

A Macroeconomic Framework for Quantifying Systemic Risk

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Abstract

Systemic risk arises when shocks lead to states where a disruption in financial intermediation adversely affects the economy and feeds back into further disrupting financial intermediation. We present a macroeconomic model with a financial intermediary sector subject to an equity capital constraint. The novel aspect of our analysis is that the model produces a stochastic steady state distribution for the economy, in which only some of the states correspond to systemic risk states. The model allows us to examine the transition from “normal” states to systemic risk states. We calibrate our model and use it to match the systemic risk apparent during the 2007/2008 financial crisis. We also use the model to compute the conditional probabilities of arriving at a systemic risk state, such as 2007/2008. Finally, we show how the model can be used to conduct a macroeconomic “stress test” linking a stress scenario to the probability of systemic risk states.

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1 Introduction

It is widely understood that a disruption in financial intermediation, triggered by losses on housing-related investments, has played a central role in the 2007 – 2009 economic crisis. Figure 2 plots the market value of equity for the financial intermediary sector, along with a credit spread, investment, and a land price index. All variables have been normalized to one in 2007Q2. The figure illustrates the close relation between reductions in the value of financial intermediary equity, rising spreads, and falling land prices and aggregate investment.

In the wake of the crisis, understanding *systemic risk*, i.e., the risk that widespread financial constraints in the financial intermediation sector trigger adverse effects for the real economy (see, e.g., Bernanke, 2009; Brunnermeier, Gorton and Krishnamurthy, 2011), has been a priority for both academics and policy-makers. To do so, it is important to not only embed a financial intermediary sector in a macroeconomic setting, but also to study a model in which financial constraints on the intermediary sector only bind in some states (“systemic states”). This is a necessary methodological step in order to study systemic risk because systemwide financial disruptions are rare, and in most cases we are interested in understanding the transition of the economy from non-systemic states into systemic states.

The first part of our paper develops such a model. The model’s equilibrium is a stochastic steady state distribution for the economy, in which systemic states where constraints on the financial sector bind correspond to only some of the possible realizations of the state variables. Moreover, in any given state, agents anticipate that future shocks may lead to constraints tightening, triggering systemic risk. As the economy moves closer to a systemic state, these anticipation effects cause banks to reduce lending and hence investment falls even though capital constraints are not binding. Relative to other papers in the literature (e.g., Bernanke, Gertler, and Gilchrist, 1999, Kiyotaki and Moore, 1997, Gertler and Kiyotaki, 2010), our approach enables us to study the global dynamics of the system, not just the dynamics around a non-stochastic steady state. Our paper belongs to a growing literature studying global dynamics in models with financial frictions (see He and Krishnamurthy 2012, He and Krishnamurthy 2013, Brunnermeier and Sannikov 2014, Adrian and Boyarchenko 2012, Maggiori 2012, DiTella 2017). Our contribution relative to these papers is quantitative: we show that our model (and by extension, this class of models) can successfully match key macroeconomic and asset pricing data. The literature thus far has explored modeling strategies that generate qualitative insights.

The second part of the paper confronts the model with data. The key feature of the model is non-linearity.

When constraints on the intermediary sector are binding or likely to bind in the near future, a negative shock triggers a substantial decline in intermediary equity, asset prices and investment. When constraints on the intermediary sector are slack and unlikely to bind in the near future, the same size negative shock triggers only a small decline in intermediary equity, asset prices and investment. In short, the model generates conditional amplification, where the state variable determining conditionality is the incidence of financial constraints in the intermediary sector. We establish that this non-linearity is present in the data. Based on U.S. data from 1975 to 2016, we compute covariances between growth in the equity capitalization of the financial intermediary sector, Sharpe ratios (i.e. economic risk premia), growth in aggregate investment, and growth in land prices, *conditional* on intermediary “distress” and “non-distress” (defined more precisely below). We choose parameters of our model based on non-distress moments of asset pricing and macroeconomic data. We then simulate the model and compute the model counterpart of the data covariances, again conditioning on whether the intermediary sector is in a distress or non-distress period. The data display a marked asymmetry in these covariances, with high volatilities and covariances in the distress periods. We show that the conditional covariances produced by the model match the magnitude of asymmetry in their data counterparts.

We should note that our model misses quantitatively on other dimensions. To keep the model tractable and analyze global dynamics, the model has only two state variables. One cost of this simplicity is that there is no labor margin in the model, and thus we are unable to address measures such as hours worked. We stress that the key feature of the model is a nonlinear relationship between financial variables and real investment, and that is the dimension on which our model is successful.

In our sample from U.S. data, the only significant financial crisis is the 2007-2009 crisis. We show that our model can replicate data patterns in this crisis. We choose a sequence of underlying shocks to match the evolution of intermediary equity from 2007 to 2009. Given this sequence, we then compute the equilibrium values of the Sharpe ratio, aggregate investment and land prices. The analysis shows that the model’s equity capital constraint drives a quantitatively significant amplification mechanism. That is, the size of the asset price declines produced by the model are much larger than the size of the underlying shocks we consider. In addition, the analysis shows that focusing only on shocks to intermediary equity results in an equilibrium that matches the behavior of aggregate investment, the Sharpe ratio, and land prices. This analysis lends further weight to explanations of the 2007-2009 crisis that emphasize shocks to the financial intermediary sector.

We then apply our model to assessing the likelihood of a systemic crisis. Our model allows us to compute counterfactuals. In early 2007, what is the likelihood of reaching a state where constraints on the financial intermediary sector bind over the next T years? What scenarios make this probability higher? We find that the odds of hitting the crisis states over the next 2 years, based on an initial condition chosen to match credit spreads in 2007Q3, is 16%. When we expand the horizon these probabilities rise to 44% for 5 years. While these numbers are moderate, it should be noted that most financial market indicators in early 2007, such as credit spreads or the VIX (volatility index), were low and did not anticipate the severity of the crisis that followed. That is, without the benefit of hindsight, in both the model and data the probability of the 2007-2009 crisis is not that high. A lesson from our analysis is that it is not possible to construct a model in which spreads are low ex-ante, as in the data, and yet the probability of a crisis is high.

The utility of our structural model is that we can compute these probabilities based on alternative scenarios, as under a stress test. That is, the model helps us understand the type of information that agents did not know ex-ante but which was important in subsequently leading to a crisis. With the benefit of hindsight, it is now widely understood that the financial sector had embedded leverage through off-balance sheet activities, for example, which meant that true leverage was higher than the measured leverage based on balance sheets. In our baseline calibration, financial sector leverage is 3.77. We perform a computation that incorporates shadow banking (structured-investment vehicles and repo financing) onto bank balance sheets, and find that leverage may be as high as 4.10. We then conduct a stress test where we increase true leverage from 3.77 to 4.10, but assume that the agents in the economy think that leverage is 3.77. The latter informational assumption captures the notion that it is only with hindsight that the extent of leveraging of the financial system has become apparent (i.e., consistent with the evidence that credit spreads and VIX were low prior to the crisis). Thus, we suppose that agents' decisions rules, equilibrium prices and asset returns are all based on an aggregate intermediary leverage of 3.77, but that actually shocks impact intermediary balance sheets with a leverage that is 4.10. We then find that the probability of the crisis over the next 2 years rises from 16% to 30%, and for 5 years it rises from 44% to 57.0%. These increases are much more muted if the agents are aware of the higher leverage, and this computation shows how much hidden leverage contributed to the crisis.

