Calculation Abilities in Young Children with Different Patterns of Cognitive Functioning

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This study examined the arithmetic calculation abilities of kindergarten and first-grade children with different patterns of cognitive functioning: children with low language but adequate spatial abilities (Low Language; n = 33, male = 42%); children with low spatial but adequate language abilities (Low Spatial; n = 21, male = 42%); children with general delays (Delayed; n = 21, male = 48%); and children with no language or spatial impairments (Nonimpaired; n = 33, male = 48%). Each child was given a series of addition and subtraction calculations presented as nonverbal problems, story problems, and number-fact problems. Story problems and number-fact problems require mastery of conventional verbal symbols, whereas nonverbal problems do not. The findings show that nonverbal, story, and number-fact problem formats are differentially sensitive to variation in cognitive ability. The Low Language group performed significantly worse than the Nonimpaired group on story problems but not on nonverbal problems or number-fact problems. The Delayed group performed significantly worse than the Nonimpaired group on nonverbal problems as well as on story problems but not on number-fact problems. The Low Spatial group did not differ significantly from the Nonimpaired group on any of the three problem types, although the overall performance of these children was weaker. When we adjusted for finger use on number-fact problems, the Nonimpaired group outperformed both the Low Language and the Delayed groups but not the Low Spatial group. Thus, the finding that the Low Language and Delayed groups perform as well as the Nonimpaired group on number-fact problems is attributable to their greater finger use.

Language deficits are common among children who have difficulty learning in school (Bashir & Scavuzzo, 1992; Levine & Jordan, 1987) and are included in the federal definition of learning disabilities under P.L. 101-476, the Individuals with Disabilities Education Act of 1990. Interdisciplinary research has shown that upward of 80% of children with early reading problems have concomitant language deficits (Tallal, 1994; Vellutino, 1986). Relatively few studies, however, have been concerned with the development of quantitative abilities in children with language and other cognitive deficits. In fact, the learning disabilities literature in general has devoted surprisingly little attention to problems in mathematics (Garnett & Fleischner, 1987). This is in contrast to the large body of research on reading disabilities that has accumulated over the past several decades. The present study systematically examines the early arithmetic calculation abilities of four subgroups of kindergarten and first-grade children: (a) children with specific language difficulties, (b) children with specific spatial difficulties, (c) children with both language and spatial difficulties, and (d) children without language or spatial difficulties. In particular, the study compares the performance of these subgroups of children on addition and subtraction tasks that vary in terms of linguistic requirements and the availability of object referents.

It has been suggested that subtle cognitive deficits in symbolic or representational thinking, temporal–sequential organization, verbal memory, and rate of verbal processing underlie language deficits in children (e.g., Rees, 1975; Stark & Montgomery, 1994; Stark & Tallal, 1981; Swisher & Hirsh, 1972). Some of these verbal-cognitive deficits also have been implicated as possible causes of certain arithmetic difficulties. Based on an extensive review of the cognitive and neuropsychological literature, Geary (1993) identified three subtypes of early mathematical disabilities, two of which involve selected verbal processes. This classification scheme extends the earlier subtyping work of Strang and Rourke (1985).

The first subtype is characterized by difficulties in the representation and retrieval of arithmetic facts in semantic memory. This type of impairment is associated with dysfunction involving posterior regions of the left hemisphere of the brain (Warrington, 1982). Such difficulties usually coexist with reading difficulties (and, by inference, language difficulties). Children in this category have specific weaknesses in learning and automatizing basic arithmetic facts, even after extensive drill and practice in school. The second subtype involves difficulties with arithmetic procedures, such as using counting strategies to solve calculation problems (Geary, 1990; Geary, Brown, & Samaranayake, 1991). The relation of procedural difficulties to neuropsychological functioning and reading and language development is unclear, although the findings suggest an association with left-hemispheric dysfunction (Ashcraft, Yamashita, & Aram, 1992). Performance characteristics of procedural deficits include the use of developmentally immature arithmetic procedures, delays in acquiring basic procedural concepts, and frequent errors when executing arithmetic procedures. Children with procedural deficits appear to have general rather than specific difficulties with the acquisition of basic number concepts. The third subtype is not associated with verbal abilities but is characterized by deficits in the spatial representation and interpretation of numerical information. In contrast to the aforementioned subtypes, the visuo-spatial subtype involves dysfunction of the posterior regions of the right hemisphere of the brain (Dahmen, Hartje, Bussing, & Strum, 1982) and is not related to reading and language difficulties. Children with visuo-spatial deficits may have difficulty reading arithmetical signs, lining up numbers in multidigit arithmetic problems, and acquiring concepts of place value.

Geary's (1993) taxonomy of mathematical disabilities indicates that children with different functional profiles (e.g., low language/adequate spatial abilities; low spatial/adequate language abilities; low language and spatial abilities) may perform differentially on arithmetic tasks that vary in their linguistic–cognitive requirements. For example, addition and subtraction calculations can be presented to young children in verbal and nonverbal formats. On a nonverbal calculation task, a child might be shown a set of objects that is then hidden. Next, the set would be transformed, by either adding or removing objects from the initial set. The final set is never in view of the child. If children can calculate, they will be able to produce an array containing the correct number of objects. This calculation task does not require conventional verbal knowledge, such as knowledge of the number words or words for operations. The terms of the problem and the addition or subtraction operations are represented physically, without accompanying verbal information. To solve a nonverbal calculation problem, a young child might construct a mental version of the original set and imagine the movement of objects being added to or removed from the set. The resultant mental set could be used to produce the correct answer (Huttenlocher, Jordan, & Levine, 1994). On problems involving larger sets, children might determine the numerosity of the initial array (by either counting the objects or perceptually abstracting its numerosity) and then count up as objects are added or count down as objects are removed. As children become more sophisticated in calculation (after formal instruction), they might solve a nonverbal calculation problem by converting it to a verbal format.

