ASSESSING EARLY ARITHMETIC ABILITIES: EFFECTS OF VERBAL AND NONVERBAL RESPONSE TYPES ON THE CALCULATION PERFORMANCE OF MIDDLE- AND LOW-INCOME CHILDREN

NANCY C. JORDAN
RUTGERS, THE STATE UNIVERSITY OF NEW JERSEY

JANELLEN HUTTENLOCHER AND SUSAN COHEN LEVINE
THE UNIVERSITY OF CHICAGO

ABSTRACT: In two studies, we compared young children's performance on three variations of a nonverbally presented calculation task. The experimental tasks used the same nonverbal mode of presentation but were varied according to response type: (1) putting out disks (nonverbal production); (2) choosing the correct number of disks from a multiple-choice array (nonverbal recognition); and (3) giving a number word (verbal production). The verbal production task required children to map numerosities onto the conventional number system while the nonverbal production and nonverbal recognition tasks did not. Study 1 showed that the performance of 3-, 4- and 5-year-old middle-income children (N = 72) did not vary with the type of response required. Children's answers to nonverbally presented addition and subtraction problems were available in both verbal and nonverbal forms. In contrast, Study 2 showed that low-income children (3- and 4-year-olds; N = 48) performed significantly better on both nonverbal response type tasks than on the verbal response type task. Analysis of individual data indicated that a number of the low-income children were successful on the completely nonverbal calculation tasks, even though they had difficulty with verbal counting (i.e., set enumeration and cardinality). The findings suggest that the ability to calculate does not depend on mastery of conventional symbols of arithmetic.

Direct all correspondence to: Nancy C. Jordan, Department of Educational Psychology, Graduate School of Education, Rutgers, The State University of New Jersey, 10 Seminary Place, New Brunswick, NJ 08903.

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The ability to calculate, or to transform sets by adding or subtracting elements, is an important foundation for later mathematical learning and is emphasized in the early grades of school. Recent research has found that this basic skill develops before children learn to solve conventional verbal problems, such as story problems (e.g., "Jill had 2 pennies. She got 3 more pennies. How many pennies did she have altogether?") and number-fact problems ("How much is 2 + 3?"). In particular, Levine, Jordan, and Huttenlocher (1992) showed that 4-year-old children can add and subtract on a nonverbal calculation task, while most children do not achieve comparable levels of success on verbally presented story problems and number-fact problems until at least 5½ years of age. The nonverbal calculation task assesses addition and subtraction abilities without requiring the child to use conventional symbols. The child is shown a set of objects that is then hidden with a cover. The set is transformed either by adding or removing objects. Following the transformation, the child's task is to construct an array that contains the same number of objects that were in the final hidden set. Number words are not used to refer to the terms of the problem (i.e., to the augend/minuend or to the addend/subtrahend) nor is the child asked to generate them. Story problems and number-fact problems, on the other hand, require knowledge of conventional verbal symbols, such as understanding of number words and words for operations, as well as an understanding of numerical transformation.

In a subsequent study with 5- to 6-year-old children from middle- and low-income families, Jordan, Huttenlocher, and Levine (1992) found that performance on the nonverbal calculation task is not sensitive to variations in socioeconomic level, whereas performance on comparable verbal tasks is highly sensitive to such variations (i.e., middle-income children perform better than low-income children). Moreover, Huttenlocher, Jordan, and Levine (1994) have found that even 2- and 3-year-olds can perform nonverbal calculations involving small numerosities. These studies suggest that the ability to calculate may not depend on formal instruction or conventional skills. Instead, it may develop through children's experiences with objects in their natural surroundings together with their ability to abstract the relevant information from these experiences.

It is possible, however, that the conventional symbols of arithmetic play some role in the ability to calculate on a nonverbal task. That is, children may spontaneously map the numerosities represented by objects onto the conventional number system, either by counting or by perceptually abstracting numerosities (subitization), even though they are not required to do so. This skill, in turn, might be essential for calculation accuracy, even with small object sets. Prior studies have shown that very young children use conventional number words to enumerate sets of objects and that they understand that the final number used in counting represents the number of objects in the set (e.g., Gelman & Gallistel 1978; Gelman & Meck 1983; Gelman, Meck, & Merkin 1986; Silverman & Rose 1980; Wynn 1990). Further, Starkey, and Gelman (1982) report that 3- to 5-year-old children use conventional counting algorithms to solve addition and subtraction problems that provide object referents and verbal labels for the terms of the problems. One problem with this study, however, is that the information reported is largely anecdotal. Although Starkey and Gelman note that "some" children used their fingers to represent hidden objects and others counted aloud, data regarding the frequency with which these strategies were employed at the various age levels tested as well as on the different calculation problems were not reported. Moreover, the authors did not indicate whether the children who used their fingers to represent the number of objects in the sum or difference also were required to respond with a number word. If children were responding with their fingers rather than with a number word, it is possible that they were performing calculations without mapping numerosities onto number words. That is, they may have mentally represented the number of objects in the final set and matched this to the number of fingers on their hands (in a manner similar to putting out disks on the previously described nonverbal calculation task).

If children accurately map the numerosities represented by objects onto the conventional number system during nonverbal calculation tasks, then responding with a number word might be just as easy as responding nonverbally (by putting out objects). In fact, responding with a number word might even be easier than responding by putting out objects where children not only have to represent an answer but also construct an array based on that representation. If they do not accurately map numerosities onto number words, however, then a verbal response type should be more difficult than a nonverbal response type.

To investigate this issue, the present research systematically varied the type of response on a set of nonverbally-presented calculation problems. Three different calculation tasks were used. For all of the tasks, children were shown sets of disks that were transformed either by adding or removing disks; they saw the initial set and the number of disks that were added or subtracted, but not the final set. Each task, however, used a different type of response. One task required the child to respond by constructing an array of disks containing the number of disks in the sum or difference. This completely nonverbal task was identical to the one described by Levine et al. (1992). Another task required the child to report verbally the number of disks that were in the final set. A third experimental task was included to see how children perform on a second nonverbal task with a nonverbal response format. This task required the child to view four arrays of varying numerosities and to select the array that contained the same number of disks that were in the final hidden set (multiple choice). Responding on a nonverbal multiple-choice task might be even easier than responding by putting out disks, since children only have to recognize (rather than to construct) the correct answer and the number of answers a child could give to each problem is constrained.