Similarly, the model allows us to ask how a stress scenario to capital, similar to the Federal Reserve's stress test, increases the probability of systemic risk. The endogenous feedback of the economy to the stress scenario

is the key economics of our model that cannot be captured in a scenario-type analysis such as the Fed’s stress tests. That is, conditional on a scenario triggering a significant reduction in the equity capital of financial firms, it is likely that the endogenous response of the economy will lead to a further loss on assets and further reduction in equity capital. Additionally, the model allows us to translate the stress test into a probability of systemic risk, which is something that the Fed’s current methodology cannot do. We illustrate through an example how to compute the probability of systemic risk based on a hypothetical stress test.

The papers that are most similar to ours are Mendoza (2010) and Brunnermeier and Sannikov (2012). These papers develop stochastic and non-linear financial frictions models to study financial crises. Mendoza is interested in modeling and calibrating crises, or sudden stops, in emerging markets. From a technical standpoint, Mendoza relies on numerical techniques to solve his model, while we develop an analytically tractable model whose equilibrium behavior can be fully characterized by a system of Ordinary Differential Equations. Our approach is thus complementary to his. Brunnermeier and Sannikov also take the differential equation approach of our paper. Their model illustrates the non-linearities in crises by showing that behavior deep in crises regions is substantially different than that in normal periods and underscores the importance of studying global dynamics and solving non-linear models. In particular, their model delivers a steady state distribution in which the economy can have high occupation time in systemic risk states. The principal difference relative to these paper is that we aim to quantitatively match the non-linearities in the data and use the model to quantify systemic risk. Finally, both Mendoza and Brunnermeier-Sannikov study models with an exogenous interest rate, while the interest rate is endogenous in our model.

The model we employ is closely related to our past work in He and Krishnamurthy (2012) and He and Krishnamurthy (2013). He and Krishnamurthy (2012) develop a model integrating the intermediary sector into a general equilibrium asset pricing model, and its empirical implications are confirmed in He, Kelly, and Manela (2017). In that model, the intermediary sector is modeled based on a moral hazard problem, akin to Holmstrom and Tirole (1997), and optimal contracts between intermediaries and households are allowed.¹ We derive the equilibrium intermediation contracts and asset prices in closed form. He and Krishnamurthy (2013) assume the form of intermediation contracts derived in He and Krishnamurthy (2012), but enrich the model so that

¹Our paper belongs to a larger literature, which has been growing given the recent crisis, on the macro effects of disruptions to financial intermediation. Papers most closely related to our work include Adrian and Shin (2010), Gertler and Kiyotaki (2010), Kiley and Sim (2011), Rampini and Viswanathan (2018), Bigio (2012), Adrian and Boyarchenko (2012), He and Kondor (2012), Maggiori (2012) and Dewachter and Wouters (2014).

it can be realistically calibrated to match asset market phenomena during the mortgage market financial crisis of 2007 to 2009. In the present paper, we also assume the structure of intermediation in reduced form. The main innovation relative to our prior work is that the present model allows for a real investment margin with capital accumulation and lending, and includes a housing price channel whereby losses on housing investments affect intermediary balance sheets. Thus the current paper speaks to not only effects on asset prices but also real effects on economic activity.

The paper is also related to the literature on systemic risk measurement. The majority of this literature motivates and builds statistical measures of systemic risk extracted from asset market data. Papers include Acharya, Pedersen, Philippon, and Richardson (2017), Adrian and Brunnermeier (2016), Billio, Getmansky, Lo, and Pelizzon (2012), and Giglio, Kelly and Pruitt (2016). Our line of inquiry is different from this literature in that we build a macroeconomic model to understand how economic variables relate to systemic risk. Acharya, Pedersen, Philippon, and Richardson (2017) is closest to our paper in this regard, although the model used in that paper is a static model that is not suited to a quantification exercise. It is ultimately important that our model-based approach meets the data-oriented approaches.

The paper is laid out as follows. Section 2 describes the model. Section 3 goes through the steps of how we solve the model. Section 4 presents our choice of parameters for the calibration. Sections 5, 6, and 7 present the results from our model. Figures and an appendix with further details on the model solution are at the end of the paper.

2 Model

Time is continuous and indexed by t . The economy has two types of capital: productive capital K_t and housing capital H . We assume that housing is in fixed supply and normalize $H \equiv 1$. We denote by P_t the price of a unit of housing, and q_t the price of a unit of capital; both will be endogenously determined in equilibrium. The numeraire is the consumption good. There are three types of agents: equity households, debt households, and bankers.

We begin by describing the production technology and the household sector. These elements of the model are a slight variant on a standard real business cycle (RBC) model. We then describe bankers and intermediaries, which are the non-standard elements of the model. In our baseline model, we assume that all of the housing and

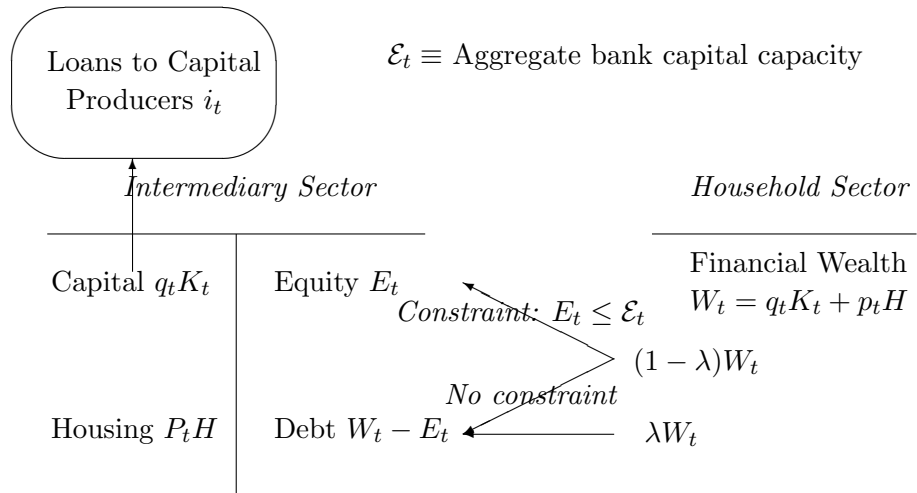


Figure 1: Schematic diagram for the model.

capital stock are owned by intermediaries that are run by bankers. Intermediaries also fund new investments. Households cannot directly own the housing and capital stock. Instead, the intermediaries raise equity and debt from households and use these funds to purchase housing and capital. We also discuss the realistic case where the households directly own a portion of the housing and capital stock, and explain how we handle this case in calibrating the model. The key assumption we make is that intermediaries face an equity capital constraint. Figure 1 presents the main pieces of the model, which we explain in detail over the next sections.

2.1 Production and Households

There is an “ AK ” production technology that generates per-period output Y_t :

$$Y_t = AK_t, \quad (1)$$

where A is a positive constant. The evolution of capital is given by:

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t. \quad (2)$$

The term i_t is the amount of new capital installed at date t . Capital depreciates by δdt , where δ is constant. The last term σdZ_t is a capital quality shock, following Gertler and Kiyotaki (2010) and Brunnermeier and Sannikov (2014). For example, K_t can be thought of as the effective quality/efficiency of capital rather than the amount of capital outstanding. Then the shock can capture variation in the quality of capital, say due to economic obsolescence. The capital quality shock is a simple device to introduce an exogenous source of variation in the value of capital. Note that the price of capital q_t and the price of housing P_t are endogenous. Thus, we will be interested in understanding how the exogenous capital quality shock translates into endogenous shocks to asset prices. Finally, the shock σdZ_t is the only source of uncertainty in the model ($\{Z_t\}$ is a standard Brownian motion, while σ is a positive constant).

We assume adjustment costs so that installing $i_t K_t$ new units of capital costs $\Phi(i_t, K_t)$ units of consumption goods where,

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t.$$

That is, the adjustment costs are assumed to be quadratic in net investment.

