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Socioeconomic Variation, Number Competence, and Mathematics Learning Difficulties in Young Children

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As a group, children from disadvantaged, low-income families perform substantially worse in mathematics than their counterparts from higher-income families. Minority children are disproportionately represented in low-income populations, resulting in significant racial and social-class disparities in mathematics learning linked to diminished learning opportunities. The consequences of poor mathematics achievement are serious for daily functioning and for career advancement. This article provides an overview of children's mathematics difficulties in relation to socioeconomic status (SES). We review foundations for early mathematics learning and key characteristics of mathematics learning difficulties. A particular focus is the delays or deficiencies in number competencies exhibited by low-income children entering school. Weaknesses in number competence can be reliably identified in early childhood, and there is good evidence that most children have the capacity to develop number competence that lays the foundation for later learning. © 2009 Wiley-Liss, Inc. Dev Disabil Res Rev 2009;15:60–68.

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Mathematics difficulties are widespread among U.S. school children. Although developmental and learning disabilities are significant sources of these difficulties, achievement in mathematics is strongly related to a child's socioeconomic status (SES) [Jordan et al., 2007]. SES is typically defined by family income, level of poverty in the child's neighborhood, and educational attainment by parents [Clements and Sarama, 2008]. On average, children from disadvantaged low-income families perform substantially worse in mathematics than their counterparts from higher income families [as reviewed by the National Mathematics Advisory Panel, 2008]. Poor children are 1.5 times more likely to have a learning disability and two times more likely to repeat a grade and eventually drop out of high school than are their non-poor counterparts [Duncan and Brooks-Gunn, 2001]. The poverty rate in the U.S. is roughly 12.5% of the population, thus affecting millions of school-age children [U.S. Census Bureau, 2007]. Minority children, such as African American, Hispanic, and Native American children, are disproportionately represented in low-income populations, resulting in significant racial and social-class disparities in mathematics learning [Royer and Walles, 2007]. The consequences of poor mathematics achievement are serious for daily functioning and for career advancement [Commission, 2000]. Strong mathematics achievement in all children is important for meeting the needs of our increasingly technological society and for workforce equity [Council, 2001]. Mathematics competence is associated with entry into the STEM (science, technology, engineering, and mathematics) disciplines in higher education, as well as STEM-related occupations [National Mathematics Advisory Panel, 2008].

The income gap in mathematics achievement is well documented in elementary and secondary school [as reviewed by the Council, 2001]. The roots of this gap are planted well before children begin school [Baroody, 1987; Klibanoff et al., 2006; Levine et al., in press]. Learning opportunities and social experiences along with basic learning and cognitive abilities all contribute to children's mathematics learning from early childhood onward. The number competencies that children bring to school set the stage for learning complex mathematics [National Mathematics Advisory Panel, 2008]. Number competence (also referred to as number knowledge or "number sense") in the context of this article involves understanding of numbers and numerical relationships [Malofeeva et al., 2004]. A child must represent collections as "sets of individuals," simultaneously representing the whole and its constituent units [Spelke, 2003]. This concept is essential to true number competence, including the ability to recognize the numerical value of small quantities without counting (i.e., subitization), to discriminate between and among quantities, to make judgments about the magnitudes of small numbers, to meaningfully count objects, and to perform simple addition and subtraction calculations. Number competence involves the ability to visualize numbers on a number line and to grasp that each number is one more than the previous number [Resnick

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Table 1. Early Math Foundations

Primary Preverbal Number Knowledge
Object file system for precise representation of small numbers (3 or less)
Analogue magnitude system for approximate representation of larger sets
Secondary Symbolic Number Knowledge
Verbal subitizing (mapping number words onto small sets)
Counting (reciting the count sequence to 10 and grasping principles of 1–1 correspondence, stable-order and cardinality to enumerate sets of objects)
Numerical magnitude comparisons (e.g., knowing that two is smaller than five or that five is larger than four)
Linear representations of number (understanding that numerical magnitudes increase linearly)
Arithmetic operations (transforming small sets through adding and subtracting in nonverbal and verbal contexts)

and Ford 1981; Siegler and Booth, 2004]. Many, perhaps all, of these competencies can be taught or improved through a wide variety of educational activities [Wynroth, 1986; Berch, 2005].

EARLY MATHEMATICS FOUNDATIONS

Most children enter school with number skills that are relevant to learning conventional mathematics [Ginsburg, 1989; Bisanz et al., 2005]. Teaching that connects with early number competencies and that builds on these competencies is likely to be most effective [Clements et al., 1999]. Even in infancy, children appear to be sensitive to numerical and related spatial representations [e.g., Antell and Keating, 1983; Cordes and Brannon, in press; Wynn, 1992]. According to a current perspective, mathematics foundations can be in viewed in terms of (a) primary preverbal number knowledge [Feigenson et al., 2004] and (b) secondary verbal or symbolic number knowledge [Levine et al., 1992; Huttenlocher et al., 1994; Geary, 1995]. A summary of these two types of knowledge can be found in Table 1.

Preverbal Number Knowledge

Preverbal number knowledge, which allows infants to represent quantity, appears to develop without verbal input or instruction [Dehaene, 1997; Feigneson et al., 2004; Berch, 2005]. Developmental theory suggests that infants have a natural capacity to represent number in a nonverbal manner [Mix et al., 2002]. According to one version of this view [Feigenson et al., 2004], infants begin with two "core" systems for representing numbers: (1) an object file system for the *precise* representation of small numbers of individual objects and (2) an analogue magnitude system for capturing *approximate* representations of larger sets [Feigenson and Carey, 2003; Feigenson et al., 2004]. These core foundations are shared by humans from differing cultural backgrounds and cognitive abilities, as well as with other species [Gordon, 2004; Pica et al., 2004].

The object file system, which is limited to small numbers (i.e., three or less), provides a precise representation for each object in the set but not of the set size. For example, a set of one might be represented as {a} and a set of two items as {a, b} [Carey, 2004; Mix et al., 2005]. The analog magnitude system, in contrast, involves an approximate representation of the numerosity of larger sets, but it does not preserve any representation of the items (i.e., it does not provide a way to distinguish between successive numbers such as 9 and 10). Thus, only when children learn the count list and the cardinal meanings of the count words, are they are able to represent numbers larger than four exactly [Wynn, 1990; Mix et al., 2002; Le Corre and Carey, 2007]. Further, it is only at this time that they have a representation of the natural numbers, which involves understanding that each number has a unique successor [Sarnecka and Carey, 2008]. According to Feigenson et al. [2004], approximate representations of large numbers and precise representations of small numbers "account for our basic numerical intuitions, and serve as the foundation for the more sophisticated numerical concepts that are uniquely human" (p. 307).