Addition and subtraction calculations also can be presented in story problem formats (e.g., "Carol and Jack like to draw. Carol has 5 crayons. Jack gives her [or takes away] 2 more crayons. How many crayons does Carol have altogether [or left]?"). In these instances, the calculations are embedded in everyday verbal contexts. Although concrete objects, such as crayons or marbles, are referred to, they are not provided. To solve a story problem, children need sufficiently developed verbal skills as well as a basic ability to calculate (Carpenter, Hiebert, & Moser, 1981; Cummins, 1991; Levine, Jordan, & Huttenlocher, 1992). For example, a child must understand number words, words referring to quantities and relations (e.g., "more"), words for operations (e.g., "take away"), and the syntax of the problem. Children with language difficulties may fail to solve story problems correctly, even though they have underlying competencies in calculation (e.g., as seen on a nonverbal task).

A third way to present addition and subtraction calculations to young children is in a verbal number-fact problem format. Number-fact problems involve decontextualized language (e.g., "How much is 5 and 2," or, "How much is 5 take away 2?"); objects are not shown or referred to (Jordan, Huttenlocher, & Levine, 1992). Learning number facts is usually emphasized in early elementary school. Children frequently are taught to retrieve addition and subtraction facts from memory. When children first learn the number facts, however, they may need to represent the numerosities of the terms of the problem with fingers or other concrete objects to reach the correct answer (Jordan, Levine, & Huttenlocher, 1994).

Over the past several years, we have investigated young children's abilities to calculate on both verbal (story and number-fact problems) and nonverbal tasks. In an initial study, Levine et al. (1992) compared the performance of 4-, 5-, and 6-year-old children on nonverbal calculation problems to their performance on orally presented story problems and number-fact problems. Calculations involving identical numerosities were used for each of the three problem types. The results showed that children can perform calculations on nonverbal tasks before they can calculate on conventional verbal tasks. Although children as young as 4 years of age were successful on nonverbal problems involving small numerosities, they did not demonstrate proficiency on story problems and number-fact problems until at least 5 years of age. More recently we have found that even 2- and 3-year-old children have some success in solving simple nonverbal calculation problems and that this skill does not depend on conventional knowledge of the number system, such as verbal counting (Huttenlocher et al., 1994; Jordan, Huttenlocher, & Levine, in press). Overall, our findings show that children bring considerable informal calculation knowledge to school.

In a related study with kindergarten children, we found that the nonverbal calculation task format is less sensitive to socioeconomic variation than are the verbal task formats (Jordan et al., 1992). That is, children of middle- and low-income families showed similar levels of performance on the nonverbal calculation task (although the two groups performed well below ceiling level). In contrast, low-income children performed much worse than middle-income children on story problems and number-fact problems, possibly because the former received less conventional verbal input at home (Kirk, Hunt, & Volkmar, 1975; Snow, 1983). A follow-up study, 1 year later, showed that many of these difficulties persisted after formal instruction in first grade (Jordan et al., 1994).

It should be noted that the low-income children in Jordan et al.'s (1992) study also performed worse than the middle-income children on a test of language abilities. However, the children's spatial or general nonverbal abilities were not measured; as a result, it is not clear whether the low-income children had specific language difficulties or more general cognitive delays.

To date, we have no data on the performance of children with different cognitive patterns on verbal and nonverbal calculation tasks. However, our previous work suggests that the calculation tasks may be differentially sensitive to variation in cognitive ability. Children with language difficulties but intact spatial abilities may perform at a normal level on nonverbal calculation problems but not on verbal calculation problems. In contrast, children with both language and spatial difficulties might perform below normal on nonverbal calculation problems as well as on verbal calculation problems. It is more difficult to predict how children with spatial difficulties but intact language abilities will perform. Children in this group might have difficulty keeping track of the objects presented on the nonverbal problems, which could contribute to performance delays. However, spatial difficulties may become less important for calculation as children learn basic number facts (Geary, 1993; Hartje, 1987).

To examine these issues, the present study directly compared the performance of kindergarten and first-grade children with specific language difficulties, with specific spatial difficulties, with both types of difficulties, and with neither difficulty on a nonverbal calculation task, as well as on two verbal calculation tasks (story problems and number-fact problems). The tasks were similar to those used in our previous studies (Jordan et al., 1992; Levine et al., 1992). Inclusion of both kindergarten and first-grade children enabled us to compare calculation performance before and after formal instruction. All of the children were drawn from regular public school classrooms; thus, our sample consisted of children with relative strengths and weaknesses in language and spatial abilities, rather than those with diagnosed clinical impairments.

In addition to examining the children's accuracy on the calculation tasks, we also observed their problem-solving strategies. We were particularly interested in how often the children used their fingers to represent numerosities on the verbal calculation tasks, for which concrete objects are not available. Previous work suggests a developmental sequence in the use of calculation strategies on story and number-fact problems (Jordan et al., 1992; Jordan, Levine, & Huttenlocher, in press). Prior to receiving formal instruction, children tend to answer story and number-fact problems without using overt calculation strategies, such as fingers, but their accuracy is low. This suggests that early on, children answer verbal calculation problems by guessing. After formal instruction, children use their fingers (or other concrete objects) frequently on both story and number-fact problems, and their performance levels increase dramatically. Eventually children learn to calculate without using their fingers or other objects for representation. Infrequent use of overt calculation strategies no longer reflects guessing but, rather, an ability to retrieve answers from memory or to use sophisticated mental calculation strategies.

