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# Differential Calculation Abilities in Young Children From Middle- and Low-Income Families

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This study examined the performance of 42 middle- and 42 low-income kindergarten children on arithmetic calculations presented in a nonverbal format as well as in 3 different verbal formats. On the nonverbal task, the child was shown an initial set of disks, which was then hidden with a cover. The set was transformed by adding or removing disks. After the transformation, the child's task was to construct an array of disks that contained the same number of disks as in the final hidden set. A significant interaction between income level and task format was obtained. Although middle-income children performed better than low-income children on each of the verbal calculation tasks, the 2 income groups did not differ in performance on the nonverbal calculation task. The findings suggest that the nonverbal task format is less sensitive to socioeconomic variation than are the verbal task formats.

A number of contextual variables can affect a young child's performance on arithmetic tasks, including the amount of verbal understanding required by the task, the nature of the quantitative terminology used in the task, and the availability of object referents (Gelman & Gallistel, 1978; Gelman & Massey 1987). In fact, our work on the development of calculation abilities in young children has shown that children's performance on addition and subtraction problems varies according to the nature of the task (Levine, Jordan, & Huttenlocher, 1992). In this prior study, middle-class children between 4 and 6 years of age were given identical addition and subtraction calculations in three different formats: nonverbal problems, story problems, and number–fact problems.

On the nonverbal calculation task, the child was shown a set of physical referents that were then hidden with a box used as a cover. The hidden set was transformed either by adding or subtracting elements through an opening in the side of the box. After the transformation, the child's task was to construct an array with the same number of elements that were in the final set. The child was never allowed to see the entire set that the experimenter had produced. This ensured that an addition or subtraction transformation was carried out to arrive at a solution. The experimenter did not provide verbal labels for any terms of the problem nor was the child asked to generate them. The story problems used a meaningful verbal context with object referents (e.g., "Mike had $m$ balls. He got $n$ more. How many balls did he have altogether?"). The number–fact problems involved decontextualized language in which no objects are referred to and the quantities remain abstract (e.g., "How much is $m$ and $n$?"). Both the story and number–fact problems were presented auditorily. Levine et al.'s (1992) results showed that children as young as 4 years of age can add and subtract on nonverbal problems involving small number sets and that their performance level increases throughout the preschool and kindergarten years. On the other hand, most children did not calculate successfully on story and number–fact problems until approximately 5½ years of age, at which point their performance increased markedly.

The findings suggest that several contextual factors should be considered in the assessment of calculation abilities in young children. For example, to solve story and number–fact problems, children must understand and generate verbal labels for numbers, understand words for operations, and comprehend various syntactic structures. A lack of any of these linguistic abilities might result in the failure to solve a story or number–fact problem correctly, despite an adequate understanding of the operations of addition and subtraction. In contrast, a lack of relevant linguistic abilities should not preclude success on the nonverbal task. The availability of object referents also appears to be an important determinant of the young child's ability to calculate. Supporting this suggestion, Levine et al. (1992) found that children perform best on nonverbal problems that provide object referents, at an intermediate level on story problems that refer to object sets that are not physically present, and most poorly on number–fact problems that do not refer explicitly to object sets.

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The verbal and nonverbal calculation tasks described in Levine et al.'s (1992) study may be differentially sensitive to environmental influences. That is, environmental factors might be less critical for solving nonverbal calculation problems than for solving story or number–fact problems. Verbal calculation tasks that rely on conventional procedures could present particular problems for children from lower-class families, even though they may have underlying competencies in addition and subtraction. Studies examining the relationship between the home environment and different cognitive abilities support this suggestion. For example, Kellaghan (1977) found that home variables, such as the quality of language use of the parents, the amount of guidance given regarding schoolwork, and the variety of "thought-provoking" toys and games available to the child are more strongly related to cognitive measures with high verbal content than to those with low verbal content. Furthermore, a number of other studies report that performance on verbal tasks, such as vocabulary, is more susceptible to environmental influences than performance on spatial tasks (e.g., MacArthur & Elley, 1963; Walberg & Marjoribanks, 1973). Thus, when examining the calculation abilities of children from different socioeconomic backgrounds, it is especially important to utilize measures using both verbal and nonverbal formats.

To date, research has not compared the performance of children from different socioeconomic levels on verbal and nonverbal calculation tasks. However, Ginsburg and Russell (1981) compared the addition calculation abilities of lower- and middle-class children at the preschool and kindergarten levels on verbally presented tasks that varied the availability of object referents. Children were asked to solve three story problems with physical objects present and three story problems with physical objects absent. Although no effect of social class was found on either task at both age levels, the findings should be interpreted in light of several methodological considerations. First, the "objects-present" story problems allowed children to view the initial array of objects and the objects to be added at the same time. As a result, they could have arrived at a correct solution by counting the total number of objects present rather than by performing an actual addition calculation. In fact, Ginsburg and Russell also found no social class differences on tasks designed specifically to assess the child's skills in counting sets of objects.

Second, for both objects-present and objects-absent problems, the calculations involved relatively large number sets (e.g., 7 + 3). This is an important consideration because studies have shown that young children perform better on arithmetic tasks involving small numbers than on those involving larger numbers (e.g., Gelman & Gallistel, 1978). It appears that the "objects-absent" story problems in Ginsburg and Russell's study were too difficult for all children regardless of social class. This seemed to be especially true at the preschool level in which the mean score for children of both social classes was less than 1. Thus, floor effects may have masked some of the differences in calculation abilities between middle- and lower-class children.