Symbolic Number Knowledge

Although the aforementioned universal preverbal number systems appear to lay the foundation for later mathematical skills [but see Rips et al., 2008 for an argument that this is not the case], subsequent development is symbolic in nature and highly dependent on the input the child receives. These secondary symbolic systems are a key concern for educators in early childhood.

Verbal subitizing

Verbal subitizing, or the ability to recognize and name small collections without counting, provides a transition from the preverbal to symbolic number knowledge. Children first map number words onto set size (cardinality) for small numbers using subitizing [Wynn, 1990, 1992; Le Corre and Carey, 2006]. They then learn to use counting to determine the cardinality of sets, and use this mechanism for sets larger than three entities [e.g., Le Corre and Carey, 2006]. Researchers argue that children reconcile the two representational systems when they learn the number words, whose meaning is not fully represented by either core system [Mix et al., 2002; Feigenson et al., 2004]. Moreover, they suggest that children learn the "cardinal principle," the principle that the last number in a count represents the set size after they map the first four numbers, one by one, onto set size [Wynn, 1990, 1992].

Counting

Counting is a critical component in learning mathematics in that it is a key basis for extending number understanding beyond the small numbers [Baroody, 1987; Ginsburg, 1989; Baroody et al., 2006]. Children begin to say the count words soon after they learn to talk [Fuson, 1988]. Initially, children might use the count words to label small quantities (e.g., "two trucks") or to recite the count list (e.g., "one, two, three, four, and five"), rather than for enumerating objects in a set. Most children develop knowledge of three important "how to count" principles before they enter kindergarten [Gelman and Gallistel, 1978], including the principles of one-to-one correspondence (each item can be counted only once), stable order (the count words must be used in a consistent order), and cardinality (the final number in the count indicates how many items are in the set). Children gradually internalize that they can count any set of objects (e.g., homogeneous or heterogeneous, concrete or abstract) in any order (e.g., left to right or right to left). Typically, children learn to recite the count sequence by rote (in a manner similar to which they recite the alphabet) and then extract counting principles through their everyday experiences with numbers [Briars and Siegler, 1984], although counting skills and counting principles also are mutually supportive [e.g., Baroody and Ginsburg, 1986; Baroody, 1992; Rittle-Johnson and Siegler, 1998].

In kindergarten and first grade, children acquire more complex counting abilities. They learn to count backward, to count by twos, and to enumerate object sets greater than 10. They also learn the words for decades and the rules for combining number words [e.g., combining 30 with 3 to make the larger number 33; Ginsburg, 1989]. Counting skills are fundamental to learning to calculate with larger numbers and for acquiring base-10 concepts.

Numerical magnitude comparisons

Children as young as 4 years of age can discriminate between quantities [Case and Griffin, 1990; Griffin, 2002; Griffin, 2004]. For example, they can tell which of two stacks of chips has more or less. As noted earlier, infants may rely on approximate analog magnitudes rather than on counting to make this judgment [Xu and Spelke, 2000]. By 6 years of age, however, most children integrate their global preverbal quantitative sensitivities and their counting schemes to develop a mental number line [Siegler and Booth, 2004]. As a result, children can reason better about their "quantitative worlds" [Griffin, 2002; Griffin, 2007]. They come to understand that numbers later in the count list have larger quantities than earlier quantities [N, n + 1, (n + 1) + 1, and so forth; Le Corre and Carey, 2006; Sarnecka and Carey, 2008; Schaeffer et al., 1974] and that numbers themselves have magnitudes, such that eight is bigger than five and that six is smaller than nine. Children use these skills in a wide range of contexts and eventually coordinate quantities to construct a linear representation of numerical magnitudes, to learn place value, and to perform mental calculations.

Estimation

Increasing reliance on linear representations of numbers is a key feature of number competence [as reviewed by the National Mathematics Advisory Panel 2008] and related to the number knowledge described previously [Siegler and Booth, 2004]. Siegler and colleagues [Siegler and Opfer, 2003; Booth and Siegler, 2006] developed a number line estimation task that presents lines with a number at each end (e.g., 0 and 100) with no other numbers in between. Children were asked to estimate the location of a number (e.g., "Where would 40 go?"). The task was designed to reflect the ratio characteristics of the number system (e.g., 40 is twice as large as 20 so the estimated location of 40 should be twice as far from 0 as the estimated location of 20). Young children's estimates tend to increase logarithmically, rather than linearly, with numerical magnitude. By second grade, however, children produce estimates consistent with a linear ruler representation [Siegler and Booth, 2004]. Linear magnitude representations are associated with a range of mathematics skills and efforts to teach children how to move along a number line show promise for increasing mathematics achievement more generally [Booth and Siegler, 2008; Ramani and Siegler, 2008; Siegler and Ramani, 2008].

Arithmetic operations

Counting and number comparisons, described previously, are relevant to learning arithmetic operations. However, children have limited success in solving verbally presented story problems ("Mike had two pennies. Jen gave him three more pennies. How many pennies does he have now?") and number combinations ("How much is two and three?") [e.g., Ginsburg and Russell, 1981; Levine et al., 1992]. Although this might indicate a lack of skill with addition and subtraction, several other factors seem to compromise the young child's ability to solve story problems and number combinations. For example, some children do not adequately understand the words and syntactic structure of a problem and/or have trouble accessing mental representations of quantities when explicit physical referents are not provided. Levine et al. [1992] developed a "nonverbal" calculation task that eliminated these sources of difficulty. The task involves calculation in that it requires a child to reach an exact solution to a problem, rather than to simply make a judgment about the effects of an addition or subtraction transformation. Young children's success in solving nonverbal calculations (with small sets) depends on their ability to hold and manipulate quantitative representations in working memory [Klein and Bisanz, 2000]. Children must form a mental model of the number representations and the act of adding or subtracting objects [Huttenlocher et al., 1994; Canobi and Bethune, 2008]. (This assumes, however, they are not simply modeling what they saw, e.g., for 3 + 1: put out 3, then put out 1 without determining the total is 4.) The ability to solve nonverbal calculation problems develops earlier than the ability to solve comparable story problems and number combinations in most children [Levine et al., 1992]. For example a three-year-old might be able to solve the calculation 2 + 1 in a nonverbal format but would not be able to solve the comparable verbal arithmetic problem until 4 years of age or later. Nonverbal calculation ability varies less across social classes than does the ability to solve verbal calculations [which clearly favors middle- over low-income children; Jordan et al., 1992, 1994]. Early performance on nonverbal calculations is significantly associated with later performance on verbal arithmetic problems, suggesting that nonverbal representations are related to calculating with number words [Levine et al., 1992].

