

arising out of concerns regarding nominal rigidities or the opportunity costs of holding money. In subsection 5.1, we expand the allocative objective (1) with a penalty for deviating from a price target for period $t = 1$. We study the resulting subgame-perfect or time-consistent equilibria. Subsections 5.2 and 5.3 examine impose a particular form of the price stability objective for period $t = 1$ or both periods directly, and analyze the resulting conflicts with allocative efficiency and financial stability. We discuss, how the interest rate policy achieves stabilizing the price level in period $t = 2$, but is ineffective in affecting allocations or the price level in period $t = 1$.

5.1 Time-Consistency

It is not plausible that central banks would commit to potentially disastrous outcomes neither with respect to the allocation nor with respect to prices and inflation, should such runs actually take place. More formally, let us analyze the subgame of the central bank liquidating y, after observing the fraction n of agents who go shopping in period 1. All impatient agents are among them. The remaining agents are patient and put their purchased goods into storage. Given n and as in (1), the allocative welfare resulting from liquidating y is

$$W(y;n) = nu\left(\frac{y}{n}\right) + (1-n)u\left(\frac{R(1-y)}{1-n}\right) \tag{18}$$

where $x_1 = y/n$ are the real resources obtained by an agent spending in period 1 and $x_2 = R(1-y)/(1-n)$ are the real resources obtained by an agent spending in period 2. Our time-consistency analysis differs from Ennis and Keister (2009) since there the bank follows a sequential service constraint, and is thus committed to paying x_1^* to each withdrawing depositor. As the bank there learns that a run is happening, that is, as she serves $n > \lambda$ agents, suspension of convertibility is imposed in an ex post efficient way taking into account the risk that some impatient types may receive zero. Here, in contrast, CBDC spending happens fast so that the central bank observes all agents n that go shopping, that is, she observes the full length of the queue and knows about the occurrence of a run before liquidating assets. The central bank is, therefore, not committed to paying x_1^* but reoptimizes via $x_1(n) = y/n$, taking into account that all shopping agents receive an equal share of the goods supply and that all impatient types must be among the observed shoppers. Second, and more importantly, demand-deposits here are nominal, whereas Ennis and Keister (2009) considers a real banking model. Here, additional asset liquidation has, therefore, a stabilizing effect on the price level.

The allocative welfare (18) should be viewed as part of a larger macroeconomic environment, where price stability is desirable. Formally and following a large monetary policy literature, we, therefore, expand the allocative objective function (18) with a concern for price stability, expressed here by a quadratic loss of the resulting price $P_1(n) = nM/y$ deviating from some target $\bar{P}$, where $\alpha \in [0, 1]$ parameterizes the importance of the allocative objective relative to the price stability objective,

$$V(y;n,\bar{P}) \quad = \quad \alpha W(y;n) - (1-\alpha)\left(P_1(n) - \bar{P}\right)^2. \tag{19}$$

The solution to the time-consistent equilibrium or subgame-perfect equilbrium is computed by maximizing this central bank objective function, given n and the price target $\bar{P}$.

Consider first the case $\alpha = 1$, neglecting a concern for price stability. The maximization problem yields

$$u'\left(\frac{y}{n}\right) = Ru'\left(\frac{R(1-y)}{1-n}\right). \tag{20}$$

Suppose that $u(c)$ is CRRA, $u(c) = (c^{1-\eta} - 1)/(1-\eta)$. Equation (20) then yields

$$y(n) = \frac{n}{n + R^{(1/\eta)-1}(1-n)} \tag{21}$$

which is neither constant nor proportional to n. The implied period-1 price level is

$$P_1(n) = \frac{Mn}{y(n)} = (n + R^{(1/\eta)-1}(1-n))M \tag{22}$$

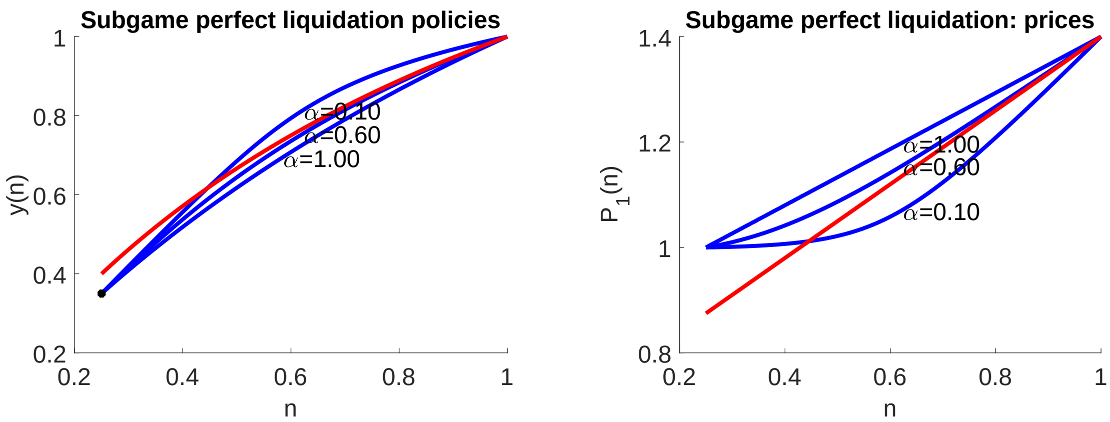
and thus affine-linear in n. The subgame-perfect solution is run-deterring for every $n < 1$[14], since patient agents always receive more, if they wait until period 2. This follows directly from (20) and the strict concavity of $u(\cdot)$, since $R > 1$ and x_1 and x_2 are the arguments of the derivative $u'(\cdot)$.

The situation changes, when a concern for price stability is included, i.e. when $\alpha < 1$. While it is straightforward to calculate the first order conditions, the solution can only be obtained numerically. We do so in figure 5 for a numerical example with $R = 2$, $\lambda = 0.25$ and picking $\eta = 3.25$ for the CRRA utility function $u(c) = c^{1-\eta}/(1-\eta)$ so that $x_1^* = 1.4$. The quantity of money $M = 1.4$ then implies the period-1 price level $P_1^* = 1$, in case that $n = \lambda$. The plot on the left in figure 5 shows the subgame-perfect liquidation policies $y_\alpha(n)$ for the three weights $\alpha = \{0.1, 0.6, 1\}$, and the period-1 price target $\bar{P} = P_1^*$. They are compared to the run-deterrence boundary $y^d(n)$, plotted in red.

All subgame-perfect liquidation policies go through the allocative optimal solution y^* at $n = \lambda$, since the price level coincides with the target $\bar{P} = P_1^*$ at that point[15]. For $\alpha = 1$, the subgame-perfect liquidation policy is below the red line and run-proof. However, as α decreases and the weight on the price stability objective increases, the liquidation policy eventually cuts through and exceeds the run-deterrence boundary at values below $n = 1$ as the left plot of figure 5 shows. This is more clearly visible in the plot on the right for period-1 prices implied by these liquidation policies. For $\alpha = 0.1$, the central bank puts a large weight on stabilizing prices. They thus drop below the

[14] At $n = 1$, full liquidation $y(n) = 1$ takes place, and $x_2 = 0 < x_1$ per (12).

[15] This is akin to the situation of "divine coincidence" of New Keynesian models when an output gap of zero coincides with achieving the inflation target.



Liquidation Policies — Prices

Figure 5: subgame-perfect liquidation policies and their pricing implication, with a comparision to the run-deterrence boundary.

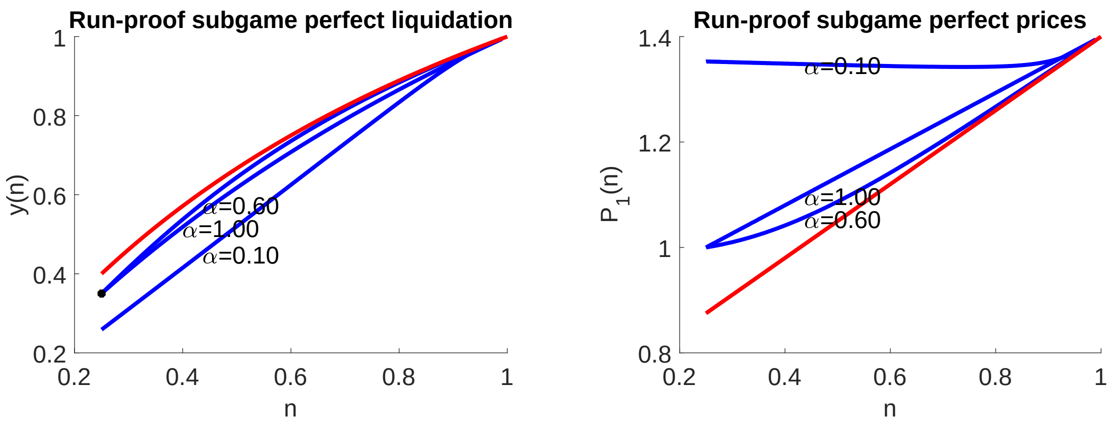
price boundary necessary to deter runs, indicated by the red line. In sum, while $\alpha = 0.6$ still yields a run-proof liquidation strategy, this is no longer the case for $\alpha = 0.1$.