We used the calculation tasks described above in two studies. The first study examined the calculation performance of preschool children from middle-income families. In the second study, we go on to examine the calculation performance of preschoolers from low-income families. Because prior research suggests that low-income children may have less experience with the conventional symbols of arithmetic than their middle-income counterparts (Jordan et al. 1992), we predicted that they would show the greatest differences on the calculation tasks involving verbal and nonverbal response types.

STUDY 1

Study 1 assessed whether 3- to 5-year-old children are differentially affected by verbal and nonverbal response types on nonverbally presented calculation tasks and whether performance varies with age. For example, older children, who have more experience mapping numerosities onto number words, may be less affected by response type than younger children. Three-year-olds were the youngest children included in the study, since calculation skill on the nonverbal task barely emerges before this age level (Huttenlocher et al. 1994). We also examined whether children used overt calculation methods (e.g., counting) and whether numerosity size has the same effect on performance across the various response types and age levels.

Further, we assessed children's ability to represent numerosities in the absence of a numerical transformation. The "nontransformation" or matching tasks also were varied according to response type. By giving the nontransformation problems prior to the respective calculation problems the child's understanding of the procedure for the calculation task is facilitated (Jordan et al. 1992). The nontransformation task also allowed us to compare children's performance on problems that do not involve a numerical transformation to their performance on comparable problems that involve a numerical transformation. Thus, we were able to examine the effects of task complexity (i.e., calculation vs. nontransformation) on children's performance.

METHOD

SUBJECTS

The sample consisted of 72 children divided equally into six age groups (years-months): (1) 3-0 to 3-5; (2) 3-6 to 3-11; (3) 4-0 to 4-5; (4) 4-6 to 4-11; (5) 5-0 to 5-5; and (6) 5-6 to 5-11. There were approximately the same number of boys and girls in each age group. The children were drawn from five preschools in central New Jersey and came from middle-income homes where English is the primary language. The children were of varying racial and ethnic backgrounds. None of the children had received formal instruction in addition or subtraction calculation in school.

MATERIALS AND PROCEDURE

Children were tested individually in their schools. Each child was given three calculation tasks and three nontransformation tasks using the same nonverbal mode of presentation. The tasks were varied according to response type: (1) nonverbal production; (2) nonverbal recognition; and (3) verbal production. The tasks were given in two or, in most cases, three sessions. The sessions were from one to five days apart. The order of task presentation was counterbalanced across subjects within each age group.

Calculation Tasks. The same calculations (N = 14; 7 addition problems and 7 subtraction problems) were used for each task format. For addition, the numerosities of the addends and augends were no greater than four and the sums were no greater than six (1 + 1, 1 + 3, 3 + 2, 2 + 2, 4 + 1, 3 + 3, 2 + 4). For subtraction, the numerosities of the minuends and subtrahends were no greater than six and the differences were no greater than four (2 − 1, 3 − 2, 4 − 1, 4 − 2, 5 − 4, 5 − 3, 6 − 2). On each task, the calculations were presented to all children in the same order, with the addition and subtraction items intermixed. The ordering of the individual calculation problems was the same for each problem type. Materials used for task presentation included two 27.9 cm × 7.6 cm cardboard mats, a set of 20 black disks (1.9 cm in diameter), a box to hold the disks and a cover to hide the disks. The size of the cover was 15.2 × 7.6 × 5.7 cm. One of the 7.6 cm sides of the cover had an opening so the experimenter could readily put in or take out the disks. The experimenter and the child sat at opposite sides of a table, each with a mat in front of himself or herself.

For addition problems, the experimenter placed the set of disks comprising the augend in a horizontal line on her own mat in full view of the child. The experimenter stated, "See these dots. Now, watch what I do." The disks were then covered. The experimenter next put the set of disks comprising the addend in a horizontal line in full view of the child and slid them under the cover, one at a time. The two terms of the problem were never in view simultaneously. A comparable procedure was used for subtraction problems, but in this case the disks comprising the subtrahend were removed from under the cover, one at a time. No verbal labels were provided on any of the problems. That is, children were not told how many disks were in the initial array or in the set that was added or removed and they were not asked to give this information.

On the *nonverbal production* calculation task, the child was asked to indicate how many disks were under the cover by placing the appropriate number of disks on his or her mat. The experimenter stated, "Make yours just like mine." On the *verbal production* calculation task, the child was asked to state the number of disks that were in the hidden final set ("How many dots are there?"). On the *nonverbal recognition* calculation task, the child was asked to point to the array with the same number of dots that were in the final hidden set from among four horizontal linear arrays of black dots with varying numerosities ("Look at each one of these pictures. Point to the one that's just like mine."). The four choice arrays were printed on a 21.6 × 27.9 cm piece of white paper, each separated by a thick black line. Foils consisted of the most common errors for individual calculation problems on the nonverbal task found in our previous studies and generally included numerosities that were adjacent to the correct answer for a particular problem (unless the answer was "1"). They also included the augend and addend in the case of addition and the minuend and subtrahend in the case of subtraction (in some cases an adjacency foil was also one of the terms of the problem). The position of the correct option was varied (1 − 4), such that the number of correct answers (of 14 problems) in a particular position was approximately the same (N = 3 or 4 for each position). These positions were presented in a random order. The printed dots were approximately the same size as the disks used in the presentation.