Several other studies, in fact, suggest that young children from different social classes perform differentially on calculation tasks. For example, Saxe, Guberman, and Gearhart (1987) found that working-class preschoolers performed more poorly than middle-class preschoolers on tasks that assess the ability to add one to sets of varying numerosities (2, 3, 7, and 9). The problems were presented verbally in a story-problem format, using pennies as props. In contrast to Ginsburg and Russell's (1981) objects-present task, the child was not allowed to view the initial array of objects and the objects to be added at the same time. Because the task was presented verbally along with physical referents, however, it is not possible to determine whether the disparity between the two groups was due to actual differences in the ability to transform sets by adding an element or to differences in verbal understanding. A similar interpretational problem exists in a study by Entwisle and Alexander (1990), who found that lower-class children lag behind their more affluent peers in arithmetic skills at the beginning of first grade. Their finding is based on performance on the California Achievement Test, a standardized test that depends on conventional verbal knowledge as well as on arithmetic ability.

Our recent pilot work has shown that low-income preschool children are successful with small-numerosity calculation problems when they are presented in a nonverbal format. In contrast, they performed poorly on a comparable set of story problems. An informal comparison of the performance of these children to that of middle-class children in our previous study (Levine et al., 1992) revealed that low-income preschoolers performed at approximately the same level on the nonverbal calculation task as middle-class preschoolers. However, it was not informative to compare the two income groups on story problems because story problems tended to be too difficult for all.

The goal of the present study was to compare middle- and low-income kindergarten children on calculation tasks that use verbal and nonverbal formats. Kindergarten rather than preschool children were assessed to avoid floor effects on the verbal calculation tasks. Identical addition and subtraction problems were presented in the form of nonverbal problems, story problems, and number–fact problems. To understand better why story problems are easier than number–fact problems (Levine et al., 1992), a fourth problem type was included. These "word problems" referred to the same object referents as the story problems but used the more formal language of the number–fact problems (e.g., "How much is two pennies and two pennies?"). If the difference between story and number–fact problems is due to the availability of referents in the story problems, then word problems should be just as easy as story problems. However, if it is the more meaningful context of story problems that facilitates calculation, then word problems should be harder than story problems and similar in difficulty to number–fact problems.

In addition to examining calculation accuracy, we analyzed children's errors on the different problem types to determine whether they reflected a basic understanding of the effects of addition and subtraction operations. In particular, we examined whether children's errors were greater than the augend for addition and less than the minuend for subtraction. We also recorded children's strategies on individual calculations. Previous studies have shown that young middle-class children use multiple strategies to solve addition and subtraction problems (Siegler, 1987, 1989). However, the use of calculation strategies has not been examined in children from low-income families. In the present study, we were particularly interested in the extent to which low-income children used their fingers during the various calculation tasks. Our prior research shows that 5- and 6-year-old middle-class children use finger strategies on the verbal calculation tasks but not on the nonverbal task (Levine et al., 1992). The object referents provided on the nonverbal task seem to obviate the use of finger strategies. On the verbal tasks, in contrast, fingers serve as object referents and reduce demands on working memory (Geary, 1990).

To assess basic competencies in counting, we gave children tasks that required them to count and state the cardinal value of sets of objects. Prior work has suggested that counting is important to calculation (Starkey & Gelman, 1982). Although studies have shown that children of all social classes develop counting skills during the preschool years (Ginsburg & Russell, 1981; Saxe et al., 1987), it nevertheless was important to rule out counting difficulties in our present sample. Finally, a measure of syntax and vocabulary was administered to determine whether middle- and low-income children differed in verbal ability.

## Method

### *Subjects*

The sample consisted of 42 kindergarten children from low-income families and a control group of 42 kindergarten children from middle-income families. All of the children were between 5 and 6½ years of age (mean age = 5.9 years, $SD = 0.32$, for the low-income group and 5.9 years, $SD = 0.34$, for the middle-income group). Within each income level, there was an equal number of boys and girls. The low-income children were drawn from two urban schools in New Brunswick, NJ. The middle-income children were drawn from three other schools in New Brunswick as well as from one school in the neighboring community of Somerset. All children came from homes in which English was the primary language. Parental permission was obtained for all of the children tested.

The schools from which the low-income children were drawn served families residing in government-subsidized housing projects. Qualification for subsidized housing is based on income for a family of a particular size. The low-income children came from families that qualified for the free- or reduced-price lunch program in school, indicating that they were at the poverty level. The schools from which the middle-income children were drawn served families from middle-income neighborhoods. The middle-income children did not reside in subsidized housing projects nor did their families qualify for the free- or reduced-price lunch program in school. Principals and teachers reported that the middle-income children's families were not characterized by economic difficulty. According to school personnel, approximately 67% of the low-income children came from single-parent homes versus 12% of the middle-income children (in several cases, a child's parental situation was not known). The ethnic composition for the middle-income children was 67% White, 21% Black, 5% Hispanic, and 7% Asian. For the low-income children, the composition was 7% White, 83% Black, and 10% Hispanic.