A central bank may thus be concerned in period 0 about setting a price target $\bar{P}$ for period 1, which might escalate to runs. The solution is to set the price target $\bar{P}$ sufficiently high ex ante in period 0 in order to deter runs[16]. For each α, compute the minimal $\bar{P}(\alpha) \geq P_1^*$ compatible with a subgame-perfect run-proof liquidation policy. Figure 6 plots the results[17]. For $\alpha = 1$ and $\alpha = 0.6$, the price level $\bar{P} = P_1^*$ works fine. However, for $\alpha = 0.1$, the price target needs to be adjusted upwards in order to assure that the run-deterrence boundary, plotted as a red line, is no longer crossed. By design, the equilibrium prices now all lie above the run-deterring price bound, plotted as a red line, as the right plot of figure 6 shows. This, however, comes at a cost. As the left plot shows, the liquidation policies $y(n; \alpha)$ no longer achieve the efficient outcome y^* for $n = \lambda$, when $\alpha = 0.1$. Note also, that the liquidations $y_\alpha(n)$ and prices $P_{1;\alpha}(n)$ are no longer monotone functions of α for intermediate values of n, in contrast to figure 5.

Figure 7 compares these run-proof liquidation policies at $n = \lambda$ and the minimal price targets $\bar{P}(\alpha)$ as a function of the weight α on the allocative objective (18). The liquidation increases and the price target declines, until they eventually hit the levels y^* and P^* compatible with the allocative efficient solution.

[16] This may, at first glance, appear to be inconsistent with a central bank concerned about price stability. However, note that this price target is already known in period 0. Thus, if the price stability objective arises out of costs for adjusting prices between the unmodelled market in period 0 and period 1, prices in period 0 simply need to be set high enough. Alternatively, the central bank can adjust the money supply to make $\bar{P}$ compatible with some a priori given price level: it is only P in relationship to M that matters.

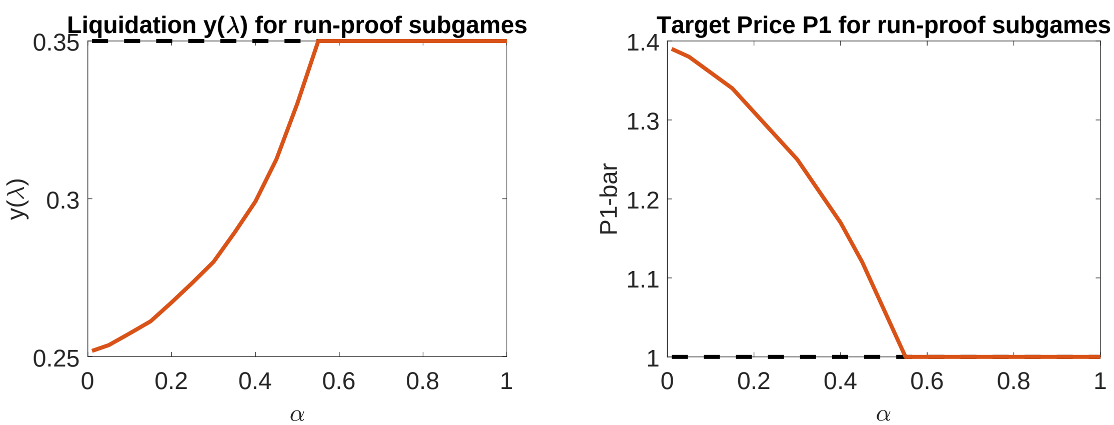
[17] For numerical reasons, we check whether the liquidation policy violates the stability limit at some $n \leq 0.99$. Economically, one may interpret this as a restriction that the central bank is not worried about a run if it takes more than $n = 0.99$ agents to spend in period 1 in the first place to make it sustainable.



Liquidation Policies

Prices $P_1(n)$

Figure 6: subgame-perfect liquidation policies and their pricing implication, with a comparision to the run-deterrence boundary, when $\bar{P}$ is set minimally so that the liquidation is run-proof for $n < 1$.



Liquidation Policies at $n = \lambda$

Price target $\bar{P}(\alpha)$

Figure 7: Adjustment of the price target $\bar{P}$ as a function of the weight on the allocative efficiency goal in order to achieve a run-deterring liquidation policy in the subgame-perfect equilibrium, provided that $n < 1$. The black dashed lines show the ex-ante efficient liquidation level $y^* = \lambda x_1^*$ and period-1 price P_1^*.

The limit $\alpha \to 0$ is particularly clean to analyze. In that case, the liquidation policies become linear until they hit full liquidation. Furthermore, the precise functional form of incorporating the price stability objective is not important as long as the same limit is reached. We analyze these policies in the next two subsections.

5.2 Full price stability

Definition 6. *We call a central bank policy*
*(i) P_1**-stable at level** $\overline{P}$, if it achieves $P_1(n) \equiv \overline{P}$ for the* ***price level target*** *$\overline{P}$, for all spending behavior $n \in [\lambda, 1]$.*
(ii) ***price-stable at level*** *$\overline{P}$, if it is P_1-stable at level $\overline{P}$ and if it achieves $P_2(n) \equiv \overline{P}$ for all spending behavior $n \in [\lambda, 1)$.*

For the definition of a *price-stable* policy, we exclude the total run $n = 1$, by absence of a demand for goods in $t = 2$, see definition 8. In our definition, price stability here is treated as a mandate and commitment to the price level $\overline{P}$ even for off-equilibrium realizations of n. From the definition, price stability at some level $\overline{P}$ implies P_1-stability at $\overline{P}$. Hence, the second price stability criterion is stronger.

What constraints does the price stability objective impose on central bank policy?

Proposition 7 (Policy under Full Price Stability)**.** *A central bank policy is:*

(i) P_1-stable at level $\overline{P}$, if and only if its liquidation policy satisfies:

$$y(n) = \frac{M}{\overline{P}} n, \text{ for all } n \in [0, 1] \tag{23}$$

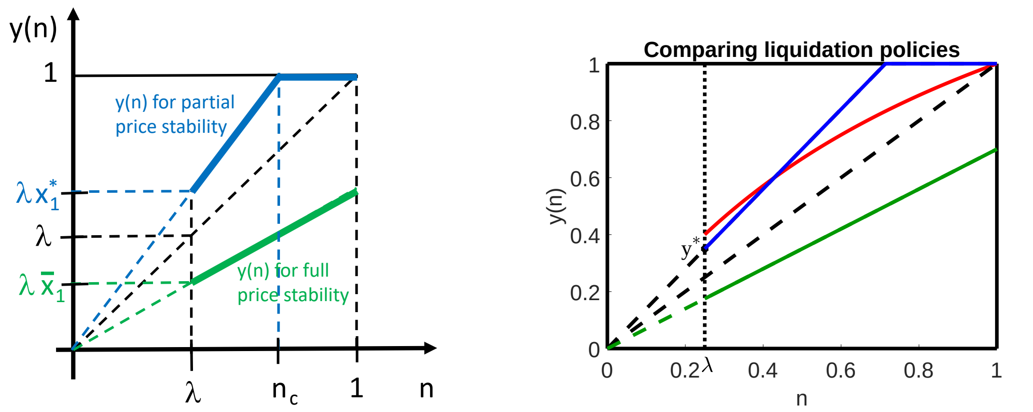
implying a real interim allocation:

$$x_1(n) \equiv \overline{x}_1 = \frac{M}{\overline{P}} \leq 1. \tag{24}$$

(ii) A central bank policy is price-stable at level $\overline{P}$, if and only if its liquidation policy satisfies equation (23), its price level satisfies (24), and its interest policy satisfies:

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1 - n} R - 1, \quad \text{for } n < 1 \tag{25}$$

A price-stable liquidation policy (23) requires asset liquidation in constant proportion to aggregate spending for all $n \in [0, 1]$; see the green line in Figure 8a, where we plot $y(n)$ for partial versus full price-stable liquidation policies. As a consequence, the individual real consumption x_1, and therefore the real value of CBDC balances are constant, regardless of aggregate spending behavior. The real allocation, however, undercuts 1 due to the resource constraint, since the central bank cannot liquidate more than the entire investment. As a consequence, a fully price stable policy can never implement the social optimum. By equation (24) and again due to the resource constraint, for a given money supply M, only price levels $\bar{P} \geq M$ can be P_1- stable or price-stable. The slope of the liquidation policy is, thus, equal to or below 1. In other words, the rationing problem shows



(a) Partial vs. full price-stable liquidation policies

(b) Price-stable versus run-deterring policy

Figure 8: Fully price-stable policies are run-deterring (below the red line) but do not reach the social optimum y^*. Partially price stable policies (which are not fully price stable) are not run-deterring but can reach the social optimum. Run-deterring policies cannot be fully price stable while reaching the social optimum, since all fully price stable policies must be linear in the spending level n while having a slope below or equal to one.

up indirectly through an upper bound on all possible price-stable central bank policies, imposing a low goods provision per realized spending level.

There is a caveat here. Should agents be able to operate the production technology on their own, then they can always assure themselves a real payoff of 1 in period $t = 1$ for every good stored in period $t = 0$. Thus, the only CBDC contract that prevails under voluntary participation would be a "green line" coinciding with the 45-degree line and a slope of 1, i.e. $\bar{P} = M$. Slopes below 1 are agreeable, however, if the central bank is the only entity capable of operating the real production technology or the only entity capable of intermediation with operators of that technology. The case $\bar{P} = M$ is further special since it is the only P_1- stable price level target at which the run equilibrium occurs since spending by all agents implies a total asset liquidation $y(1) = 1 = y^d(1)$.