CHARACTERISTICS OF MATHEMATICS LEARNING DIFFICULTIES IN ELEMENTARY SCHOOL

As noted previously, core components of number (e.g., exact representations of small numerosities and approximate representation of larger numerosities) develop without formal instruction [Dehaene, 1997; Feigenson et al., 2004; Berch, 2005]. These preverbal foundations are thought to provide a basis for learning more complex number skills involving number words, number comparisons, and counting. However, these preverbal foundations are not sufficient. Most children with mathematics difficulties in first grade and later seem to have particular problems with the verbal or symbolic systems of number, which are heavily influenced by early experiences and instruction.

Weaknesses with Counting Procedures

Children use a variety of counting strategies to solve number combinations in first through third grades [Siegler and Robinson, 1982; Svenson and Sjoberg, 1983; Geary and Burlinghman-Dubree, 1989; Siegler and Shipley, 1995]. Early on, they might represent the first part of the problem with their fingers, then physically add on the second part, and then count the total number of objects (i.e., count all). They eventually become proficient in counting on or up from addends, a more efficient and accurate approach for combinations with larger set sizes [Baroody, 1999], and eventually solve these problems without counting.

Young children who develop mathematics learning difficulties rely on the more basic "count all" finger strategies for extended periods, do not use more effective counting procedures (e.g., counting on from the larger addend), and thus make frequent counting errors while adding and subtracting [Geary, 1990]. Children with mathematics learning difficulties also are less accurate than their normally achieving counterparts in estimating the placement of numbers on a number line [Geary et al., 2007].

Poor Calculation Fluency

Poor calculation fluency is a key characteristic of children with mathematics learning difficulties throughout elementary school [e.g., Russell and Ginsburg, 1984; Hasselbring, et al., 1988; Jordan and Montani, 1997; Ostad, 1998; Jordan et al., 2003a,b; Geary, 2004]. Calculation fluency refers to fast, accurate, and effortless computation with basic operations as well as appropriate and flexible application [Council, 2001]. Poor "fact mastery" interferes with problem solving for learning advanced mathematics [Goldman and Pellegrino, 1987]. For example, a child might be spending so much effort computing that few cognitive resources are left for understanding a multi-step arithmetic problem. Algebra and even geometry also depend on basic computational facility. Calculation fluency deficits can be diagnosed reliably in primary school and, if not addressed, may continue to have negative impacts on mathematics achievement throughout elementary [Jordan et al., 2003b] and middle school [Ostad, 1999].

Associated Reading and Language Difficulties

More than half of the children who experience mathematics difficulties also experience reading and language difficulties [Barbaresi et al., 2005]. The percentage of co-morbid mathematics and reading difficulties is even higher among children from low-income backgrounds [Jordan et al., 2002]. Jordan and colleagues [Hanich et al., 2001; Jordan et al., 2002; Jordan et al., 2003a], as well as other researchers [e.g., Geary et al., 2000; Landerl et al., 2004], suggest that most of the characteristics of mathematics deficits are similar to those children who only have mathematical difficulties and for those who have mathematical and accompanying reading difficulties. However, children with both reading and mathematics difficulties show particular weaknesses on mathematics word problems, which depend on language comprehension and procedural facility (e.g., "Jill has four marbles. Then Mike gives her some more marbles. Now she has seven marbles. How many marbles did Mike give her?") [Jordan et al., 2003a]. Moreover, children with both mathematics and reading difficulties achieve at a slower rate than those with circumscribed mathematics problems who are good readers. The latter group can compensate to a certain extent for their mathematics weaknesses with language, that is, they can "talk their way through" complex mathematics problems. Thus, children from low SES backgrounds, who tend to have both mathematics and reading difficulties, are at particular risk for experiencing persistent mathematics difficulties.

Roots of Mathematics Learning Difficulties

Mathematics learning difficulties are related to fundamental weaknesses in number, number relationships, and number operations, or in other words, number sense or number competence [e.g., Gersten et al., 2005; Geary et al., 2007]. Poorly developed counting procedures, slow fact retrieval, and inaccurate computation all reflect weak number competence [Geary et al., 2000; Jordan et al., 2003b]. It is difficult to master mathematics facts, without understanding linear representations of number and number relations (Booth and Siegler, 2008). Mathematics difficulties are explained more by domain specific impairments in number processing than by general deficits related to memory, spatial processing, or language [Butterworth and Reigosa, 2007].

SES AND MATHEMATICS ACHIEVEMENT

As noted earlier, virtually all children bring foundational knowledge of mathematics to school. However, the child's income status and associated early home and preschool experiences, in addition to their general cognitive capacity, heavily influence the level of this knowledge. Even in preschool, there is a large gap between low- and middle-income children in mathematics-related skills [Sarama and Clements, 2009; Klibanoff et al., 2006]. On tests of mathematics readiness, low-income preschoolers who attend Head Start Programs perform worse than their counterparts who attend preschools serving middle-income children [Klibanoff et al., 2006]. Parental social class and educational level predicts mathematics achievement throughout elementary and secondary school. However, research findings are inconsistent with respect to the strength of these associations, whether only select aspects of mathematics are affected [e.g., Russell and Ginsburg, 1984], and the mechanisms that underlie the associations, such as a lack of opportunity for out-of-school learning and parenting characteristics, [Clements and Sarama, 2008; Blevins-Knabe and Musun Miller, 1996].

