

Tariffs as Cost-Push Shocks: Implications for Optimal Monetary Policy*

Iván Werning^(r)

Guido Lorenzoni^(r)

Veronica Guerrieri

May 2025

We study the optimal monetary policy response to the imposition of tariffs in a model with imported intermediate inputs. In a simple open-economy framework, we show that a tariff maps exactly into a cost-push shock in the standard closed-economy New Keynesian model, shifting the Phillips curve upward. We then characterize optimal monetary policy, showing that it partially accommodates the shock to smooth the transition to a more distorted long-run equilibrium—at the cost of higher short-run inflation and a depreciation in the exchange rate.

1 Introduction

“We may find ourselves in the challenging scenario in which our dual-mandate goals are in tension.”

Jerome H. Powell (Federal Reserve Chair, Speech at the Economic Club of Chicago, April 16, 2025)

A central bank confronted with a significant shift toward protectionist policies faces two immediate risks. First, tariffs exert inflationary pressure by directly raising the cost of imported goods—affecting both final consumer products and intermediate inputs used by domestic producers. Second, by distorting consumption and production decisions, tariffs generate efficiency losses that lower the economy’s potential output. Together, these effects amount to a “supply shock” that raise the specter of slower growth alongside rising inflation—in other words, a stagflation scenario.

*We thank Javier Bianchi and Tommaso Monacelli for useful comments.

In this paper, we formalize this intuition and shed further light on existing results using a simple model that highlights how tariffs distort the use of intermediate inputs—highly relevant in a world with large volumes of trade in intermediate goods and highly integrated supply chains. Our analysis yields a sharp conclusion: a tariff in an open economy is exactly equivalent to a labor wedge shock in a closed economy. In effect, our equivalence result shows that a tariff can be studied *as if* it were a cost-push shock in a standard New Keynesian (closed economy) model.

One benefit of our result is that cost-push shocks—such as those that shift the Phillips curve upward—are well studied and understood. They force monetary policy to navigate a trade-off between its dual objectives.¹ Viewed through the lens of our result, tariffs introduce a wedge-type distortion that creates an unavoidable conflict for monetary policy. The set of feasible inflation-output combinations deteriorates, leaving a central bank unable to simultaneously achieve both price stability and natural output.

Behind our result lie two distinct effects of tariffs: (i) a fall in labor productivity, and (ii) a sharper fall in firm profitability and, consequently, in the natural wage consistent with zero inflation. It is this discrepancy that generates the positive labor wedge—or cost-push shock. In fact, the decline in productivity turns out to be zero to a first-order approximation, while the decline in the required wage is not. As a result, the labor wedge has a first-order impact on the economy, whereas the drop in productivity—perhaps surprisingly—does not.²

Is a tariff expansionary or contractionary? The answer depends on how monetary policy responds, underscoring the value of an optimal policy framework.³ Based on our equivalence result, a tariff is best understood as a supply-side disturbance—not a demand-side disturbance—that introduces a trade-off for monetary policy. If the central bank prioritizes price stability, the tariff will lead to a sharp short-run contraction. Alternatively, the central bank can offset this contraction by pursuing an expansionary

¹It is sometimes less appreciated, however, that not all “supply shocks” take the same form. For example, shocks to productivity do not necessarily create the same tension between central bank objectives as do “wedge” shocks. Indeed, we show that tariffs create a drop in productivity and a labor wedge, but only the latter shapes the monetary policy response.

²Of course, the second-order decline in productivity is still welfare-relevant and has economic effects that merit policy attention. Fortunately, the impact of these productivity shocks is straightforward to analyze. However, productivity shocks do not generate the same policy trade-offs as labor wedge shocks.

³The traditional beggar-thy-neighbor view of older Keynesian models saw unilateral tariffs as having an expansionary effect on the domestic economy, due to their expenditure-switching effect, moving domestic demand away from foreign goods and towards domestic goods. [Krugman \(1982\)](#) questioned the direction of this effect and explored the potential contractionary effect of tariffs through the exchange rate. In recent years, there is a growing literature that uses new Keynesian models to analyze the effect of tariffs. This literature has also pointed out that tariffs can be contractionary. This is a central point of [Barattieri et al. \(2021\)](#), who also show empirically the contractionary effects of tariff shocks.

response. In sum, the notion of a “neutral” monetary policy is not well defined. This highlights the central importance of understanding the optimal policy response.

Having established that a tariff acts as a labor wedge shock, we characterize the welfare-maximizing monetary policy response. We begin with the textbook case of price rigidity and flexible wages, then extend the analysis to include nominal wage rigidity. In both settings, optimal policy allows inflation to rise temporarily and keeps output above its natural (flexible-price) level in the short run. This inflation arises precisely because output is held above the new, lower natural level, which falls due to the efficiency loss induced by the tariff. Over time, output gradually declines toward the distorted steady state. Maintaining zero inflation would require an abrupt drop in activity—an outcome optimal policy avoids by smoothing the adjustment path. Consumption and labor even rise modestly on impact, but both eventually contract. Thanks to the tractability offered by our equivalence result and our continuous-time formulation, we are able to derive all these results analytically. When wages are sticky, the cost of disinflation rises, and the central bank has less flexibility to engineer a smooth adjustment—though the core logic of the optimal response remains unchanged.

The backbone of our model is relatively standard, but to obtain our sharp equivalence results, we make a few important modeling choices. Specifically, we study a small open economy that takes world prices as given and is subject to a trade balance constraint. These simplifying assumptions allow us to focus squarely on the core trade-off between output stabilization and inflation control. In particular, the small open economy assumption abstracts from terms-of-trade manipulation, directing attention to more traditional monetary policy concerns. Of course, terms-of-trade effects are important in their own right and have been extensively studied in both trade and international macroeconomics. The trade balance assumption plays a role in delivering our exact equivalence result, but we believe the result remains a good approximation for monetary policy responses even when trade balance is not imposed. On balance, we view these assumptions as a worthwhile trade-off to gain clarity and insight into a central economic policy question.

Our paper contributes to the growing literature on the implications of tariffs for monetary policy, including [Barattieri et al. \(2021\)](#), [Jeanne \(2021\)](#), [Bergin and Corsetti \(2022\)](#), [Auray et al. \(2024\)](#), [Cuba-Borda et al. \(2024\)](#), [Bianchi and Coulibaly \(2025\)](#), [Monacelli \(2025\)](#), [Sebnem Kalemli-Ozcan \(2025\)](#) and [Auclert et al. \(2025\)](#). The two closest papers, that also focus on optimal monetary policy responses of the domestic monetary authority to a domestic tariff shock are [Bianchi and Coulibaly \(2025\)](#) and [Monacelli \(2025\)](#). These papers show, using simulations, that a short-run increase in inflation and output above

its natural level tend to arise at the optimal monetary policy.⁴ Our results agree with this conclusion and shed further economic light on the positive mechanisms and normative forces behind it by providing analytical results that characterize generally the optimal policy response. In terms of interpretation, [Bianchi and Coulibaly \(2025\)](#) focus on the fiscal revenue coming from the tariff and on the fact that consumers buying foreign goods do not internalize the effect on their purchases on fiscal revenue. This fiscal externality explains why monetary policy may choose a more inflationary path. Our interpretation is closer to traditional analysis of monetary policy that focuses on the trade-off between output gaps and inflation. In doing so, we trace a direct connection between our results and the classic analysis of optimal monetary policy with a distorted steady state of [Benigno and Woodford \(2004\)](#), since the tariff induces a new distorted steady state.

2 Tariffs as Cost-Push Shocks in Static Economy

We begin with a simple single-period neoclassical economy without nominal rigidities. This setup allows us to isolate two inefficiencies arising from the imposition of tariffs. In the sections that follow, we embed this static framework into a dynamic setting with nominal rigidities. The inefficiencies identified here will then serve as driving shocks in that framework.

The main result of this section is that a tariff is equivalent to two distortions: (i) a fall in productivity and (ii) a rise in the labor wedge (akin to a tax). Although both distortions are present and affect welfare, perhaps surprisingly, only the labor wedge matters to a first-order approximation.

2.1 Primitives: Technology, Utility and Trade

The Home economy consumes and produces a single final good. A representative agent has utility function

$$U(C, L),$$

assumed concave, differentiable, increasing in C and decreasing in L .

Production combines labor with an imported intermediate input by

$$Y = F(L, M),$$

⁴[Bergin and Corsetti \(2022\)](#) obtain somewhat different results, but study a different model and policy experiments—notably the effects of a retaliatory trade war and of coordinated monetary policy.

where F is concave, differentiable and has constant returns to scale. Here Y denotes the final output, L labor and M the imported intermediate input.⁵

The Home country exports the domestic good to foreign countries. Letting X denote exports, we then have the feasibility condition

$$Y = C + X,$$

where C denotes consumption.

The Home country takes world prices as given and must satisfy trade balance.⁶ We normalize prices $P_Y = P_M = 1$ so the trade balance condition becomes $X = M$. Domestic consumption then satisfies $C = F(L, M) - M$, which we write as

$$C = A(\mu)L \tag{1}$$

where we define

$$A(\mu) \equiv F(1, \mu) - \mu$$

and $\mu \equiv M/L$ represents the input-to-labor ratio. Equation (1) indicates that for any given ratio μ we have a linear production possibility frontier (PPF) between labor and consumption with $A(\mu)$ acting as a productivity parameter.⁷

2.2 Equilibrium with and without Tariffs

We now study the equilibrium and its dependence on tariffs, starting with the benchmark without tariffs.