This previous argument provides the second part of our trilemma:

Corollary 8 (Trilemma part II - No optimal risk sharing)**.** *If the central bank commits to a P_1-stable policy, then:*
(i) Optimal risk sharing is never implemented.
(ii) If $\bar{P} > M$, then the no-run equilibrium is implemented in dominant strategies. There is a unique equilibrium in which only impatient agents spend, $n^* = \lambda$. There are no central bank run equilibria.
(iii) If the central bank commits to a price-stable policy, then the nominal interest rate increases in n and is non-negative $i(n) \geq 0$ for all $n \in [\lambda, 1]$.

Intuitively, no runs take place under a P_1-stable policy since the real allocation in $t = 1$ is too low, causing all patient agents to prefer spending late.

5.3 Partial price stability

While price stability and the absence of central bank runs are desirable, the slope constraint (24) and the consequent failure to implement optimal risk sharing allocations is not. The implementation of the social optimum is impossible under full price stability. Recall that optimal risk sharing at $x_1^* > 1$ triggers potential bank runs in models of the Diamond-Dybvig variety: thus, part (ii) of the proposition above should not be a surprise. Demanding price stability for all possible spending realizations of n is thus too stringent. For attaining the social optimum, we therefore examine a more modest goal: a central bank may still wish to ensure price stability, but may deviate from its goal in times of crises. We capture this with the following definition.

Definition 9. *A central bank policy is*
(i) ***partially** P_1**-stable at level** $\overline{P}$, if for all spending behavior $n \in [\lambda, 1]$, either the policy attains the target $P_1(n) = \overline{P}$ for some* ***price level target*** *$\overline{P}$, or aggregate spending satisfies $n > \bar{P}/M$. In the latter case, we require full liquidation, $y(n) = 1$.*
(ii) ***partially price-stable at level*** *$\overline{P}$, if for all spending behavior $n \in [\lambda, 1]$, either the policy achieves $P_1(n) = P_2(n) = \overline{P}$ for some* ***price level target*** *$\overline{P}$, or aggregate spending satisfies $n > \bar{P}/M$. In the latter case, we require $y(n) = 1$.*

The idea of this definition is, for a given spending realization the central bank tries to attain the target price level whenever possible. When spending is, however, too high, the price target can no longer be reached in which case the central bank liquidates all assets. For a graphical illustration, see the blue line in Figure 8a. Obviously, P_1-stable central bank policies are also partially P_1-stable, and price-stable central bank policies are also partially price-stable.

Recall that only price levels above the money supply $\overline{P} \geq M$ can attain full price stability. We therefore now concentrate on lower price levels $M > \overline{P}$, since attaining optimality requires $1 < x_1^* = M/\bar{P}$. We additionally encounter a (weaker) feasibility constraint for partially price-stable policies. Since the central bank cannot liquidate more than the entire asset, $y(n) = x_1 n \in [0, 1]$ for all $n \in [\lambda, 1]$, it faces the constraint $\lambda x_1 \leq 1$. Feasibility, therefore, implies a lower bound on all possible partially stable price levels, $\overline{P} \geq \lambda M$. Partial price stability restricts central bank policies the following way:

Proposition 10 (Policy under Partial Price-Stability)**.** *Suppose that $M > \overline{P} \geq \lambda M$.*

(i) A central bank policy is partially P_1-stable at level $\overline{P}$, if and only if its liquidation policy satisfies:

$$y(n) = \min\left\{\frac{M}{\overline{P}}n, 1\right\}. \tag{26}$$

(ii) For every partially P_1-stable central bank policy at level $\overline{P}$, there exists a critical aggregate spending level $n_c \equiv \frac{\overline{P}}{M} \in (0,1)$ such that

(ii.a) For all $n \leq n_c$, the price level is stable at $P_1(n) = \overline{P}$ and the real goods purchased per agent in period $t = 1$ equal $x_1(n) = \overline{x}_1 = \frac{M}{\overline{P}} > 1$ while real goods purchased per agent in period $t = 2$ equal $x_2(n) = R(1 - \overline{x}_1 n)/(1 - n)$.

(ii.b) For spending $n > n_c$, the real goods purchased per agent in period $t = 1$ equal $x_1(n) = 1/n$ while $x_2(n) = 0$ and the price level $P_1(n)$ proportionally increases with total spending n: $P_1(n) = Mn$.

(iii) A central bank policy is partially price-stable at $\overline{P}$, if and only if its liquidation policy satisfies equation (26) and its interest rate policy satisfies:

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1 - n} R - 1, \qquad \text{for all } n \leq n_c. \tag{27}$$

For $n > n_c$, there is no supply of real goods in $t = 2$. Thus, $P_2 = \infty$ and $i(n)$ is irrelevant.

(iv) For a partially price-stable central bank policy at $\overline{P}$, there exists a spending level

$$n_0 = \frac{R\frac{\overline{P}}{M} - 1}{R - 1} = \frac{Rn_c - 1}{R - 1} \in [0, n_c), \tag{28}$$

such that the nominal interest rate turns negative for all $n \in (n_0, n_c)$. For $R < M/\overline{P}$, the nominal interest rate is negative for all $n \in [0, n_c)$.

Proposition 10 reflects the central bank's capacity to keep the price level and the real interim allocation x_1 stable for spending behavior below the critical level n_c. Indeed, the partial price stability policy may arise not from a concern regarding keeping prices stable, but rather from a commitment of the central bank to offering the optimal allocation x_1^* to all n agents shopping in period $t = 1$: the liquidation policy is then $y(n) = \min\{1, nx_1^*\}$. The stabilization of the price level requires the liquidation of real investment proportionally to aggregate spending by factor $M/\overline{P}$. At the critical spending level n_c, the central bank is forced to liquidate the entire asset to maintain the price level P_1 at the target. Since the central bank cannot liquidate more than its entire investment, price level stabilization via asset liquidation becomes impossible as spending exceeds the critical level n_c. For all spending behavior $n > n_c$, the real allocation to late spending agents is thus zero. Since liquidation can no longer increase, rationing of real goods occurs in $t = 1$, meaning that the price level has to rise in aggregate spending. Since the goods supply in $t = 2$ is zero, the price level in $t = 2$ explodes. One could argue here, that the price level in $t = 2$ can be maintained when setting a negative nominal interest rate at $i(n) = -1$. That would imply that zero CBDC balances meet zero goods in the market. But that would just be window dressing.

The spending level $n_0 < n_c$ is the level at which the real allocation to early and late spenders is just equal

$$x_1(n_0) = x_2(n_0) = \bar{x}_1. \tag{29}$$

Therefore, n_0 is the spending level at which the red and the blue line in Figure 8b intersect, and thus a partial run equilibrium exists. Notice that $x_2(n)$ declines in n for $n \in [0, n_c]$. Thus, if fewer than measure n_0 of agents spend, then not spending early, i.e. 'roll over' is optimal for patient agents. But for all spending realizations $n > n_0$, the allocation at $t = 2$ undercuts the allocation at $t = 1$: $x_2(n) < x_1(n)$, turning the real interest rate on the CBDC negative, and causing "spend early" to be a patient agent's optimal response to an aggregate spending behavior in excess of n_0. Consequently, self-fulfilling runs are possible as in Diamond and Dybvig (1983), and we obtain the following result as a corollary of Proposition 10:

Corollary 11 (Trilemma part III- Runs on the Central Bank (Fragility))**.** *Under every partially P_1-stable central bank policy with $M > \overline{P} \geq \lambda M$, there is multiplicity of equilibria:*

(i) There exists a good equilibrium in which only impatient agents spend, $n^* = \lambda$. In that case, there is no run on the central bank, the social optimum is attained and the price level target is attained, $P_1 = \overline{P}$.

(ii) There also exists a bad equilibrium in which a central bank run occurs, $n^* = 1$, the social optimum is not attained, and the price level target is missed.

Proposition 10 is in marked contrast to Proposition 7. One could argue that when banking is interesting, i.e., $x_1^* > 1$, then the goal of price stability induces the possibility of runs on the central bank, the necessity for negative nominal interest rates, and the abolishment of the price stability goal, if a run indeed occurs. In the context of banking with real contracts, Ennis and Keister (2009) already point out that the depositors' anticipation of too lenient but potentially ex post efficient regulatory policies may give rise to bank runs. Here, these runs have additional implications for the price level.

6 Money supply policy or suspension of spending

It is natural to ask why the central bank cannot resort to a more classic monetary policy to resolve the trilemma and attain price stability: expansion or reduction of the money supply. In this section, let us then allow for the possibility that M is state-contingent, i.e., M is chosen as a function of aggregate spending $M = M(n)$ at $t = 1$. Therefore, a central bank policy consists of three functions $(M(\cdot), y(\cdot), i(\cdot))$.