Questionnaires regarding the kindergarten arithmetic curriculum were administered to the teachers of the participating children. The data indicated that the instructional time and sequence were essentially the same for all subjects. Teachers reported that none of the children had received any formal instruction in addition and subtraction. However, all children engaged in various counting activities in their classrooms (e.g., enumerating sets of objects).

### *Materials and Procedure*

Each child was given a set of seven addition and seven subtraction problems presented in four formats: (a) nonverbal problems, (b) story problems, (c) word problems, and (d) number–fact problems. The same calculations were given in each format. For addition problems, the numerosities of the augends and addends were no greater than 5 and the sums were no greater than 7 ($1 + 1, 2 + 2, 1 + 3, 2 + 4, 4 + 1, 3 + 2$, and $3 + 4$). For subtraction problems, the numerosities of the minuends and subtrahends were no greater than 7 and the differences were no greater than 4 ($2 - 1, 4 - 1, 4 - 2, 5 - 4, 5 - 3, 6 - 2$, and $7 - 4$). On each task, the calculations were presented to all children in the same order, with the addition and subtraction items intermixed. The ordering of the individual problems was random except that $1 + 1$ and $2 - 1$, the problems with the smallest number sets, were given first and second, respectively. Furthermore, addition or subtraction problems never were presented more than twice in a row.

All of the children were tested in school during late February and March. The method of presenting each of the four calculation tasks as well as the other arithmetic and language tasks are described next.

*Matching task and nonverbal calculation problems.* Materials for the matching task as well as the nonverbal calculation problems included two 28-cm × 15-cm cardboard mats, a set of 20 black disks (1.9 cm in diameter), a box for the disks, and a cover for the disks. One of the sides of the cover had an opening so the experimenter could easily put in or take out disks. The experimenter and the child sat at opposite sides of a table, each with a mat in front of her or himself.

Before the nonverbal calculation problems, the child was given a matching task. On this task, the experimenter took a disk from the box and placed it on her mat in full view of the child. The disk was then hidden under a cover. The experimenter then put a disk on the child's mat and lifted the cover from her own mat. Thus, the child could see that the two mats had the same number of disks on them. The experimenter stated, "See, yours is just like mine," pointing to the disks on both mats. The demonstration item was presented again, following the same procedure, except this time the child was asked to place the appropriate number of disks on his or her mat after the experimenter's disks were hidden. The child was then asked to do this with six other sets of disks (varying in number from two to seven), which were presented in a random order. Each set was displayed in a horizontal linear array. The experimenter did not show the child the correct answer after the trial was completed. The matching task was designed to help the child understand the procedure for the nonverbal calculation problems. Additionally, the task assessed whether children could reproduce a hidden numerosity of a single set of disks without an addition or subtraction transformation. The total possible score on the matching task ranged from 0 to 6.

The nonverbal calculation task was presented immediately after the matching task. For addition, the experimenter placed the set of disks comprising the augend in a horizontal line on her mat and then covered it. The experimenter then put the set of disks comprising the addend in a horizontal line in full view of the child and slid them under the cover one at a time. The two terms of the problem were never simultaneously in view. The child then indicated how many disks were hiding under the cover by placing the appropriate number of disks on his or her mat. A comparable procedure was used for subtraction, but in this case the disks comprising the subtrahend were removed from under the cover one at a time. No verbal labels were provided on any of the problems, nor was the child asked to generate them.

*Story problems.* The story problems were presented auditorily. The verbal content was intended to be as simple as possible (Hiebert, Carpenter, & Moser, 1982; Riley, Greeno, & Heller, 1983). The addition story problems required subjects to join two sets of objects (e.g., "Beth has *m* balloons. Steve gives her *n* more balloons. How many balloons does Beth have altogether?"). The subtraction story problems required subjects to separate a set of objects (e.g., "Jack has *m* balloons. Diane takes away *n* of his balloons. How many balloons does Jack have left?"). The same verbs and syntactic structures were used for all of the problems. The following objects were referred to once in an addition story problem and once in a subtraction story problem: apples, pennies, cookies, balloons, oranges, crayons, and marbles. The names of the actors were varied to sustain children's interest.

*Word problems.* The word problems used the same object referents as the story problems. For example, both story problems and word problems referred to pennies for the calculation 2 + 2. However, the word problems used more decontextualized language without actors (e.g., "How much is *n* pennies and *m* pennies?" for addition or "How much is *m* pennies take away *n* pennies?" for subtraction).

*Number-fact problems.* The experimenter read the addition number-fact problems as "How much is *m* and *n*?" and the subtraction number-fact problems as "How much is *m* take away *n*?" Unlike the story problems and word problems, no reference was made to objects.

For the story, word, and number-fact problems, the child responded to each item with a number word. Physical props were not provided. The experimenter did not suggest strategies to children on any of the calculation tasks, allowing them to choose their own methods for solving the problems. For each calculation task, the total possible score ranged from 0 to 7 for addition and 0 to 7 for subtraction.