No Tariff, First-Best Equilibrium. Without a tariff, the competitive equilibrium outcome coincides with the first best allocation. The first-best allocation maximizes $U(C, L)$ subject to (1). One can solve this allocation in two steps. First, we maximize the input-to-labor ratio to achieve the highest productivity

$$A^* = \max_{\mu} A(\mu)$$

⁵The production function $F(L, M)$ describes the efficient final output given (L, M) after possibly optimizing over many other production choices, such as the production and use of domestic inputs. For example, consider a roundabout domestic input Z , based on the final good, then $Y + Z = \hat{F}(L, M, Z)$ and $F(L, M) = \max_Z \{\hat{F}(L, M, Z) - Z\}$. We can similarly collapse an economy with multiple sectors that produce various inputs from labor and each other into the final good production function $F(L, M)$.

⁶We set initial net foreign assets to zero.

⁷The linearity of the frontier follows from constant returns to scale and the fact that labor is the only primary factor.

Second, we choose consumption and labor to maximize utility subject to $C = A^*L$.

Equilibrium with Tariffs. We now study the distorted competitive equilibrium when the domestic economy imposes a tariff τ on its imports. We begin with firms and then turn to households.

Thanks to constant returns to scale, firms' profits can be written as $(A(\mu) - \tau\mu - W)L$. Firms choose the input-to-labor ratio μ to maximize $A(\mu) - \tau\mu$. This yields the function $\mu(\tau)$. At an interior equilibrium firms make zero profits, and the wage is then given by

$$W(\tau) = \max_{\mu} \{A(\mu) - \tau\mu\} = A(\mu(\tau)) - \tau\mu(\tau). \quad (2)$$

Economically, the dependence of the wage on the tariff represents moving along the iso-cost curve for the firm: if the tariff rises, the wage must fall to compensate firms so that they are still willing to produce at a unit price.

Evaluating the production possibility frontier (1) with the equilibrium ratio $\mu(\tau)$ gives

$$C = A(\mu(\tau))L, \quad (3)$$

a linear production possibility frontier with $A(\mu(\tau))$ acting as a productivity parameter.

Turning to households, they maximize their utility subject to a budget constraint $C = W(\tau)L + T$ where T is a lump sum transfer that households take as given, equal, in equilibrium, to the revenue from the tariff: $T = \tau M$. The first-order optimality condition is

$$-\frac{U_L(C, L)}{U_C(C, L)} = W(\tau). \quad (4)$$

Putting firms and households together, given the constructed productivity $A(\mu(\tau))$ and wage $W(\tau)$, an equilibrium is entirely characterized by two equations in two unknowns: the optimal labor supply condition (4) and the production possibility frontier (3).

2.3 Two Inefficiencies from Tariffs

A positive tariff $\tau > 0$ introduces two sources of inefficiency relative to the first best.

First, the tariff introduces productive inefficiency. This occurs because firms maximize $A(\mu) - \tau\mu$ rather than $A(\mu)$. As a result, for $\tau \neq 0$, we have $A(\mu(\tau)) < A^*$. The tariff distorts the use of the imported input, reducing the level of consumption that can

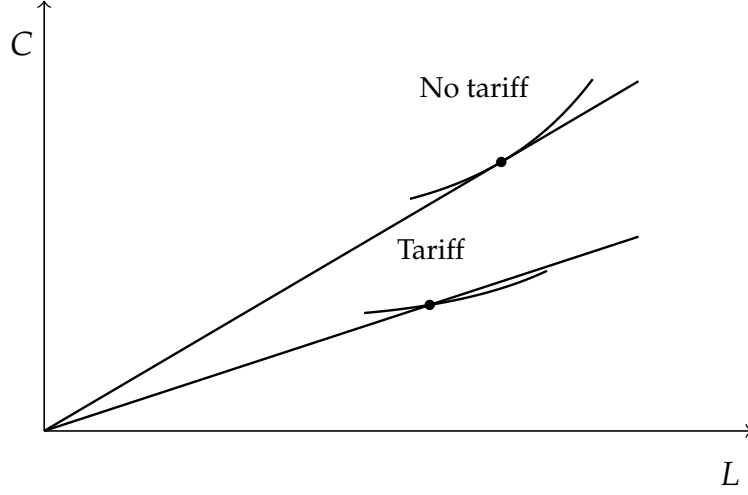


Figure 1: Equilibrium with and without a tariff

be achieved for any given level of labor L . This represents a downward shift in the production possibility frontier (3).

Second, the tariff lowers the wage, so that

$$W(\tau) < W(0)$$

for $\tau > 0$, since we have

$$W'(\tau) = -\mu(\tau) < 0 \quad (5)$$

by the Envelope Theorem applied to equation (2). As mentioned above, the lower wage reflects reduced firm profitability under a higher τ .

The second distortion arises from the fact that the wage no longer equals productivity, so that $W(\tau) \neq A(\mu(\tau))$ for $\tau \neq 0$. The resulting labor wedge is given by

$$A(\mu(\tau)) - W(\tau) = \tau\mu(\tau) \geq 0,$$

which follows directly from (2). Note that the labor wedge is strictly positive unless the tariff is zero ($\tau = 0$) or so prohibitively high that it induces zero imports ($\mu(\tau) = 0$).

These two effects are illustrated in Figure (1). The two straight lines represent the linear possibility frontier $C = A(\mu(\tau))L$, with and without the tariff. We also plot household indifference curves tangent to the wage $W(\tau)$ in each case. Without a tariff $A(0) = W(0)$ so the household chooses the optimum along the frontier at the point of tangency. With

the tariff, $W(0) < A(0)$ and the equilibrium is no longer tangent to the frontier; labor is inefficiently lower.

If we consider small tariffs starting from free trade then the production inefficiency is initially small. Formally, it is zero to a first order. To see this, note that from the definition of $A(\mu)$ we have

$$A'(\mu(0)) = 0 \tag{6}$$

and thus the marginal effect of the tariff on productivity is $A'(\mu(0))\mu'(0) = 0$.

On the other hand, the tariff does have a first-order effect on the wage and the labor wedge. To see this note that (6) implies that $W'(0) = -\mu(0) < 0$ so that $A'(\mu(0))\mu'(0) - W'(0) = \mu(0) > 0$.

Since, to first order approximation, productivity is unaffected while the labor wedge is, this represents a (local) pure substitution effect that reduces both labor and consumption. Letting $L(\tau)$ denote equilibrium labor, we have $L'(0) < 0$.

We summarize these results in the next proposition.

Proposition 1. *A tariff $\tau > 0$ leads to lower productivity, a lower wage, and a higher labor wedge: $A(\mu(\tau))$ and $W(\tau)$ are both decreasing, while $A(\mu(\tau)) - W(\tau)$ is increasing. The productivity effect is zero to a first order, but the effects on the wage and labor wedge are non-zero. Labor is affected negatively to a first order.*

The economic consequences of this characterization go beyond this static neoclassical setting and play out in a dynamic economy with nominal price and wage rigidities, as we shall see in the next section.

2.4 Quadratic Approximation to Welfare

We now briefly discuss the effects of the tariff on welfare. Let $\mathcal{U}(\tau)$ denote household utility as a function of the tariff. As usual, starting from a zero tariff, the first-order effect on household welfare is zero: $\mathcal{U}'(0) = 0$. Since we know that $\mathcal{U}(\tau) \leq \mathcal{U}(0)$, it follows that $\mathcal{U}''(0) \leq 0$. Indeed, computing the second-order effects, we find a utility loss (expressed in consumption units) of

$$\frac{\mathcal{U}''(0)}{U_C} = L(0)A''(\mu(0))\mu'(0)^2 - W'(0)L'(0) < 0, \tag{7}$$

where U_C is the marginal utility of consumption at the zero tariff allocation.⁸

⁸The equation is derived in the appendix.

The first term captures the direct loss in productive efficiency due to the distortion in the choice of the ratio μ .⁹ The second term captures the indirect distortionary effect of the labor wedge on labor. Both terms are negative, so both sources of inefficiency contribute to the reduction in welfare.

A similar expression to (7) reemerges in the next section, when we analyze optimal monetary policy using a second-order approximation of the welfare function.

2.5 Towards Adding Nominal Rigidities

We briefly explore an alternative approach to deriving the results above. This method is particularly useful when prices do not fully adjust and firms produce to meet demand at a given price. At this stage, what follows is best interpreted as a partial equilibrium analysis.

For any given output level Y , firms minimize costs $WL + (1 + \tau)M$ subject to $F(L, M) = Y$. This leads to the familiar condition

$$\frac{F_L(1, \mu)}{F_M(1, \mu)} = \frac{W}{1 + \tau}, \quad (8)$$

with solution $\hat{\mu}(\frac{1+\tau}{W})$. We see that both τ and W influence the choice of μ . Economically, this reflects the fact that it is the relative price of the two inputs $(1 + \tau)/W$ that matters and that we are considering any wage W because we are not constraining ourselves to the iso-cost curve as before.