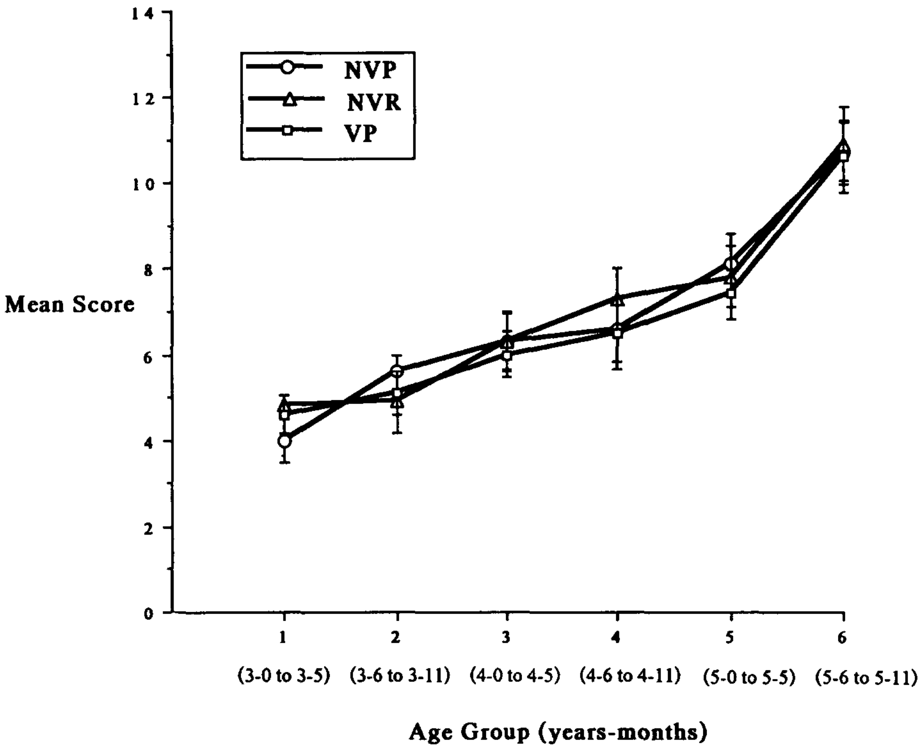
Immediately preceding each calculation task, children were given a nontransformation task using the corresponding response type and the same experimental materials. The experimenter placed one disk on her mat in full view of the child. She then hid the disk under a cover. For the nontransformation task with the nonverbal production response type, the experimenter put a disk on the child's mat and lifted the cover from her own mat. In this way, the child could see that the two mats had the same number of disks on them. The experimenter stated, "See, yours is just like mine," pointing to the disks on both mats. The demonstration item was presented again, following the same procedure, except this time the child was asked to place the disk on his or her mat after the experimenter's disk was hidden. The child was then asked to do this with five other sets of disks (varying in numerosity from 2 to 6). The experimenter did not show the correct answers to the test items. The same procedure was used prior to the nonverbal recognition and verbal production calculation tasks, only on these nontransformation tasks the response types corresponded to those used for the respective calculation tasks (i.e., the nontransformation task with the nonverbal recognition response type required children to select the array that contained the correct numerosity and the nontransformation task with the verbal production response type required children to state the correct numerosity).

In addition to scoring the number of items answered correctly on the calculation tasks, we observed children's solution methods. After each trial, the experimenter noted whether the child used either finger strategies or counting strategies without fingers. Children were classified as using a *finger* strategy if they explicitly counted on their fingers or if they held up their fingers for any term of the problem without counting them in an overt manner. Children were classified as using a *counting* strategy if they displayed explicit counting behaviors without using their fingers (e.g., subvocalizing the number sequence, moving head, etc.).

RESULTS

The mean calculation scores broken down by response mode and age group are graphically displayed in Figure 1. A preliminary analysis revealed no significant

FIGURE 1
Study 1: Mean calculation scores by age group and response type (bars denote standard error; NVP denotes nonverbal production, NVR nonverbal recognition, and VP verbal production).



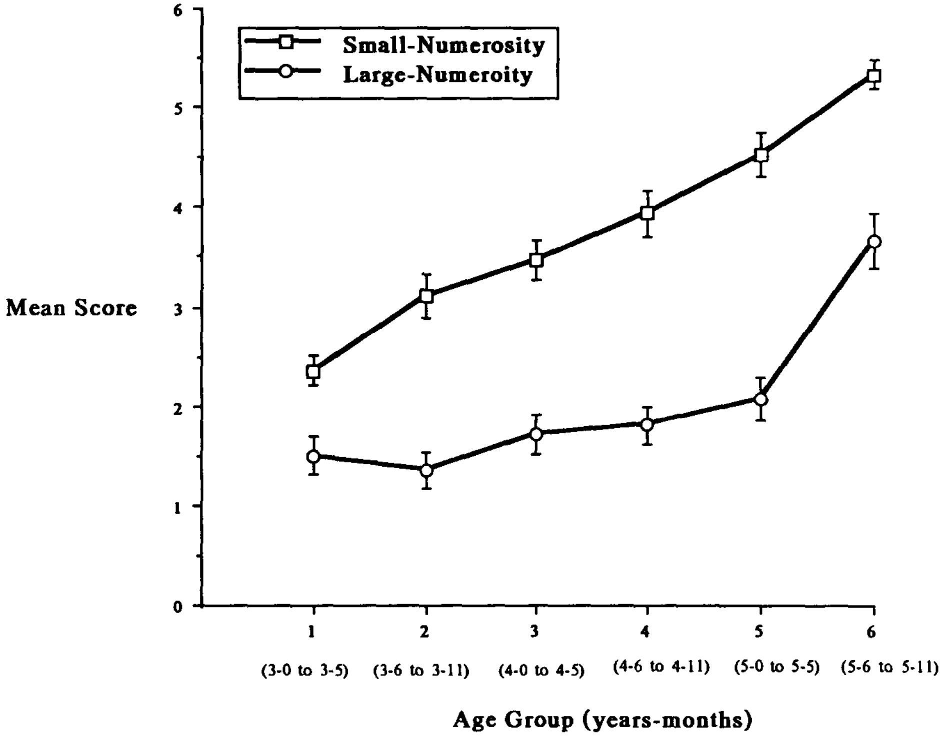
effects of sex. Thus this factor was not used in subsequent analyses. An analysis of variance (ANOVA) with age group (1–6) as a between-subjects factor and response type (nonverbal production, nonverbal recognition, and verbal production) and operation (addition and subtraction) as within-subjects factors was performed. Important for the purpose of Study 1, the analysis showed no main effect of response type ($p < .52$). This finding was true for each age group and operation. A significant main effect of age group, $F(5, 66) = 17.65$, $p < .0001$, indicated that children's performance got better with age. Tukey tests revealed a significant difference between the mean scores of younger and older 5-year-olds for all problem types ($p < .01$), and no significant differences for any of the other adjacent age groups. Three-year-olds performed worse than both younger and older 5-year-olds ($p < .01$ in each case) and 4-year-olds performed worse than older 5-year-olds ($p < .01$ in each case). A significant main effect of operation, $F(1, 66) = 7.21$, $p < .01$, indicated that subtraction problems (mean = 3.60) were slightly easier than addition problems (mean = 3.25). There was no interaction between age and operation.