During the testing, the experimenter recorded children's calculation strategies on each trial. Children's strategies were classified into the following categories, similar to those described by Siegler and Shrager (1984): (a) counting-fingers strategy, (b) fingers strategy, (c) counting strategy, and (d) unobserved strategy. Children were classified as using a counting-fingers strategy if they explicitly counted on their fingers either orally or by moving their fingers or head. A fingers strategy was recorded if children held up their fingers for any term of the problem without counting. Children were classified as using a counting strategy if they displayed counting behaviors without counting their fingers (e.g., subvocalizing the number sequence or pointing with fingers or head or both). An unobserved strategy was recorded when children answered without using their fingers and without counting overtly. In this case, children may have been retrieving the answer from memory, using some kind of covert algorithm (e.g., silent counting), or simply guessing.

A fifth category, imitation, was relevant only on the nonverbal problems (Levine et al., 1992). An imitation strategy was recorded when the child appeared to be copying the experimenter's actions on the nonverbal calculation task. For example, on the nonverbal addition problem 2 + 4, a child who is imitating would put two disks on one part of the mat and four disks on another part. The child would then slide the four disks over to the two disks, copying the experimenter's transformation. On the nonverbal subtraction problem 6 − 2, a child using such a strategy would put six disks on the mat and then would remove two of them.

*Counting/cardinality tasks.* These tasks were adapted from previous studies in the literature (Gelman & Gallistel, 1978; Ginsburg & Russell, 1981; Saxe et al., 1987; Schaeffer, Eggelston, & Scott, 1974). First, the child was shown a horizontal linear display of black dots (1.25 cm in diameter) printed on white cardboard. Separate displays of 4, 9, 13, 18, and 21 dots were presented one at a time. As the experimenter presented each display, she said: "Here are some dots. I want you to count each dot. Touch each dot as you count." Immediately after the child finished counting each set of dots, the experimenter hid the display and asked, "How many dots were there?" In this way, it was possible to assess whether the child understood that the final number used in the count sequence represents the number of objects in the set. Each of the five counting items was scored as correct if the child counted the sequence accurately. The total possible counting score ranged from 0 to 5. The cardinality items were scored as correct if the child stated the final number he or she used in the count sequence, even if a counting mistake had been made. One point was given for each correct response, with a total possible cardinality score ranging from 0 to 5.

*Language task.* The verbal subtest of the Primary Test of Cognitive Skills (PTCS) was given to each child (Huttenlocher & Levine, 1990). The PTCS verbal subtest is a standardized measure of syntax and vocabulary skills that uses a multiple-choice format.

All of the arithmetic tasks were administered to children individually. The order in which the four calculation tasks were presented was counterbalanced for boys and girls within each income level (Latin square design). The counting/cardinality tasks always were presented after the calculation tasks. The arithmetic tasks were given in two 10- to 15-min sessions: Two calculation tasks were given during the first session, and two calculation tasks as well as the counting/cardinality tasks were given during the second session. In most cases, the sessions took place on separate days, usually 24 to 48 hr apart. For logistical reasons (e.g., child going on vacation), however, a few of the children received all of the tasks on the same day. In these instances, a sufficient break was given between sessions. The PTCS verbal subtest was administered several weeks after the children were given the arithmetic tasks. This test was administered to groups of 4 or 5 children. Two examiners were present during the PTCS testing to help children attend to the tasks and follow directions.

## Results

Children's performance on each of the four calculation tasks was scored for the number of items answered correctly. The mean addition and subtraction scores broken down by problem type and income level are displayed in Figure 1. A preliminary analysis of variance (ANOVA) showed no significant effects of sex, $F(1, 68) = .04$, $p < .84$, or order of presentation on any of the four calculation tasks, $F(3, 68) = .902$, $p < .44$. Thus, these factors were excluded from subsequent analyses.

A mixed-design ANOVA with income level (low income and middle income) as a between-subjects factor and problem type (nonverbal, story, word, and number-fact) and operation (addition and subtraction) as within-subjects factors was performed. The results showed significant main effects of income level, $F(1, 82) = 26.75$, $p < .0001$, problem type, $F(3, 246) = 50.99$, $p < .0001$, and operation, $F(1, 82) = 30.04$, $p < .0001$. The Income Level × Problem Type interaction also was significant, $F(3, 246) = 13.91$, $p < .0001$. Simple effects analyses revealed no effect of income level on nonverbal problems ($p < .229$) but significant effects of income level on story problems, word problems, and number-fact problems, respectively ($p < .0001$ in each case).

The analysis also revealed a significant Problem Type × Operation interaction, $F(3, 246) = 3.18$, $p < .02$. Neither the Income × Operation interaction ($p < .11$) nor the Income × Operation × Problem Type interaction ($p < .37$) were significant. Simple effects analyses showed that addition items were significantly easier than subtraction items on nonverbal problems ($p < .01$), word problems ($p < .01$), and number-fact problems ($p < .001$) but not on story problems ($p < .32$). Simple effects analyses also showed a significant effect of problem type for each operation ($p < .0001$ in both cases). Contrasts indicated that, for both addition and subtraction, nonverbal problems were performed better than each of the other problem types ($p < .01$ in each case). The three verbal problem types did not differ significantly from each other on addition items. On