Delays in Number Competence

Jordan and colleagues [Jordan et al., 2006; Jordan et al., 2007; Jordan et al., 2008] investigated performance and growth in kindergarten number competence in relation to mathematics achievement through third grade. They found that low-income children (i.e., children who live in urban, low-income communities and qualify for their school's reduced-lunch program) enter kindergarten well behind their middle-income peers on tasks assessing number competence, which include knowledge of counting, numerical relationships (e.g., recognizing which of two numbers is smaller), and numerical operations (e.g., adding and subtracting with small numbers). Longitudinal assessment over six time points, from the beginning of kindergarten to the middle of first grade, revealed three empirically distinct growth trajectories in number competence: (1) children who started kindergarten with low number competence and showed little growth; (2) children who started kindergarten with relatively low number competence but showed good growth; and (3) children who started kindergarten with high number competence and remained at a high level. Discouragingly, low-income children were over-represented in the low performance, flat growth group and under-represented in the other two groups. In fact, an analysis by sub-tasks revealed that low-income children were four times more likely than middle-income children to fall in the low-flat group on arithmetic story problems, an area that is particularly sensitive to variations in SES. For example, many children in the low-income group could not solve simple problems, such as "Paul had five oranges. Maria takes away two of his oranges. How many oranges does Paul have now?"

Jordan et al. [2007] also found that *level* of performance in number competence in kindergarten and *rate* of growth between kindergarten and first grade accounted for 66% of the variance in mathematics learning at the end of first grade. Income status (as well as gender, age, and reading ability) did not add explanatory variance over and above performance and growth in number competence. The predictability of number competence remained strong through at least third grade (Jordan et al., in press). Kindergarten number competence predicted rate of growth in mathematics achievement between first and third grade as well as achievement level in third grade, while controlling for income status. Number competence also has been shown to be uniquely predictive of mathematics outcomes when IQ was considered in the analyses [Locuniak and Jordan, 2008]. These data suggest that number competence, which can be taught and learned, could be a key factor in bridging the income gap in mathematics achievement.

Difficulties with Verbal Versus Nonverbal Aspects of Mathematics

Although young children from low-income backgrounds perform at a lower level than their higher income counterparts on mathematics-related tasks, findings are not consistent [Ginsburg and Pappas, 2004]. The nature of the tasks and the skills they are tapping are key factors. Most important, early influences of SES appear to be greatest on verbal aspects of mathematics [Jordan et al., 1994; Dowker, 2005].

Knowledge of number words can facilitate performance on numerosity matching tasks, even those that are nonverbal. On tasks that involve matching sets that are of equivalent numerosity, either from memory or with both target and choices simultaneously present, 3-year-olds who have strong knowledge of the meaning of the number words perform better than those whose knowledge of number word meaning is weak [Mix, Huttenlocher, & Levine, 1999; Mix, 2008]. Thus, low income preschool children, whose number word knowledge is weak compared to their middle-income peers, are likely to perform worse on a wide variety of numerical tasks, even those that are nonverbal in format such as matching sets on the basis of numerical equivalence.

Consistent with this conclusion, by kindergarten entry, middle-income children have a strong advantage over low-income children on verbally-presented number combinations ("How much is four and three) and story problems (Jack had four marbles. Beth gave him three more marbles. How many marbles does Jack have now?") [Jordan et al., 1992, 1994; Jordan et al., 2006], skills that are highly predictive of later mathematics outcomes [Jordan et al., in press; Mazzocco and Thomson, 2005]. Middle-income kindergartners also achieve at a much faster rate on number combinations and story problems in kindergarten [Jordan et al., 2006, 2007]. In contrast, income differences are attenuated if the same calculations are presented in a nonverbal format (e.g., the child is shown four disks that are then hidden with a cover. The tester slides three more disks under the cover. The child must indicate how many disks are now hidden.) Low-income children's relatively strong ability to calculate on tasks that provide representation without number words could serve as a starting point for mathematics instruction in preschool and kindergarten.

Low-income children are four times more likely than their middle-income counterparts to start school at a low level and to show flat growth between kindergarten and first grade in key areas of number competence.

One reason why low-income children lag behind middle-income children on accuracy with number combinations in kindergarten seems to be that they do not use their fingers adaptively to represent and manipulate the quantities that are represented with number words [Jordan et al., 2008]. Fingers can facilitate the transition between early nonverbal representations and conventional representations with number words. Jordan et al. [2008] found that low-income children show developmental trajectories in finger use (between kindergarten and second grade) that are different from middle-income children. Middle-income children frequently used their fingers in kindergarten, but they gradually decreased their finger use between first and second grades. Low-income children started using their fingers about a year later (i.e., in first grade) and used them more often than middle-income children by the end of second grade. Notably, in kindergarten there is a strong positive association between finger use and accuracy on number combinations, but this correlation diminishes over time and even becomes negative by the end of second grade (i.e., children who use their fingers are less accurate). Although Jordan et al. [2008] did not analyze the types of finger strategies children used at the various grade levels, they suggested that using fingers to count on or up from a number helps them form an association between a particular combination and the correct answer, which in turn leads to mental calculation and fact mastery. Low-income children, like children with mathematics difficulties more generally, use fingers later and seem to stick with less mature "counting all" finger strategies for longer periods, reflecting delayed development in number competence.

At least some of the variation in children's knowledge of number words and symbols seems to be tied to differential exposure to the language and symbol system of mathematics. This system extends the universal starting points of children's quantitative knowledge, allowing them to represent number exactly for sets beyond the subisitizing range. In both home and preschool school settings, exposure to the language of mathematics varies widely. For example, in a longitudinal project in which investigators visited families every 4 months from 14 months to 30 months for five 90-min sessions (total of 7.5 hr), caregiver use of number words ranged from a low of three instances to a high of 175 instances [Levine et al., in press]. Similarly, in preschools, the amount of teacher number talk teachers engaged in during a one-hour period that encompassed a single observation of 45 min, including "circle time" ranged widely, from 1 to 104 coded instances [Klibanoff et al., 2006]. Importantly, these variations in amount of number input, both at home and at preschool, have a significant impact on preschool children's mathematics knowledge. The characteristics of the speakers' language also seem to influence young children's knowledge of number words; [Miura, 1987]. Interestingly, adults in cultural groups with few number words perform worse than adults from cultural groups with more elaborated number systems in matching set sizes, performing arithmetic operations, and on other cognitive tasks requiring knowledge of exact number [Gordon, 2004; Pica et al., 2004].