Previously, the wage was endogenous and pinned down by the iso-cost condition, to ensure the firm was willing to produce at a unit price. With the present formulation, this amounts to evaluating $\mu(\tau) = \hat{\mu}(\frac{1+\tau}{W(\tau)})$.¹⁰

This cost-minimization formulation with the ratio $\hat{\mu}(\frac{1+\tau}{W})$ will be useful in the next section where we consider firms that, due to price rigidities, do not optimize over output for a given price, but instead produce to meet demand. In that economy, the wage is also endogenous—determined by households—and is influenced by monetary policy and potentially subject to nominal rigidities. For both reasons, the real wage does not necessarily equal the flexible equilibrium real wage found earlier $W(\tau)$. Thus, the central relation we

⁹The term can be interpreted as a Harberger triangle. Think of the equation $1 + \tau = A'(\mu)$ as representing the inverse demand function for imports (since $1 + \tau$ is the after-tax price). The slope of the inverse demand function is $-A''(\mu)$, so the triangle is $-\frac{1}{2}A''(\mu) d\mu \cdot d\mu = -\frac{1}{2}A''(\mu) (\mu')^2 (d\tau)^2$.

¹⁰The wage itself solves

$$W(\tau) = A\left(\hat{\mu}\left(\frac{1+\tau}{W(\tau)}\right)\right) + \tau\hat{\mu}\left(\frac{1+\tau}{W(\tau)}\right).$$

But we saw it was more directly obtained from $W(\tau) = \max_{\mu}\{A(\mu) + \tau\mu\}$.

will use to determine imports of intermediates will be an analog of $\hat{\mu}(\frac{1+\tau}{W})$ rather than of $\mu(\tau)$.

3 Dealing with Cost-Push Shocks in a Dynamic Economy

We now extend our simple economy to a dynamic setting with nominal rigidities. We begin with constraints on price changes and later incorporate wage rigidities.

With flexible prices, there is nothing the monetary authority can do to offset the inefficient decline in output. However, nominal rigidities allow the central bank to keep output and consumption temporarily above their flexible-price levels—that is, to smooth the transition to a lower level of activity. This can be beneficial, as it keeps both labor and intermediate inputs temporarily closer to their efficient levels. As we shall see in this section, this comes at the cost of higher inflation, so an optimizing central bank must balance these two considerations.

3.1 The Dynamic Model

The economy operates in continuous time $t \geq 0$. Households have utility

$$\int_0^\infty e^{-\rho t} U(C_t, L_t) dt,$$

with $\rho > 0$. We now specialize the utility function to

$$U(C, L) = \log C - \frac{1}{1+\phi} L^{1+\phi}.$$

Households face an intertemporal budget constraint with the nominal interest rate set by the central bank.

The domestic final good is produced competitively by combining a continuum of varieties

$$Y_t = \int_0^1 \left(Y_{it}^{1-\frac{1}{\varepsilon}} di \right)^{\frac{1}{1-\frac{1}{\varepsilon}}} di,$$

where Y_{it} is variety i . The price of the final good equals its cost given by $P_t = \int_0^1 \left(P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} di$.

Each variety i is produced by a monopolistically competitive firm with production function

$$Y_{it} = F(L_{it}, M_{it}) = \left(a_L^{\frac{1}{\eta}} L_{it}^{1-\frac{1}{\eta}} + a_M^{\frac{1}{\eta}} M_{it}^{1-\frac{1}{\eta}} \right)^{\frac{1}{1-\frac{1}{\eta}}},$$

where η is the constant elasticity of substitution between labor and the imported input.

We retain our simplifying assumptions with regards to international trade. The relative price of the final good relative to the imported input is given and independent of domestic policy.¹¹ Moreover, we assume closed financial markets so that trade balance holds in all periods. The resource constraint for the country is again

$$C_t = Y_t - M_t.$$

Each monopolistic competitor producer faces a downward sloping demand schedule $Y_{it} = (P_{it}/P_t)^{-\varepsilon} Y_t$ for their variety. They reset their price at random intervals with Poisson arrival rate $\theta > 0$. They always meet demand.

Before time $t = 0$ the economy finds itself in its free trade steady state. At $t = 0$, a sequence of tariffs are announced $\{\tau_t\}$ unexpectedly (an “MIT shock”).¹² As in the previous section, the tariff alters the quantities and relative prices that would prevail under flexible prices—in exactly the same way, in each period, as in the static model. We refer to this as the flexible-price allocation. However, due to nominal rigidities, monetary policy is non-neutral and can affect the equilibrium allocation. In particular, the flexible-price allocation is attainable, but other allocations that deviate from it are as well.¹³

3.2 Equivalence: Equilibrium and Welfare

We now present our equivalence result.

Proposition 2. *The open economy subject to a tariff shock $\{\tau_t\}$ is equivalent to a standard closed-economy New Keynesian model subject to a cost-push shock $\{\epsilon_t\}$ in the Phillips curve*

$$\pi_t = \int_0^\infty e^{-\rho s} (\kappa y_{t+s} + \epsilon_{t+s}) ds$$

where ϵ_t proportional to τ_t and standard welfare function

$$-\frac{1}{2} \int_0^\infty e^{-\rho t} (y_t^2 + \lambda \pi_t^2) dt$$

¹¹Thus, our economy is a limit case of the open-economy model of [Gali and Monacelli \(2005\)](#), when all country goods become perfect substitutes.

¹²The permanence of the tariff is not crucial to any of our arguments; we adopt it as a benchmark because it helps highlight the temporary nature of the optimal response. In particular, our characterization in terms of a cost-push shock applies to any sequence of tariffs $\{\tau_t\}$, which would then imply a corresponding sequence of cost-push shocks.

¹³As is common in this class of models, we introduce a subsidy on production that offsets the monopolistic markup distortion. This ensures that the initial steady state—absent shocks—coincides with the first-best allocation, allowing us to focus on fluctuations and stabilization policy.

In addition, we have an Euler equation for output

$$\dot{y}_t = \frac{1}{\hat{\sigma}}(i_t - \pi_t) + u_t$$

with the “demand shock” u_t proportional to $\dot{\tau}_t$; if $\tau_t = \tau$ then demand shocks are zero $u_t = 0$.

Below we prove this result by deriving the equilibrium conditions and then the welfare function. Together these provide our equivalence result, connecting our open economy to a standard closed economy model subject to a cost-push shock.

Equilibrium. Intuitively, the driving force behind inflation in New Keynesian models is the gap between the nominal marginal costs and the prices at which goods are sold, adjusted for desired markups. When the gap is positive, firms have an incentive to raise prices, and inflation follows. Using lowercase variables to denote log deviations from the initial steady state, the gap between nominal marginal costs is

$$s_L w_t + s_M(p_t^M + \tau_t) - p_t,$$

where w_t is the nominal wage, p_t^M is the price of imported inputs in domestic currency, τ_t is the tariff, and s_L and s_M are the cost shares of the labor input and the imported intermediate good, at the initial steady state, with $s_L + s_M = 1$.¹⁴

Because of fixed terms of trade $p_t^M = p_t$ and the gap between nominal marginal costs and the domestic price is

$$s_L \omega_t + s_M \tau_t,$$

where ω_t denotes the real wage $w_t - p_t$. Following standard derivations for the firm price setting problem and its aggregation, we then obtain the Phillips curve

$$\rho \pi_t = \theta(\rho + \theta)(s_L \omega_t + s_M \tau_t) + \dot{\pi}_t \quad (9)$$

where π_t is inflation, ρ is the household discount factor, and θ is the arrival rate of the Calvo process. In integral form

$$\pi_t = \theta(\rho + \theta) \int e^{-\rho s} (s_L \omega_{t+s} + s_M \tau_t) ds$$

The direct effect of the tariff on inflation is captured by the term $s_M \tau_t$. Inflation can be kept at bay only if the real wage falls enough $\omega_t < 0$ so that $s_L \omega_{t+s} + s_M \tau_t = 0$. The model intuition is in line with the standard narrative: a tariff increases costs for the firms

¹⁴With a slight abuse of notation in all log-linearized expressions we use τ_t to denote $\ln(1 + \tau_t)$.

using imported intermediates; for a given real wage ω_t , these cost pressures translate into inflation.

To complete the equilibrium characterization we need four additional log-linearized static conditions. Output and consumption are given by

$$y_t = s_L l_t + s_M m_t, \quad (10)$$

$$c_t = l_t. \quad (11)$$

Equation (10) is standard and log linearizes production in terms of the inputs and their shares, while equation (11) log-linearizes consumption $C_t = Y_t - M_t$.¹⁵ Note that equation (11) echoes our observation that productivity in the linear frontier (3) is unaffected to a first order. Turning to optimality conditions, the household labor supply condition (4) becomes

$$c_t + \phi l_t = \omega_t. \quad (12)$$

Finally, the firm cost minimizing input-to-labor ratio condition (8) becomes

$$m_t - l_t = \eta(\omega_t - \tau_t). \quad (13)$$

Substituting (10)–(13), the Phillips curve (9) can be rewritten compactly in the traditional form

$$\rho \pi_t = \kappa y_t + \epsilon_t + \dot{\tau}_t, \quad (14)$$

where the term ϵ_t is proportional to τ_t .¹⁶ In integral form $\pi_t = \int e^{-\rho s} (\kappa y_{t+s} + \epsilon_t) ds$. A positive tariff $\tau_t > 0$ generates a cost-push shock $\epsilon_t > 0$ in equation (14). This form of a supply shock shifts the Phillips curve upward, without affecting productivity.

To keep inflation at zero, a central bank must allow output to contract so that $\kappa y_t + \epsilon_t = 0$. This corresponds to the flexible-price allocation discussed in Section (2). Using a tilde to denote flexible-price allocations, we need

$$y_t = \tilde{y}_t \equiv - \left(\frac{1}{1 + \phi} + \eta \right) \frac{s_M}{s_L} \tau_t < 0.$$

¹⁵Log linearizing this expression gives $c_t = \frac{1}{1-s_M} y_t - \frac{s_M}{1-s_M} m_t$. Substituting (10) then yields $c_t = l_t$.