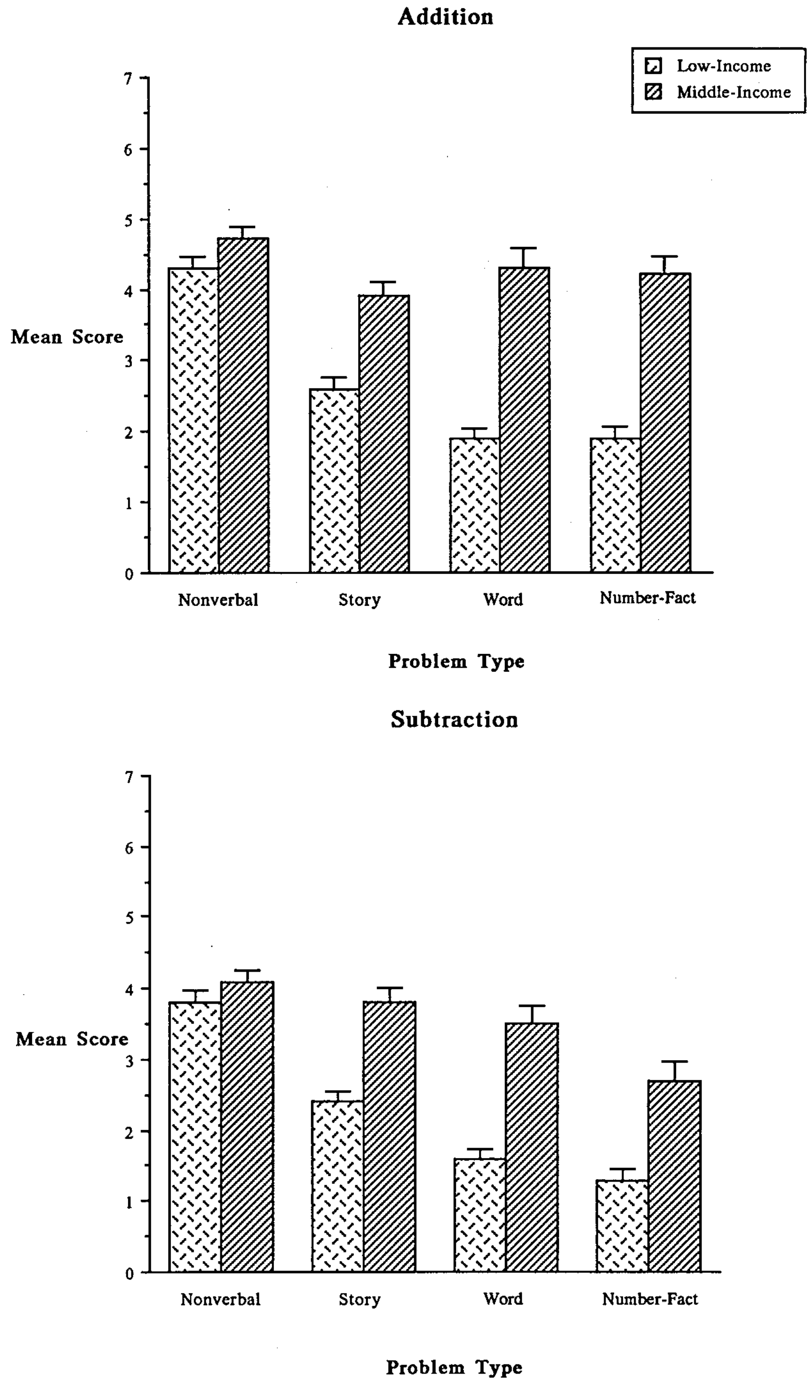


*Figure 1.* Mean calculation scores by problem type, operation, and income group. (Bars denote standard errors.)

subtraction items, performance on story problems was significantly better than performance on number–fact problems ($p < .001$). Subtraction word problems, although at an intermediate level, did not differ significantly from subtraction story problems or number–fact problems.

A chi-square analysis showed that the distributions of scores for nonverbal addition problems were not different for the two income groups, $\chi^2$ (5, $N$ = 84) = 3.01, $p < .70$, nor were the distributions of scores for nonverbal subtraction problems, $\chi^2$ (6, $N$ = 84) = 5.65, $p < .46$. In general, the individual data mirrored the group means on the nonverbal problems.

Children in the two income groups did not differ significantly on the quantity-matching task that was given before the nonverbal calculations. The mean score was 4.4 of 6 ($SD$ = 1.3) for the low-income children and 4.8 of 6 ($SD$ = 0.87) for the middle-income children.

### Calculation Strategies

Table 1 summarizes the percentage of trials (of a total of 294 for each income group and operation) on which the various strategies were used on each problem type as well as the percentage of trials on which the strategy produced a correct answer. It should be noted that for both income groups the majority of strategies fell into the unobserved category. However, the data do reveal that middle-income children used the finger-counting strategy on more trials than low-income children on the three types of verbal calculation problems. Moreover, the use of this strategy was associated with more accurate performance on these problems for middle-income children. Thus, we performed an analysis of covariance on each verbal calculation problem type, with the number of trials on which children counted on their fingers as a covariate and the respective calculation score as the dependent variable. The results showed that middle-income children still performed significantly better than low-income children on each of the three verbal problem