There is a unit measure of households. Define a consumption aggregate as in the Cobb-Douglas form,

$$C_t = (c_t^y)^{1-\phi} (c_t^h)^\phi,$$

where c_t^y is consumption of the output good, c_t^h is consumption of housing services, and ϕ is the expenditure share on housing. The household maximizes utility,

$$\mathbb{E} \left[\int_0^\infty e^{-\rho t} \frac{1}{1-\gamma_h} C_t^{1-\gamma_h} dt \right],$$

(i.e. CRRA utility function, with the log case when $\gamma_h = 1$), and the constant ρ is the discount rate.

Given the intratemporal preferences, the optimal consumption rule satisfies:

$$\frac{c_t^y}{c_t^h} = \frac{1-\phi}{\phi} D_t, \tag{3}$$

where D_t is the endogenous rental rate on housing to be determined in equilibrium. In equilibrium, the parameter ϕ affects the relative market value of the housing sector to the goods producing sector.

2.2 Bankers and Equity Capital Constraint

We assume that all productive capital and housing stock can only be held directly by “financial intermediaries.” When we go to the data, we calibrate the intermediaries to include commercial banks, broker/dealers, and hedge funds. In the data, intermediaries hold loan claims on housing and capital. However, and contrary to the model’s assumptions, households also directly own housing and capital. But, as discussed in the calibration section of the paper (and Appendix B), we can extend our model to allow households to directly own claims on housing and capital without much difficulty. Indeed, if the relative shares of housing and capital held by intermediaries and that held directly by households are the same, then the equations of the model are a slightly relabeled version of the ones we are deriving under the assumption that households have no direct holdings of housing and capital. Thus the main complication with allowing for direct holdings is not in the modeling but in the calibration. We discuss the issue further when calibrating the model.

There is a continuum of intermediaries. Each intermediary is run by a single banker who has the know-how to manage investments. That is, we assume that there is a separation between the ownership and control of an intermediary, and the banker makes all investment decisions of the intermediary.

Consider a single intermediary run by a banker. The banker raises funds from households in two forms, equity and debt, and invests these funds in housing and capital. To draw an analogy, think of equity raised as the assets under management of a hedge fund and think of debt financing as money borrowed in the repo market. At time t , a given banker has a type of ϵ_t that parameterizes the equity capital constraint. The banker can issue equity up to ϵ_t at zero issuance cost, but faces infinite marginal issuance cost in issuing equity above ϵ_t . Thus, faced with an ϵ_t -banker, households invest up to ϵ_t to own the equity of that intermediary. The banker can also raise funds in the form of short-term (from t to $t + dt$) debt financing (see Figure 1). The debt issuance is not subject to constraints.²

Denote the realized profit-rate on the intermediary’s assets (i.e. holdings of capital and housing) from t to $t + dt$, net of any debt repayments, as $d\tilde{R}_t$. This is the return on the shareholder’s equity of the intermediary. The return is stochastic and depends on shocks during the interval $[t, t + dt]$. We assume that the equity capital

²Note that we place no restriction on the raising of debt financing by the intermediary. Debt is riskless and is always over-collateralized so that a debt constraint would not make sense in our setting. It is clear in practice that there are times in which debt or margin constraints are also quite important. Our model sheds light on the effects of limited equity capital (e.g., limited bank capital) and its effects on intermediation.

capacity of a given banker evolves based on the banker’s return on equity:

$$\frac{d\epsilon_t}{\epsilon_t} = d\tilde{R}_t. \quad (4)$$

Poor investment returns reduce ϵ_t and thus reduce the maximum amount of equity a given intermediary can raise going forward.

The dynamics of the equity capital capacity ϵ_t resembles the dynamics of “net-worth” of productive agents in many macroeconomic models. For instance, in Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997), the “net worth” of productive agents plays a key role in macroeconomic dynamics. In these papers, net worth fluctuates as a function of the past performance and profits of the productive agent, just as in (4). In He and Krishnamurthy (2012) we consider a setting with bankers who intermediate investments on behalf of households, but are subject to a moral hazard problem. We derive an incentive contract between bankers and households and find that the banker’s net worth plays a similar role as ϵ_t in our current setting. In particular, we find that bankers’ net-worth, which is proportional to the bankers equity capital constraint, evolves with the banker’s past performance just as (4).

In the the afore-mentioned literature, the net worth is equal to the wealth of a class of agents called bankers, who have preferences over consumption, and consume and invest out of their net worth. We depart from this typical modeling. We assume that the bankers are a class of agents who do not derive utility from consumption, but derive utility directly from ϵ_t , similar to the warm-glow preferences of Andreoni (1990). We refer to ϵ_t as the banker’s “reputation,” and assume that a banker makes investment decisions to maximize utility derived from reputation. In particular, we assume that a banker solves a simple mean-variance objective,

$$\mathbb{E}_t\left[\frac{d\epsilon_t}{\epsilon_t}\right] - \frac{\gamma}{2}\text{Var}_t\left[\frac{d\epsilon_t}{\epsilon_t}\right] = \mathbb{E}_t[d\tilde{R}_t] - \frac{\gamma}{2}\text{Var}_t[d\tilde{R}_t]. \quad (5)$$

where $\gamma > 0$ parameterizes the “constant relative risk aversion” of the banker.

To relate this modeling to that of the existing literature, in He and Krishnamurthy (2012) bankers have log utility over consumption. With log preferences, bankers effectively maximize $\mathbb{E}_t[d\tilde{R}_t] - \frac{1}{2}\text{Var}_t[d\tilde{R}_t]$, which is identical to (5) with $\gamma = 1$. Thus one change is that our modeling allows us to have risk aversion different than one. Additionally, in He and Krishnamurthy (2012), given the banker wealth w_t , the endogenous equity capital

constraint implies that the banker can issue equity upto $m w_t$ where m is a constant that parameterizes the moral hazard problem. Thus the equity constraint in our model is equivalent to saying that $\epsilon_t \equiv m w_t$. But in He and Krishnamurthy (2012), the bankers also choose consumption, equal to a fraction of w_t , so that they consume part of the aggregate output of the economy. In our current model, the banker does not consume. Having the bankers not consume is convenient as the economy resembles a representative agent economy in which the households consume all of the economy's output, as in typical macroeconomic models with no frictions. Note that we can imagine rewriting the He and Krishnamurthy (2012) model, taking w_t towards zero and m towards infinity in a manner that keeps $\epsilon_t = m w_t$ the same (in a given state). Doing so shrinks the consumption of bankers toward zero, but does not alter the equity capital constraint, or its dynamics. However, if we were to write such a model, there would be a non-zero probability that a string of good shocks arises which increases the bankers' wealth w_t and hence his/her consumption share in the economy, so that a global solution of the model would have to account for such a possibility that bankers fund all investments out of their own wealth. The advantage of our current modeling is that this counterfactual possibility is ruled out.³

Finally, we assume that a given banker may exit at a constant Poisson intensity of $\eta > 0$. We also allow for entry of new bankers, which we discuss further when explaining the boundary conditions.

To summarize, a given intermediary can raise at most ϵ_t of equity capital. If the intermediary's investments perform poorly, then ϵ_t falls going forward, and the equity capital constraint tightens. The banker in charge of the intermediary chooses the intermediary's investments to maximize the mean excess return on equity of the intermediary minus a penalty for variance multiplied by the "risk aversion" γ . Relative to He and Krishnamurthy (2012, 2013) in which bankers make consumption-portfolio decisions, in this model bankers make portfolio decisions only, but not intertemporal consumption decisions. Hence, in our model the equilibrium Sharpe ratio is determined by the bankers' risk-return trade-off, and the equilibrium interest rate is set purely by the household's Euler equation (since the bankers do not consume goods).