¹⁶The expressions for κ and ϵ are

$$\begin{aligned} \kappa &= \theta(\rho + \theta) \frac{s_L(1 + \phi)}{1 + \eta s_M(1 + \phi)}, \\ \epsilon_t &= \theta(\rho + \theta) \frac{1 + \eta(1 + \phi)}{1 + \eta s_M(1 + \phi)} s_M \tau_t. \end{aligned}$$

There are two underlying forces causing $\tilde{y}_t < 0$, which, once again, reflect the two inefficiencies identified in the previous section: a reduction in the input-to-labor ratio, $m_t - l_t$, and a reduction in labor, l_t .¹⁷

Finally, note that $y_t \neq c_t$, but that combining equations (10)–(13) we see that y_t and c_t are in a one-to-one linear relationship,

$$y_t = (1 + s_M \eta (1 + \phi)) c_t - s_M \eta \tau_t.$$

Differentiating and using that $\dot{c}_t = \frac{1}{\sigma}(i_t - \pi_t)$ then gives the Euler equation for output with $\hat{\sigma} = \sigma / (1 + s_M \eta (1 + \phi))$.

Welfare Function. In contrast, the effects of the tariff and of monetary policy do appear once we turn to the second-order approximation of welfare,

$$-\frac{1}{2} \int_0^\infty e^{-\rho t} \left[\frac{1}{\eta} s_M (m_t - l_t)^2 + (1 + \phi) l_t^2 + \Psi_P \pi_t^2 \right] dt, \quad (15)$$

with

$$\Psi_P = \frac{\varepsilon}{\theta(\rho + \theta)}.$$

The first two terms in this welfare function directly reflect the two inefficiencies identified in the previous section and derived formally in equation (7). The first term captures productive inefficiency arising from a distorted input-to-labor ratio $m_t - l_t \neq 0$. Later, we will use equation 13 to rewrite this term in terms of the real wage. The second term is the inefficiency in employment. With balanced growth preferences, the efficient level of l_t is equal to 0 regardless of productivity, which explains why this term is simply proportional to l_t^2 ; the natural rate of employment is constant. Finally, the last term reflects the cost of nonzero inflation, which is standard in models with nominal rigidities. It arises from the inefficient price dispersion that inflation induces across varieties.

Surprisingly, the welfare function (15) can also be rewritten as

$$-\frac{1}{2} \int_0^\infty e^{-\rho t} \left(\Psi_Y y_t^2 + \Psi_P \pi_t^2 \right) dt, \quad (16)$$

where we have dropped additive terms that only involve the exogenous shock τ_t . Although this welfare expression is simpler, the first two terms in (15) contribute towards the $\Psi_Y y_t^2$ term here. In particular, Ψ_Y depends on η and s_M . We can also express welfare in terms of c_t or l_t (they are equivalent); this version, however, contains a linear term.

¹⁷Note that, as discussed, the effect of tariffs on the frontier between consumption and labor is not first order, yet its effect on the relation between output and labor is first order.

The expression for welfare, together with the Phillips curve (14), highlights the core tension in the model. The level of output that maximizes welfare remains at the initial steady state, $y_t = 0$. However, the level of output consistent with zero inflation is now $\tilde{y} < 0$. The central bank can no longer achieve both objectives simultaneously.

The tension just highlighted is precisely what happens with a textbook cost-push shock, like one due to an increase in the firms' market power. It is useful to recall that the same tension is not present with other supply-side shocks. In particular, if we had a structurally similar model with $C = AL$, but where A was a fully exogenous technological parameter, then there would be no trade-off following a negative shock to A , as the central bank would just let output fall as in the flexible price allocation and zero inflation would be the optimal response. The crucial reason for the monetary policy trade-off here is in the distortions associated to the tariff shock.

4 Optimal Monetary Policy

We now turn to the optimal response to the tariff shock. The planning problem is

$$\min_{\{\pi_t, y_t\}} \int_0^\infty e^{-\rho t} \left(\Psi_Y y_t^2 + \Psi_P \pi_t^2 \right) dt,$$

subject to $\rho\pi_t = \kappa y_t + \epsilon_t + \dot{\pi}_t$ with π_0 free. When we study optimal policy we abstract from the zero lower bound on interest rates $i_t \geq 0$ and, hence, drop the Euler equation. Thus, we directly study the choice of $\{y_t, \pi_t\}$ letting i_t be determined by the Euler equation.

Given the mapping to the familiar closed economy setting with a cost push shock, our results are fairly intuitive. We find that policy manages the trade-off by keeping output temporarily above its flexible-price level immediately after the tariff is imposed, and then gradually adjusting towards it. Doing so comes at the cost of some positive inflation.

A more detailed characterization is given in the next proposition.

Proposition 3. *The optimal policy following the imposition of a permanent tariff $\tau_t = \tau > 0$ yields an equilibrium allocation with output given by*

$$y_t = (1 - e^{-rt})\tilde{y}, \tag{17}$$

and implies:

(i) *unchanged output on impact, $y_0 = 0$, followed by decreasing output converging to its new, lower steady state: $y_t \rightarrow \tilde{y} < 0$;*

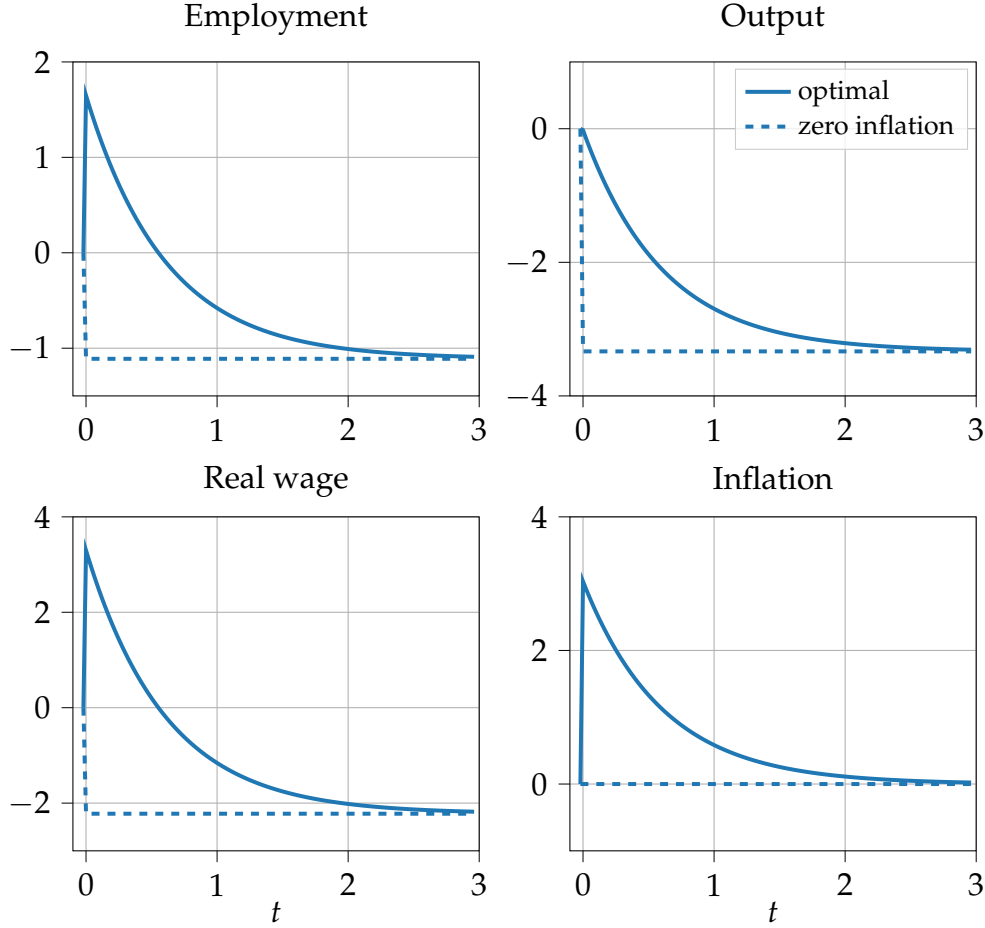


Figure 2: Responses to the tariff shock, under optimal policy and zero-inflation policy
All variables in log deviations from steady state (times 100)

(ii) an increase in employment on impact $l_0 > 0$, followed by decline in employment toward its new, lower steady state: $l_t \rightarrow \tilde{l} < 0$;

(iii) an increase in inflation on impact, $\pi_0 > 0$, followed by a gradual return to zero: $\pi_t \rightarrow 0$.

The coefficient r in equation (17), which governs the speed of adjustment of y_t , is defined in the appendix.

The proposition is illustrated in Figure 2 for a numerical example. The unit of time is a year and parameters are set to

$$\rho = 0.03, \quad \phi = 1, \quad s_M = 0.1, \quad \eta = 1, \quad \varepsilon = 2, \quad \theta = 1.$$

The tariff shock is set to $\tau = 0.20$. The figure shows the responses of real variables and inflation following the tariff shock.

The result that the central bank only temporarily keeps y_t above $\tilde{y}$ is a classic feature

of economies with a distorted steady state (Benigno and Woodford, 2004). In this case, the initial steady state is undistorted, but the new steady state—following the permanent tariff—is distorted. The key intuition is that maintaining output above its flexible-price level far into the future imposes inflationary distortions in all prior periods, as price-setters anticipate the impact of future policy on marginal costs. As the horizon extends, the cumulative inflation costs from earlier periods grow, eventually outweighing the short-term benefits of boosting output. This makes it too costly for the central bank to sustain such a policy indefinitely.