The analysis is now straightforward and easiest to explain for the case where the liquidation policy is not state-contingent, $y(n) \equiv y^*$. To maintain price stability at some level $\overline{P}$, market clearing demands

$$nM(n) = \overline{P}y^*. \tag{30}$$

As a result, the total money balances spent in $t = 1$ stay constant in n, implying

$$n\,M(n) \equiv \lambda\,M(\lambda), \quad \text{for all } n \in [\lambda, 1]. \tag{31}$$

But spending per agent alters, as does the total money supply $M(n)$. That is, the central bank would have to commit itself to **reducing** the quantity of money in circulation in response to a demand shock encapsulated in n: the more people go shopping, the lower are individual money balances. With policy (30), $y(n) \equiv y^*$ and $i(n) \equiv i^*$ chosen so that $P_2 = \overline{P}$, the central bank can now achieve full price stability, efficiency, and financial stability. The CBDC trilemma appears to be resolved. There are several ways of thinking about this.

State-contingent money supply. A first approach is to make the amount of CBDC balances available for shopping in $t = 1$ state-contingent. Having such CBDC accounts with random balances is an intriguing possibility: it is quite impossible with paper money but fairly straightforward with electronic forms of currency. A different interpretation of this approach is to think in terms of a state-contingent nominal interest rate paid on CBDC accounts between $t = 0$ and $t = 1$. One should recognize that both of these routes are a bit odd, and contrary to how we usually treat money and interest rates. As for money, a dollar today is a dollar tomorrow: changing that amount in a state-contingent fashion probably risks severely undermining the trust in the monetary system, and trust is key for maintaining a fiat currency. As for interest rates, it is commonly understood that interest rates are agreed upon before events are realized in the future. A state-contingent interest rate turns accounts into risky and equity-like contracts, likewise undermining trust in the safety of the system (see, nonetheless, Section 7 for trade in equity).

Helicopter drops. A third way to think about the state-contingent nature of M corresponds to a classic monetary injection in the form of state-contingent lump-sum payments ("helicopter drops") $M(n) - \bar{M}$ (or taxes, if negative), compared to some original baseline $\bar{M}$. If one wishes to insist that $M(n) - \bar{M} \geq 0$, i.e., only allowing helicopter drops, then the central bank would choose $\bar{M} \leq M(1)$ as payment for goods in period $t = 0$ and thus always distribute additional helicopter money in the "normal" case $n = \lambda$ in period 1. Notice that distributional issues would arise in richer models, where agents are not coordinating on the same action, thereby distorting savings incentives.

Suspension of spending. With an account-based CBDC, there is an additional and rather drastic policy tool at the disposal of the central bank: the central bank can simply disallow agents to spend (i.e., transfer to others) more than a certain amount of their account. In other words, the bank can impose a "corralito" and suspend spending. This policy is different from the standard suspension of liquidation, as the amount of goods made available is a policy-induced choice that still exists separately from the suspension-of-spending policy. Notice also that "suspension of spending" should perhaps not be called "suspension of withdrawal." Since there are only CBDC accounts and they cannot be converted into something else, the amounts can only be transferred to another account. With the suspension-of-spending policy, the central bank could arrange matters in such a way that not more than the initially intended amount of money $\lambda M(\lambda)$ will be spent in period 1; see equation (31). In practice, the central bank would then either take all spending requests at once and, if the total spending requests exceeded the overall threshold, impose a pro-rata spending limit. Alternatively, it could arrange and work through the spending requests in some sequence (first-come-first-served), thereby possibly imposing different limits depending on the position of a request in that queue.

Monetary neutrality. Last but not least, a state-contingent money supply cannot replace the central bank's liquidation policy as the active policy variable. Not only price targeting, but also the deterrence of runs is an objective of the central bank for attaining optimal risk sharing.

A state-contingent money supply, however, does not impact the agent's spending behavior: the individual agents exclusively care for their individual real allocation at $t = 1$, y/n, versus $t = 2$, $R(1-y)/(1-n)$. These allocations are independent of nominal quantities (M, P_1). That is, money is neutral. Given a realization of an individual real allocation y/n, the identity:

$$\frac{y}{n} = \frac{M(n)}{P_1} \tag{32}$$

pins down a relationship that needs to hold between the money supply and the price level that clears the market. The central bank can implement all money supplies and price level pairs (M, P_1) that satisfy equation (32). And as soon as the price level goal P_1 is pinned down, contingent on the realization $\frac{y}{n}$, the money supply that solves equation (32) is unique. But in equation (32) the classic dichotomy holds, and the choice of the right-hand side (M, P_1) cannot alter the left-hand side, i.e., cannot alter incentives to run. Consequently, if the central bank wants to impact consumers' behavior to run on the central bank to implement the social optimum, it can only do so by altering the real goods supply y through adjustment of its liquidation policy.

In summary. Given the previous discussion, a state-contingent money supply strikes us as odd monetary policy. First, the usual inclination for central banks is to accommodate an increase in demand with a rise, rather than a decline in the money supply. A central bank that reacts to an increase in demand by making money scarce may undermine trust in the monetary system. In particular, and needless to say, a spending suspension might create considerable havoc; the experience in Argentina at the end of 2001 provides ample proof. Even if this was not the case, monetary neutrality implies that adjusting the money supply does not affect the run decisions of agents. Therefore, we think that this particular escape route from the CBDC trilemma needs to be treated with considerable caution.

7 Nominal Jacklin (1987): CBDC balances as equity shares

For the real banking model by Diamond and Dybvig (1983), Jacklin (1987) demonstrates that optimal risk-sharing can be implemented in a run-proof way if banks offer shares in equity instead of demandable deposits. For Jacklin's mechanism to work, the bank pays real dividends in $t = 1$ and $t = 2$ to all agents. The dividend payments are predetermined in $t = 0$, and therefore imply a specific amount of asset liquidation, and thus aggregate consumption in $t = 1$. To optimize intertemporal consumption, the agent's in Jacklin can trade claims on future dividends in return for claims on contemporaneous dividends, which, by the design of the dividends, happens in an incentive-compatible way depending on each agent's type realization.

The Jacklin solution will work here, too, if the central bank provides agents with claims to the real resources, circumventing nominal currencies altogether. In fact, the liquidation policy discussed around equation (17), which implements the social optimum in dominant strategies via CBDC demand deposits, equals the real allocation that is implemented in Jacklin (1987) via his proposed dividend policy

$$D_1 = \lambda\, c_1^* = y^*, \quad \text{for all } n \in [0,1]. \tag{33}$$

That is, the dividend policy proposed in Jacklin (1987) is a special case of a run-deterring liquidation policy.

The focus of this paper, however, is on a world where a (nominal) CBDC is a priori necessary for acquiring goods. What if Jacklin's dividend payments were nominal? Must inflation arise there too for deterring runs? And what is a run on a bank under trade in equity shares?

To answer these questions, assume the extreme case where agents hand over their real goods endowment in $t = 0$ in return for nominal equity shares in the central bank.[18] All agents receive a nominal dividend payment D_1 in units of CBDC at $t = 1$ and a dividend D_2 units of CBDC in $t = 2$. The central bank predetermines and thus fixes both nominal dividends in $t = 0$. Call (D_1, D_2) the central bank's dividend policy. We impose for now that D_1 can only be spent in $t = 1$ and D_2 can only be spent in $t = 2$, but we will relax this restriction later. One can implicitly assume here that D_1 expires and, unlike a demand-deposit, cannot be rolled over to $t = 2$. As before, the central bank pools the real goods for investment in the real technology in $t = 0$. In $t = 1$, types realize and impatient types want to consume as much as possible in $t = 1$. Given their type, the agents decide whether to consume in $t = 1$ or $t = 2$. As in Jacklin (1987), a market for trading claims on dividends is assumed to exist. Unlike in Jacklin (1987), dividends are nominal here and therefore cannot be consumed directly. Instead, we assume the market for claims opens prior to the goods market. Agents who want to consume in $t = 1$ can can sell their claims on a late nominal dividend

[18] Recall that, historically, many central banks sold shares to the public at large that paid dividends. Even today, one can buy shares of the central banks of Japan and Switzerland. Note that claims on fixed nominal future payments are commonly also called bonds rather than equity. We continue with the terminology 'equity share' and 'dividends' to stay in line with Jacklin (1987).