³As bankers do not consume goods, we also need to discuss what happens to any profits made by bankers. We assume that a given intermediary-banker is part of larger intermediary-conglomerate (i.e., to draw an analogy, think of each intermediary as a mutual fund, and the conglomerate as a mutual fund family). In equilibrium, the intermediary-bankers make profits which then flow up to the conglomerate and are paid out as dividends to households, who are the ultimate owners of the conglomerates. It will be clearest to understand the model under the assumption that the households' ownership interest in these conglomerates is not tradable. That is, it is not a part of the household's investable wealth (which we denote as W_t).

2.3 Aggregate Intermediary Capital

Consider now the aggregate intermediary sector. We denote by \mathcal{E}_t the maximum equity capital that can be raised by this sector, which is just the aggregate version of individual banker's capital constraint ϵ_t . The maximum equity capital \mathcal{E}_t will be one of the state variables in our analysis, and its dynamics are given by,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = d\tilde{R}_t - \eta dt + d\psi_t. \quad (6)$$

The first term here reflects that all intermediaries are identical, so that the aggregate stock of intermediary reputation/capital constraint evolves with the return on the intermediaries' equity.⁴ The second-term, $-\eta dt$, captures exit of bankers at the rate η . Exit is important to include; otherwise, $d\mathcal{E}_t/\mathcal{E}_t$ will have a strictly positive drift in equilibrium, which makes the model non-stationary. In other words, without exit, intermediary capital will grow and the capital constraint will not bind. The last term, $d\psi_t \geq 0$ reflects entry. We describe this term fully when describing the boundary conditions for the economy. In particular, we will assume that entry occurs when the aggregate intermediary sector has sufficiently low equity capital.

2.4 Capital Goods Producers

Capital goods producers, owned by households, undertake real investment. As with the capital stock and the housing stock, we assume that capital goods must be sold to the intermediary sector. Thus, q_t , based on the intermediary sector's valuation of capital also drives investment. Given q_t , i_t is chosen to solve,

$$\max_{i_t} q_t i_t K_t - \Phi(i_t, K_t) \Rightarrow i_t = \delta + \frac{q_t - 1}{\kappa}. \quad (7)$$

Recall that $\Phi(i_t, K_t)$ reflects a quadratic cost function on investment net of depreciation.

2.5 Household Members and Portfolio Choices

We make assumptions so that a minimum of λW_t of the household's wealth is invested in the debt of intermediaries. We may think of this as reflecting household demand for liquid transaction balances in banks, although

⁴The model can accommodate heterogeneity in equity capital capacity, say ϵ_t^i where i indexes the intermediary. Because the optimal decision rules of a banker are linear in ϵ_t^i , we can aggregate across bankers and summarize the behavior of the aggregate intermediary sector with the average capital capacity of \mathcal{E}_t .

we do not formally model a transaction demand. The exogenous constant λ is useful to calibrate the leverage of the intermediary sector, but is not crucial for the qualitative properties of the model.

The modeling follows the representative family device introduced in Lucas (1990). Each household is comprised of two members, an “equity household” and a “debt household.” At the beginning of each period, the household consumes, and then splits its W_t between the household members as $1 - \lambda$ fraction to the equity household and λ fraction to the debt household. We assume that the debt household can only invest in intermediary debt paying the interest rate r_t , while the equity household can invest in either debt or equity. Thus households collectively invest in at least λW_t of intermediary debt. The household members individually make financial investment decisions. The investments pay off at period $t + dt$, at which point the members of the household pool their wealth again to give wealth of W_{t+dt} .

Collectively, equity households invest their allocated wealth of $(1 - \lambda) W_t$ into the intermediaries subject to the restriction that, given the stock of banker reputations, they do not purchase more than \mathcal{E}_t of intermediary equity. When $\mathcal{E}_t > W_t(1 - \lambda)$ so that the intermediaries reputation is sufficient to absorb the households’ maximum equity investment, we say that the capital constraint is not binding. But when $\mathcal{E}_t < W_t(1 - \lambda)$ so that the capital constraint is binding, the equity household restricts its equity investment and places any remaining wealth in bonds. In the case where the capital constraint does not bind, it turns out to be optimal – since equity offers a sufficiently high risk-adjusted return – for the equity households to purchase $(1 - \lambda)W_t$ of equity in the intermediary sector. We verify this statement when solving the model. Let,

$$E_t \equiv \min(\mathcal{E}_t, W_t(1 - \lambda))$$

be the amount of equity capital raised by the intermediary sector. The households’ portfolio share in intermediary equity, paying return $d\tilde{R}_t$, is thus, $\frac{E_t}{W_t}$.

The debt household simply invests its portion λW_t into the riskless bond. The household budget constraint implies that the amount of debt purchased by the combined household is equal to $W_t - E_t$.

2.6 Riskless Interest Rate

Denote the interest rate on the short-term bond as r_t . Given our Brownian setting with continuous sample paths, the short-term debt is riskless. Consider at the margin a household that cuts its consumption of the

output good today (the envelope theorem allows us to evaluate all of the consumption reduction in terms of the output good), investing this in the riskless bond to finance more consumption tomorrow.⁵ The marginal utility of consumption of the output good is $e^{-\rho t}(1-\phi)(c_t^y)^{(1-\phi)(1-\gamma_h)-1}(c_t^h)^{\phi(1-\gamma_h)}$, which, in equilibrium, equals $e^{-\rho t}(1-\phi)(c_t^y)^{(1-\phi)(1-\gamma_h)-1}$ as $c_t^h = H \equiv 1$ in equilibrium. Let $\xi \equiv 1 - (1-\phi)(1-\gamma_h)$. The equilibrium interest rate r_t satisfies:

$$r_t = \rho + \xi \mathbb{E}_t \left[\frac{dc_t^y}{c_t^y} \right] - \frac{\xi(\xi+1)}{2} \text{Var}_t \left[\frac{dc_t^y}{c_t^y} \right]. \quad (8)$$

Here, $1/\xi$ can be interpreted as the elasticity of intertemporal substitution (EIS).⁶

2.7 Intermediary Portfolio Choice

Each intermediary chooses how much debt and equity financing to raise from households, subject to the capital constraint, and then makes a portfolio choice decision to own housing and capital. The return on purchasing one unit of housing is,

$$dR_t^h = \frac{dP_t + D_t dt}{P_t}, \quad (9)$$

where P_t is the pricing of housing, and D_t is the equilibrium rental rate given in (3). Let us define the risk premium on housing as $\pi_t^h dt \equiv \mathbb{E}_t[dR_t^h] - r_t dt$. That is, by definition the risk premium is the expected return on housing in excess of the riskless rate. Then,

$$dR_t^h = (\pi_t^h + r_t)dt + \sigma_t^h dZ_t.$$

Here, the volatility of investment in housing is σ_t^h , and from (9), σ_t^h is equal to the volatility of dP_t/P_t .

For capital, if the intermediary buys one unit of capital at price q_t , the capital is worth q_{t+dt} next period

⁵There are some further assumptions underlying the derivation of the Euler equation. If the household reduces consumption today, a portion of the foregone consumption is invested in riskless bonds via the debt member of the household and a portion is invested in equity via the equity member of the household. We assume that equity households are matched with bankers to form an intermediary, and that bankers have a local monopoly with the equity households, such that the households receive their outside option, which is to invest in the riskless bond at rate r_t , on any marginal funds saved with intermediaries. Thus, the Euler equation holds for riskless bonds paying interest rate r_t , and equation (8) has the appealing property that it is a standard expression for determining interest rates. It is straightforward to derive an expression for interest rates under the alternative assumption that the equity household receives an excess return from his investment in the intermediary. In this case, households will have an extra incentive to delay consumption, and the equilibrium interest rate will be lower than that of (8).