Table 1
*Children's Calculation Strategies by Problem Type, Income Level, and Operation*

| Strategy | Addition: Trials on which strategy was used (%) | Addition: Correct answers (%) | Subtraction: Trials on which strategy was used (%) | Subtraction: Correct answers (%) |
|---|---|---|---|---|
| **Middle-income children** | | | | |
| Nonverbal problems | | | | |
| Unobserved | 77 | 68 | 77 | 58 |
| Counting | 10 | 53 | 10 | 39 |
| Count fingers | 7 | 80 | 7 | 76 |
| Fingers | 0 | — | 0 | — |
| Imitation | 6 | 74 | 6 | 67 |
| Story problems | | | | |
| Unobserved | 80 | 54 | 83 | 52 |
| Counting | 3 | 70 | 2 | 50 |
| Count fingers | 14 | 71 | 12 | 64 |
| Fingers | 3 | 25 | 3 | 57 |
| Word problems | | | | |
| Unobserved | 73 | 55 | 68 | 46 |
| Counting | 7 | 45 | 8 | 27 |
| Count fingers | 19 | 87 | 22 | 58 |
| Fingers | 1 | 100 | 2 | 80 |
| Number–fact problems | | | | |
| Unobserved | 72 | 49 | 71 | 34 |
| Counting | 4 | 83 | 4 | 42 |
| Count fingers | 23 | 87 | 24 | 56 |
| Fingers | 1 | 50 | 1 | 33 |
| **Low-income children** | | | | |
| Nonverbal problems | | | | |
| Unobserved | 68 | 61 | 62 | 54 |
| Counting | 23 | 64 | 28 | 42 |
| Count fingers | 0 | — | 0 | — |
| Fingers | 0 | — | 0 | — |
| Imitation | 9 | 50 | 10 | 46 |
| Story problems | | | | |
| Unobserved | 93 | 37 | 89 | 32 |
| Counting | 1 | 40 | 2 | 43 |
| Count fingers | 2 | 0 | 2 | 20 |
| Fingers | 4 | 21 | 7 | 38 |
| Word problems | | | | |
| Unobserved | 93 | 24 | 93 | 23 |
| Counting | 2 | 50 | 2 | 17 |
| Count fingers | 4 | 69 | 4 | 0 |
| Fingers | 1 | 33 | 1 | 0 |
| Number–fact problems | | | | |
| Unobserved | 90 | 24 | 90 | 17 |
| Counting | 3 | 44 | 2 | 17 |
| Count fingers | 6 | 67 | 6 | 27 |
| Fingers | 1 | 50 | 2 | 57 |

types when we covaried for the use of finger counting ($p < .0005$ in each case).

On the nonverbal task, low-income children used overt counting strategies without fingers (e.g., pointing to dots) on more trials than did middle-income children on both addition and subtraction items, $F(1, 82) = 8.19$, $p < .005$. However, finger and finger-counting strategies were rarely used by children in either income group on the nonverbal task (7% of all trials for middle-income children, no trials for low-income children). The explicit object referents used on the nonverbal task made the use of fingers unnecessary. Neither middle- nor low-income children used the imitation strategy on more than 10% of the addition trials or 10% of the subtraction trials. However, middle-income children used imitation strategies somewhat more effectively than low-income children (see Table 1). When children reached an incorrect solution by imitating, it was usually because the they incorrectly matched either the first or second term of the problem.

### Error Analysis

The direction of errors provides a means of assessing whether children have knowledge of the effects of operations even if they cannot arrive at correct answers to problems (Levine et al., 1992). Right-direction errors may reflect more knowledge of addition and subtraction operations than wrong-direction errors. Thus, we noted whether children's errors were in the right or the wrong direction. An incorrect response was coded as being in the right direction if it was greater than the augend for addition (e.g., $2 + 4 = 7$) or less than the minuend for subtraction (e.g., $6 - 2 = 3$). An incorrect response was coded as being in the wrong direction if it was less than the augend for addition (e.g., $4 + 1 = 3$) or greater than the minuend for subtraction (e.g., $5 - 4 = 6$). Errors that were the same as the augend or minuend (e.g., $2 + 4 = 2$) or that were the same as the addend or subtrahend (e.g., $6 - 2 = 2$) were placed in separate categories. In these instances, the child simply may have copied or parroted one of the terms of the problem.

Table 2 shows the percentage of children's errors for each problem type and operation at each income level that were in the right direction, in the wrong direction, the same as the first term of the problem, and the same as the second term of the problem. (For $1 + 1$ an answer of "1" and for $2 + 2$ an answer of "2" would be a repetition of both the augend and the addend. Such errors were coded as augend repetitions.) On the nonverbal calculation task, the pattern of errors was similar for both income groups; the majority of children's errors were in the right direction. On the three verbal problem types, children also tended to make more right-direction errors than wrong-direction errors. However, this discrepancy was relatively narrow on word and number-fact subtraction problems. In fact, on subtraction number-fact problems, the low-income children made more errors in the wrong direction (40%) than in the right direction (28%). It should be noted that the boundary of zero increases the possibility of being in the wrong direction on subtraction problems and the right direction on addition problems. Thus, a comparison of the direction of errors on the two operations was not appropriate.