We also examined the rank orders of calculation items for each response type. The ranking of items was determined on the basis of overall number correct for each item across all subjects within a particular response mode. Spearman rank-order correlations were significant between each pair of tasks: $r_s = .79$ ($p < .01$) between the nonverbal production and nonverbal recognition tasks; $r_s = .95$ ($p < .01$) between the nonverbal production and verbal production tasks; and $r_s = .88$ ($p < .01$) between the recognition and naming tasks. For all response modes at each age level, problems with sums or minuends involving four or less (small-numerosity problems) tended to be in the easier half of the ordering. Conversely, sums or minuends of five or six (large-numerosity problems) tended to be in the harder half.

The finding that young children perform somewhat better on subtraction problems than on addition problems contrasts with a number of studies, which show no difference between the two operations (Levine et al. 1992; Jordan et al. 1992) or that addition problems are solved at an earlier age than subtraction problems (Klein & Starkey 1988). However, Starkey (1992) also found young children perform better on subtraction problems than on addition problems. Because of these inconsistent findings, we investigated the operation difference further. Our finding that small-numerosity problems tend to be easier than large-numerosity problems led us to examine more closely the set sizes involved in our addition and subtraction problems. This analysis revealed that more large-numerosity problems were used for addition than for subtraction. That is, 4 of 7 addition problems involved sums that were greater than 4 whereas only 3 of 7 subtraction problems involved minuends that were greater than 4. In view of this finding, we removed the calculations, $3 + 3$ and $3 - 2$, the two items that did not have a parallel subtraction or addition problem (e.g., $6 - 3$ or $2 + 1$), and redid the ANOVA. The effect of operation was eliminated for all response modes at every age group. Numerosity size, rather than operation, appears to be the major determinant of problem difficulty for all of the calculation tasks.

We also examined whether the numerosity size involved in the problems differentially affects performance on the nonverbal production, nonverbal recognition, and verbal production calculation tasks. We separated the calculation problems into two groups, large numerosity (i.e., sums or minuends of 5 or more) and small numerosity (i.e., sums or minuends of 4 or less), and examined performance across age groups and response modes. Once again, $3 + 3$ and $3 - 2$, the two items that did not have a parallel addition or subtraction problem, were removed from the analysis. Since a preliminary analysis revealed no effect of operation for either small- or large-numerosity problems, we did not include this factor in the analysis. An ANOVA with age group as a between-subjects factor and response type and numerosity as within-subjects factors showed no effect of response type for either small-or large-numerosity problems for any age group. There were significant main effects of age group, $F(5, 66) = 16.64$, $p < .0001$, and numerosity, $F(1, 66) = 309.51$, $p < .0001$, as well as an Age × Numerosity interaction, $F(5, 66) = 4.76$, $p < .001$. Figure 2 reveals that children's performance on small-numerosity problems increased steadily with age, whereas their performance on large-numerosity problems remained relatively flat until approximately 5½ years of age. This difference in slopes accounts for the interaction

FIGURE 2
Study 1: Mean calculation scores by age group and numerosity (bars denote standard error).

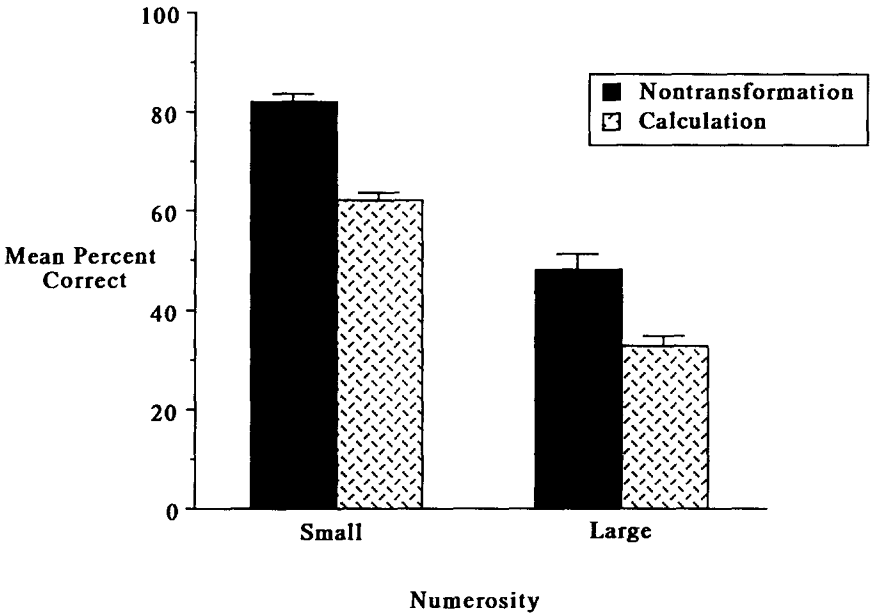


between age and numerosity. Supporting this observation, trend analyses showed a significant interaction of numerosity with a linear contrast ($p < .01$) as well as with a quadratic contrast ($p < .01$).