By examining children with language and/or spatial difficulties in both kindergarten and first grade, we can determine whether different cognitive patterns affect the development of calculation strategies. For example, children with language difficulties may use their fingers to represent numerosities more frequently and for more extended developmental periods than children without such difficulties. As noted earlier, difficulty in retrieving answers to number-fact problems is associated with selected verbal deficits.

Finally, we analyzed children's errors on the calculation tasks to determine whether they reflected a basic understanding of addition and subtraction operations even though their answer was not correct (Levine et al., 1992). Specifically, we examined whether children's errors were at least in the right direction, that is, greater than the augend for addition or less than the minuend for subtraction.

Method

Subjects

Prior to subject selection, we screened the language and spatial abilities of 98 kindergarten children and 98 first-grade children. All were in regular education classrooms, which were not grouped according to ability level. They came from two public schools in the same community in central New Jersey. Each of the schools serves children of varying social classes and ethnic backgrounds. Children participating in bilingual or English as a Second Language programs were not included in the screening pool. None of the children had been diagnosed as having primary hearing, vision, or emotional problems. The same mathematics program (Mathematics Their Way; Baratta-Lorton, 1976) was used to teach all of the participating children; the program uses concrete materials to develop an understanding of early mathematics concepts. The kindergarten children had not received formal instruction in addition and subtraction calculation in school.

To screen language abilities, we administered the short form of the Test of Language Development–Primary (TOLD-P) (Newcomer & Hammill, 1988) to each child individually. The TOLD-P short form is presented orally and includes the Picture Vocabulary and Grammatic Completion subtests. To screen spatial abilities, children were given the Test of Nonverbal Intelligence–Second Edition (TONI-2) (Brown, Sherbenou, & Johnsen, 1990). On the TONI-2, subjects are asked to complete a series of nonverbal matrices, one at a time. No listening, speaking, reading, or writing is required. Salvia and Ysseldyke (1991) observed that the TONI-2 is especially useful for examining the cognitive abilities of children with language impairments.

Children's raw scores on each test were converted to percentile scores, which were based on the raw scores of the total number of children who were tested at each grade level. Children who scored below the 30th percentile on the TOLD-P but above the 30th percentile on the TONI-2 were assigned to the Low Language group (n = 19 kindergartners and 14 first graders). Children who scored below the 30th percentile on the TONI-2 but above the 30th percentile on the TOLD-P were assigned to the Low Spatial group (n = 11 kindergartners and 10 first graders). Children who scored below the 30th percentile on both the TOLD-P and the TONI-2 were assigned to the Delayed group (n = 10 kindergartners and 11 first graders). Finally, a group of children who scored above the 30th percentile on both the TOLD-P and the TONI-2 were assigned to the Nonimpaired group (n = 19 kindergartners and 14 first graders). Children in the Nonimpaired group were matched as closely as possible to children in the Low Language group in terms of nonverbal ability and gender. The mean TOLD-P and TONI-2 percentile scores broken down by ability group are presented in the first column of Table 1. These scores are close to the scores obtained with national norms (based on age; see column 2 of Table 1).

The mean spatial (TONI-2) percentile score of the Nonimpaired group did not differ significantly from the mean spatial percentile score of the Low Language group, $F(1, 62) = 1.7$, $p < .19$. The mean language (TOLD-P) percentile score of the Nonimpaired group was significantly higher than the mean language percentile score of the Low Spatial group, $F(1, 50) = 5.1$, $p < .03$. Neither of these findings varied according to grade level. Some of the demographic characteristics of the children in the four ability groups are presented in Table 2.

Materials and Procedure

Children were tested individually in school. Identical addition and subtraction calculations were presented in three formats: (a) nonverbal problems, (b) story problems, and (c) number-fact problems. For each problem type, there were two practice items (1 + 1; 2 − 1) and 14 experimental items (1 + 3; 5 − 3; 7 − 4; 4 + 2 − 1; 4 − 1; 5 + 4; 2 + 1; 9 − 5; 3 +4; 3 − 2; 2 + 6 − 3; 2 + 3; 6 + 5 − 3; 8 + 4). The calculation items were presented in the same order for each problem type. On all of the tasks, the experimenter said, "We are going to play some number games. I am going to ask you some questions, and you can do anything you want that will help you find the answer. Take your time. You don't have to answer quickly. Some questions might be easy for you and others might be hard. Don't worry if you don't get them all right. You can figure them out any way you want. You can count, you can use your fingers, or you can do it in your head. Let's try one."

Nonverbal Problems. Materials for the nonverbal problems included two 28-cm × 15-cm cardboard mats, a set of 20 black disks (1.9 cm in diameter),

TABLE 1
Mean Language and Spatial Percentile Scores for Children in the Four Ability Groups

Group	Language[a] Study sample		Language[a] National norms		Spatial[b] Study sample		Spatial[b] National norms	
Nonimpaired	72	(18)	69	(20)	65	(19)	60	(23)
Low Language	17	(68)	15	(10)	60	(18)	55	(21)
Low Spatial	61	(17)	59	(21)	13	(07)	18	(10)
Delayed	14	(09)	14	(13)	16	(08)	20	(10)

Note. Standard deviations are given in parentheses.
[a]Test of Language Development–Primary. [b]Test of Nonverbal Intelligence.