D_2 in return for additional early nominal dividends D_1. Let $1+q$ be the price of one CBDC unit in $t=1$ denominated in terms of CBDC units in $t=2$. Note, q can be interpreted as a market interest rate. Let $n \in [0,1]$ denote the measure of agents who choose to trade their late CBDC dividends D_2 for early dividends. The market interest rate q is pinned down per the bond market-clearing condition

$$nD_2 = (1-n)D_1(1+q). \tag{34}$$

We assume that an agent can visit the goods market only once, either early in $t=1$ or late in $t=2$.[19] Therefore, agents that are indifferent between consuming early or late will not spend early CBDC dividends on goods without trading their claims on late dividends for additional early dividends. Therefore, and because D_1 can only be spent in period $t=1$, all agents that trade claims on late dividends for early dividends also spend the entire balance of early dividends on goods, that is, n is also the share of agents who spend their nominal CBDC dividends in the goods market. The central bank observes the share $n \in [0,1]$ of spending agents and liquidates a share $y(n) \in [0,1]$ of investment to provide goods in the market. In the goods market, the market-clearing price $P_1 = P_1(n)$ satisfies

$$D_1 = P_1(n)\, y(n). \tag{35}$$

In contrast to the nominal demand-deposit model (7), the quantity of money spent in period $t=1$ is now fixed and does not rise in proportion with n. Likewise, in $t=2$ the market-clearing price $P_2 = P_2(n)$ satisfies

$$D_2 = P_2(n)R(1-y(n)). \tag{36}$$

The $t=1$ real allocations per agent equals,

$$x_1 = \frac{y(n)}{n} = \frac{D_1}{P_1 n} \tag{37}$$

and in $t=2$

$$x_2 = \frac{R(1-y)}{1-n} = \frac{D_2}{P_2(1-n)}. \tag{38}$$

As before, the real allocation only depends on the liquidation policy y and not on nominal quantities. The definition of a run on the central bank is unchanged but reveals itself differently. Define

Definition 7.1 (Run on equity shares)**.** *A run on nominal equity shares is the incidence where also patient types shop for goods early $n > \lambda$.*

That is, some patient types are unwilling to trade their early nominal dividends for late nominal dividends, meaning that trade in dividends between the patient and impatient agent groups partially

[19] In the nominal demand-deposit contract model, this assumption was implicit in the assumption that an agent can either spend CBDC deposit balances early or late, but not both.

collapses. An **equilibrium** is now defined analogously to definition 2, but where the central bank policy announced in $t = 0$ equals $(D_1, D_2, y(\cdot))$.

Recall the run-deterrence boundary $y^d(n)$ per definition 4. Similar to proposition 5 and as before, the following result then follows immediately.

Proposition 12 (Deterring runs on equity shares)**.** *In the nominal equity share model,*

(i) A run on the central bank, $n > \lambda$, cannot be an equilibrium, if $y(n) < y^d(n)$ for all $n \in [0, 1]$.

(ii) A run on the central bank $n > \lambda$ is an equilibrium, iff $y(n) \geq y^d(n)$.

Why would a run on nominal equity shares occur here? In Jacklin (1987), dividends are real and predetermined in $t = 0$. Therefore, the real value of dividends is fixed there at one-to-one. By contrast, with nominal dividends, as in our main nominal demand-deposit model, asset liquidation is here decoupled from the money supply, meaning that the real value of dividends varies with the liquidation policy. Generically, predetermined nominal dividends offer no solution for deterring runs, all that matters is the liquidation policy $y(n)$.

The requirement on a run-proof liquidation policy in Proposition 12 (i) implies a particular design on the real value of early dividends via equation (35).

Remark 7.1 (Run-deterring price-dividend pairs)**.** *A price-dividend pair $(D_1, P_1(\cdot))$ deters runs on equity shares if*

$$\frac{D_1}{P_1(n)} < \frac{nR}{1 + n(R - 1)}, \quad \text{for all } n \in (\lambda, 1]. \tag{39}$$

Note, because the nominal dividend payments are predetermined in $t = 0$, they cannot depend on the share of shoppers n. Define the constant liquidation policy

$$\hat{y} := \frac{\lambda R}{1 + \lambda(R - 1)} \in (0, 1) \tag{40}$$

as the minimum of the right-hand side of (39).

Comparing the nominal dividend model to the nominal demand-deposit model, there exist different consequences for prices when comparing equation (35) to equation (7). Because dividends are predetermined in $t = 0$ and cannot be stored, we obtain

Lemma 7.1 (Price stability)**.** *Consider the central bank policy $(D_1, D_2, y(\cdot))$ with $D_1, D_2 > 0$. Every constant (demand-insensitive) liquidation policy $y(n) \equiv y \in (0, 1)$ for all $n \in [0, 1]$ implies constant price levels in $t = 1$ and $t = 2$, $P_1(n) = \bar{P}_1$, $P_2(n) = \bar{P}_2$ for all $n \in [0, 1]$.*

This result holds via market clearing (35) and (36). By contrast, if agents hold a nominal CBDC demand-deposit contract and total liquidation is constant in CBDC spending, $y(n) = const$, the price level still varies with the measure of goods shoppers because CBDC deposits can be rolled over to $t = 2$; see equations (7) and (8).

In particular, the trilemma can now be avoided by fixing the extent of asset liquidation at the socially efficient level.

Proposition 7.1 (No trilemma with nominal dividends)**.** *Consider the central bank policy* $(D_1, D_2, y(\cdot))$ *with* $D_1, D_2 > 0$*:*
(i) [run-deterrence and price-stability]: If the central bank sets a constant liquidation policy $y(n) = \tilde{y} \in (0, \hat{y}]$ *for all* $n \in [0,1]$*, she implements the stable price level* $P_1(n) \equiv \frac{D_1}{\tilde{y}} =: \bar{P}$ *in* $t = 1$ *for all* $n \in [0,1]$ *and simultaneously deters runs .*
(ii) [run-deterrence, price-stability, and social optimality]: If the central bank sets the constant liquidation policy $y(n) = y^*$ *for all* $n \in [0,1]$*, not only runs are deterred but the social optimum is implemented in dominant strategies. In addition, the price target* $P_1 = \bar{P}$ *is attained in* $t = 1$*. The trilemma vanishes.*
(iii) If the late dividend payment D_2 *additionally satisfies*

$$D_2 = \bar{P}R\,(1 - \hat{y}) \tag{41}$$

then the price target is also implemented in $t = 2$*.*

Result (i), follows directly from Lemmata 7.1, 7.1, bound (40) and market clearing (35). Part (ii) follows since $y^* < \hat{y}$ is feasible, and since $y^* = \lambda x_1^*$.

To conclude, generically, the nominal version of Jacklin (1987) is prone to runs. Therefore, whether agents receive CBDC via demand-deposit contracts or dividends makes no difference as to whether runs occur: all that matters is the liquidation policy $y(n)$. Runs are deterred, if $y(n) < y^d(n)$, for all $n \in [0,1]$. However, unlike in the nominal demand-deposit model, in the nominal dividend model the amount of CBDC spent in period $t = 1$ no longer depends on the spending fraction of agents. Therefore, price stability can be assured if the liquidation policy is constant. In particular, constant liquidation $y(n) \equiv y^*$ now avoids the trilemma entirely. This, however, corresponds to a very particular monetary system where CBDC is designed such that it can only be spent in the period when it is obtained as a dividend payment, that is, CBDC would require an expiration date.

8 Voluntary participation in CBDC and competition by private banks

The main model assumes that all consumers invest in a CBDC. It remains to clarify whether agents may be better off using the investment technology on their own, rather than relying on the central bank. This is an important question: if agents were to decide to stay in autarky and invest in the investment technology directly, they might have incentives to supply goods at the interim stage, thus, potentially undermining the central bank's liquidation policy. Similarly, if the outside option is not autarky but investing in deposits with a different, private bank, then the liquidation policy of that private bank has implications for the aggregate real goods supply at the interim stage, again impairing the effectiveness of the central bank's policy. We now discuss both.

8.1 Autarky and voluntary participation in a CBDC

Assume all but one agent invest in a CBDC. Assume that this single agent invests in the real technology at $t = 0$, yielding storage between $t = 0$ and $t = 1$, and yielding $R > 1$ when held between $t = 0$ and $t = 2$. Then, at $t = 1$, she would learn her type. If she is impatient, she will liquidate the technology, yielding 1 unit of the real good, and she would consume her good. She would not sell the good against nominal CBDC deposits, since she only cares about consumption at $t = 1$. In the case where she is impatient, she is worse off in comparison to an agent who invested in CBDCs with the central bank if the central bank offers optimal risk sharing and manages to implement a run-deterring policy. This is so, since under the latter, an individual impatient agent gets $x_1^* > 1$ real goods.

If the individual agent is patient, she will stay invested in the technology until time two. There, the technology yields $R > 1$ units of the good. The agent will, thus, be better off than under investment in a CBDC since $x_2^* < R$; see Section 2.1. But, in particular, also in the patient case, the individual agent will not offer goods for sale in the interim period, since liquidation and selling against a CBDC will only yield x_2^* in $t = 2$. Thus, in either case, patient or impatient, the agent who invests in autarky will not have an incentive to undermine the central bank's policy by increasing the goods supply in the interim period.

Does the agent prefer to remain in autarky rather than participating in the CBDC? *Ex-ante*, the risk-averse agent cannot know whether she will turn out to be patient or impatient. Diamond and Dybvig (1983) show that pooling of resources via banking can attain the social optimum under an absence of runs, while investment under autarky cannot. That is, the single agent is always better off investing in the CBDC account if the central bank offers optimal risk sharing and implements a run-deterring policy. Thus, participation in the CBDC account is individually rational.

What if the central bank runs a policy of full price stability at goal $\bar{P}$? In that case, our second main result, Corollary 8, shows that runs on the central bank do not occur but $x_1 \leq 1$. Thus, for all $x_1 < 1$, investing in a CBDC is dominated by investing in autarky. Voluntary participation thus requires $x_1 = 1$ or $M = \bar{P}$, implying $x_2 = R$. The agent is then indifferent between investing in a CBDC and staying in autarky. Yet, if she stayed in autarky, she will not undermine the central bank's liquidation policy for the reasons above.

In the case of a partial price-stable policy, the situation is as in Diamond and Dybvig (1983). *Ex-ante*, the agent cannot know whether a run occurs or not. Conditional on the no-run equilibrium, we implement the social optimum and the agent is better off investing in a CBDC. But conditional on the run equilibrium, she was better off in autarky. From within the model, it is not possible to attach likelihoods for each equilibrium.