We also compared children's performance on the calculation tasks to their performance on the nontransformation tasks. Because the calculation and nontransformation tasks did not have the same number of items, children's raw scores were converted to percent correct scores. An ANOVA was performed with age level as a between-subjects factor and response type, task (nontransformation vs. calculation), and numerosity as within-subjects factors. As expected, there was not a significant main effect of response type. The nontransformation task was easier than the calculation task, $F(1, 66) = 90.28$, $p < .0001$. This finding did not vary according to age, numerosity or response type. Figure 3 shows the mean percentage of scores broken down by numerosity size and task. Of particular interest is the finding that children's scores are higher on calculation tasks involving small numerosities than on nontransformation tasks involving larger numerosities, $t(71) = 4.75$, $p < .0001$ (2-tailed).

Finally, we examined children's overt calculation strategies. Most notably, overt calculation strategies were rarely observed on the calculation tasks at any

FIGURE 3
Study 1: Mean percentage correct by task and numerosity (bars denote standard error).



age level. Only one child in the sample of 72 used his fingers on at least one trial on the nonverbal production task and only four children used their fingers on at least one trial on the nonverbal recognition and verbal production tasks. Finger strategies were used on no more than 1% of the total number of trials on any of the three calculation tasks. Counting strategies (without fingers) also were used infrequently. Eighteen children in the sample counted overtly on at least one trial on the production task, 16 children on the recognition task, and 14 children on the naming task (distributed across the age range tested). Counting strategies were used on 8% of the total number of trials on the production task, on 7% of the trials on the recognition task, and on 5% of the trials on the naming task.

DISCUSSION

The results of Study 1 show that response type has no significant effect on the calculation performance of 3- to 5-year-old children from middle-income families. This finding was true for each age level tested, for both small and large numerosity problems, and for both addition and subtraction operations. The relative ease of the nonverbal task, described by Levine et al. (1992), does not depend on whether children respond by putting out objects, by choosing the correct number of objects from a multiple choice array, or by giving a number word. Young middle-income children's answers to nonverbally presented calculation problems are available in both verbal and nonverbal forms.

It should be noted that children's calculation performance on the nonverbal recognition task might have been somewhat inflated. That is, the probability of reaching a correct answer by guessing might have been greater for this task than for the other two tasks, since the number of possible responses on each problem was four. In fact, when we corrected for guessing on the nonverbal recognition task, children's performance was significantly worse on the nonverbal recognition task than on either the nonverbal production or verbal production tasks. Thus, the nonverbal recognition task actually may be harder for young children than the nonverbal production task or the verbal production task, possibly because they lose track of their answer when they have to evaluate several options. However, guessing also may have affected performance on the nonverbal production and verbal production tasks, especially since the correct responses covered a relatively small range of numerosities (1 to 6). Unfortunately, it is not possible to determine what chance level guessing may have been on the nonverbal production and verbal production tasks.

The finding that children rarely used overt strategies on any of the calculation tasks suggests that the object referents used in the type of presentation (i.e., disks) obviated their need to use their fingers to represent numerosities (Levine et al. 1992). The object referents also may encourage some kind of covert counting (recall that the disks were added or taken away from the box one at a time). The absence of a difference in performance level between verbal and nonverbal response types suggests that children may have been using covert counting algorithms. That is, when an answer is obtained through verbal counting, the number word would be available immediately (Brissiaud 1992). However, children also may have accessed numerosities and performed addition or subtraction calculations without counting by subitizing or apprehending the size of a set as a whole (Klahr & Wallace 1973). This would be especially true for small-numerosity problems (Klein & Starkey 1988). For larger numerosity problems, where subitization would be more difficult, children may need to count to reach correct solutions. It also is possible that children solved some of the problems by retrieving answers to previously memorized number facts (Siegler & Robinson 1982). However, the ability to solve number-fact problems develops later than the ability to solve nonverbal problems and usually depends on formal instruction (Levine et al. 1992). Thus, it is unlikely that the children in the present study, at least at the younger age levels, used this kind of "retrieval" strategy to reach correct solutions.

On both the calculation and nontransformation tasks, children performed better on small-numerosity problems than on large-numerosity problems. The results show that numerosity size is an important determinant of performance on quantitative tasks. This finding was true for both addition and subtraction problems, which did not differ from each other when we controlled for numerosity. The finding that quantitative problems involving smaller number sets are solved earlier than those involving larger number sets supports previous reports (e.g., Gelman & Gallistel 1978; Levine et al. 1992; Starkey 1992; Starkey & Gelman 1982). Interestingly, children performed better on small-numerosity calculation tasks than they did on large-numerosity nontransformation tasks. Nontransformation tasks do not require children to perform an operation that alters the numerosity of a set. Instead, they simply have to abstract and represent the numerosity of a single set of objects. These results suggest that numerosity size may limit children's performance on arithmetic tasks more than task complexity.

In sum, Study 1's finding that 3- to 5-year-olds do not differ in performance on calculation tasks involving verbal and nonverbal response types shows that they can verbally code the numerosities of the disk sets during nonverbally presented calculation and nontransformation tasks. It is still unclear, however, whether this verbal coding ability is necessary for successful calculation. For example, we do not know whether children label numerosities with number words and access a verbal answer when they are not required to do so (e.g., on the nonverbal production and nonverbal recognition tasks).

STUDY 2

To investigate whether proficiency in calculation depends on verbal coding of number words, we conducted a second study in which the nonverbal production, nonverbal recognition, and verbal production tasks were given to preschool children from low-income families. Study 2 focuses on low-income children because prior research indicates that they may experience more difficulties on conventional verbal number tasks than middle-income children. For example, Jordan et al. (1992) found that middle-income kindergarten children perform better than a comparable group of low-income children on verbal story problems and number-fact problems but not on nonverbal calculation problems. Further, Ginsburg and Russell (1981) found that low-income preschoolers perform worse than middle-income preschoolers on tasks involving knowledge of the conventional counting words and the cardinality principle. These findings suggest that we might find children who have difficulties mapping numerosities onto the conventional number words in a sample of low-income preschoolers.