TABLE 2
Demographic Characteristics of Children in the Four Ability Groups

Group	*n*	Age	Gender (M/F)	Ethnicity: White	Ethnicity: African American	Ethnicity: Other[a]	SES (Low income[b])
Nonimpaired	33	6.0 (0.6)	16/17	18	9	6	1
Low Language	33	6.1 (0.7)	14/19	10	12	11	9
Low Spatial	21	6.0 (0.5)	9/12	14	4	3	2
Delayed	21	6.1 (0.7)	10/11	6	14	1	10
Total	108	6.1 (0.6)	49/59	48	39	21	22

Note. Standard deviations are given in parentheses.
[a]"Other" includes children of Asian and Hispanic backgrounds. [b]Children who participated in the free and reduced-price lunch program in school were considered to be low income.

a box, and a cover. One of the sides of the cover had an opening so the experimenter could easily put in or take out disks. The experimenter and the child sat on opposite sides of a child-size table, each with a mat in front of herself or himself.

To teach the procedure for the nonverbal task, the experimenter placed three disks on her mat, in full view of the child, and said, "See this? Now watch what I do." The disks were then hidden under the cover. Next, the experimenter placed three disks in a horizontal line on the child's mat and lifted the cover to show the three disks on her own mat, saying, "See, yours is just like mine." The demonstration item was presented again, following the same procedure, except this time the child was asked to place the appropriate number of disks on his or her mat after the original set of disks was hidden. The experimenter said to the child, "Make yours just like mine." If the child placed the wrong number of disks on the mat, the response was corrected and the demonstration was repeated one more time.

For the addition problems, the experimenter placed the set of disks constituting the augend in a horizontal line on her mat and asked the child to watch what she did next. The disks were then hidden under the cover. The experimenter next put the set of disks constituting the addend in a horizontal line in full view of the child and then slid them under the cover through the side opening, one at a time. The two terms of the problem were never simultaneously in view. After the transformation had been made, the experimenter said, "Make yours just like mine." The child then indicated how many disks were under the cover by placing the appropriate number of disks on his or her mat. A comparable procedure was used for the subtraction problems, but in this case the disks constituting the subtrahend were removed from under the cover, one at a time. The three-term problems combined the addition and subtraction procedures. Verbal labels were not provided on any of the nonverbal problems, nor was the child asked to generate them. However, if children responded with a number word, rather than by putting down disks, their answer was accepted.

Verbal Problems. The experimenter read the story problems to the children. The verbal content was intended to be as simple as possible. The addition story problems required children to join two sets of objects (e.g., "Lydia and Robert like snack time when they can eat crackers. Lydia has 1 cracker. Robert gives her 3 more crackers. How many crackers does Lydia have altogether?"); the subtraction story problems required them to separate a set of objects (e.g., "Kim and Andrew are playing games with their marbles. Kim has 5 marbles. Andrew takes away 3 of her marbles. How many marbles does Kim have left?"). The three-term problems required children to join and then separate a set of objects (e.g., "Beth and Steve go to the circus. They see the clowns and get some balloons. Beth has 4 balloons. Steve gives her 2 more balloons. Then he takes 1 balloon back. How many balloons does Beth have now?"). The child was asked to respond to each item with a number word. All of the problems were parallel in structure. The names of the actors and the objects were varied to sustain children's interest.

The number-fact problems also were presented orally. The experimenter read the addition number-fact problems as "How much is x and y?", the subtraction number-fact problems as "How much is x take away y?", and the three-term problems as "How much is *x* and *y* take away *z*?" As on the story problems, the child was asked to respond to each item with a number word.

Upon request, or if the child clearly did not appear to be attending, the experimenter repeated a problem once. For each of the three experimental calculation tasks, the total possible score ranged from 0 to 14. The order of presentation for the three calculation tasks was counterbalanced for each ability group and grade level.

During the testing, the experimenter recorded the child's answer as well as the general strategy used to solve each problem. We coded two overt strategies: fingers and counting. Children were classified as using a *finger strategy* if they explicitly counted with their fingers, either orally or by moving their fingers or head. Children were classified as using a *counting strategy* if they displayed counting behaviors without using their fingers (e.g., subvocalizing the number sequence, moving lips). When children answered without using their fingers or counting overtly, they were classified as using an *unobserved strategy*. In these instances, they may have been using a mental strategy, retrieving the answer from memory, or guessing (Geary & Burlingham-Dubree, 1989; Siegler & Robinson, 1982). Prior to receiving formal instruction, children probably are guessing or using rough estimation skills when unobserved strategies are coded on story and number-fact problems, whereas after formal instruction it is more likely that they are using retrieval or mental computational strategies (Jordan et al., 1994).

Results

Children's performance on each of the three calculation tasks was scored for the number of items answered correctly (out of 14). To examine whether children in the four ability groups performed differentially on the verbal and nonverbal calculation tasks, we performed a repeated-measures multivariate analysis of variance (MANOVA), with ability group (Nonimpaired, Low Language, Low Spatial, and Delayed) and grade (kindergarten and first grade) as between-subjects factors, and problem type (nonverbal problems, story problems, and number-fact problems) as a within-subjects factor. The results showed a significant main effect of ability group, $F(1, 100) = 13.88$, $p < .0001$. Contrasts (planned) indicated that on the three problem types combined, the Nonimpaired group performed better than the Delayed group ($p < .0001$), the Low Language group ($p < .004$), and the Low Spatial group ($p < .02$).