8.2 Can private banks undermine the central bank's policy?

The question of under what circumstances consumers prefer investing in a CBDC account with the central bank rather than investing in demand deposits with private banks, with implications for how both types of banks can coexist, is addressed in Fernández-Villaverde et al. (2020). In this section, we will analyze private banks' incentives to provide goods at the interim stage, *conditional on the coexistence of private banks with the central bank.*

Goods supply. If the central bank coexists with private banks, it controls the market of goods only partially, with the remainder of the real goods being supplied by commercial banks. As before, the measure of agents is normalized to one, divided between a share $\alpha \in (0,1)$ of agents who are CBDC customers at the central bank and a share $1-\alpha$ who are customers at private banks. Assume that all agents invest their 1 unit endowment in their corresponding bank and that the private banks invest in the same asset as the central bank does. Then, at $t=1$, the central bank can supply up to α goods via liquidation, while private banks can supply up to $1-\alpha$ goods.

Assume that there is one centralized goods market to which customers and banks have access. That is, CBDC depositors can spend CBDC balances on goods supplied by private banks and private bank customers can spend their private deposit balances on goods supplied by the central bank. Let n denote the total measure of spending agents across both customer groups at the central bank and private banks, given by

$$n = \alpha\, n_{CB} + (1-\alpha)\, n_P, \tag{42}$$

where n_{CB} is the total share of consumers at the central bank who spend, while n_P is the total share of consumers at the private bank who spend. Given total spending n in period $t=1$, let $y_P(n)$ be the share of assets liquidated by private banks. In contrast, let $y_{CB}(n)$ be the central bank's liquidation policy, i.e., the share of assets liquidated by the central bank. The total goods supply y in the centralized market at the interim stage is then:

$$y(n) = \alpha\, y_{CB}(n) + (1-\alpha)\, y_P(n). \tag{43}$$

Private deposit making. To collect investment in $t=0$, the private banks offer a nominal demand-deposit account in return for 1 unit of the real good. The private nominal accounts are denominated in units of the CBDC. Due to competition with the central bank, the private contract also offers M units of the CBDC in $t=1$ or $M(1+i(n))$ units in $t=2$.

To service withdrawals in terms of the CBDC, private banks first observe their customers' CBDC withdrawal needs n_P, and borrow the required amount $(1-\alpha)n_P M$ of the CBDC from the central bank at the beginning of period $t = 1$. The central bank creates the CBDC quantity $(1 - \alpha)n_P M$ on demand for the private banks. Private banks observe CBDC spending at the central bank n_{CB}, yielding aggregate spending n. During period one, the private banks sell the share $y_P(n)$ of their real goods investment at price P_1 in the centralized market to all consumers, thus receiving proceeds of $P_1 y_P(n)(1 - \alpha)$ units of the CBDC in return, where P_1 satisfies market clearing:

$$M\Big((1-\alpha)n_P + \alpha n_{CB}\Big) = P_1\Big(y_P(n)(1-\alpha) + y_{CB}(n)\,\alpha\Big). \tag{44}$$

The private banks use these CBDC proceeds to (partially) repay their loan to the central bank at zero interest within period one. Since the central bank retains only partial control over the goods market, it generically no longer holds $n_{CB}M = P_1 y_{CB}(n)$. As a consequence, the private banks can hold positive or negative CBDC balances $(1 - \alpha)(P_1 y_P(n) - n_P M)$ with the central bank between $t = 1$ and $t = 2$.

We seek to examine a range of possibilities for the private bank withdrawals n_P as well as liquidation choices y_P. Thus, it is useful to impose the condition that private banks make zero profits, regardless of the "circumstances" n_P or their choice for y_P. This requires some careful calculation, which we provide in Appendix 13, and only summarize here.

We assume that the central bank charges or pays the nominal interest rate $z = (RP_2/P_1) - 1$ on the excess or deficit CBDC balances of private banks, to be settled at the end of $t = 2$. Note that $z > i$, if $x_1 > 1$ and equals the internal nominal shadow interest rate regarding private bank liquidation choices. Moreover, we impose a market share tax at the end of period $t = 2$ in order to compensate for profits or losses due to circumstances n_P.

At $t = 2$, the remaining private customers spend the quantity $(1 - \alpha)(1 - n_P)M(1 + i(n))$ of private CBDC accounts that the private banks borrow from the central bank at the beginning of $t = 2$. The private banks sell their returns on the remaining investment $R(1 - y_P(n))(1 - \alpha)$ at price P_2, where P_2 satisfies market clearing

$$M(1+i(n))\Big((1-\alpha)(1-n_P) + \alpha(1-n_{CB})\Big) = \\ P_2 R\Big((1-y_P(n))(1-\alpha) + (1-y_{CB}(n))\,\alpha\Big). \tag{45}$$

At the end of $t = 2$, the private banks settle their accounts with the central bank, taking into account the remaining balances at $t = 1$ adjusted for interest, the end-of-period tax compensating for circumstances n_P, the loan at the beginning of $t = 2$, and the sales proceeds at $t = 2$.

Joint liquidation policies. The actions of private banks and the central bank may not be perfectly aligned when it comes to the liquidation of assets and the supply of goods at the interim stage. Private banks can have various objectives depending on their ownership structure and may be subject to regulation of their liquidation policy, both shaping y_P. Independently of whether private banks maximize depositor welfare as in Diamond and Dybvig (1983), or pursue some other objective, the prevention of runs is key. We have seen above that runs occur if the provision of real goods at the interim stage is high. Since the market is centralized, for the spending incentives of bank customers it is irrelevant whether these goods are provided by the central bank's or the private bank's liquidation of assets.

Hence, as before, a run-deterring liquidation policy $y(\cdot)$ is a function of aggregate spending n such that the real allocation at $t = 1$ undercuts the real allocation at $t = 2$:

$$\frac{y(n)}{n} < R\frac{(1 - y(n))}{1 - n}, \quad \text{for all } n \in [\lambda, 1]. \tag{46}$$

Thus, again, a run-deterring policy satisfies

$$y(n) < \frac{nR}{1 + n(R - 1)}, \quad \text{for all } n \in [\lambda, 1]. \tag{47}$$

Perfect coordination. If the central bank and the private banks coordinate perfectly, i.e., act as one entity, and have full control over the asset liquidation, then all run-deterring policies are possible, as in the case where the central bank is a monopolist. But why would they coordinate perfectly? By the market's centralization, the destiny of the central bank is intertwined with the destiny of the private banks and both types of banks have an interest in deterring runs. In particular, the private bank will, therefore, not undermine a central bank's run-deterring policy by supplying additional goods when, for instance, prices are high, since this might cause a run not only on the central bank but also on the private bank. Coordination is therefore among the equilibrium outcomes.

Runs under imperfect coordination. The following example shows how, for general liquidation policies y_P of private banks, runs can occur. Assume that the private bank, for some reason, follows a liquidation rule $y_P(n) \in [0, 1]$ where $y_P(n_b) = 1$ for all $n \geq n_b$ where $n_b \in (0, 1)$. For instance, $n_b = 1 - \alpha$, i.e., the private bank is subject to regulation and has to liquidate all assets if a fraction of its customers equal to its market share spends. In that case, as we show next, the central bank's capacity to deter runs depends on the size of the private banking sector, i.e., its market power α. Since the central bank can only control the liquidation of its own investment y_{CP}, via (46) and (43), a run-deterring policy y_{CB} needs to satisfy:

$$y_{CB}(n) < \frac{Rn - (1 - \alpha)y_P(n)(Rn + 1 - n)}{\alpha(Rn + 1 - n)}, \quad \text{for all } n \in [\lambda, 1]. \tag{48}$$

Now assume $n > n_b$, such that $y_P(n) = 1$. If in addition the central bank has a small market share $\alpha \to 0$, then the numerator converges to $-(1 - n)$, while the denominator goes to zero, $\alpha(1 + (R - 1)n) \to 0$. That is, for $n_b < n < 1$, the right-hand side in (48) goes to minus infinity such that (48) cannot hold. This implies that the run equilibrium exists.

A sufficient condition: Run-deterrence under imperfect coordination. The example above makes clear that the central bank's share in the deposit market needs to be large enough in order to prevent runs. The following proposition provides the appropriate bound under which the central bank can ensure the absence of a run, regardless of the private bank's liquidation schedule $y_P : [\lambda, 1] \to [0, 1]$.

Proposition 13. *Suppose that the central bank's share in the deposit market satisfies*

$$\alpha > \frac{1 - \lambda}{(1 - \lambda + R\lambda)}. \tag{49}$$

Then the central bank can always find a run-deterring liquidation policy $y_{CB} : [\lambda, 1] \to [0, 1]$, regardless of the private bank's liquidation policy $y_P : [\lambda, 1] \to [0, 1]$.

Such an $\alpha \in (0, 1)$ exists since $\frac{1-\lambda}{(1-\lambda+R\lambda)} \in (0, 1)$. Thus, the right-hand side $\frac{1-\lambda}{(1-\lambda+R\lambda)}$ of equation (49) imposes a lower bound on the balance-sheet size of the central bank as a percentage of the total demand deposit market, such that run-deterring policies remain possible despite coexisting private banks that are subject to liquidation restrictions.