⁶Note that with two goods, the intratemporal elasticity of substitution between the goods enters the household's Euler equation. Piazzesi, Schneider and Tuzel (2007) clarify how risk over the composition of consumption in a two-goods setting with housing and a non-durable consumption good enters into the Euler equation.

and pays a dividend equal to Adt . However, the capital depreciates at the rate δ and is subject to the capital quality shocks σdZ_t . Thus, the return on capital investment, accounting for the Ito quadratic variation term, is as follows:

$$dR_t^k = \frac{dq_t + Adt}{q_t} - \delta dt + \sigma dZ_t + \left[\frac{dq_t}{q_t}, \sigma dZ_t \right]. \quad (10)$$

We can also define the risk premium and risk on capital investment suitably so that,

$$dR_t^k = (\pi_t^k + r_t)dt + \sigma_t^k dZ_t.$$

We use the following notation in describing an intermediary's portfolio choice problem. Define α_t^k (α_t^h) as the ratio of an intermediary's investment in capital (housing) to the equity raised by an intermediary. Here, our convention is that when the sum of α s exceed one, the intermediary is taking on leverage (i.e., shorting the bond) from households. For example, if $\alpha_t^k = \alpha_t^h = 1$, then an intermediary that has one dollar of equity capital will be borrowing one dollar of debt (i.e. $1 - \alpha_t^k - \alpha_t^h = -1$) to invest one dollar each in housing and capital. The intermediary's return on equity is,

$$d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1 - \alpha_t^k - \alpha_t^h)r_t dt. \quad (11)$$

From the assumed objective in (6), a banker solves,

$$\max_{\alpha_t^k, \alpha_t^h} \mathbb{E}_t[d\tilde{R}_t] - \frac{\gamma}{2} Var_t[d\tilde{R}_t]. \quad (12)$$

The optimality conditions are,

$$\frac{\pi_t^k}{\sigma_t^k} = \frac{\pi_t^h}{\sigma_t^h} = \gamma \left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h \right). \quad (13)$$

The Sharpe ratio is defined to be the risk premium on an investment divided by its risk (π/σ). Optimality requires that the intermediary choose portfolio shares so that the Sharpe ratio on each asset is equalized. Additionally, the Sharpe ratio is equal to the riskiness of the intermediary portfolio, $\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h$, times the risk aversion of γ . This latter relation is analogous to the CAPM. If the intermediary sector bears more risk in its portfolio, and/or has a higher γ , the equilibrium Sharpe ratio will rise.

2.8 Market Clearing and Equilibrium

1. In the goods market, the total output must go towards consumption and real investment (where we use capital C to indicate aggregate consumption)

$$Y_t = C_t^y + \Phi(i_t, K_t). \quad (14)$$

Note again that bankers do not consume and hence do not enter this market clearing condition. Households receive all of the returns from investment.

2. The housing rental market clears so that

$$C_t^h = H \equiv 1. \quad (15)$$

3. The intermediary sector holds the entire capital and housing stock. The intermediary sector raises total equity financing of $E_t = \min(\mathcal{E}_t, W_t(1 - \lambda))$. Its portfolio share into capital and housing are α_t^k and α_t^h .⁷ The total value of capital in the economy is $q_t K_t$, while the total value of housing is P_t . Thus, market clearing for housing and capital are:

$$\alpha_t^k E_t = K_t q_t \text{ and } \alpha_t^h E_t = P_t. \quad (16)$$

These expressions pin down the equilibrium values of the portfolio shares, α_t^k and α_t^h .

4. The total financial wealth of the household sector is equal to the value of the capital and housing stock:

$$W_t = K_t q_t + P_t.$$

An equilibrium of this economy consists of prices, (P_t, q_t, D_t, r_t) , and decisions, $(c_t^y, c_t^h, i_t, \alpha_t^k, \alpha_t^h)$. Given prices, the decisions are optimally chosen, as described by (3), (7), (8) and (12). Given the decisions, the markets clear at these prices.

⁷Keep in mind that while we use the language “portfolio share” as is common in the portfolio choice literature, the shares are typically larger than one because in equilibrium the intermediaries borrow from households.

3 Model Solution

We derive a Markov equilibrium where the state variables are K_t and \mathcal{E}_t . That is, we look for an equilibrium where all the price and decision variables can be written as functions of these two state variables. We can simplify this further and look for price functions of the form $P_t = p(e_t)K_t$ and $q_t = q(e_t)$ where e_t is the aggregate reputation/capital-capacity of the intermediary sector scaled by the outstanding physical capital stock:

$$e_t \equiv \frac{\mathcal{E}_t}{K_t}.$$

That P_t is linear in K_t is an important property of our model and greatly simplifies the analysis (effectively the analysis reduces to one with a single state variable). To see what assumptions lead to this structure, consider the following. In equilibrium, aggregate consumption of the non-housing good is,

$$C_t^y = Y_t - \Phi(i_t, K_t) = K_t \left[A - i_t + \frac{\kappa}{2} (i_t - \delta)^2 \right],$$

given the adjustment cost specification. From the Cobb-Douglas household preferences, we have derived in equation (3) that $\frac{C_t^y}{C_t^h} = \frac{1-\phi}{\phi} D_t$. Since $C_t^h = H = 1$, the rental rate D_t can be expressed as

$$D_t = \frac{\phi}{1-\phi} \left[A - i_t + \frac{\kappa}{2} (i_t - \delta)^2 \right] K_t.$$

As the price of housing is the discounted present value of the rental rate D_t , and this rental rate is linear in K_t , it follows that P_t is also linear in K_t .

In summary, K_t scales the economy while e_t describes the equity capital constraint of the intermediary sector. The equity capital constraint, e_t , evolves stochastically. The appendix goes through the algebra detailing the solution. We show how to go from the intermediary optimality conditions, (13), to a system of ODEs for $p(e)$ and $q(e)$.

3.1 Capital Constraint, Amplification, and Anticipation Effects

The solution of the model revolves around equation (13) which is the optimality condition for an intermediary. The equation states that the required Sharpe ratio demanded by an intermediary to own housing and capital is

linear in the total risk borne by that intermediary, $(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h)$. If intermediaries hold more risky portfolios, which can happen if α_t^k and α_t^h are high, and/or if σ_t^h and σ_t^k are high, they will require a higher Sharpe ratio to fund a marginal investment. Equilibrium conditions pin down the α s (portfolio shares) and the σ s (volatilities). Consider the α s as they are the more important factor. The variable α_t^k is the ratio of the intermediary's investment in capital to the amount of equity it raises. Market clearing dictates that the numerator of this ratio must be equal to $q_t K_t$ across the entire intermediary sector, while the denominator is the equity capital raised by the intermediary sector, E_t (see (16)).

Let us first consider the economy without an equity constraint. Then, the household sector would invest $(1 - \lambda)W_t$ in equity and λW_t in debt. That is, from the standpoint of households and given the desire for some debt investment on the part of households, the optimal equity/debt mix that households would choose is $(1 - \lambda)W_t$ of equity and λW_t of debt. In this case, α_t^k is equal to $\frac{q_t K_t}{(1 - \lambda)W_t}$. Moreover, because $W_t = K_t(q_t + p_t)$, i.e., the aggregate wealth is approximately proportional to the value of the capital stock, this ratio is near constant. A negative shock that reduces K_t also reduces W_t proportionately with no effects on α_t^k . A similar logic applies to α_t^h . This suggests that the equilibrium Sharpe ratio would be nearly constant if there was no equity capital constraint. While we have not considered the σ s in this argument (they are endogenous objects that depend on the equilibrium price functions), they turn out to be near constant as well without a capital constraint. Thus, without the capital constraint, shocks to K_t just scale the entire economy up or down, with investment, consumption, and asset prices moving in proportion to the capital shock.