The result that output starts exactly at its initial steady state $y_0 = 0$ reflects the fact that, in continuous time, the inflationary costs are negligible over an arbitrarily short interval following the shock. This result would not be apparent in discrete time, although it would be approached for sufficiently small time periods.

Maintaining output unchanged on impact requires a temporary increase in employment above its initial steady state to compensate for the reduction in imported inputs caused by the tariff. In this model, raising employment above its initial level improves efficiency because it increases the real wage ω_t . Since the input-to-labor ratio, $m_t - l_t$, is increasing in $\omega_t - \tau$, keeping real wages elevated partially offsets the inefficient reduction in $m_t - l_t$ induced by the tariff.

Adding Nominal Wage Rigidities

The model so far features sticky prices but flexible wages, so the real wage adjustment can occur instantly and without cost. This implies that a zero-inflation outcome can be achieved by replicating the flexible-price allocation. However, this path is suboptimal. Thus, inflation resulted from the central bank's choice to smooth the economy's adjustment toward the more distorted equilibrium.

Whether the so-called “divine coincidence” holds in this context is a matter of semantics. If we take the flexible-price allocation as the reference point, then divine coincidence holds, since the central bank can achieve that allocation while maintaining zero inflation. If, instead, we take the efficient allocation $y = 0$ as reference, divine coincidence fails. What is unambiguous is that a trade-off exists between the central bank's welfare objectives.

If we add sticky wages, the needed real wage adjustment can no longer occur without some combination of wage deflation and price inflation, both of which are undesirable. Indeed, if the central bank replicates the quantities from the flexible-price allocation, inflation still arises—unlike in the previous model. This is because nominal wages adjust



Figure 3: Sticky wage model, with employment at its flexible-price level

All variables in log deviations from steady state (times 100)

only gradually. As a result, nominal marginal costs exceed firms' prices along the transition path, generating price inflation. The details of this argument are provided in the appendix. An example is illustrated in Figure 3. The parameters are the same as in the example in Figure 2, with the addition of $\theta_W = 1/2$ and $\varepsilon_L = 6$, the Calvo parameter for wage setting and the elasticity of substitution among varieties of labor services.

Welfare now takes the form

$$-\frac{1}{2} \int_0^\infty e^{-\rho t} \left[\eta s_M (\omega_t - \tau)^2 + (1 + \phi) l_t^2 + \Psi_P \pi_t^2 + \Psi_W \pi_{Wt}^2 \right] dt. \quad (18)$$

The first two terms are as in equation (15), but we have substituted the demand for imported inputs $m_t - l_t = \eta(\omega_t - \tau)$ into the first term. Previously, with flexible wages, ω_t was a function of current allocation—through the labor supply condition—which is why we were able to combine the first term with the second term. Now, with sticky nominal

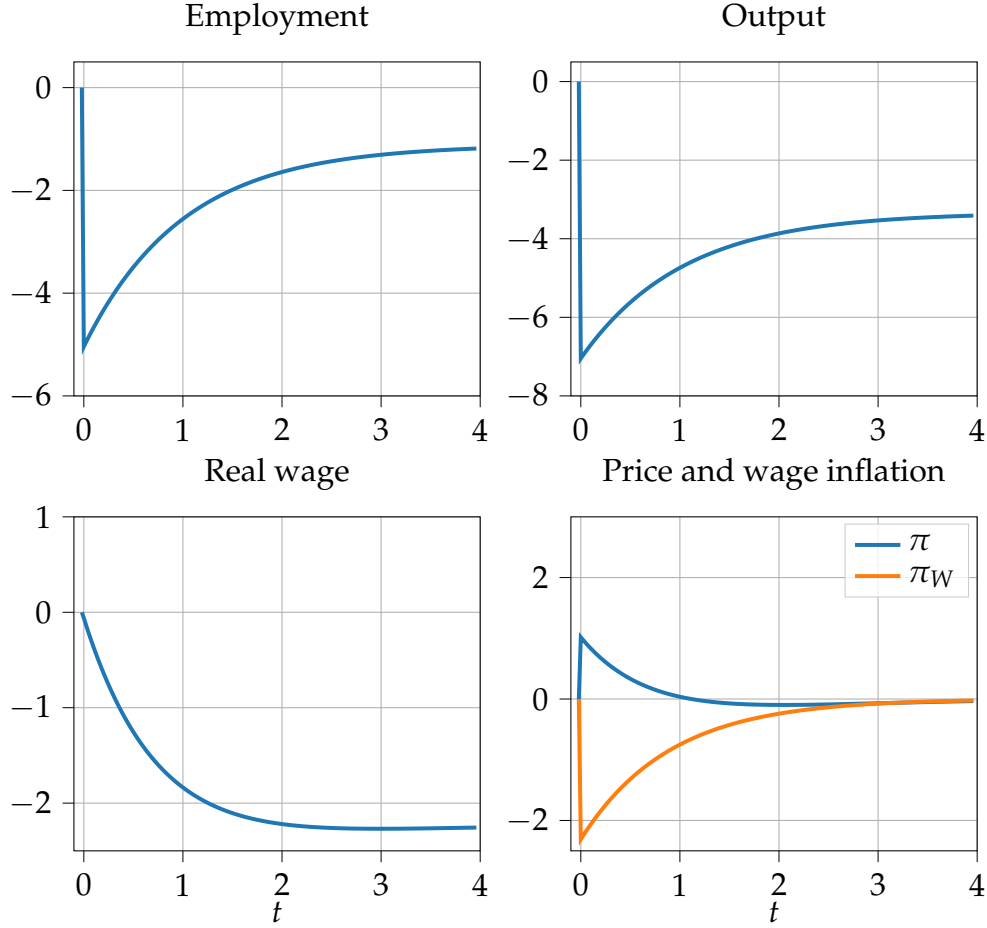


Figure 4: Sticky wage model, achieving a lower inflation path

All variables in log deviations from steady state (times 100)

wages, ω_t is a state variable that is not a function of the allocation. This fact has implications for optimal policy. The second and third term are unchanged relative to equation (15). The last term penalizes non-zero wage inflation, π_{Wt} , and is analogous to the penalty for price inflation.

Imagine now that the central bank attempts to achieve a lower rate of price inflation than the one shown in Figure 3. This requires a contraction in employment larger than $\tilde{l}$, along with greater wage deflation. This is the only way to generate the reduction in real wages needed to offset the price pressure from imported inputs. This response is illustrated in Figure (4), which shows a contraction in employment and output that overshoots (from below) the level needed to achieve zero inflation in the long run.

The first result here is that, with sticky wages, containing the inflationary effects of the tariff shock is more costly, as it requires a contraction in real activity larger than under flexible wages.

However, the contraction shown in Figure (4) is not the optimal response. As in the case with flexible wages, optimal policy moves in the opposite direction: it entails a smoother path for quantities, keeping the economy temporarily above its flexible-price allocation, and gradually converging to lower activity. We illustrate this in the example in Figure 5 and prove the result in the following proposition.

Proposition 4. *With both sticky prices and sticky wages, optimal policy in response to a permanent tariff $\tau > 0$ yields an equilibrium with:*

- (i) *a fall in output on impact, $y_0 < 0$, followed by a larger output decline in the long run: $y_t \rightarrow \tilde{y} < y_0 < 0$;*
- (ii) *unchanged employment on impact, $l_0 = 0$, and lower employment in the long run: $l_t \rightarrow \tilde{l} < 0$;*
- (iii) *positive average price inflation along the transition, $\int_0^\infty \pi_t dt > 0$, with inflation converging to zero in the long run: $\pi_t \rightarrow 0$.*

Unlike in Proposition 3, we do not have a closed-form characterization of the transition path for inflation. However, we can still show that inflation is, on average, positive along the transition path.

Whether wage inflation is positive at the optimum is ambiguous and generally depends on the model parameters. In particular, it reflects a trade-off between the welfare costs of sustained wage and price inflation and the benefits of keeping quantities closer to their efficient levels for longer. In the example shown in Figure 5, both price inflation and wage inflation are positive.

While the two cases—flexible wages and sticky wages—are similar in requiring a gradual adjustment of quantities, there is one qualitative difference between Propositions 3 and 4, which is visible when comparing Figures 2 and 5. In the first model, we have stable output and higher employment on impact, $y_0 = 0, l_0 > 0$; in the second, we have stable employment and lower output, $l_0 = 0, y_0 < 0$. The input-to-labor ratio chosen by cost-minimizing firms is a function of $\omega_t - \tau$. The difference between the two cases arises because, with nominal wage rigidities, the real wage is a slow moving state variable and the central bank has more limited powers to prevent a fall in the input-to-labor ratio by pushing up real wages. This explains why the central bank has a weaker incentive to keep activity higher.

A way of describing the difference between the two versions of our economy is that in the first case the central bank can immediately contrast both sources of inefficiency by stimulating real activity, while in the second case the central bank can directly contrast the inefficiency in labor allocation—the term $(1 + \phi)l_t^2$ —but can only indirectly contrast the productive efficiency—the term $\eta s_M(\omega_t - \tau)^2$.



Figure 5: Sticky wage model, optimal policy
All variables in log deviations from steady state (times 100)

Summing up the results so far, with nominal wage rigidities it becomes more costly to pursue a contractionary policy (output below its flexible-price level) in order to reduce inflation, but there are also lower gains from keeping the economy above its flexible-price benchmark.