Proof. [Proposition 13] We need to show that for any private bank liquidation policy $y_P : [\lambda, 1] \to [0, 1]$, there is a central bank liquidation policy $y_{CB} : [\lambda, 1] \to [0, 1]$ so that (48) is satisfied. To derive a sufficient condition on the central bank's market share α under which it can nevertheless implement a run-deterring policy, note that by $R > 1$, the right-hand side in (48) declines in the value y_p for all $\alpha \in (0, 1)$. Thus, if a central bank policy y_{CP} is run-deterring for $y_P = 1$ for all $n \in [0, 1]$, then y_{CP} is also run-deterring for a private bank policy $y_P(n) \leq 1$ for all $n \in [0, 1]$. Thus, assume $y_P = 1$ for all $n \in [0, 1]$. Then, a sufficient condition for a run-deterring policy against all private bank policies y_P is:

$$y_{CB}(n) < \frac{Rn - (1 - \alpha)(Rn + (1 - n))}{\alpha(1 + (R - 1)n)} = 1 - \frac{1 - n}{\alpha(1 + (R - 1)n)}, \quad \text{for all } n \in [\lambda, 1]. \tag{50}$$

The right-hand side is increasing in n and $y_{CB}(n)$ cannot undercut zero. Thus, a sufficient condition for the existence of a policy $y_{CB} \in [0, 1]$ that satisfies (50) is an α such that:

$$0 < 1 - \frac{1 - \lambda}{\alpha(1 + (R - 1)\lambda)}. \tag{51}$$

□

9 The financial system

Our model abstracts from many features of the financial system. In our baseline setting, we only have households and the central bank interacting with each other, dropping the financial intermediary sector entirely. This can appear as rather different from the institutional framework seen in practice and the risk-sharing framework in place.

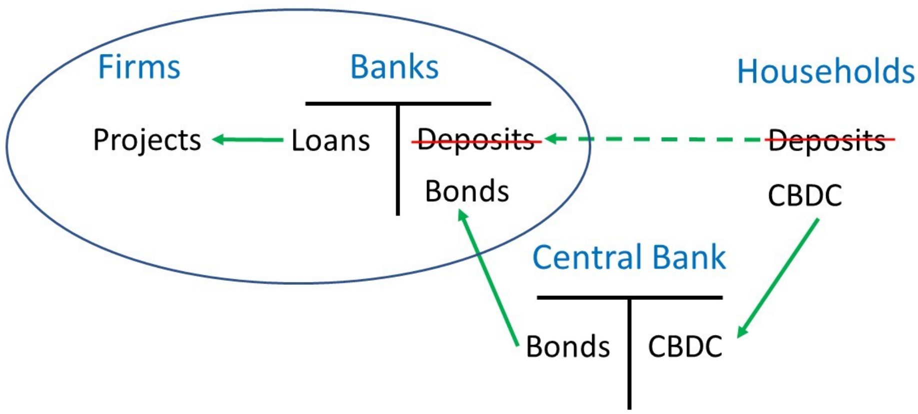


Figure 9: The Financial System: Households, firms, and banks.

Consider then the financial system as depicted in figure 9, containing firms, banks, households and a central bank. Before the introduction of a central bank digital currency, households hold deposits at banks. Banks use these deposits to provide loans to firms, who in turn use them to finance investment projects. These projects are as described in our model above.

With the introduction of a CBDC, households may become inclined to hold CBDC rather than deposits, given the rather similar functionality. Without further action, this would then lead to a disintermediation of the banks and impair their ability to make loans to firms. This issue does not disappear in a hybrid system either, where banks handle the "front end" of the CBDC accounts: in order to assure that the nature of the money does not depend on the handling bank, these cannot be treated as deposit accounts. However, the disintermediation can be avoided if the central bank engages in "pass through," funneling the funds deposited by households back to the retail banks, as Brunnermeier and Niepelt (2019) have argued. In figure 9, this is indicated by the central banks refunding the banks with loans in the form of bank-issued bonds. If done properly, the financing of firms remains unchanged.

With this new structure, however, the central bank is exposed to the intermediation risks inherent in banking and firm financing. In the environment as envision in figure 9, the central bank becomes the main source of bank funding: deposit finance has disappeared. In particular, the central bank can encourage or discourage production by increasing or decreasing the amount of bank bonds it holds. One could enrich this structure by assuming that households may also hold bank bonds or bank equity. What is key to our considerations, however, is that CBDC rather than bank bond holdings or bank equity holdings of households will be the substitute for their original deposits, and that deposits originally are the lion share of stable bank funding. With that and with the introduction of a CBDC, the central bank will now provide the lion share of stable bank funding.

One way to think through the consequences is to model firms, banks, households and central banks as well as their contractual interplay explicitly. It is quite common in the banking literature to assume that banks run these projects directly, rather than explicitly model the relationship between banks and firms. Here and in analogy, we go a step further, and now assume that it is the central banks running these projects directly.

It should be clear that we do not mean to imply that we envision the central bank to run the entire economy. Rather, this is meant to be a useful abstraction of a richer environment as envisioned in figure 9, with the aim of presenting our analysis as clearly as possible.

10 Extensions

10.1 Token-based CBDC

With a token-based CBDC, a central bank issues anonymous electronic tokens to agents in period 1, rather than accounts.[20] These electronic tokens are more akin to traditional banknotes than to deposit accounts. Trading with tokens only requires trust in the authenticity of the token rather than knowledge of the identity of the token holder. Thus, token-based transactions can be made without the knowledge of the central bank.

With appropriate software, digital tokens can be designed in such a way that each unit of a token in $t = 1$ turns into a quantity $1 + i$ of tokens in $t = 2$, with i to be determined by the central bank at the beginning of period $t = 2$: even a negative nominal interest rate is possible.[21]

With that, the analysis in the previous sections still holds, since nothing of essence depends on the identity of the spending agents other than total CBDC tokens spent in the goods market. With a token-based CBDC, agents obtain M tokens in period $t = 0$, and decide how much to spend in periods $t = 1$ and $t = 2$. Thus, the same allocations can be implemented except for those that require the suspension of spending, as discussed in Subsection 6.

For the latter, the degree of implementability depends on technical details outside the scope

[20] This can be done with or without a blockchain. In the second case, a centralized ledger to record transactions can be kept by a third party that is separate from the central bank. That third party could also potentially pay interest or impose a suspension of spending. For the purpose of this paper, we do not need to worry about the operational details of such a third party or to specify which walls should exist between it and the central bank to guarantee the anonymity of tokens.

[21] Historically, we have examples of banknotes bearing positive interest (for instance, during the U.S. Civil War, the U.S. Treasury issued notes with coupons that could be clipped at regular intervals) and negative interest (demurrage-charged currency, such as the prosperity certificates in Alberta, Canada, during 1936). Thus, an interest-bearing electronic token is novel only in its incarnation, but not in its essence.

of this paper. Note that even with a token-based system, the transfer of tokens usually needs to be registered somewhere, e.g., on a blockchain. It is technically feasible to limit the total quantity of tokens that can be transferred on-chain in any given period. A pro-rata arrangement can be imposed by taking all the pending transactions waiting to be encoded in the blockchain, taking the sum of all the spending requests, and accordingly dividing each token into a portion that can be transferred and a portion that cannot. It may be that off-chain solutions arise circumventing some of these measures, but their availability depends on the precise technical protocol of the CBDC token-based system. In the case where the token-based CBDC is operated by a centralized third party, such an implementation is even easier.

10.2 Synthetic CBDC and retail banking

With a synthetic CBDC, agents do not hold the central bank's digital money directly. Rather, agents hold accounts at their own retail bank, which in turn holds a CBDC not much different from current central bank reserves. This may be due to tight regulation by the monetary authority. The retail banks undertake the real investments envisioned for the central bank in our analysis above. A synthetic CBDC, therefore, corresponds to the model sketched in Section 8.2 with $\alpha = 0$.

The key difference from the current cash-and-deposit-banking system is that cash does not exist as a separate central bank currency or means of payment. That is, in a synthetic CBDC system, agents can transfer amounts from one account to another, but these transactions are always observable to the banking system and, thereby, the central bank. Likewise, agents (and banks) cannot circumvent negative nominal interest, while they could do so in a classic cash-and-deposit banking system by withdrawing cash and storing it.

For the purpose of our analysis, observability is key. Our analysis is relevant in the case of a systemic bank run, i.e., if the economy-wide fraction of spending agents exceeds the equilibrium outcome. Much then depends on the interplay between the central bank and the system of private banks. For example, if the liquidation of long-term real projects is up to the retail banks, and these retail banks decide to make the same quantity of real goods available in each period, regardless of the nominal spending requests by their depositors, then the aggregate price level will have to adjust. The central bank may seek to prevent this either by imposing a suspension of spending at retail banks or by forcing banks into higher liquidation of real projects: both would require considerable authority for the central bank. Proposition 13, for instance, says that with $\alpha = 0$, the central bank alone cannot implement a run-deterring policy when offering a synthetic CBDC. Run deterrence then requires retail banks to control liquidation in a particular way.