If young low-income children can represent numerosities and perform numerical transformations despite difficulties mapping numerosities onto number words, we would expect them to perform better on the nonverbal response types than on the verbal response type. If, on the other hand, the representation of numerical transformations depends on verbal coding of number words, low-income children, like the middle-income children in Study 1, may perform no better on the nonverbal response types than on the verbal response type.

In Study 2, we confined our experimental sample to 3- and 4-year-olds because children in this age range would have the least amount of skill on conventional verbal arithmetic tasks. Previous research has shown that low-income 5-year-olds have considerable skill in verbal counting (Ginsburg & Russell 1981; Jordan et al. 1992). Because large numerosity items were very difficult for children in Study 1 until about 5 years of age, we used problems involving only relatively small numerosities (sums or minuends of 4 or less). We also gave children a counting task that required them to enumerate sets of objects and to state how many objects were in each set. This allowed us to examine individual differences in children's counting abilities and their relation to calculation performance.

METHOD

SUBJECTS

The sample consisted of 48 children divided equally into four age groups (years-months): (1) 3-0 to 3-5; (2) 3-6 to 3-11; (3) 4-0 to 4-5; and (4) 4-6 to 4-11. There were approximately the same number of boys and girls in each age group. The children were drawn from two Headstart programs serving impoverished children in central and northern New Jersey. The children came from homes where English is the primary language and were of varying racial and ethnic backgrounds. Prior to the experimental testing, none of the children had received formal instruction in addition or subtraction calculation in their Headstart programs.

MATERIALS AND PROCEDURE

The materials and procedures for the calculation and nontransformation tasks were identical to those describe in Study 1. However, the individual nontransformation and calculation items involved only small numerosities (sums and minuends of four or less). There were three experimental nontransformation items (2, 3, and 4) and eight experimental calculation items ($1 + 1$, $2 - 1$, $2 + 1$, $4 - 2$, $1 + 2$, $3 - 1$, $2 + 2$, $3 - 2$). The addition and subtraction calculations were matched according to numerosity. After completing the nontransformation and calculation tasks, each child was given a counting task. This task was adapted from previous studies in the literature (Gelman & Gallistel 1978; Ginsburg & Russell 1981; Schaeffer, Eggelston, & Scott 1974). First, the child was shown a horizontal linear display of black dots (1.9 cm in diameter) printed on white cardboard. Separate displays of 1, 2, 3 and 4 dots were presented, one at a time and in a fixed random order. As the experimenter presented each display, she said: "Here are some dots. I want you to count each dot. Touch each dot as you count." Immediately after the child finished counting each set of dots, the experimenter hid the display and asked, "How many dots were there?" In this way, it was possible to assess whether the child understood that the final number used in the count sequence represents the number of objects in the set. Each enumeration item was scored as correct if the child counted the sequence of dots with the conventional count words. Each cardinality item was scored as correct if the child stated the final number he or she used in the count sequence, even if a counting mistake had been made. One point was given for each correct enumeration and cardinality response with a total possible counting score ranging from 0 to 8.

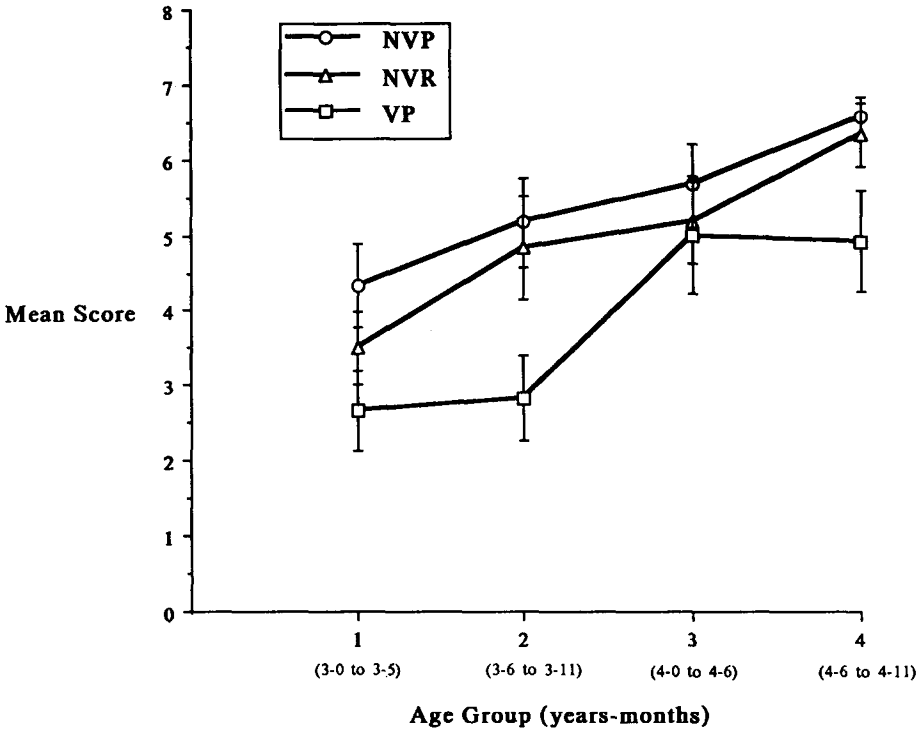
RESULTS

The mean calculation scores for each response type and age group are shown in Figure 4. A preliminary analysis revealed no significant effects of sex or operation. Thus these factors were not included in subsequent analyses. An ANOVA with age as a between-subjects factor and response type as a within-subjects factor was performed. In contrast to the findings of Study 1, there was a significant main effect of response type, $F(2, 88) = 14.96$, $p < .0001$. Tukey tests showed that the low-income children performed worse on the verbal production task than on either the nonverbal production task or the nonverbal recognition task ($p < .01$ in each case). Children's performance on the nonverbal production task and the nonverbal recognition task did not differ significantly. A significant main effect of age, $F(3, 44) = 6.83$, $p < .001$, indicates that the low-income children's performance on the calculation tasks increased with age. Tukey tests showed that younger 3-year-olds performed worse than both younger ($p < .05$) and older ($p < .05$) 4-year-olds and older 3-year-olds perform worse than older 4-year-olds ($p < .05$). The performance of younger and older 3-year-olds did not differ significantly nor did the performance of younger and older 4-year-olds. The interaction between age and response type was not significant.