5 Connections to Policy Debate

We now discuss and extend the results above, drawing more connections to the policy debate.

Commitment. A prominent concern for central bankers is commitment. After a spell of high inflation, this issue takes on added importance in current policy debates. What guidance does our simple model offer?

Note that every optimal policy path above assumes the central bank can commit, targeting the entire trajectory of real variables. The commitment problem becomes more acute under discretion, when outcomes hinge on the central bank's reputation and on how private agents form expectations.

Second, models with a distorted steady state imply that the central bank does not wish to exploit the trade-off between output and inflation in the long run. This general result is evident in all of the simulations above, which show real variables converging to their flexible-price levels—even though those levels are now inefficient. The implication is often invoked to support policy rules that disregard persistent structural distortions, such as those stemming from market power or the tax system. In other words, in a stable environment, monetary policy should not systematically attempt to expand the economy simply because equilibrium output falls short of the first-best benchmark.

Woodford (2003) addresses this issue by advocating for a “timeless perspective” in policy design. Imagine a central bank able to commit to its policy rule in advance, before any tariff shock occurs. Would such a central bank respond to the shock as in the optimal paths above—or would it have preferred to precommit to a lower inflation response?

Perhaps counterintuitively, the optimal responses we characterized are also optimal from the timeless perspective—provided the ex ante probability of the tariff shock was small. In other words, a central banker in 2005 who placed very low probability on a trade war occurring in 2025 would have committed to a reaction function that closely resembles the one illustrated above. This result is proved formally in the appendix (7.4). The intuition is that when the shock is perceived as unlikely, the inflation response it triggers has only a minor effect on pre-shock price-setting behavior.

What about the Exchange Rate? The analysis above did not explicitly model the exchange rate. However, the exchange rate is implicitly present in the background. We now explicitly draw out these implications.

Our model can be viewed as a limiting case of the open economy framework in Gali and Monacelli (2005), under the assumption that goods produced by different countries are perfect substitutes. As shown in the appendix, domestic output is given by

$$y_t = \omega \frac{C}{Y} (c_t - \epsilon (p_{Ht} - p_t)) - \left(1 - \omega \frac{C}{Y}\right) \zeta (p_{Ht} - e_t).$$

Here, all variables are in log-deviations from their steady states. The endogenous variables are: y_t domestic output; p_{Ht} the price of the home-produced goods, p_t the domestic consumer price index; e_t the nominal exchange rate. The coefficients depend on structural

parameters: the steady-state consumption-output ratio C/Y , the home bias in consumption ω , a parameter that captures home bias in consumption, ϵ , the elasticity of substitution between home and foreign goods, and the elasticity of substitution across country varieties ζ .

In our model, we take the limit $\zeta \rightarrow \infty$, implying perfectly elastic demand for the home good. In this case, the above condition simplifies to the purchasing power parity relation:

$$p_{Ht} = e_t.$$

This equation then fully characterizes exchange rate movements in our setting.

We therefore conclude that, in our model, the nominal exchange rate tracks domestic inflation: as tariffs raise domestic prices, the currency depreciates gradually. While this may not always be a realistic prediction at high-frequency—where exchange rates are likely dominated by financial market forces—it may still capture the relevant relative price dynamics that matter for allocations and inflationary pressures. In this sense, high-frequency exchange rate volatility may play only a secondary role.

Seeing Through the Import Price Shock? A widely held tenet in the practice of central banking is that a shock like a tariff, that entails a mechanical increase in some prices, is a shock that requires the central bank to “see through it”. In other words, the aim is to let the mechanical price effects play out while preventing any broader ramifications. To be fair, this is a rule of thumb which, as far as we know, does not correspond to any result in economic theory. However, it is useful to compare this practical precept to the optimal responses derived above.

A simple exercise is to take the cumulated effect of the shock on the price level

$$p = \lim_{t \rightarrow \infty} p_t = \int_0^{\infty} \pi_t dt$$

and compare it with the mechanical effect that would come from simply increasing import prices by τ , which, given the model structure, gives an effect proportional to the share of imported inputs in costs

$$s_M \tau.$$

Comparing these two quantities tells us whether the “see-through” principle holds exactly or approximately, given the model parameters.

In the model with sticky prices and flexible wages, the cumulated price response is

derived in the proof of Proposition (3) and is

$$p = \frac{1}{\varepsilon s_L^2} \left(\frac{1}{1 + \phi} + \eta \right) s_M \tau.$$

This expression can be in general smaller or larger than $s_M \tau$. For plausible parameter choices, and, in particular when ε is large enough, the principle seems to fail and the central bank wants less total inflation than see-through inflation. Notice that ε is the parameter that controls the welfare cost of inflation. The larger the parameter, the smaller the optimal cumulated inflation response. A somewhat surprising finding is that stickiness (the parameter θ) does not matter for this conclusion, which applies both in economies with low θ , which display a small and long-lasting increase in inflation, and in economies with high θ , which display larger but short-lived inflation.

However, the conclusion above is easily reversed when we turn to the model with sticky wages. With sticky wages, reaching a see-through outcome or a less-than-see-through outcome for prices becomes more costly, as it requires nominal wages to fall. In particular, the long-run price level response p always satisfies

$$p = s_L w + s_M (p + \tau) > s_M \tau$$

where w is the long-run effect on nominal wages. It is easy to show that the optimal policy will require more than see-through inflation if and only if

$$w > -\frac{s_M^2}{s_L} \tau,$$

that is if cumulated wage inflation is not too negative. In particular, if positive wage inflation is part of an optimal policy, as in the example of Figure(5), optimal inflation will always exceed see-through inflation.

In summary, the see-through principle is not generally supported by economic principles. In particular, when the economy displays both sticky prices and sticky wages, it seems violated in the direction that more than see-through inflation appears to be optimal. Whether this result leads to quantitatively significant deviations from the see-through principle, is an open question for future work.

6 Conclusions

We analyze the implications of tariffs for optimal monetary policy in an open economy with imported intermediate inputs. Our main result shows that tariffs can be studied as

if they were cost-push shocks in a closed economy.

This equivalence provides a powerful analytical lens for evaluating the trade-off between stabilizing inflation and supporting output. Our continuous-time framework allows us to characterize the optimal policy response sharply: a temporary accommodation of the shock, with inflation rising above target and output held above its new, lower natural level, before both variables gradually converge to the distorted steady state.

Our analysis highlights the core tension that tariffs create for monetary policy: the trade-off between inflation control and output stabilization. The standard intuition that central banks should “see through” supply shocks does not hold unambiguously—optimal policy often calls for a transitory inflation overshoot to mitigate output losses. Moreover, we show that even when the steady state is distorted, the central bank’s optimal response under commitment (including from the timeless perspective) supports a gradual transition.

Our analysis deliberately abstracts from channels such as tariff and policy uncertainty, consumer confidence, and financial or currency instability. The integration of these additional, but potentially important, factors is a task for future work.

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7 Appendix

7.1 Additional Derivations for Section (2)

First, we derive properties of the functions $W(\tau)$ and

$$\mathcal{A}(\tau) \equiv A(\mu(\tau)) = F(1, \mu(\tau)) - \mu(\tau).$$

Differentiating W gives

$$W'(\tau) = -\mu(\tau), \quad W''(\tau) = -\mu'(\tau).$$

Differentiating $\mathcal{A}(\tau)$ gives

$$\mathcal{A}'(\tau) = A'(\mu(\tau)) \mu'(\tau) = \tau \mu'(\tau),$$

$$\mathcal{A}''(\tau) = A''(\mu(\tau)) (\mu'(\tau))^2 + A'(\mu(\tau)) \mu''(\tau) = \mu'(\tau) + \tau \mu''(\tau). \quad (19)$$

Evaluated at $\tau = 0$, these yield

$$\mathcal{A}'(0) = 0, \quad \mathcal{A}''(0) = \mu'(0).$$

Next, we can look at the effects on equilibrium welfare, defined as

$$\mathcal{U}(\tau) = U(\mathcal{A}(\tau)L(\tau), L(\tau)).$$

The first derivative of $\mathcal{U}$ is

$$\mathcal{U}'(\tau) = U_C \mathcal{A}'(\tau) L(\tau) + (U_C \mathcal{A}(\tau) + U_L) L'(\tau)$$

or, using the labor supply condition $-U_L/U_C = W$,

$$\mathcal{U}'(\tau) = U_C \cdot [\mathcal{A}'(\tau) L(\tau) + (\mathcal{A}(\tau) - W(\tau)) L'(\tau)]. \quad (20)$$

Evaluating this expression at $\tau = 0$ we have $\mathcal{U}'(0) = 0$ because $\mathcal{A}'(0) = 0$ and $\mathcal{A}(0) - W(0) = 0$.