Now consider the effect of the capital constraint. If $\mathcal{E}_t < W_t(1 - \lambda)$, then the intermediary sector only raises $E_t = \mathcal{E}_t$ of equity. In this case, α_t^k and α_t^h must be higher than without the capital constraint. In turn, the equilibrium Sharpe ratios demanded by the intermediary sector must rise relative to the case without the capital constraint because the amount of risk borne in equilibrium by intermediaries, $(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h)$, rises. In this state, consider the effect of negative shock. Such a shock reduces W_t , but reduces $E_t = \mathcal{E}_t$ more since the intermediary sector is levered (i.e. in equilibrium the sum of α s are larger than one simply because some households only purchase debt which is supplied by the intermediary sector), and the return on equity is a multiple of the underlying return on the intermediary sector's assets. Thus negative shocks are amplified and cause the equilibrium α s to rise when the capital constraint binds. The higher α s imply a higher Sharpe ratio on capital and housing investment, which in turn implies that the price of capital and housing must be lower in

order to deliver the higher expected returns implied by the higher Sharpe ratios. This means in turn that the capital constraint is tighter, further reducing equity capital. This effect also amplifies negative shocks. There is a further amplification mechanism: since the price of housing and capital are more sensitive to aggregate equity capital when such capital is low, the equilibrium volatility (i.e., σ s) of housing and capital are higher, further increasing Sharpe ratios and feeding through to asset prices and the equity capital constraint. All of these effects reduce investment, because investment depends on q_t which is lower in the presence of the equity capital constraint.

Next consider how the economy can transit from a state where the equity capital constraint does not bind to one where the constraint binds. Even when the constraint is not active, returns realized by the intermediaries affect the capital capacity \mathcal{E}_t , as in equation (4). If there is a series of negative shocks causing low returns, \mathcal{E}_t falls, and as described above, the fall is larger than the fall in W_t . Thus, a series of negative shocks can cause \mathcal{E}_t to fall below $W_t(1 - \lambda)$, leading to a binding capital constraint.

Last consider how the effect of an anticipated constraint may affect equilibrium in states where the constraint is not binding. We can always write the price of housing at time t , $p(e_t)$, recursively as the risk-adjusted expected discounted value of dividends from time t to time $t + \tau$ and the (discounted) value of housing at time $t + \tau$, $p(e_{t+\tau})$. Now we have observed that in the constrained region, asset prices are low. Thus as the economy moves closer to states where the constraint binds and $p(e_{t+\tau})$ is low, the asset price at time t , $p(e_t)$, will fall to anticipate the possibility that the constraint may bind in the future. Through this channel, the equilibrium is affected by \mathcal{E}_t even in cases where it is larger than $W_t(1 - \lambda)$. This is an anticipation effect that emerges from solving for the global dynamics of the model.

The anticipation effect is important in empirically verifying the model. It is likely that widespread financial constraints in the intermediary sector were only present during the 2007-2009 crisis. Our analysis shows that even when such constraints are not binding, if agents anticipate that they are likely to bind in the near future (what we label below as “distress”), then financial friction effects will be present.

3.2 Boundary Conditions

The equilibrium prices $p(e_t)$ and $q(e_t)$ satisfy a system of ODEs based on (13), which are solved numerically subject to two boundary conditions. First, the upper boundary is characterized by the economy with $e \rightarrow \infty$

so that the capital constraint never binds. We derive exact pricing expressions for the economy with no capital constraint and impose these as the upper boundary. Appendix A provides details.

The lower boundary condition describes entry, the term $d\psi_t$ in equation (6). We assume that new bankers enter the market when the Sharpe ratio reaches B , which is an exogenous parameter in the model. This captures the idea that the value of entry is high when the Sharpe ratio of the economy is high. We can also think of entry as reflecting government intervention in the financial sector in a sufficiently adverse state.

Entry alters the evolution of the state variables e and K . In particular, the entry point \underline{e} is endogenous and is a reflecting barrier. We assume that entry increases the aggregate intermediary reputation (and therefore the aggregate intermediary equity capital), but is costly. In order to increase \mathcal{E}_t by one unit, the economy must destroy $\beta > 0$ units of physical capital. Thus, we adjust the capital evolution equation (2) at the entry boundary.

Since the entry point is a reflecting barrier it must be that the price of a unit of capital, $q(e)$, and the price of a unit of housing, $p(e_t)K_t$, have zero derivative with respect to e at the barrier (if not, an investor can make unbounded profits by betting on an almost sure increase/decrease in the asset price). This immediately implies $q'(\underline{e}) = 0$. For the housing price, imposing that $p(e_t)K_t$ has zero derivative implies the lower boundary condition $p'(\underline{e}) = \frac{p(\underline{e})\beta}{1+\underline{e}\beta} > 0$. The derivative is positive, as entry uses up capital, K_t falls at the entry boundary, and hence p must rise in order to keep pK constant. In economics terms, the positive derivative $p'(\underline{e}) > 0$ implies that at the entry point \underline{e} a negative shock lowers the land price. Intuitively, a falling K_t reduces the aggregate housing rental income which is proportional to the aggregate consumption, leading to a lower land price. See Appendix A.5 for the exact argument and derivation.

4 Calibration

The model is a mix between a relatively standard stochastic real business cycle (RBC) model and an intermediation model. For the model parameters that correspond to the RBC aspects of the model, we choose either standard values from the literature or use targets based on simulating our model and computing moments in states where e is high. In particular, we define $e_{distress}$ as the value of e such that 67% of the mass of the steady state distribution of e has $e > e_{distress}$. We generate targets by computing averages from our simulation in the states where $e > e_{distress}$ (“non-distress” states).

RBC parameters: The parameters, ρ (household time preference), δ (depreciation), and κ (adjustment cost) are relatively standard RBC parameters. We use conventional values for these parameters (see Table 1). Note that since our model is set in continuous time, the values in Table 1 correspond to annual values rather than the typical quarterly values one sees in discrete time DSGE parameterizations.

We set $A = 0.133$. This parameter most directly affects the investment to capital ratio. This ratio is 9% in the non-distress states of the simulation, which is typical of values in the literature.

We set σ , which governs the volatility of the only exogenous shock in our model, to 3%. It should be clear that increasing σ increases the volatility of all quantities and asset prices in our model. The choice of 3% leads to a volatility of investment growth in the non-distressed states of the model of 5.23%, a volatility of consumption growth of 1.66%, and volatility of output growth of 3.9%. In the data, the volatility of investment growth from 1973 to 2015 is 6.90% while the volatility of consumption growth is 1.47%. We also present results for a variation with higher σ .

The parameter ϕ governs housing demand and hence the aggregate value of land relative to the value of capital. We earlier discussed how our model simplifies by assuming that only intermediaries own all housing and capital while in reality, households directly hold housing and capital. This issue enters into the choice of ϕ .