We also examined children's performance on the nontransformation tasks. The mean nontransformation scores (of 3) were 2.3 (SD = .9) for the nonverbal production response type, 1.8 (SD = .8) for the nonverbal recognition response type, and 1.5 (SD = 1.0) for the verbal production response type. An ANOVA with age as a between-subjects factor and response type as a within-subjects factor showed a significant effect of response type, $F(2, 34) = 9.65$, $p < .001$. Tukey tests indicated that children performed significantly better on the nonverbal production task than either the nonverbal recognition ($p < .05$) or the verbal production ($p < .01$) task. The difference between nonverbal recognition and verbal production did not reach significance, although there was a tendency for children to perform better on the nonverbal recognition task than on the verbal production task. There was a significant effect of age, $F(3, 44) = 4.87$, $p < .01$, but no interaction between age and response type.

Analysis of performance on the counting task showed a significant effect of age, $F(3, 44) = 12.46$, $p < .0001$. The mean counting scores (of 8) were 2.5 (SD = 2.6) for the younger 3-year-olds, 3.2 (SD = 1.6) for the older 3-year-olds, 6.4 (SD = 1.9) for the younger 4-year-olds and 6.9 (SD = 1.4) for the older 4-year-olds.

FIGURE 4
Study 2: Mean calculation scores by age group and response type (bars denote standard error; NVP denoted nonverbal production, NVR nonverbal recognition, and VP verbal production).



Examination of individual differences on the calculation and counting tasks was most revealing. We were especially interested in children who clearly could calculate on one or both of the nonverbal response type tasks but who had difficulty on the verbal response type task. We decided a priori that such "discrepant" children must solve at least 5 of 8 items correctly on the nonverbal production task and/or the nonverbal recognition task but 3 or less on the verbal production task, with a discrepancy of 3 or more points between performance on the verbal response type and at least one of the nonverbal response types. Eighteen of the 48 children (38%) fell into this category: twelve 3-year-olds and six 4-year-olds (10 children met the criterion for both nonverbal production and recognition, 7 for nonverbal production only, and 1 for nonverbal recognition only). We also identified 19 "high performing" children (40%) who demonstrated proficiency on both the nonverbal and verbal response type tasks (four 3-year-olds and fifteen 4-year-olds). These children answered at least 5 of 8 items correctly on the verbal production calculation task and at least 5 of 8 items correctly on the nonverbal production task and/or the nonverbal recognition task. The performance of the remaining children was either generally low (N = 4;

scores of 3 or less on all three response types, all 3-year-olds) or ambiguous (N = 6; not clearly high, low or discrepant). One child performed proficiently on the calculation task with the verbal response type but not on either of the calculation tasks with the nonverbal response types.

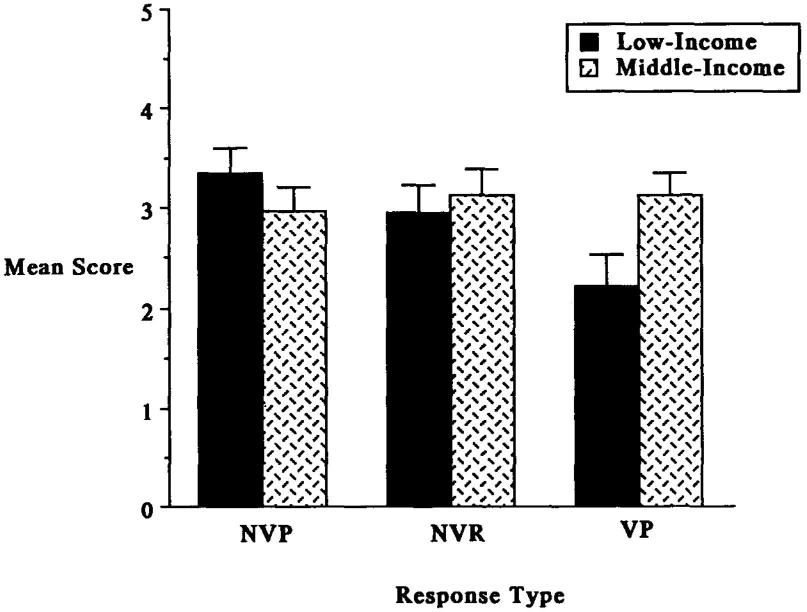
We were especially interested in how the children in the discrepant and high performing subgroups performed on the counting task. The mean counting score was 3.18 (SD = 2.3) for the discrepant children vs. 7.16 (SD = 1.3) for the high performing children, $t(17) = 7.01$, $p < .0001$ (2-tailed). In the Discrepant subgroup, half of the children could not count a set greater than 2 (4 of the children could not count any of the sets). Examples of counting errors included using the incorrect number words (e.g., "8, 2, 7" for a set of 3), substituting letter names for number words (e.g., "i" for a set of one, "3, 2, G" for a set of 4), and failing to make one-to-one correspondences (e.g., "1, 2, 3, 4, 5, 6, 7" for a set of three). More strikingly, 72% of the discrepant children could not give the cardinal number for more than one set (in almost all cases the set responded to correctly was the set of one). In contrast, 99% of the high performing children could count sets to at least 3 and 84% could give the cardinal value for at least three sets. It should be noted that the pattern of performance on the non-transformation task with the verbal response (verbal production), which also assesses understanding of the cardinal number principle, mirrors the pattern found on the counting task (mean out of 3 = 0.9 for discrepant children and 2.5 for high performing children). Clearly, as children learn conventional counting principles (i.e., set enumeration and the cardinal number rule), their ability to perform a calculation task with a verbal response increases.