Of particular interest for this study was the significant Ability Group × Problem Type interaction, $F(6, 200) = 4.89$, $p < .0001$ (see Figure 1). Contrasts showed a significant interaction of problem type with the Low Language versus the Nonimpaired group ($p < .0001$), the Low Language versus the Low Spatial group ($p < .0001$), and the Low Language versus the Delayed group ($p < .004$). These findings reflect a larger discrepancy between performance on nonverbal problems and verbal problems for the Low Language group than for children in any of the other ability groups (i.e., the Low Language group showed better performance on nonverbal problems than on story problems or number-fact problems, whereas children in each of the other ability groups showed equivalent levels of performance across the three problem types).

Univariate tests were performed to determine whether the performance of children in the four ability groups differed on each of the three problem types. (Using the Bonferroni procedure, alpha was adjusted to .01 by dividing .05 by the number of problem types.) On nonverbal problems, there was a significant effect of ability group, $F(3, 104) = 7.39$, $p < .0002$. Tukey tests

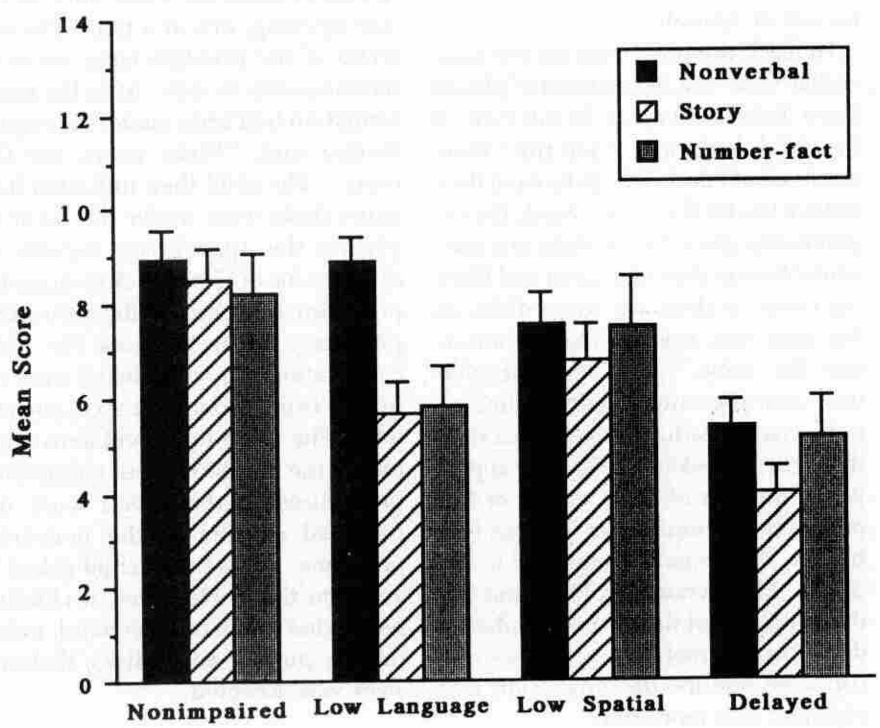

FIGURE 1. Mean calculation scores by problem type and ability group (bars denote standard error).

showed that children in the Nonimpaired group and the Low Language group performed significantly better than those in the Delayed group ($ps < .01$ in each case). On story problems, there also was a significant effect of ability group, $F(3, 104) = 7.35$, $p < .0002$. Tukey tests showed that the children in the Nonimpaired group performed better than children in the Low Language group and the Delayed group ($ps < .01$ in each case). On number-fact problems, the effect of ability group did not reach significance, $F(3, 104) = 2.59$, $p < .05$.

Finally, the main MANOVA model also showed significant main effects of grade, $F(1, 100) = 119.58$, $p < .0001$, and problem type, $F(2, 200) = 14.9$, $p < .0001$, as well as a Grade × Problem Type interaction, $F(2, 200) = 27.6$, $p < .0001$. Separate analyses were performed at each grade level (alpha adjusted to .01). There was a significant effect of problem type in both kindergarten, $F(2, 116) = 33.78$, and first grade, $F(2, 96) = 9.1$, $p < .0002$. However, there were different patterns of performance at the two grade levels (see Figure 2). In kindergarten, nonverbal problems were significantly easier than either story problems or number-fact problems; in first grade, number-fact problems were significantly easier than story problems but not nonverbal problems ($p < .01$, Tukey tests).

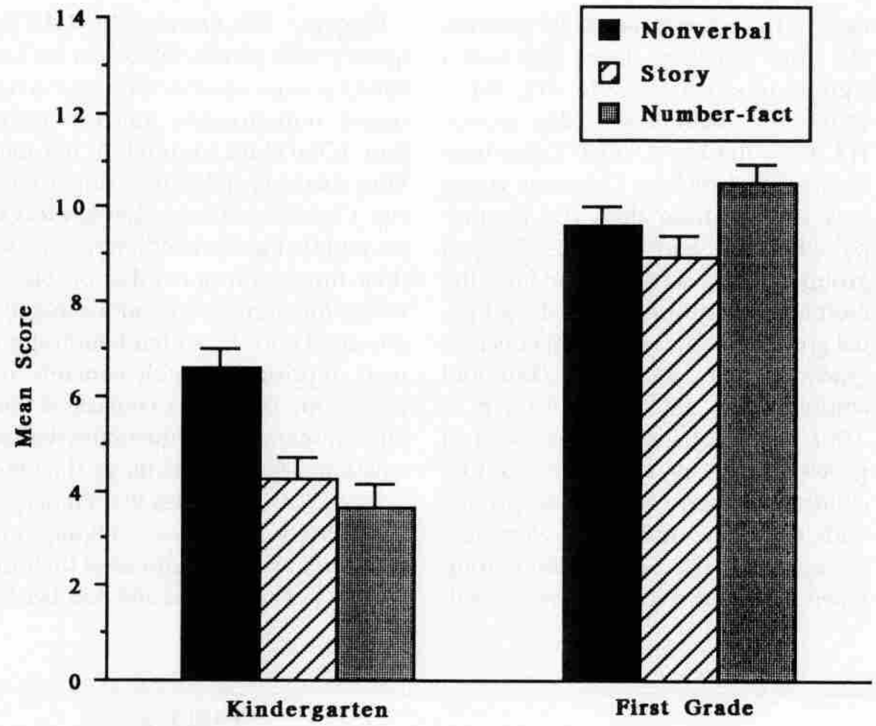

FIGURE 2. Mean calculation scores by problem type and grade (bars denote standard error).