An error that is the same as the addend/subtrahend (second term) may reflect more knowledge of calculation than one that is the same as the augend/minuend (first term). A repetition of the augend or minuend is never in the right direction or the wrong direction. In contrast, a repetition of the subtrahend always is in the right direction and may even be the correct answer (e.g., $4 - 2 = 2$). A repetition of the addend could be in the right direction or the wrong direction depending on the problem ($1 + 3 = 3$, $2 + 4 = 4$, and $3 + 4 = 4$ are in the right direction; $4 + 1 = 1$ and $3 + 2 = 2$ are in the wrong direction). Thus, we calculated how many addend repetition errors were in the right direction for each problem type. For low-income children, 100% of the addend repetition errors were in the right direction for nonverbal problems, 81% for story problems, 46% for word problems, and 53% for number-fact problems. For middle-income children, 89% of the addend repetition errors were in the right direction for nonverbal problems, 59% for story problems, 65% for word problems, and 50% for number-fact problems.

### Counting/Cardinality Tasks

Children in both income groups performed well on the counting task. The mean counting scores for the low-income children (4.62 of 5, $SD = 0.79$) and middle-income children (4.67 of 5, $SD = 0.65$) did not differ significantly from each other, although these results should be interpreted in light of near-ceiling performance for both groups.

Children in both groups also performed well on the cardinality task, although middle-income children performed significantly better than low-income children, $t(82) = 2.23$, $p < .05$, two-tailed. The mean cardinality score was 4.5 of 5 ($SD = 1.2$) for the low-income group and 4.9 of 5 ($SD = 0.29$) for the middle-income group. With the exception of 2 low-income subjects, both of whom received scores of 0, all of the children could give the cardinal value for at least two of the five sets of dots. Moreover, a perfect score of 5 was the modal performance for children in both income groups.

### Performance on the PTCS Verbal Subtest

The mean scaled score on the verbal subtest of the PTCS was 301 ($SD = 77$; seventh stanine based on national norms) for the middle-income children and 183 ($SD = 64$; fourth stanine based on national norms) for the low-income children. The difference between the two income groups was significant, $t(82) = 7.8$, $p < .001$. To determine whether linguistic factors contribute to the differences between the two income groups on the verbal calculation tasks, we performed an analysis of covariance on each verbal calculation problem type, with the PTCS score as a covariate and the respective calculation score as the dependent variable. For story problems and word problems, the effect of income group was not significant, nor was there an interaction between income group and PTCS scores. For number-fact problems, the difference between the two groups was reduced (relative to the unadjusted means) but still significant ($p < .04$).

## Discussion

We have examined the calculation abilities of kindergarten children from middle- and low-income families. Identical ad-

Table 2
*Percentage of Total Errors That Were in the Right Direction, in the Wrong Direction, or Repetitions by Income Level, Problem Type, and Operation*

| Income level | Addition | | | | Subtraction | | | |
|---|---|---|---|---|---|---|---|---|
| | Right | Wrong | Augend | Addend | Right | Wrong | Minuend | Subtrahend |
| | | | | Nonverbal problems | | | | |
| Low | 76 | 05 | 05 | 13 | 51 | 09 | 21 | 19 |
| Middle | 82 | 02 | 07 | 09 | 49 | 09 | 15 | 26 |
| | | | | Story problems | | | | |
| Low | 62 | 11 | 15 | 12 | 50 | 20 | 18 | 11 |
| Middle | 72 | 07 | 06 | 15 | 63 | 11 | 11 | 15 |
| | | | | Word problems | | | | |
| Low | 45 | 12 | 19 | 24 | 34 | 32 | 14 | 20 |
| Middle | 54 | 14 | 12 | 19 | 44 | 28 | 15 | 13 |
| | | | | Number–fact problems | | | | |
| Low | 54 | 12 | 16 | 17 | 28 | 40 | 19 | 13 |
| Middle | 69 | 07 | 14 | 10 | 41 | 31 | 19 | 09 |

dition and subtraction calculations were presented in four problem-type formats: nonverbal problems, story problems, word problems, and number–fact problems. The most striking finding was the significant interaction between income level and problem type. Although middle- and low-income children performed equally well on the nonverbal problems, the middle-income children performed significantly better than the low-income children on the three verbal problem types. This pattern was observed on both addition and subtraction calculations.

Our findings support Ginsburg and Russell's (1981) claim that certain mathematical skills develop in a "robust fashion" despite environmental influences related to socioeconomic status. Although the low-income child's environment may not provide as many opportunities for learning conventional verbal arithmetic skills as that of the middle-income child, it nevertheless seems to provide the necessary experiences with combining and separating sets of objects to permit the development of calculation ability. Children's skill on the nonverbal calculation task may reflect knowledge that has been constructed directly from their own actions on objects as well as from their observations of the world (i.e., seeing objects being added or taken away; Piaget, 1971). Such skills do not appear to be sensitive to environmental factors, such as socioeconomic variation.

Observations of problem-solving strategies revealed that children in both income groups rarely counted overtly on the nonverbal calculation task. However, this does not preclude the possibility that they were counting silently to arrive at solutions to problems. First, our data show that both middle- and low-income children have skill in counting. In particular, they successfully counted single sets of objects and, for the most part, could state the cardinal value of sets. It should be noted that children in both income groups received some instruction in counting in kindergarten. Second, the tendency for addition problems to be easier than subtraction problems indicates that children may have been counting. That is, for nonverbal addition problems, the child could obtain the correct answer by determining the size of the augend and then counting each additional item as it is slid under the cover (Fuson, 1982). A similar approach for subtraction is more difficult because it involves counting backward (Fuson & Willis, 1988; Thornton, 1990).