Differentiating (20) we get the second derivative of $\mathcal{U}$,

$$\begin{aligned} \mathcal{U}''(\tau) = & U_C \cdot [L(\tau) \mathcal{A}''(\tau) + \mathcal{A}'(\tau) L'(\tau) + (\mathcal{A}'(\tau) - W'(\tau)) L'(\tau) + (\mathcal{A}(\tau) - W(\tau)) L''(\tau)] + \\ & + \frac{\partial}{\partial \tau} (U_C (\mathcal{A}(\tau) L(\tau), L(\tau))) [\mathcal{A}'(\tau) L(\tau) + (\mathcal{A}(\tau) - W(\tau)) L'(\tau)]. \end{aligned}$$

Evaluated at $\tau = 0$ the second line is zero and recalling that $\mathcal{A}'(0) = 0$ we are left with

$$\begin{aligned} \mathcal{U}''(0) &= U_C (\bar{C}, \bar{L}) [\bar{L} \mathcal{A}''(0) - W'(0) L'(0)] = \\ &= U_C (\bar{C}, \bar{L}) [\bar{L} A''(\mu(0)) (\mu'(0))^2 - W'(0) L'(0)] \\ &= U_C (\bar{C}, \bar{L}) [\bar{L} \mu'(0) + \mu(0) L'(0)], \end{aligned}$$

where bars denote variables at the zero tariff equilibrium. The second line is equation (7) in the text. The last line exploits (19).

Finally, we prove $L'(0) < 0$. Labor supply satisfies

$$U_C (\mathcal{A}(\tau) L(\tau), L(\tau)) W(\tau) + U_L (\mathcal{A}(\tau) L(\tau), L(\tau)) = 0.$$

Differentiating gives

$$(U_{CC} A + U_{CL}) L' + U_C W' + (U_{LL} A + U_{LC}) L' = 0$$

The expression

$$U_{CC} A + 2U_{CL} + U_{LL} A$$

is negative at $\tau = 0$ because it is the second-order condition of the optimization problem

$$\max_L U(\mathcal{A}(0)L, L)$$

and $L(0)$ is the solution to this problem. Therefore, we have

$$L'(0) = -\frac{U_C W'(0)}{U_{CC}A + 2U_{CL} + U_{LL}A} < 0$$

because $W'(0)$.

7.2 Proof of Proposition 3

Substituting $\omega_t = (1 + \phi) l_t$, the per-period objective can be written as

$$-\frac{1}{2} \left[s_M \eta ((1 + \phi) l_t - \tau)^2 + (1 + \phi) l_t^2 + \frac{\varepsilon}{\theta (\rho + \theta)} \pi_t^2 \right].$$

Ignoring constant terms unaffected by policy this expression can be rewritten as

$$-\frac{1}{2} \left[\Psi_L (l_t - l^o)^2 + \Psi_P \pi_t^2 \right],$$

where

$$\Psi_L = (1 + \phi) (1 + \eta s_M (1 + \phi)),$$

$$\Psi_P = \frac{\varepsilon}{\theta (\rho + \theta)},$$

and

$$l^o = \frac{\eta s_M}{1 + \eta s_M (1 + \phi)} \tau.$$

We can also rewrite compactly the Phillips curve (9) as

$$\rho \pi_t = \kappa (l_t - \tilde{l}) + \dot{\pi}_t$$

where

$$\kappa = \theta (\rho + \theta) s_L (1 + \phi)$$

and

$$\tilde{l} = -\frac{1}{1 + \phi} \frac{s_M}{s_L} \tau.$$

The Hamiltonian for the central bank optimization problem can then be written com-

pactly as

$$-\frac{1}{2} \left[\Psi_L (l_t - l^o)^2 + \Psi_P \pi_t^2 \right] - \kappa (l_t - \tilde{l}) \nu_t + \pi_t \dot{\nu}_t$$

where ν_t denotes the Lagrange multiplier on the Phillips curve. The optimality conditions are

$$-\Psi_L (l_t - l^o) - \nu_t \kappa = 0, \quad (21)$$

$$-\Psi_P \pi_t + \dot{\nu}_t = 0. \quad (22)$$

Given that the initial π_0 is a free variable, we have the initial condition $\nu_0 = 0$ for the Lagrange multiplier ν_t . From (21), the initial value for l_t is then

$$l_0 = l^o.$$

Differentiating (21) we have

$$\Psi_L \dot{l}_t + \kappa \dot{\nu}_t = 0,$$

which, combined with (22), yields

$$-\Psi_P \pi_t + \dot{\nu}_t = 0$$

gives

$$\Psi_L \dot{l}_t + \kappa \Psi_P \pi_t = 0. \quad (23)$$

Differentiating the Phillips curve we have

$$\rho \dot{\pi}_t = \kappa \dot{l}_t + \ddot{\pi}_t$$

and substituting from (23) we obtain a second order ODE in π_t :

$$\rho \dot{\pi}_t = -\kappa^2 \frac{\Psi_P}{\Psi_L} \pi_t + \ddot{\pi}_t.$$

The solution takes the form

$$\pi_t = e^{-rt} \pi_0,$$

where r is the positive solution of the quadratic equation

$$r(r + \rho) = \kappa^2 \frac{\Psi_P}{\Psi_L} = \theta(\rho + \theta) \frac{s_L^2 (1 + \phi) \varepsilon}{1 + \eta s_M (1 + \phi)}. \quad (24)$$

The value of π_0 comes from the Phillips curve at date 0

$$\rho \pi_0 = \kappa (l^o - \tilde{l}) - r \pi_0$$

which gives

$$\pi_0 = \frac{\kappa}{r + \rho} (l^o - \tilde{l}) > 0. \quad (25)$$

The closed form solution for l_t is derived from (23) and the initial condition $l_0 = l^o$ and is

$$l_t - \tilde{l} = e^{-rt} (l^o - \tilde{l}).$$

The solution for output is

$$y_t = l_t + \eta s_M (\omega_t - \tau) = (1 + \eta s_M (1 + \phi)) l_t - \eta s_M \tau,$$

and the impact effect on output is

$$y_0 = (1 + \eta s_M (1 + \phi)) \frac{\eta s_M}{1 + \eta s_M (1 + \phi)} \tau - \eta s_M \tau = 0.$$

Cumulated inflation is

$$\int_0^\infty \pi_t dt = \int_0^\infty e^{-rt} \pi_0 = \frac{1}{r} \frac{\kappa}{r + \rho} (l^o - \tilde{l}) = \frac{\Psi_L}{\kappa \Psi_P} (l^o - \tilde{l}) = \frac{1 + \eta s_M (1 + \phi)}{\varepsilon s_L} (l^o - \tilde{l})$$

where we used 24, 25 and the definitions of Ψ_L, Ψ_P, κ . Substituting

$$l^o - \tilde{l} = \frac{\eta s_M}{1 + \eta s_M (1 + \phi)} \tau + \frac{1}{1 + \phi} \frac{s_M}{s_L} \tau,$$

we get

$$\int_0^\infty \pi_t dt = \frac{1}{\varepsilon s_L^2} \left(\frac{1}{1 + \phi} + \eta \right) s_M \tau.$$

The total effect on the price level is larger than the direct effect $s_M \tau$ iff

$$\frac{1}{1 + \phi} + \eta > \varepsilon s_L^2.$$

7.3 Proof of Proposition (4)

7.3.1 Characterization of optimal path l_t

If we introduce sticky wages the model is characterized by two Phillips curves

$$\rho \pi_t = \Theta (s_L \omega_t + s_M \tau) + \dot{\pi}_t,$$

$$\rho \pi_{Wt} = \Theta_W [(1 + \phi) l_t - \omega_t] + \dot{\pi}_{Wt},$$

where

$$\Theta = \theta (\rho + \theta), \Theta_W = \theta_W (\rho + \theta_W)$$

and θ_W is the frequency of wage adjustment. The dynamics of the real wage are given by

$$\dot{\omega}_t = \pi_{Wt} - \pi_t.$$

A second-order approximation of the welfare function takes the form

$$-\frac{1}{2} \int_0^\infty e^{-\rho t} \left[\eta_{SM} (\omega_t - \tau)^2 + (1 + \phi) l_t^2 + \Psi_P \pi_t^2 + \Psi_W \pi_{Wt}^2 \right] dt,$$

where

$$\Psi_P = \frac{\varepsilon}{\theta(\rho + \theta)}, \Psi_W = \frac{\varepsilon_L}{\theta_W(\rho + \theta_W)},$$

and ε_L is the elasticity of substitution across different varieties of labor services.

The central bank problem is to choose $\{l_t\}$ to maximize the objective function, given $\omega_0 = 0$, subject to the constraints above.

The Hamiltonian can be written as follows

$$\begin{aligned} & -\frac{1}{2} \left[\eta_{SM} (\omega_t - \tau)^2 + (1 + \phi) l_t^2 + \Psi_P \pi_t^2 + \Psi_W \pi_{Wt}^2 \right] + \\ & \quad -\Theta_P (s_L \omega_t + s_M \tau) \nu_t + \pi_t \dot{\nu}_t + \\ & \quad -\Theta_W [(1 + \phi) l_t - \omega_t] \nu_{Wt} + \pi_{Wt} \dot{\nu}_{Wt} + \\ & \quad (\pi_{Wt} - \pi_t) \mu_t - \rho \omega_t \mu_t + \omega_t \dot{\mu}_t, \end{aligned}$$

where ν_t, ν_{Wt}, μ_t are Lagrange multipliers on the law of motions, respectively, of nominal prices, nominal wages, and of the real wage.