In the data, households own land and capital, directly and indirectly through the non-financial and financial sectors, while in the model they own no real assets directly. In our model, intermediaries own land and capital directly, while in the data, the intermediary sector owns financial claims on land and capital, such as mortgage loans and corporate loans. Thus our model would appear to overstate the importance of intermediary balance sheets. However, we show in Appendix B that we can rewrite our model as follows. We introduce a class of sub-households that are housing/capital owners. These households own a share of the stock of housing and capital directly, in contrast to the equity and debt households who own these two assets indirectly through the banking system. We will show shortly that in the data, collectively the relative share between housing and capital assets differs little whether we look inside or outside the banking system. This motivates us to assume that these sub-households own a share, $1 - \chi < 1$, of the stock of both housing and capital. Otherwise, the households behave exactly as do the equity and debt households, pooling resources at the end of every period. Then, the intermediaries own χ of the stock of housing and capital. Thus, this change brings our model closer

to reality. We show in Appendix B that the change does not alter any of the equations of the model. The reason is two-fold: (1) since the households own the equity and debt of the intermediaries and hence the entire wealth of the economy, their total wealth dynamics do not depend on χ ; and (2), the percentage change in the intermediary equity, which is the key driver of dynamics, does not depend on χ .⁸

Table 1: Parameters and Simulated Moments

| Panel A: Intermediation Parameters | | | |
|--|----------------------------|--------|----------------------------------|
| | Parameter | Choice | Target |
| γ | Banker risk aversion | 2 | Mean non-distress Sharpe ratio |
| λ | Debt ratio | 0.75 | Measured Intermediary leverage |
| η | Banker exit rate | 15% | Probability of crisis |
| B | Entry barrier | 6.5 | Highest Sharpe ratio |
| β | Entry cost of intermediary | 2.8 | Land price volatility |
| Panel B: Technology Parameters | | | |
| σ | Capital quality shock | 3% | Consumption volatility |
| δ | Depreciation rate | 10% | Literature |
| κ | Adjustment cost | 3 | Literature |
| A | Productivity | 0.133 | Investment-to-capital ratio |
| Panel C: Other Parameters | | | |
| ρ | Time discount rate | 2% | Literature |
| γ_h | 1/EIS | 0.13 | Interest rate volatility |
| ϕ | Housing share | 0.6 | Measured Housing-to-wealth ratio |
| Panel D: Unconditional Moments from Simulation | | | |
| Probability of Crisis | | 3.3% | |
| Mean Sharpe Ratio | | 52.8% | |
| Mean Interest Rate | | 1.8% | |
| Mean Intermediary Leverage | | 4.0 | |
| Panel E: Conditional Moments in the Non-Distress Periods From Simulation | | | |
| Mean $\left(\frac{\text{Land Value}}{\text{Total Wealth}}\right)$ | | 46.6 % | |
| Volatility(Cons. Growth) | | 1.7% | |
| Volatility(Inv. Growth) | | 5.2% | |
| Volatility(Output Growth) | | 3.9 % | |
| Volatility(Land Price Growth) | | 8.2% | |
| Volatility(Equity Growth) | | 5.7% | |
| Volatility(Interest Rate) | | 0.2% | |

Given these results, there are two possible targets to use to pin down ϕ . From Federal Reserve Flow of

⁸Increasing χ , leaving all other parameters unchanged, results in a steady-state distribution of the state variable, e , that is the same but with a redefined state variable e/χ . That is, the CDF, $F(e)$ is shifted so that $F(e) = F(e/\chi)$ for all e . But, as we pin down the location of the distribution by choosing the banker exit rate η so that $F(e = e_{crisis}) = 3\%$, the value of χ has no effect on our numerical results.

Funds data from 2014, we compute the share of real estate in net worth owned by households, non-profits, and business. The share, computed from B.101, B.103, and B.104, is equal to 45.1%. The other possibility is to compute the share of mortgage loans in the portfolios of the financial sector. From Flow of Funds data from 2014, we compute this ratio across the commercial banking sector and the broker/dealer sector (L.110 and L.129). The share is 45.5%. By coincidence the shares are almost identical, so that we choose to set ϕ to target a share of 45%.⁹ We set $\phi = 0.6$ which delivers an average housing to total wealth share in the non-distress region of the simulation of 46.6%.

Intermediation parameters: The main intermediation parameters are γ and λ . The parameter γ governs the “risk aversion” of the banker. As we vary γ , the Sharpe ratio in the model changes proportionately (see (13)). The choice of $\gamma = 2$ gives an average Sharpe ratio in the non-distress states of the model of 45%, which is in the range of typical consumption-based asset pricing calibrations. As another benchmark, He, Kelly, and Manela (2017) empirically implement an intermediary asset pricing model using the measured equity-to-asset ratio (“capital ratio”) of the primary dealers to proxy for the intermediary stochastic discount factor.¹⁰ Their factor successfully prices returns on a cross-section of assets, particularly the complex derivatives and fixed-income securities that are most traded by sophisticated financial institutions. Their estimates imply a Sharpe ratio (see the caption of Table 17 of the paper) on these intermediated assets of 48%.

The parameter λ is equal to the financial intermediary sector’s debt/assets ratio when the capital constraint does not bind. We base this on data from the year 2007, which is plausibly a time when the capital constraints did not bind. We calibrate our model to data for the commercial bank, broker/dealer, and hedge fund sector. The Federal Reserve’s Flow of Funds for calendar year 2007 reports commercial bank (L.110) assets of \$13.1 tn and debt of \$10.5 tn. For the broker/dealer sector (L.129), assets are \$4.7tn and debt is \$4.1tn. The total assets under management of the hedge fund sector is \$1.98tn as of December 2007 (Barclay Hedge, Hedge Fund Flow Reports 2009). Ang, Gorovyy and van Inwegen (2011) report average hedge fund leverage of 2.1. Then the leverage ratio across these sectors $\left(\frac{\text{assets}}{\text{assets} - \text{debt}}\right)$ is 3.77. Our choice of $\lambda = 0.75$ produces an average leverage ratio in the non-distress states of the simulation of 4.1.

⁹We can also work out a case of our model where the shares differ in the financial sector versus the household sector. This change will alter the dynamics of our model; see Appendix B for details. But we decide not to pursue this case in our calibration because the data does not suggest such a case.

¹⁰Primary dealers serve as counterparties of the Federal Reserve Bank of New York in its implementation of monetary policy. Primary dealers are large and sophisticated financial institutions that operate in virtually the entire universe of capital markets, and include the likes of Goldman Sachs, JP Morgan, and Deutsche Bank.

Crisis parameters: We set η (the bankers' exit rate; see equation (6)) equal to 15% based on considerations of the historical incidence of financial crises. The choice of η affects the mean of the steady state distribution of e . A high value of η implies that the aggregate capital of the banking sector falls faster and thus increases the probability mass for low values of e in the steady state distribution. We choose η so that the probability of the capital constraint binding in the steady state is 3%, which is chosen as a target based on observing three major financial crises in the US over the last 100 years. In later comparative static analyses when we vary parameters we also vary η so as to keep the crisis probability at 3%.

The entry boundary condition (i.e. lower boundary) is determined by B and β . We set $B = 6.5$, so that new entry occurs when the Sharpe ratio is 650%. Based on movements in credit spreads, as measured by Gilchrist and Zakrajsek (2010)'s excess bond premium (see the data description in Section 6.1), we compute that Sharpe ratio of corporate bonds during the 2008 crisis was roughly 15 times the average. Since in our simulation the average Sharpe ratio is around 45%, we set the highest Sharpe ratio to be 650%. Although a high entry threshold is crucial for our model, the exact choice of B is less important because the probability of reaching the entry boundary is almost zero. Our choice is principally motivated by setting B sufficiently high that it does not affect the model's dynamics in the main part of the distribution.

The value of β determines the slope of the housing price function at the entry boundary, and therefore the slope all through the capital constrained region. The volatility of land prices is closely related to the slope of the price function. In the data, the empirical volatility of land price growth from 1975 to 2015 is 11.9%. The choice of $\beta = 2.8$ produces unconditional land price volatility of 11.3%.¹¹

Other parameters: We set $\xi = 0.15$. This choice implies an EIS of 6.66 which is unusually high. Our parameterization is based on attempting to match the empirical volatility of real interest rates (1%). More conventional values of the EIS produces an overly volatile interest rate.