Although middle- and low-income children performed at a similar level on the nonverbal calculation task, we obtained large differences between the two groups on all of the verbal calculation tasks. To understand why middle-income children perform better than low-income children on verbal calculation tasks, we first must consider the skills that are necessary to solve story, word, and number–fact problems (barring rote memorization of particular answers). For both verbal and nonverbal problems, the child must represent and retain the operation and numerosities of the two terms of the problem. The child must also create a numerical outcome based on the addition or subtraction transformation. Additional abilities, however, are required to solve verbal problems. For example, verbally presented calculation problems require the child to construct representations of the numerosities and operations involved from the linguistic input. For nonverbal problems, on the other hand, the numerosities involved in the terms of the problem as well as the physical act of combining or separating sets are provided. Thus, they may be easier for the child to represent. Furthermore, the ability to solve verbal problems requires particular linguistic knowledge, including the ability to understand relevant vocabulary (e.g., the number words, "and," "take away," "altogether," "left," and so on) and the ability to comprehend the syntax of the problem (Aiken, 1971; Carpenter, Hiebert, & Moser, 1981; Riley et al., 1983).

Our findings showed that middle-income children used their fingers to represent numerosities on verbally presented problems more often than low-income children. In fact, finger-counting strategies were rarely observed among low-income children on any problem type. Because no children in the study were taught to use their fingers for calculation in school, it seems likely that middle-income children acquired this skill at home. This suggestion is supported by Saxe et al. (1987), who found that middle-class mothers engage their children in more complex number activities than working-class mothers, including adding and subtracting with fingers and other concrete objects.

The use of finger-counting strategies was associated with higher performance levels on the verbal calculation tasks for middle-income children. However, middle-income children still performed significantly better than low-income children on each of the verbal calculation tasks when we adjusted for the use of finger-counting strategies. When we adjusted for language abilities, on the other hand, the differences between the two income groups were eliminated on the story and word problems and reduced on the number–fact problems. These findings suggest that the disparity between the middle- and low-income children on the verbal calculation tasks is strongly associated with general linguistic knowledge, such as vocabulary and syntax. However, it also is possible that the low-income children understood the verbal input but could not produce a correct answer without object referents. That is, the low-income children might have performed better on the verbal calculation problems if they had been given objects to represent numerosities.

It should be noted that the majority of middle-income children in this study were White and the majority of low-income children were Black. This pattern reflects general ethnic differences in the poverty rate in the United States (i.e., Black children are disproportionately represented among the total population of children who are poor; McLoyd, 1990). However, there is no reason to believe that ethnicity is relevant to the observed differences between the two income groups in the present study. That is, prior research has shown that Black and White kindergarten children do not differ on mathematics tasks when socioeconomic status is taken into account (Ginsburg & Russell, 1981).

Finally, let us turn to some strictly cognitive issues that come out of the present study. Our previous research showed that subtraction was significantly harder than addition on number–fact problems but not on story problems, in which performance level was approximately the same for the two operations (Levine et al., 1992). The data suggested that the term "take away" may have been problematic for young children. This term was used on number–fact problems but not on story problems, which used the verb "lost" (e.g., "Kim had 4 crayons. She lost 3. How many crayons did she have left?"). In the present study, we used "take away" on all of the verbal subtraction problems. However, we still replicated the finding that subtraction items are harder than addition items on number–fact problems but not on story problems. Moreover, children performed significantly better on subtraction story problems than on subtraction number–fact problems, a difference that was not found in addition. It appears that the absence of a context contributes more to the child's relative difficulty in solving subtraction number–fact problems than an inadequate understanding of the term "take away."

To examine this issue more closely, we assessed children's performance on an intermediate verbal problem type: word problems. Recall that word problems referred to the same objects as the story problems but were presented in a number–fact format, which is relatively free of context. For subtraction, performance level on word problems was between performance level on story problems and number–fact problems (see Figure 1), although it did not differ significantly from either of these problem types. This finding suggests that referring to objects may have a slight facilitatory effect on problems presented in a number–fact format but that embedding calculation problems in a meaningful context is even more helpful.

In conclusion, our results indicate that low-income kindergarten children can transform sets by adding and subtracting elements on a nonverbal calculation task. However, such calculation knowledge is not always apparent on tasks that rely on language and conventional mathematical procedures. The findings underscore the importance of assessing calculation skills in a variety of contexts, both verbal and nonverbal, to determine the abilities that young children have as well as the abilities they lack. Prior research has indicated that some of the difficulty children experience during the early grades in solving mathematical problems is attributable to changes in problem format (i.e., from formats with object referents to formats that rely increasingly on verbal mediation; Carpenter & Moser, 1982; Ginsburg & Allardice, 1984). For low-income children, who may have had less experience with verbal arithmetic tasks, it might be particularly advantageous to supplement verbal calculation problems with concrete referents. In this way, they would be allowed to incorporate rather than abandon the calculation abilities they have developed before formal instruction.

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