Children's Calculation Strategies

These analyses focus on the relative frequency with which children in the four ability groups used unobserved, finger, and counting (without fingers) strategies on the calculation tasks. We also examined the effectiveness of these strategies. Table 3 shows the mean percentage of trials on which children used the various strategies. It also shows the mean percentage of times a particular strategy produced a correct answer. The data are broken down by grade, ability group, and problem type. (Note that the standard deviations can be substantially larger than the means, especially when the means are very low. This indicates a positively skewed distribution of the children's scores, because percentages necessarily have a minimum of 0 and a maximum of 100. Therefore the comparison of two standard deviations can provide a description of relative variability; however, the standard deviation cannot be used in the same manner as it can with a quantitative variable having a less restrictive range.) Because of the number of statistical analyses performed in this section, we used the .01 alpha level to guard against Type I errors.

Unobserved Strategies. We first examined whether there were ability group differences in the frequency with which children used unobserved strategies. A repeated-measures MANOVA on the percentage of trials on which unobserved strategies were used with grade and ability group as between-subjects factors and problem type as a within-subjects factor showed a significant Grade × Ability group interaction, $F(3, 100) = 5.63$, $p < .001$. Simple effects analyses showed a significant effect of ability group in kindergarten ($p < .01$) but not in first grade. This effect was due to the Delayed group, who, in contrast to the other groups, used unobserved strategies almost exclusively in kindergarten.

Our next question concerned the calculation accuracy of children in the four ability groups when strategies were not observed. Thus, we performed ANOVAs on children's percentage-correct scores when strategies were unobserved. (Children who did not use unobserved strategies on a particular problem type were excluded from the respective analyses.) On nonverbal problems, there were significant effects of grade (kindergartners less accurate than first graders), $F(1, 96) = 38.01$, $p < .0001$, and ability group, $F(3, 96) = 6.52$, $p < .0005$. Tukey tests ($p < .01$) showed that the Delayed group was less accurate than each of the other three ability groups. On story problems, there also was a significant effect of grade, $F(1, 96) = 24.09$, $p < .0001$, and ability group, $F(3, 96) = 10.61$, $p < .0001$. Tukey tests showed that the Low Language group was less accurate than the Nonimpaired group and that the Delayed group was less accurate than both the Nonimpaired group and the Low Spatial group. Finally, significant effects of grade, $F(1, 81) = 18.89$, $p < .0001$, and ability group, $F(3, 81) = 9.15$, $p < .0001$, were obtained on number-fact problems. Tukey tests showed that children in each of the three groups with cognitive difficulties were less accurate than the Nonimpaired group when unobserved strategies were used.

Fingers. We also examined the frequency with which children in the four ability groups used their fingers to represent numerosities during calculation. (One child counted on her toes! This also was coded as a finger strategy.) Table 3 indicates that kindergarten and first-grade children rarely used their fingers on nonverbal problems, where the numerosities of the augend/minuend and the addend/subtrahend were represented with concrete objects. This finding is consistent with other research using the nonverbal calculation task (e.g., Levine et al., 1992).

Table 3 also indicates that kindergarten children in the Low Language and Delayed groups seldom used their fingers on story problems and number-fact problems. In first grade, children in every ability group increased their use of finger strategies on the verbal problems. Among first graders, there were no significant effects of ability group on the frequency or effectiveness of finger use for either problem type.

Recall that there was no effect of ability group on number-fact problems (number correct) in our initial analysis. However, because finger strategies were associated with accurate performance, and because children used them frequently in first grade, we performed an analysis of covariance with children's accuracy on number-fact problems as the dependent variable and percentage of trials on which fingers were used as a covariate. This

TABLE 3
Mean Percentage of Trials on Which a Strategy Was Used and
Mean Percentage of Trials on Which a Strategy Produced a Correct Answer