Cultural Differences

Understanding cultural differences between social classes provides some clues to why there are persistent mathematics achievement disparities and why low-income children may be less responsive to reforms in mathematics education recommended by the National Council for Teachers of Mathematics [2001] (e.g., exploration and discussion of mathematical ideas). Children from lower-income families may have different cultural beliefs about mathematics than their higher-income counterparts. Lubienski [2000] found that low-income middle schoolers, even those who were relatively high achieving, preferred direct teacher instruction to more open-ended discussion, focused on finding the correct answer, and reasoned in more idiosyncratic ways based on their own experience and specific contexts (as opposed to thinking about connections to other problems and generalizable mathematical principles). Lubienski argues that characteristics of "discussion-intensive" mathematics classrooms may be more supportive of children from middle-class cultures and that educators, beginning in early childhood, need to "find ways to help lower-SES students gain the knowledge, skills, and beliefs necessary to become critical thinkers and actors in society" (p. 400).

REDUCING THE MATHEMATICS GAP

Young children from low-income families receive less support for mathematics in their home environment than do their middle-income peers [Saxe et al., 1987; Holloway et al., 1995; Blevins-Knabe and Musun-Miller, 1996; Starkey et al., 1999; Levine et al., in press; Jordan et al., 2006]. The input they receive may also differ qualitatively. For example, Saxe et al. [1987] report that working class mothers set less complex goals than middle-class mothers when interacting with their children about number. Further, Levine et al. [in press] find that middle-income mothers refer to number with objects present and talk about the cardinality of sets more often than lower income mothers when interacting with their 14-month-olds. Making matters worse, public preschool programs serving children from low-income families provide fewer learning opportunities and supports for mathematical development than ones serving middle-income families [Clements and Sarama, 2008]. However, quality childcare programs can help level the playing field for all children entering school.

Helping Children in Preschool and Kindergarten

Recent research on early mathematical development has shown that instructional programs can prepare disadvantaged, low SES children for school mathematics and reduce the SES-related mathematics gap [Starkey and Klein, 2008]. More generally, it has been shown that Montessori education has significant positive effects on urban, low-income children's ability to solve applied mathematics problems during the preschool and kindergarten years [Lillard and Else-Quest, 2006]. Principles of Montessori education include multi-age classrooms, individual and small group instruction in academic as well as social skills, collaboration, and student-chosen activities. More specifically, the PreK Mathematics curriculum [Klein and Starkey, 2004] was developed for use in preschools serving low-income families. PreK Mathematics is a supplemental program to develop number concepts and skills in small groups. The program also provides home activities to help parents provide support outside of school. Studies using randomized controlled trials have shown positive effects on children's early mathematics learning [What Works Clearinghouse, 2007].

Although these curricula show positive effects, they do not provide information about what elements of instruction are particularly effective. Recently, researchers have identified specific number activities that can improve numerical understanding in young children from low-income families. Based on work demonstrating the importance of children's understanding of estimation and numerical magnitudes, Siegler and colleagues [Siegler, in press] have found positive effects of playing board games that involve linear number representations (i.e., board games with consecutively numbered, equal-size spaces). Such games also involve the counting, one-to-one correspondence, and number identification. Using random assignment, Siegler and Ramani [2008] taught children from Head Start centers to play either a number-board game or a color board game that served as a control. Children played the games on four occasions, each lasting about 20 min, over a two-week period. Posttesting revealed that children in the number board game condition made moderate and significant gains in their number line estimates, whereas children in the color board control condition did not. The gains from learning to play the board game also improved children's ability to compare which of two numbers is bigger, to identify written numbers by name, and to count from 1 to 10. Strikingly, these gains held at least 9 weeks after the final game playing session suggesting that the benefits of the intervention were relatively long lasting. Why do number board games produce sustainable gains in mathematical knowledge? Siegler (in press) suggests it is likely that the children had few direct mathematics experiences to begin with and that the intervention provided an effective environment to trigger development in number skills.

Based on the work of her research team, which points to the importance of early facility with comparing and manipulating numbers, Jordan [2007] recommends that at-risk kindergartners be given explicit help in comparing, combining, and separating sets, starting with totals of five or fewer. Children should manipulate quantities by adding and taking away with their fingers or other concrete objects. After working on nonverbal calculation tasks, they also might be encouraged to visualize set transformations in their heads (e.g., the caregiver might say "Imagine three dots in your head. Now take away two of the dots. How many dots are left?"). These activities could then be connected to learning simple number combinations without explicit referents (e.g., "How much is three take away two."). For problems involving larger numbers, children should be encouraged to count on from the larger addend for an addition problem or to count on from the number that is being taken away for a subtraction problem [Fuson, 1982]. Counting-on can be promoted by helping children devise the number-after rule for adding one (e.g., the sum of 5 + 1 is the number after "five" when we count). This seems to serve as a scaffold for constructing counting-on [e.g., if sum of 5 + 1 is the number after "five," then 5 + 2 must be two numbers after "five," and 5 + 3 must be three numbers after "five" [Baroody, 1995]. This method also appears to apply with children with a range of learning difficulties, including those diagnosed with learning disabilities.

Helping Children at Home

Children's number competence is supported by their daily experiences in the world [Saxe et al., 1987]. Parents of young children report they spend more time with their children on reading-related activities than on number-related activities, and low-income caregivers spend less time teaching number skills to their children than middle-income caregivers [Jordan et al., 2006]. It has been observed that parents of low-income children think that math learning in early childhood is the responsibility of the school whereas middle-income parents see more value in home input [Clements and Sarama, 2008]. Although the educational level attained by parents and income status clearly have direct effects on the child, these effects can be moderated by supportive parent/child interactions [e.g., support for problem solving; Blevins-Knabe and Musun Miller, 1996; Clements and Sarama, 2008]. As noted earlier, activities as simple as playing board games can improve children's understanding of foundational mathematical abilities [Siegler and Ramani, 2008]. For example, Chutes and Ladders, which requires children to move up and down a number list from 1 to 100, may help children focus on numbers and learn the count sequence, one-to-one correspondences, and number magnitudes. Games that use dice or number cubes and play money may help children recognize and combine quantities. Board games may also develop more general problem-solving skills related to taking turns, delaying gratification, and planning ahead.