We next analyzed children's calculation strategies. Children used their fingers on 3% of the total number of trials on the nonverbal production task, on less than 1% of the trials on the nonverbal recognition task and on 34% of the trials on the verbal production task. Eighteen of the 48 children used their fingers on at least half of the trials on the verbal production task making up almost the entire 34%. In all of the cases in which a fingers strategy was recorded, fingers were used only to represent the answer (finger counting during calculation was not observed). Interestingly, children who used the correct number of fingers to represent a sum or a difference on the verbal production task often could not give the appropriate number word. For example, on the problem, $4 - 2$, a child held up two fingers but said "one" when asked how many there were; another child, when shown $2 + 2$, held up four fingers but stated that "three" was the answer. Such behavior was displayed by 13 children (9 of whom were in the discrepant subgroup).

Overt counting, without fingers, was observed rarely on the calculation tasks. Only 3, 7 and 1 subject(s) counted without fingers on at least one of the trials on the nonverbal production, nonverbal recognition, and verbal production tasks, respectively. The total number of trials on which children showed counting behaviors without fingers was 3% on nonverbal production, 5% on nonverbal recognition, and < 1% on verbal production.

Finally, we compared the calculation performance on the low-income 3- and 4-year-olds in Study 2 to the calculation performance of the middle-income 3-

FIGURE 5
Study 2: Mean calculation scores by income group and response type (bars denote standard error; NVP denotes nonverbal production, NVR nonverbal recognition, and VP verbal production).



and 4-year-olds in Study 1. To make a direct comparison, we separated the calculations that were common to both studies (N = 5) and computed individual calculation scores across the three response types. (It should be noted that the middle-income children were given a larger set of problems than the low-income children; thus the following findings should be interpreted cautiously). An ANOVA with income level and age as a between-subjects factor and response type as a within-subjects factor showed a significant Income Level × Response Type interaction, $F(2, 176) = 9.48$, $p < .0001$. Simple effects analyses showed no effect of income level on the nonverbal production task or on the nonverbal recognition task, but a highly significant effect of income level (favoring the middle-income children) on the verbal production task ($p < .001$). This finding did not vary with age. The mean calculation scores (age groups combined) for the middle- vs. low-income children are shown in Figure 5.

DISCUSSION

In contrast to the middle-income children in Study 1, the low-income children in Study 2 were differentially affected by type of response on a nonverbally presented calculation task. That is, for the group as a whole, low-income children performed significantly better on calculation problems with a nonverbal response than on calculation problems with a verbal response.

Further data analyses revealed interesting individual differences in the performance of low-income preschoolers. Among the sample of low-income 3- and 4-year-olds, 38% of the children showed calculation proficiency on the nonverbal response types but not on the verbal response type. Most interestingly, these "discrepant" children successfully performed simple addition and subtraction calculations even though they had difficulty with verbal counting tasks, that is, the ability to enumerate sets of objects with the conventional count words or the ability to state the cardinal number of a set or both (half of the children could count no more than 2 items and 72% of the children could not give the cardinal number for a set of 2 or more). When asked to give a verbal response to a nonverbally presented calculation, a number of these children could not respond with the correct number word even though they spontaneously held up the correct number of fingers. On the other hand, children who performed well on the verbal production response type also showed good skills on our counting task. Thus, the data suggest that children develop basic calculation abilities with small number sets before they acquire proficiency with conventional counting.

The findings of Study 2 add to recent work suggesting that the development of calculation abilities (at least with tasks involving small numerosities) does not depend on verbal coding of number words. Huttenlocher et al. (1994) have shown that the ability to perform addition and subtraction calculations on the same nonverbal task used in the present study (nonverbal production response type) emerges between two and three years of age and that this ability is related to overall intellectual competence. They posit that the ability to calculate on nonverbal tasks is based on a mental version of the initial set of objects and of the movement of objects into or out of the set. The resultant mental array allows the child to produce the correct numerosity of the hidden set. Such a mental model, while symbolic in nature, should not depend on mastery of number words or conventional counting. Using different nonverbal calculation tasks, the claim has been made that even younger children can perform addition and subtraction calculations (Starkey 1992; Wynn 1992).

A comparison of the middle-income 3-year-old children in Study 1 with the low-income 3-year-old children in Study 2 on a subset of problems that was given to children in both studies showed that low-income children perform at about the same level as middle-income children on the nonverbal production and nonverbal recognition calculation tasks. Children in both income groups demonstrated competencies on nonverbal calculation items involving small numerosities. The data suggest that, regardless of income level, children develop important numerical abilities at a very early age, abilities that can serve as underpinnings for later mathematical learning in school (Ginsburg & Allardice 1984; Siegler & Jenkins 1989). However, the finding that low-income children perform worse than middle-income children on the calculation task requiring a verbal response suggests that they may develop certain conventional arithmetic skills later than their middle-income counterparts. In helping young children learn more conventional verbal methods of calculation it may be useful to begin with number tasks in which the numerosities of the terms of the problem are represented with objects. However, further research should examine how concrete objects can be used most effectively in structured settings and how to facilitate the transition from using concrete objects in problem solving to using the more abstract symbols of arithmetic.

Finally, our findings have implications for the assessment of early arithmetic abilities. They show that the nature of the calculation task can affect performance levels, at least with young low-income children. For example, a low-income preschooler might perform poorly on arithmetic tasks depending on conventional verbal knowledge, even though he or she has a good understanding of addition and subtraction operations. Thus, nonverbal calculation tasks, which do not rely on knowledge of number words, would be useful for assessing basic arithmetic skills in early childhood.

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