The optimality conditions are

$$-(1 + \phi) l_t - \nu_{Wt} \Theta_W (1 + \phi) = 0, \quad (26)$$

$$-\Psi_P \pi_t + \dot{\nu}_t - \mu_t = 0, \quad (27)$$

$$-\Psi_W \pi_{Wt} + \dot{\nu}_{Wt} + \mu_t = 0, \quad (28)$$

$$-\eta_{SM} (\omega_t - \tau) - \Theta_{PSL} \nu_t + \Theta_W \nu_{Wt} - \rho \mu_t + \dot{\mu}_t = 0. \quad (29)$$

We have initial conditions $\nu_0 = \nu_{W0} = 0$ because the initial values π_0 and π_{W0} are free variables. Initial condition $\nu_{W0} = 0$ implies $l_0 = 0$. The optimality condition for μ_0 , given $\omega_0 = 0$ and $\nu_0 = \nu_{W0} = 0$, gives

$$\dot{\mu}_0 = \rho \mu_0 - \eta_{SM} \tau. \quad (30)$$

Differentiating (26) gives

$$\dot{l}_t + \dot{\nu}_{Wt} \Theta_W = 0,$$

and substituting for $\dot{v}_{Wt}$ gives

$$\dot{i}_t + (\Psi_W \pi_{Wt} - \mu_t) \Theta_W = 0. \quad (31)$$

Differentiating (29) gives

$$-\eta s_M \dot{\omega}_t - \Theta_P s_L \dot{v}_t + \Theta_W \dot{v}_{Wt} - \rho \dot{\mu}_t + \ddot{\mu}_t = 0,$$

and substituting for $\dot{\omega}_t, \dot{v}_t, \dot{v}_{Wt}$ gives the second order ODE for μ

$$-\eta s_M (\pi_{Wt} - \pi_t) - \Theta_P s_L (\Psi_P \pi_t + \mu_t) + \Theta_W (\Psi_W \pi_{Wt} - \mu_t) - \rho \dot{\mu}_t + \ddot{\mu}_t = 0$$

or

$$(\Theta_W \Psi_W - \eta s_M) \pi_{Wt} - (s_L \Theta_P \Psi_P - \eta s_M) \pi_t - (s_L \Theta_P + \Theta_W) \mu_t - \rho \dot{\mu}_t + \ddot{\mu}_t = 0.$$

The stable solution of this ODE satisfies

$$\dot{\mu}_t = r_1 \mu_t + \int_t^\infty e^{-r_2(s-t)} ((\Theta_W \Psi_W - \eta s_M) \pi_{Ws} - (s_L \Theta_P \Psi_P - \eta s_M) \pi_s) ds \quad (32)$$

where r_1 and r_2 are the two roots of the quadratic equation

$$r(r - \rho) = s_L \Theta_P + \Theta_W,$$

ordered so that $r_2 > \rho > 0 > r_1$. Evaluating (32) at $t = 0$ and using (30) gives us the initial condition for μ

$$\begin{aligned} \mu_0 &= \frac{1}{\rho - r_1} \left[\eta s_M \tau + \int_0^\infty e^{-r_2 t} [(\Theta_W \Psi_W - \eta s_M) \pi_{Wt} - (s_L \Theta_P \Psi_P - \eta s_M) \pi_t] dt \right]. \\ \mu_t &= e^{r_1 t} \mu_0 + \int_0^\infty H(t, s) [(\Theta_W \Psi_W - \eta s_M) \pi_{Ws} - (s_L \Theta_P \Psi_P - \eta s_M) \pi_s] ds \end{aligned} \quad (33)$$

where

$$H(t, s) = \frac{1}{r_2 - r_1} \left(e^{\min\{r_1(t-s), -r_2(s-t)\}} - e^{r_1 t - r_2 s} \right).$$

Steady state The new steady state is characterized by the terminal conditions for employment and real wages, equal to their flexible-price equilibrium levels:

$$\tilde{\omega} = -\frac{s_M}{s_L} \tau, \quad \tilde{l} = -\frac{1}{1 + \phi} \frac{s_M}{s_L} \tau.$$

Inflation and the Lagrange multiplier μ converge to zero

$$\pi = \pi_W = \mu = 0.$$

However, the Lagrange multipliers on the inflation equations converge to positive values, reflecting the tension due to the distorted steady state, namely, we have

$$\nu_W \Theta_W = -l > 0,$$

$$\Theta_{PSL} \nu = \eta \frac{s_M}{s_L} \tau + \Theta_W \nu_W > 0.$$

It is easy to check that all the equilibrium conditions and optimality conditions are satisfied given the variables and Lagrange multipliers above, with $\dot{\pi} = \dot{\pi}_W = \dot{\mu} = \dot{\nu} = \dot{\nu}_W = 0$.

Solution To solve for an equilibrium requires making a conjecture for l_t that satisfies the terminal conditions $l_0 = 0$ and $\lim_{t \rightarrow \infty} l_t = \tilde{l}$, solving for price and wage inflation, substitute in 33 and checking the optimality condition 31. To compute numerical solutions we use a finite difference method to solve for time derivatives and integrals, which yields a single linear equation in the vector representing the path l_t . The details are in the Jupyter notebook available online.

7.3.2 Proof of the results in the proposition

The initial condition $l_0 = 0$ and terminal condition $\lim_{t \rightarrow \infty} l_t = \tilde{l} < 0$ are proved in the characterization above. The results for output follow directly from the terminal conditions for l_t and ω_t and from $y_t = l_t + \eta s_M (\omega_t - \tau)$.

Combining the optimality conditions (27) and (28) yields

$$\Psi_P \pi_t + \Psi_W \pi_{Wt} = \dot{\nu}_t + \dot{\nu}_{Wt}.$$

Substituting $\pi_{Wt} = \pi_t + \dot{\omega}_t$ we obtain

$$(\Psi_P + \Psi_W) \pi_t = \dot{\nu}_t + \dot{\nu}_{Wt} - \Psi_W \dot{\omega}_t.$$

Given that the paths for $\nu_t, \nu_{Wt}, \omega_t$ are continuous and differentiable and satisfy the terminal conditions $\nu_0 = \nu_{W0} = \omega_0 = 0$ and

$$\lim_{t \rightarrow \infty} \nu_t > 0, \lim_{t \rightarrow \infty} \nu_{Wt} > 0, \lim_{t \rightarrow \infty} \omega_t < 0,$$

we have

$$(\Psi_P + \Psi_W) \int_0^\infty \pi_t dt > 0.$$

7.4 Optimal Response from the Timeless Perspective

Let $V(\pi)$ denote optimum welfare when the tariff τ is present and the central banker has committed to deliver the inflation rate π in the current period. The function V satisfies the Hamilton-Jacobi-Bellman equation

$$\rho V(\pi) = \max_l f(l, \pi; \tau) + V'(\pi) (\rho\pi - \kappa(l - \tilde{l})),$$

where is the linear-quadratic expression

$$f(l, \pi) = -\frac{1}{2} [\Psi_L (l - l^o(\tau))^2 + \Psi_P \pi^2]$$

and κ, Ψ_L, Ψ_P and l^o are defined as in the proof of Proposition (3).

The presence of a distorted flexible price allocation is captured by the fact that $f_l(\tilde{l}, 0) > 0$.

As shown in Proposition (3), the economy eventually reaches a long-run steady state with zero inflation in which

$$V'(0) = \frac{1}{\kappa} f_l(\tilde{l}, 0) > 0.$$

The function V is concave and has a maximum at $\pi = \pi_0 > 0$, which correspond to the optimal inflation rate on impact after the introduction of the tariff, as characterized in Proposition 3. That is, the problem with commitment and no past implies the choice of an initial inflation rate π_0 , that satisfies

$$V'(\pi_0) = 0.$$

Consider now the policy problem in a situation preceding the introduction of tariffs. With Poisson arrival α there is a tariff shock and tariffs go from 0 to τ . The Phillips curve is now

$$\rho\pi = \kappa l + \alpha(\pi' - \pi) + \dot{\pi}$$

because agents are aware of the possible switch to a regime with tariffs and π' denotes the inflation on impact when the tariff is introduced. Also, the current allocation is undistorted so only l appears on the right-hand side.

Denote the value in the pre-tariff regime as $J(\pi)$. The Hamilton-Jacobi-Bellman equa-

tion is

$$\rho J(\pi) = \max_{l, \pi'} g(l, \pi) + J'(\pi) (\rho\pi - \kappa l - \alpha (\pi' - \pi)) + \alpha (V(\pi') - J(\pi)),$$

where

$$g(l, \pi) = -\frac{1}{2} [\Psi_L l^2 + \Psi_P \pi^2].$$

This problem includes in every period: the commitment to satisfy the inflation promise π from the past, the optimal choice of real activity l , and the optimal choice of promised inflation in case the tariff shock were to hit, π' .

The optimal choice of π' yields the condition

$$J'(\pi) = V'(\pi').$$

The remaining optimality conditions are optimality for l

$$g_l(l, \pi) = \kappa J'(\pi),$$

and the envelope condition

$$\rho J'(\pi) = g_\pi(l, \pi) + (\rho + \alpha) J'(\pi) - \alpha J'(\pi) + J''(\pi) (\rho\pi - \kappa l - \alpha (\pi' - \pi)),$$

which boils down to

$$g_\pi(l, \pi) + J''(\pi) (\rho\pi - \kappa l - \alpha (\pi' - \pi)) = 0.$$

If the economy remains in the pre-tariff state long enough, it is easy to show that it will converge to an allocation with zero inflation (from $g_\pi(l, \pi) = 0$), with the pair l, π' characterized by the two conditions

$$0 = \kappa l + \alpha \pi'$$

and

$$g_l(l, 0) = \kappa V'(\pi').$$

As $\alpha \rightarrow 0$ is then easy to show that the optimal solution converges to $l = 0$ and π' that satisfies

$$V'(\pi') = 0.$$

This is the same condition that defines the inflation rate on impact in the commitment model with no past, we have showed that solution in Proposition (3) coincides with the timeless solution if $\alpha \rightarrow 0$.