Early interventions have potential to help all children develop the foundations they need to learn school mathematics. Without such help, many disadvantaged learners are almost certain to experience a "cascade of mathematics failure" in elementary school and may have great difficulty catching up to their peers who have had more mathematical input in early childhood (Jordan et al., in press). Circumscribed problems early on can trigger wider problems as children later in development [Karmiloff-Smith, 1998]. A child's likelihood for developing mathematics difficulties is greatest if the child has more than one risk factor (e.g., coming from a low-income background in addition to a learning or developmental disability).

Early interventions have potential to help all children develop the foundations they need to learn school mathematics. Without such help, many disadvantaged learners are almost certain to experience a "cascade of mathematics failure" and may have great difficulty catching up to their peers who have had more mathematical input during early childhood.

CONCLUSIONS

Foundational number competencies develop early in life. These competences include preverbal number knowledge, which is present in infancy and shared by humans of differing cultural backgrounds, and symbolic or verbal knowledge, which depends on the input the child receives. Children with learning difficulties in mathematics seem to have particular problems with symbolic number knowledge, which is influenced heavily by early experiences and instruction. Number competence in kindergarten, related to counting, number comparisons, and addition and subtraction, is highly predictive of later mathematics achievement, over and above income status and general cognitive deficits. Weaknesses in number competence can be reliably identified in early childhood, and there is good evidence that most children have the capacity to develop foundational number competence in preschool and kindergarten as well as through home experiences.

Further research is needed to examine the long-term effectiveness of evidence-based early mathematics interventions, to determine the kinds of interventions that are most sustainable, and to develop meaningful supports at critical junctures during development. It is also important to develop principles that differentiate more and less effective ways of increasing young children's number competence. Early educational programs for families need to be designed and evaluated. Research in early mathematics interventions is important not only for addressing fundamental theoretical questions about the development of mathematical foundations but also for addressing issues of job equity and societal needs. ■

REFERENCES

Antell S, Keating DP. 1983. Perception of numerical invariance in neonates. Child Dev 54: 695–701.

Barbaresi MJ, Katusic SK, Colligan RC, et al. 2005. Math learning disorder: incidence in a population-based birth cohort, 1976–1982 Vol. 5. Rochester, Minnesota. Ambulatory Pediatrics 5:281–289.

Baroody AJ. 1987. The development of counting strategies for single-digit addition. J Res Math Educ 18:141–157.

Baroody AJ. 1992. The development of preschoolers' counting skills and principles. In: Bideaud J, Meljac C, Fischer JP, editors., Pathways to number. Hillsdale NJ: Erlbaum. p 99–126.

Baroody AJ. (1995). The role of the number-after rule in the invention of computational short-cuts. Cogn Instruct 13:189–219.

Baroody AJ. 1999. Children's relational knowledge of addition and subtraction. Cognition and Instruct 17:137–175.

Baroody AJ, Ginsburg HP. 1986. The relationship between initial meaningful and mechanical knowledge of arithmetic. In: Hiebert J., editors. Conceptual and procedural knowledge: the case of mathematics Hillsdale, NJ: Erlbaum. p 75–112.

Baroody AJ, Lai ML, Mix KS. 2006. The development of young children's early number and operation sense and its implications for early childhood education. In: Spodek B, Saracho O. editors., Handbook of research on the education of young children. Mahwah, NJ: Lawrence Erlbaum Associates. p 187–221.

Berch DB. 2005. Making sense of number sense: implications for children with mathematical disabilties. J Learn Disabil 38:333–339.

Bisanz J, Sherman J, Rasmussen C, et al. 2005. Development of arithmetic skills and knowledge in preschool children. In: Campbell JID editor. Handbook of mathematical cognition. New York: Psychology Press. p 143–162.

Blevins-Knabe B, Musun-Miller L. 1996. Number use at home by children and their parents and its relationship to early mathematical performance. Early Dev Parent 5:35–45.

Booth JL, Siegler RS. 2006. Developmental and individual differences in pure numerical estimation. Dev Psychol 41:189–201.

Booth JL, Siegler RS. 2008. Numerical magnitude representations influence arithmetic learning. Child Dev 79:1016–1031.

Briars D, Siegler RS. 1984. A featural analysis of preschoolers' counting knowledge. Dev Psychol 20:607–618.

Butterworth B, Reigosa V. 2007. Information processing deficits in dyscalculia. In: Berch MMD, editor. Why is math so hard for some children? Baltimore MD: Paul H. Brookes, p 65–81.

Canobi KH, Bethune NE. 2008. Number words in young children's conceptual and procedural knowledge of addition, subtraction and inversion. Cognition 108:675–686.

Carey S. 2004. Bootstrapping and the origins of concepts. Daedalus 133:59–68.

Case R, Griffin S. 1990. Child cognitive development: the role of central conceptual structures in the development of scientific and social thoughts. In: Hauert CH, editor. Advances in psychology-developmental psychology: Cognitive, perception-motor, and neurological perspectives. Amsterdam: North Holland.

Clements D, Swaminathan S, Hannibal MAZ, et al. 1999. Young children's concepts of shape. J Res Math Educ 30:192–212.

Clements DH, Sarama J. 2008. Experimental evaluation of the effects of a research-based preschool mathematics curriculum. Am Educ Res J 45:443–494.

Commission G. 2000. Before it's too late: a report to the nation from the National Commission on Mathematics and Science Teaching for the 21st Century: U.S. Department of Education.

Cordes S, Brannon EM. 2008. The relative salience of discrete and continuous quantity in Young Infants. Developmental Science 11: 803–808.

Council NR. 2001. In: Kilpatrick J, Swafford J, Findell B, editors. Adding it up: helping children learn mathematics. Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Dehaene S. 1997. The number sense: how the mind creates mathematics. New York: Oxford University Press.

Dowker A. 2005. Individual differences in arithmetic: implications for psychology, neuroscience, and education. Hove and NY: Psychology Press.

Duncan GJ, Brooks-Gunn J. 2001. Poverty, welfare reform, and children's achievement. In: Biddle, editor. Social class, poverty, and education: Policy and practice. New York: Routledge Almer. p 49–76.

Education, U.S. Department of 2007. Pre-K mathematics. What Works Clearninghouse intervention report. Washington, DC: Institute of Education Sciences.

Feigenson L, Carey S. 2003. Tracking individuals via object-files: evidence from infants' manual search. Dev Sci 6:568–584.

Feigenson L, Dehaene S, Spelke E. 2004. Core systems of number. Trends Cogn Sci 8:307–314.