	Unobserved				Finger				Counting (without fingers)			
	Mean % trials		Mean % correct		Mean % trials		Mean % correct		Mean % trials		Mean % correct	
	K	1	K	1	K	1	K	1	K	1	K	1
Nonimpaired												
Nonverbal problems	65	51	83	59	02	15	68	70	34	35	43	58
	(25)	(21)	(17)	(32)	(07)	(16)	(02)	(35)	(20)	(21)	(37)	(32)
Story problems	70	51	43	86	21	47	61	79	09	02	21	75
	(33)	(38)	(34)	(16)	(31)	(39)	(32)	(28)	(21)	(03)	(37)	(50)
Number-fact problems	67	51	58	92	21	38	58	66	12	11	60	75
	(33)	(37)	(62)	(14)	(30)	(36)	(38)	(33)	(23)	(21)	(37)	(42)
Low Language												
Nonverbal problems	68	42	86	60	01	14	58	64	31	44	39	70
	(28)	(24)	(17)	(22)	(03)	(20)	(33)	(45)	(28)	(23)	(29)	(32)
Story problems	93	40	22	57	06	58	81	69	01	02	50	83
	(17)	(33)	(19)	(39)	(17)	(34)	(24)	(17)	(03)	(04)	(70)	(29)
Number-fact problems	82	24	14	60	15	68	18	84	03	07	06	80
	(33)	(31)	(19)	(43)	(30)	(37)	(30)	(14)	(07)	(15)	(13)	(45)
Low Spatial												
Nonverbal problems	56	62	80	51	06	10	72	64	38	29	28	48
	(29)	(25)	(19)	(24)	(19)	(17)	(40)	(17)	(21)	(23)	(28)	(32)
Story problems	89	60	37	64	10	39	44	69	01	01	00	67
	(26)	(30)	(29)	(31)	(03)	(30)	(51)	(35)	(03)	(02)	(00)	(47)
Number-fact problems	57	50	16	75	40	49	43	88	04	01	33	00
	(39)	(38)	(19)	(33)	(41)	(39)	(22)	(13)	(11)	(02)	(58)	(00)
Delayed												
Nonverbal problems	96	38	63	29	00	02	—	00	04	60	00	40
	(07)	(29)	(23)	(16)	(00)	(03)		(00)	(07)	(27)	(00)	(24)
Story problems	100	51	18	24	00	45	—	46	00	04	—	00
	(00)	(37)	(14)	(30)	(00)	(40)		(26)	(00)	(13)		(00)
Number-fact problems	97	25	14	22	02	70	00	69	01	04	100	50
	(07)	(40)	(14)	(16)	(05)	(45)	(00)	(12)	(02)	(13)	(00)	(00)

Note. Standard deviations are given in parentheses.

analysis showed a significant effect of ability group, $F(3, 100) = 5.60$, $p < .001$. Least-square-means comparisons revealed that the adjusted mean for Nonimpaired children was significantly higher than the adjusted mean for Low Language children ($p < .001$) and Delayed children ($p < .001$), but not for Low Spatial children. Thus, Nonimpaired children perform better than Low Language children and Delayed children on number-fact problems when frequency of finger use is taken into account.

Counting. Finally, we examined children's use of counting strategies (without fingers). Because this strategy was used infrequently on story problems and number-fact problems (see Table 3), we analyzed the data only for nonverbal problems. When children counted on the nonverbal problems, they usually counted the first set of disks and/or the disks that were added or taken away. An ANOVA on the number of trials on which counting strategies were used on nonverbal problems with grade and ability group as between-subjects factors showed a significant Grade × Ability group interaction, $F(3, 100) = 8.55$, $p < .0001$. Tukey tests ($p < .01$) showed a significant increase in counting strategies between kindergarten and first grade only for children in the Delayed group.

We also analyzed children's accuracy when counting strategies were used on nonverbal problems. Because counting strategies were almost never used by kindergarten children in the Delayed group, we performed ability group comparisons only on first-grade children. There was no effect of ability group on the accuracy of children's counting strategies on nonverbal problems.

Children's Errors

The direction of children's errors indicates whether children have some knowledge of the effects of addition and subtraction operations even if they cannot arrive at correct answers (Jordan et al., 1992; Levine et al., 1992). An incorrect response was coded as being in the right direction if it was greater than the augend for addition (e.g., 4 + 2 = 7) or less than the minuend for subtraction (e.g., 7 − 4 = 4). Problems involving three terms were excluded from this analysis. The mean percentage of children's errors that were in the right direction, broken down by ability group and problem type, are presented in Table 4. An ANOVA on the percentage of errors that were in the right direction for nonverbal problems showed a significant effect of ability group, $F(3, 87) = 5.16$, $p < .003$. Tukey tests ($p < .01$) showed that the Nonimpaired group made more right-direction errors than did the Delayed group. No significant effects of ability group for story problems or number-fact problems were found. It should be noted that children who did not make any errors on a particular problem type were excluded from the above analyses (ns = 13 for nonverbal problems, 5 for story problems, and 13 for number-fact problems).

Discussion

This study examined the calculation abilities of kindergarten and first-grade children with different patterns of cognitive functioning: children with low language but adequate spatial abilities, children with low spatial but adequate language abilities, children with general delays, and children with no language or spatial impairments. Each child was given identical addition and subtraction calculations presented in a nonverbal format as well as in two verbal formats (i.e., story problems and number-fact problems). To solve a nonverbal calculation problem correctly, a child must be able to transform sets through addition or subtraction operations. Story problems and number-fact problems, on the other hand, require mastery of the conventional verbal symbols of arithmetic, general verbal comprehension, and quantitative knowledge. The findings show that the three problem formats are differentially sensitive to variation in cognitive ability.

In particular, children in the Low Language group showed a larger discrepancy between their performances on nonverbal and verbal calculation problems than did children in the other three ability groups. The Low Language group performed as well as the Nonimpaired group on nonverbal problems but performed significantly worse than the Nonimpaired group on story problems. These findings suggest that children with specific language difficulties develop basic nonverbal calculation skills at a rate similar to that of their nonimpaired peers. However, their quantitative competence may not be reflected when calculations are embedded in verbal contexts, such as story problems.