11 Conclusion

Diamond and Dybvig (1983) have taught us that the implementation of the social optimum via the financial intermediation of banks comes at the cost of making these banks prone to runs. This dilemma becomes a trilemma when the central bank acts as the intermediary offering a CBDC because central banks are additionally concerned about price stability. As our main result, a central bank that wishes to simultaneously achieve a socially efficient solution, price stability, and financial stability (i.e., absence of runs) will see its desires frustrated. We have shown that a central bank can only realize two of these three goals at a time.

12 Appendix A: Proofs

Proof. [Proposition 7] Proof (i): Via the market clearing condition (7), setting $P_1(n) \equiv \bar{P}$ for all n requires $y(n) = \frac{M}{\bar{P}}n$, for all $n \in [0,1]$. Thus, via (11), $x_1(n) = y(n)/n = \frac{M}{\bar{P}}$ is constant for all n. Last, since the central bank cannot liquidate more than the entire investment in the real technology, $y(n) \in [0,1]$ for all n, together with x_1 constant requires, in particular, $\frac{M}{\bar{P}} = x_1 = x_1(1) = y(1) \leq 1$. Thus, $M \leq \bar{P}$. Proof (ii): When additionally requiring price stability, $P_1(n) = P_2(n) \equiv \bar{P}$, the market clearing condition (8) together with (23) yields (25). □

Proof. [Corollary 8] Proof (i): We know that price stability demands $x_1 \leq 1$ but the social optimum satisfies $x_1^* > 1$. Proof (ii): $\overline{x}_1 \leq 1$ implies $x_2(n) = \frac{1-y(n)}{1-n}R = \frac{1-n\overline{x}_1}{1-n}R \geq R > 1 \geq \overline{x}$. Since the real value of the allocation at $t = 2$ always exceeds the real value of the time one allocation at $t = 1$, patient agents never spend at $t = 1$; thus, there are no runs. Proof (iii): By equation (24), $\frac{\overline{P}}{M} \geq 1$, implies $i(n) = \frac{\frac{\overline{P}}{M} - n}{1-n}R - 1 \geq R - 1 > 0$ for all $n \in [\lambda, 1]$ by $R > 1$. Further, $\frac{\overline{P}}{M} \geq 1$ implies that $i(n)$ increases in n. □

Proof. [Proposition 10] Proof (i): Equation (26) follows immediately from (7) and the constraint $y(n) \leq 1$. Proof (ii): In $n = n_c$, we have $\frac{M}{\bar{P}}n = 1$. Therefore, $n_c > 0$. By assumption $\bar{P} < M$, thus $n_c < 1$, with $n_c \in (0,1)$. Equation (26) implies that $x_1(n) = y(n)/n$ is constant at the level $\overline{x} = M/\overline{P}$, as long as $y(n) < 1$: this is the case for $n < n_c$. For $n \geq n_c$, $y(n) \equiv 1$. All goods are liquidated, so $x_1(n) = 1/n$. Equation $P_1(n) = Mn$ follows from equation (7). Proof (iii): Equation (27) follows from (8) combined with (26). Proof (iv): This is straightforward, when plugging in (26) into $P_2(n)$ and observing that n_0 is positive only for $R > M/\overline{P}$. □

13 Appendix B: Private bank accounting

Consider the collective of private banks with market share $(1-\alpha) \in (0,1)$. For the sake of brevity, we refer to the collective as "the private bank." A fraction n_P of the private bank's customers spend in $t = 1$, while a fraction n_{CB} of the central bank's customers do so, for a total fraction n of all agents $n = (1-\alpha)n_P + \alpha n_{CB}$. Agents are promised M units of the CBDC, when spending in $t = 1$, or $M(1+i)$ units, when spending in $t = 2$. The central bank liquidates y_{CB} goods in period $t = 1$, while the private bank liquidates y_P, for total liquidation $y = (1-\alpha)y_P + \alpha y_{CB}$. For accounting, we introduce some notation. The private bank borrows CBDC L_1 from the central bank to meet withdrawals at the beginning of each period, repaying the loan at the end of the period with the sales proceed S_1 from selling real goods. No interest is charged for the within-period loan.

The difference D_1 at the end of period $t = 1$ is kept on account at the central bank, earning or paying the nominal interest rate z, to be settled at the end of period $t = 2$. Further, the bank has to pay a tax $\tau(1-\alpha)$ denoted in CBDC at the end of period 2 (or receive this as a subsidy, if $\tau < 0$). The interest rate z and the tax τ are chosen by the central bank (CB in the accounting below), and may depend on n_P and choices y_P of the private bank. We seek to calculate x and τ so that the private bank makes zero profits, i.e., is left with zero CBDC balances D_2 at the end of period 2, after having liquidated and sold all its remaining goods at the end of period 2. Then:

Accounting in period $t = 1$:

$$\begin{aligned} \text{Loan from CB:} \quad L_1 &= (1-\alpha)n_P M \\ \text{Sales proceeds:} \quad S_1 &= (1-\alpha)P_1 y_P \\ \text{Difference:} \quad D_1 &= S_1 - L_1 = (1-\alpha)(P_1 y_P - n_P M) \end{aligned}$$

Accounting in period $t = 2$:

$$\begin{aligned} \text{Loan from CB:} \quad L_2 &= (1-\alpha)(1-n_P)(1+i)M \\ \text{Sales proceeds:} \quad S_2 &= (1-\alpha)P_2 R(1-y_P) \\ \text{CB account:} \quad A_2 &= (1+z)D_1 - \tau(1-\alpha) \\ \text{Difference:} \quad D_2 &= A_2 + S_2 - L_2 \\ &= (1-\alpha)\Big(P_2 R + ((1+z)P_1 - P_2 R)y_P - (1+i)M - (z-i)n_P M - \tau\Big) \end{aligned}$$

Market clearing:

$$\begin{aligned} \text{In } t = 1: \quad P_1 y &= nM \\ \text{In } t = 2: \quad P_2 R(1-y) &= (1-n)(1+i)M \end{aligned}$$

Sum $(1+i)$ times the market clearing equation for P_1 with the equation for P_2 to obtain $P_2 R + ((1+i)P_1 - P_2 R)y = (1+i)M$. Use the latter equation to replace $(1+i)M$ in the last expression for D_2 to find

$$\frac{D_2}{P_1(1-\alpha)} = (i-s)(y_P - y) + (z-i)(y_P - n_P x_1) - \frac{\tau}{P_1} \tag{52}$$

where, as usual, $x_1 = \frac{M}{P_1}$ is the amount of real goods acquired by agents in period $t = 1$ and where we introduce:

$$s = \frac{P_2}{P_1} R - 1 \tag{53}$$

to denote the "shadow" nominal interest rate for private banks, equating liquidating a unit of the good in $t = 1$, selling at P_1 and investing at the shadow nominal return $1 + s$ to keeping the unit of good and thus selling R units at price P_2. Notice that $y = nx_1$ and the market clearing equations imply

$$1 + s = (1 + i)\frac{1 - n}{1 - x_1 n}x_1 \tag{54}$$

and, thus, $s > i$, whenever $x_1 > 1$. In particular, this is the case at the efficient outcome. We note that $s = i$, if and only if $x_1 = 1$, which is the maximal full price-stable solution as well as the market allocation, when agents engage in self-storage.

Suppose now that the private bank sells exactly as many goods as purchased by its withdrawing customers, i.e., $y_P = n_P x_1$. Absent τ, equation (52) reveals that the private bank will make a loss or profit, if $x_1 \neq 1$ and if $y_P \neq y$, i.e., $n_P \neq n$. For example, if the share of private-bank customers who go shopping in $t = 1$ is larger than the average share of customers who shop economy-wide, $n_P > n$, and if the allocation achieves $x_1 > 1$ and thus $s > i$, then the private bank incurs a loss $D_2 < 0$, absent τ, as the opportunity costs for servicing agents in $t = 1$ are high. We shall use these observations to fix the tax τ to compensate for these losses or profits, and assume that

$$\tau = P_1(i - s)(n_P - n)x_1 \tag{55}$$

from here onward. This τ depends on the specifics of the bank only via the "circumstances" n_P and does not depend on the choice y_P. To take care of the case where $y_P \neq n_P x_1$, we use the central bank-account interest rate z. Solving for z per setting $D_2 = 0$ in (52) and imposing (55) yields the following result, which we formulate as a proposition.

Proposition 14. *Suppose τ satisfies (55). Then, $D_2 \equiv 0$ for all $1 \leq n_P \leq 1$ and all $y_P \in [0, 1]$, iff $z = s$.*

In sum, taxing the "circumstance" profits per (55) and paying an internal interest rate z on central bank balances equal to the shadow nominal interest rate s achieves the objective that private banks make zero profits, regardless of their circumstances n_P and regardless of their liquidation choice y_P.

Alternatively, one could envision some kind of regulation or other policy tool, that enforces $y_P \equiv y_{CB}$. In that case, the next proposition is useful. It shows that one can implement the zero profit solution without a tax, provided the interest rate on reserves D_1 coincides with the nominal interest rate offered on CBDC accounts.

Proposition 15. *Suppose the private bank always sets $y_P \equiv y_{CB}$ and suppose that $\tau \equiv 0$. Then, final balances are zero, $D_2 = 0$, if the interest rate on reserves satisfies $z = i$.*

Proof. This follows directly from equation (52). □

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