Fuson KC. 1982. An analysis of the counting-on solution procedure in addition. In: Carpenter TP, Moser JM, Romberg TA, editors. Addition and subtraction: A cognitive perspective. Hillsdale, NJ: Erlbaum. p 67–81.

Fuson KC. 1988. Children's counting and concepts of number. New York: Springer-Verlag.

Geary DC. 1990. A componential analysis of an early learning deficit in mathematics. J Exp Child Psychol 49:363–383.

Geary DC. 1995. Reflections of evolution and culture in children's cognition: implications for mathematical development and instruction. Am Psychol 50:24–37.

Geary DC. 2004. Mathematics and learning disabilities. J Learn Disabil 37:4–15.

Geary DC, Burlingham-Dubree. 1989. External validation of the strategy choice model for addition. J Exp Child Psychol 47:175–192.

Geary DC, Hamson CO, Hoard MK. 2000. Numerical and arithmetical cognition: a longitudinal study of process and concept deficits in children with learning disability. J Exp Child Psychol 77:236–263.

Geary DC, Hoard MK, Byrd-Craven J, et al. 2007. Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. Child Dev 78:1343–1359.

Gelman R, Gallistel CR. 1978. The child's understanding of number. Cambridge, MA: Harvard University Press.

Gersten R, Jordan NC, Flojo JR. 2005. Early identification and interventions for students with mathematics difficulties. J Learn Disabil 38:293–304.

Ginsburg HP. 1989. Children's arithmetic. Austin, TX: PRO-ED.

Ginsburg HP, Pappas S. 2004. SES, ethnic, and gender differences in young children's informal addition and subtraction: a clinical interview investigation. Appl Dev Psychol 25:171–192.

Ginsburg HP, Russell RL. 1981. Social class and racial influences on early mathematical thinking. Monogr Soc Res Child Dev 46:1–69.

Goldman SR, Pellegrino JW. 1987. Information processing and educational microcomputer technology: where do we go from here? J Learn Disabil 20:144–154.

Gordon P. 2004. Numerical cognition without words: evidence from Amazonia. Science 306:496–499.

Griffin S. 2002. The development of math competence in the preschool and early school years: cognitive foundations and instructional strategies. In: Roher JM, editor. Mathematical cognition Greenwich, CT: Information Age Publishing. p 1–32.

Griffin S. 2004. Building number sense with number worlds: a mathematics program for young children. Early Child Res Q 19: 173–180.

Griffin S. 2007. Early intervention for children at risk of developing mathematical learning difficulties. In: Berch DB, Mazzocco MMM. editors. Why is math so hard for some children? Baltimore, MD: Paul H. Brookes p 373–395.

Hanich L, Jordan NC, Kaplan D, et al. 2001. Performance across different areas of mathematical cognition in children with learning difficulties. J Educ Psychol 93:615–626.

Hasselbring TS, Goin LI, Bransford JD. 1988. Developing math automaticity in learning handicapped children: the role of computerized drill and practice. Focus Except Child 20:1–7.

Holloway SD, Rambaud MF, Fuller B, et al. 1995. What is "appropriate practice" at home and in child care?: low-income mothers' views on preparing their children for school. Early Child Res Q 10:451–473.

Huttenlocher J, Jordan NC, Levine SC. 1994. A mental model for early arithmetic. J Exp Psychology General 123:284–296.

Jordan NC. 2007. The need for number sense. Educ Leadersh 65:63–66.

Jordan NC, Montani TO. 1997. Cognitive arithmetic and problem solving: a comparison of children with specific and general mathematics difficulties. J Learn Disabil 30:624–634.

Jordan NC, Hanich LB, Kaplan D. 2003a A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with co-morbid mathematics and reading difficulties. Child Dev 74:834–850.

Jordan NC, Hanich LB, Kaplan D. 2003b. Arithmetic fact mastery in young children: a longitudinal investigation. J Exp Child Psychol 85:103–119.

Jordan NC, Huttenlocher J, Levine SC. 1992. Differential calculation abilities in young children from middle- and low-income families. Dev Psychol 28:644–653.

Jordan NC, Huttenlocher J, Levine SC. 1994. Assessing early arithmetic abilities: effects of verbal and nonverbal response types on the calculation performance of middle- and low-income children. Learn Ind Differences 6:413–432.

Jordan NC, Kaplan D, Hanich LB. 2002. Achievement growth in children with learning difficulties in mathematics: findings of a two-year longitudinal study. J Educ Psychol 94:586–597.

Jordan NC, Kaplan D, Olah L, et al. 2006. Number sense growth in kindergarten: a longitudinal investigation of children at risk for mathematics difficulties. Child Dev 77:153–175.

Jordan NC, Kaplan D, Locuniak MN, et al. 2007. Predicting first-grade math achievement from developmental number sense trajectories. Learn Disabil Res Pract 22:36–46.

Jordan NC, Kaplan D, Ramineni C, et al. 2008. Development of number combination skill in the early school years: when do fingers help? Dev Sci 11:662–668.

Jordan NC, Kaplan D, Ramineni C, et al. in press. Early math matters: kindergarten number competencies and later math achievement. Dev Psychol

Jordan NC, Levine SC, Huttenlocher J. 1994. Development of calculation abilities in middle- and low-income children after formal instruction in school. J Appl Dev Psychol 15:223–240.

Karmiloff-Smith A. 1998. Development itself is the key to understanding developmental disorders. Trends Cogn Sci 2:39–398.

Klein A, Starkey P. 2004. Fostering preschool children's mathematical knowledge: findings from the Berkeley Math Readiness Project. In: Clements DH, Sarama J, editors. Engaging young children in mathematics: findings of the 2000 national conference on standards for preschool and kindergarten mathematics education. Mahway, NJ: Erlbaum.

Klein JS, Bisanz J. 2000. Preschoolers doing arithmetic: the concepts are willing but the working memory is weak. Can J Exp Psychol 54:105–115.

Klibanoff RS, Levine SC, Huttenlocher J, et al. 2006. Preschool children's mathematical knowledge: the effect of teacher "Math Talk". Dev Psychol 42:59–69.

Landerl K, Bevan A, Butterworth B. 2004. Developmental dyscalculia and basic numerical capacities: a study of 8–9-year-old students. Cognition 93:99–125.