Children with general delays showed weak performance on all three problem types, most likely because of difficulties with higher order cognitive processes. They performed significantly worse than the Nonimpaired group on both nonverbal problems and story problems. In contrast to the Low Lan-

TABLE 4
Mean Percentage of Right-Direction Errors by Ability Group and Problem Type

Group	Nonverbal problems	Story problems	Number-fact problems
Nonimpaired	89 (17)	75 (33)	61 (34)
Low Language	84 (22)	62 (30)	43 (33)
Low Spatial	84 (18)	78 (22)	52 (41)
Delayed	70 (20)	66 (25)	49 (25)

Note. Standard deviations are given in parentheses.

guage group, children in the Delayed group seemed to have problems that involved the understanding of calculation. This is evidenced by their relatively poor performance on a task with no verbal requirements. This finding was corroborated by our error analysis, which showed that the Nonimpaired group made more right-direction errors on the nonverbal calculation task (i.e., greater than the augend for addition or less than the minuend for subtraction) than did the Delayed group. However, the majority of the Delayed group's calculation errors were in the right direction, indicating a general awareness of the effects of addition and subtraction operations. Children in the Delayed group may have had difficulties using counting procedures or mental calculation strategies to arrive at exact quantitative solutions on the nonverbal task. These difficulties may also have resulted in an increase in wrong-direction errors, compared to children in the Nonimpaired group.

It should be noted that children in the Low Spatial group performed significantly worse than those in the Nonimpaired group when the scores on the three problem types were combined. However, when each calculation task was considered separately, there were no significant differences between the Low Spatial and Nonimpaired groups. These data suggest that spatial difficulties (with intact language) may lower overall performance but do not lead to specific calculation problems in kindergarten and first grade. This is in contrast to verbal difficulties (with intact spatial skills), which lead to problems on verbal calculation tasks but not on nonverbal ones. Children with spatial difficulties may be able to use their verbal strengths to compensate for their relative weaknesses in the spatial domain. It is possible, for example, that children in the Low Spatial group solved the nonverbal problems by converting them to a verbal format. The compensatory abilities of the Low Spatial group in kindergarten and first grade may be associated with the heavy emphasis on verbal learning in school; however, children with specific spatial difficulties may have more problems compensating for their cognitive weaknesses in subsequent grades, when the spatial demands of mathematics calculation increase (e.g., arranging columns of numbers, using place value, performing written calculations, reading mathematical signs; Badian, 1983; Hartje, 1987; Rourke, 1993). It is possible that nonverbal calculation tasks, like the one used in the present study, may be more sensitive to specific spatial difficulties in toddlers or preschool children, who have not yet mastered the number words or verbal counting.

We were surprised that children in the Low Language and Delayed groups did not differ significantly from children in the Nonimpaired group on number-fact problems. However, our observations of the children's problem-solving strategies showed that children in the Low Language and Delayed groups used their fingers frequently and accurately in first grade, especially on number-fact problems (e.g., the Low Language group used fingers on 68% of number-fact trials with 84% accuracy, and the Delayed group used fingers on 70% of number-fact trials with 69% accuracy; in contrast, the Nonimpaired group used fingers on 38% of the trials with 66% accuracy). It is interesting to note that when we adjusted for the frequency of finger use, the Nonimpaired group outperformed both the Low Language and the Delayed groups on number-fact problems. Thus, the finding that the Low Language and Delayed groups perform as well as the Nonimpaired group on number-fact problems may be attributable to their greater reliance on finger strategies.

It is noteworthy that children with specific language difficulties and children with general cognitive delays benefit a great deal from using their fingers to represent numerosities on verbal calculation problems. This observation suggests that children with such impairments should not be discouraged from using this compensatory strategy in the early elementary grades. Although skilled performance in arithmetic calculation depends on automatic retrieval (Ackerman, Anhalt, & Dykman, 1986), the use of fingers seems to serve as an intermediate "self-scaffolding" step for these children. Whether these children can master the basic facts or perform mental calculations efficiently after extended practice remains an open question.

Overall, the present study suggests that verbal facility may be helpful but not necessary for developing calculation skills. Performance on the nonverbal task also does not seem to be affected by specific language or spatial difficulties in young children; however, it does seem to be sensitive to more general cognitive delays. This is consistent with Huttenlocher et al.'s (1994) claim that the ability to calculate on nonverbal tasks, such as the one used in the present study, involves symbolic processes that are related to overall intellectual competence. These findings add to our previous work showing that children can succeed on nonverbal calculation problems at an earlier age than on conventional verbal calculation problems (Huttenlocher et al., 1994; Jordan, Huttenlocher, & Levine, in press; Levine et al., 1992).

In the present study, children's cognitive abilities were classified on the basis of screening tests alone. Future work should examine the cognitive mechanisms that underlie young children's language and spatial weaknesses (e.g., verbal or visual memory, concept formation, etc.: Rourke, 1993) and determine their relation to performance on nonverbal, story, and number-fact problems. For example, some language difficulties may be associated with more circumscribed calculation problems (e.g., difficulties with number-fact retrieval only; Geary, 1993; Warrington, 1982), while others may be related to more general deficits in verbal calculation (e.g., difficulties understanding verbally presented problems). It also might be fruitful to examine the development of verbal and nonverbal calculation abilities in children with clinically diagnosed neuropsychological impairments in the language and/or spatial domains.

Our findings have educational implications for young children with specific and more general cognitive problems. Calculation is emphasized in the early elementary grades, and demonstrated facility in it frequently portends a young child's success in mathematics. Calculation also provides a foundation for later mathematical learning (Siegler & Jenkins, 1989). Early calculation difficulties can erode children's confidence in their mathematical abilities. Our data show that children with different cognitive patterns may have different instructional needs in beginning calculation skills. The use of verbal and nonverbal calculation tasks in educational assessments may prove to be helpful in characterizing young children's mathematical strengths and weaknesses.

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