Le Corre M. Carey S. 2006. Re-visiting the competence/performance debate in the acquisition of the counting principles. Cogn Psychol 52:139–169

Le Corre M, Carey S. 2007. One, two, three, four, nothing more: an investigation of the conceptual sources of the verbal counting principles. Cognition 105:395–438.

Levine SC, Gunderson EA, Huttenlocher J. (In press). Number development in context: variations in home and school input during the preschool years. In: Stein N, Raudenbush S, editors. Developmental science goes to school. New York: Taylor and Francis, Inc.

Levine SC, Jordan NC, Huttenlocher J. 1992. Development of calculation abilities in young children. J Exp Child Psychol 53: 72–103.

Lillard A, Else-Quest N. 2006. The early years: evaluating Montessori Education. Science 313:1893–1894.

Locuniak MN, Jordan NC. 2008. Using kindergarten number sense to predict calculation fluency in second grade. J Learn Disabi 41:451–459.

Lubienski ST. 2000. A clash of social class cultures? Students' experiences in a discussion-intensive seventh-grade mathematics classroom. Elem Sch J 100:377–403.

Malofeeva E, Day J, Saco X, et al. 2004. Construction and evaluation of a number sense test with Head Start children. J Educ Psychol 96:648–659.

Mazzocco MM, Thompson RE. 2005. Kindergarten predictors of math learning disability. Learn Disabil Res Pract 20:142–155.

Miura IT. 1987. Mathematics achievement as a function of language. J Educ Psychol 79: 79–82.

Mix KS, Huttenlocher J, Levine SC. 2002. Multiple cues for quantification in infancy: is number one of them? Psychol Bull 128: 278–294.

Mix KS, Huttenlocher J, Levine SC. 2002. Quantitative development in infancy and early childhood. New York, NY: Oxford University Press.

Mix KS, Sandhofer CM, Baroody AJ. 2005. Number words and number concepts: the interplay of verbal and nonverbal quantification in early childhood. Adv Child Dev Behav 33:305–346.

National Council of Teachers of Mathematics. 2001. Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Mathematics Advisory Panel. 2008. Foundations for success: the final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education.

National Research Council. 2001. Adding it up: Helping children learn mathematics. In: Kilpatrick J, Swafford J, Findell B, editors. Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Ostad SA. 1998. Developmental differences in solving simple arithmetic word problems and simple number-fact problems: a comparison of mathematically normal and mathematically disabled children. Math Cogn 4:1–19.

Ostad SA. 1999. Developmental progression of subtraction strategies: a comparison of mathematically normal and mathematically disabled children. Eur J Spec Needs Educ 14: 21–36.

Pica P, Lemer C, Izard V, et al. 2004. Exact and approximate arithmetic in an Amazonian Indigene group. Science 306:499–503.

Ramani GB, Siegler R. 2008. Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. Child Dev 79:375–394.

Resnick LB, Ford WW. 1981. The psychology of mathematics for instruction. Hillsdale, NJ: Erlbaum

Rips LJ, Bloomfield A, Asmuth J. 2008. From numerical concepts to concepts of number. Behav Brain Sci 31:623–42; discussion 642–87.

Rittle-Johnson R, Siegler RS. 1998. The relations between conceptual and procedural knowledge in learning mathematics: a review. In: Donlan C, editor. The development of mathematical skill. Hove, UK: Psychology Press. 75–110.

Royer JM, Walles R. 2007. Influences of gender, ethnicity, and motivation on mathematical performance. In: Berch DB, Mazzocco MMM, editors. Why is math so hard for some children? Baltimore, MD: Paul H. Brookes. 349–367.

Russell RL, Ginsburg HP. 1984. Cognitive analysis of children's mathematic difficulties. Cogn Instruct 1:217–244.

Sarama J, Clements DH. 2009. Early childhood mathematics education research: learning trajectories for young children. New York: Routledge.

Sarnecka BW, Carey S. 2008. How counting represents number: what children must learn and when they learn it. Cognition 108: 662–674.

Saxe GB, Guberman SR, Gearhart M. 1987. Social processes in early number development. Monogr Soc Res Child Dev 52.

Schaeffer B, Eggleston V, Scott J. 1974. Number development in young children. Cogn Psychol 6:357–379.

Siegler RS. in press. Improving numerical understanding of children from low-income families. Child Dev Perspect

Siegler RS, Booth JL. 2004. Development of numerical estimation in young children. Child Dev 75:428–444.

Siegler RS, Opfer JE. 2003. The development of numerical estimation: evidence for multiple representations of numerical quantity. Psychol Sci 14:237–243.

Siegler RS, Ramani GB. 2008. Playing board games promotes low-income children's numerical development. Dev Sci 11:655–661.

Siegler RS, Robinson M. 1982. The development of numerical understandings. In: Reese HW, Lipsitt LP, editors. Advances in child development and behavior, Vol 16. New York: Academic Press. 242–312.

Siegler RS, Shipley C. 1995. Variation, selection, and cognitive change. In: Halford G, Simon T, editors. Developing cognitive competence: new approaches to process modeling. Hillsdale, NJ: Erlbaum. p 31–76.

Spelke ES. 2003. What makes us smart? Core knowledge and natural language In: Gentner D, Goldin-Meadow S, editors. Language in mind: advances in the investigation of language and thought. Cambridge, MA: MIT Press. 277–311.

Starkey P, Klein A. 2008. Sociocultural influences on young children's mathematical knowledge. Contemporary perspectives on mathematics in early childhood education. Charlotte, NC: Information Age Publishing.

Starkey P, Klein A, Chang I, et al. 1999. Environmental supports for young children's mathematical development in China and the United States. Albuquerque, NM: Society for Research in Child Development.

Svenson O. Sjoberg K. 1983. Evolution of cognitive processes for solving simple addtions during the first three school years. Scandinavian J Psychol 24:117–124.

U.S. Census Bureau 2007. Poverty: 2007 highlights. Available at: http://www.census.gov/hhes/www/poverty/poverty07/pov07hi.html.

Wynn K. 1990. Children's understanding of counting. Cognition 36:155–193.

Wynn K. 1992. Addition and subtraction by human infants. Nature 27:749–750.

Wynroth L. 1986. Wynroth math program-the natural numbers sequence. Ithaca, NY: Wynroth math program.

Xu F, Spelke E. 2000. Large number discrimination in 6- month-old infants. Cognition 74: B1–B11.