¹¹It is tautological within our model that at the entry barrier the household sector is willing to pay exactly βK units of capital to boost wealth (i.e. P and q) by increasing e . That is, the value of β cannot be independently pinned down from this sort of computation.

5 Results

5.1 Price and Policy Functions: “Anticipation” Effects

Figures 4 and 5 plot the price and policy functions for the baseline parameterization and a variation with a higher σ . Consider the baseline in Figure 4 first. The X-axis in all of the graphs is the scaled intermediary capital $e = \mathcal{E}/K$. The equity capital constraint binds for points to the left of 0.396. The lower-right panel plots the steady-state distribution of the state variable e . The two vertical lines mark the points where the capital constraint binds ($e_{crisis} = 0.396$) and the boundary of the distress region ($e_{distress} = 0.657$). Most of the weight is on the part of the state space where the capital constraint does not bind. That is, a systemic crisis, defined as periods where the capital constraint binds, is rare in the model (calibrated to be probability of 3%).

The top row, third panel is the Sharpe ratio. The Sharpe ratio is about 45% in the unconstrained region and rises rapidly upon entering the constrained region. The interest rate (second row, left panel) also falls sharply when the economy enters the constrained region. Both effects reflect the endogenous increase in “risk aversion,” operating through the binding constraint rather than preferences, of the intermediary sector during a systemic crisis.

The first two panels on the top row are $p(e)$ (housing price divided by capital stock) and $q(e)$. Both price functions are increasing in equity capital as one would expect. It is worth noting that going from right-to-left, prices fall before entering the capital constrained region. This occurs through *anticipation effects*. As the economy moves closer to the constraint, the likelihood of falling into the constrained region rises and this affects asset prices immediately. Moreover, note that if the model had no capital constraint, these price functions $p(e)$ and $q(e)$ would be flat lines. The crisis-states, even though unlikely, affect equilibrium across the entire state space, resembling a result from the rare-disasters literature (Rietz, 1988; Barro, 2006). Relative to this work, crises in our model are endogenous outcomes.

Comparing the first two panels for $p(e)$ and $q(e)$, the main difference is that the range of variation for q is considerably smaller than that for p . This is because housing is in fixed supply while physical capital is subject to adjustment costs. With the $\kappa = 3$ parameterization, the adjustment costs are sufficiently small that capital prices do not vary much. It may be possible to arrive at higher volatility in q if we consider higher adjustment costs or flow adjustment costs as in Gertler and Kiyotaki (2010). As noted earlier, q will also vary more if we

allowed for shocks to A instead of directly shocking K_t .¹² The graph illustrates that the aggregate asset price volatility in the economy is substantially driven by housing volatility. The middle and right panel of the second row are for return volatility of q and p . Housing volatility is much higher than q volatility. Note also that the actual price of housing is equal to p times K , and since K is also volatile, housing prices are more volatile than just p .

The first panel in the bottom row graphs the investment policy function. Since investment is driven by $q(e)$, investment also falls before the intermediary sector is constrained. The second panels in the bottom row graphs the consumption policy function. Investment-to-capital falls as q falls. The resource constraint implies that $C/K + \Phi(I, K)/K = A$. Thus, consumption-to-capital rises as the constraint becomes tighter. Note that aggregate consumption depends on this policy function and the dynamics of capital. In the constrained region, capital falls so that while C/K rises, K falls, and the net effect on aggregate consumption depends on parameters. For our baseline parameters, consumption growth in the non-distress states averages -0.02% while it is -0.84% in the distress states.

Figure 5 plots the baseline plus a variation with higher sigma ($\sigma = 4\%$). The results are intuitive. With higher exogenous volatility, Sharpe ratios, return volatility and risk premia are higher. Thus asset prices and investment are lower.

5.2 Model Nonlinearity and Impulse Response Function

An important feature of the model, apparent in the figures, is its nonlinearity. A reduction to intermediary equity, conditional on a low current value of intermediary equity, has a larger effect on the economy than the same size shock, conditional on a high value of intermediary equity. Figure 6 illustrates this feature. We study the effect of -1% shock in σdZ_t , so that the fundamental shock leads capital to fall exogenously by 1% . We consider the effect of this shock in a “crisis” state (at the boundary $e = e_{crisis}$) and a “high” state ($e = 15$) which is a very good state, even above the 99% quantile of the steady-state distribution of e . We trace out the effect on investment (first panel), the Sharpe ratio (second panel), and the price of land (third panel).

¹²In investigating the model, we have also found that increasing the intertemporal elasticity of substitution (IES) for the household increases the range of variation of q . This appears to be through an effect on the interest rate. In the current calibration, interest rates fall dramatically in the constrained region, which through a discount rate effect supports the value of q . Dampening this effect by increasing the IES increase the range of variation of q . This observation also suggests to us that introducing nominal frictions that bound the interest rate from falling below zero will increase the range of variation of q .

Because the impact of a shock depends on future shocks in a nonlinear model, and our stochastic economy is always subject to shocks, we adopt the following procedure to calculate impulse response functions. First, we compute the benchmark path of these variables without any shocks, but still subject to the endogenous drift of the state variable in our model. In other words, we calculate the benchmark path for the realizations of $dZ_{t+s} = 0$ for $s \geq 0$. Second, we compute the “shocked” path of these variables given this initial shock $\sigma dZ_t = -1\%$, but setting future realizations of shocks to be zero, i.e. $dZ_{t+s} = 0$ for $s > 0$. We then calculate and plot the (log) difference between the path with the shock and the benchmark path without any shocks. This computation is meant to mimic a deviation-from-steady-state computation that is typically plotted in impulse response functions in linear-non-stochastic models. Therefore, the effect illustrated in Figure 6 should be thought of as the marginal effect of the shock on the *mean path* for the variables plotted.¹³

As e goes to infinity, the economy approaches an RBC model. For such a model, the -1% shock would lower investment by 1%, have no effect on the Sharpe ratio, and lower land prices by 1%. The dotted line (“high”) illustrates an impulse response that resembles the RBC benchmark. The solid line (“crisis”) illustrates an impulse response from an initial condition of $e = e_{crisis}$. In this case, investment falls by 2.5%, the Sharpe ratio rises by 0.4, and the land price falls by 6%. There is clear amplification of the shock. Moreover, the effects die out over time. The panels illustrate the nonlinear response of the economy to shocks, depending on how close e is to e_{crisis} .

6 Matching Nonlinearity in Data

Guided by the nonlinearity present in the model, we first ask if such nonlinearity is present in historical data, and second, we ask how well our model can quantitatively match the empirically observed nonlinearity.

¹³Another reasonable way to calculate impulse response functions in our stochastic nonlinear models is to calculate the expected impact of the initial shock $\sigma dZ_t = -1\%$ on the variable at $t + s$ by integrating over all possible future paths. Here, we only focus on the mean path by shutting future shocks to zero. Note that traditional linear models are free from this issue, as the impulse response functions in linear models are independent of future shocks. For more on the difference between impulse responses in linear models with a non-stochastic steady state and those non-linear models with a stochastic steady state, see Koop, Pesaran, and Potter (1996) and a recent contribution by Borovička, Hansen, Hendricks, and Scheinkman (2011).

6.1 Data

We compute covariances in growth rates of intermediary equity, investment, consumption, the price of land, as well as the level of a credit risk spread, using quarterly data from 1975Q1 to 2015Q4. We sample the data quarterly but compute annual log changes in the series. We focus on annual growth rates because there are slow adjustment mechanisms in practice (e.g., flow adjustment costs to investment) that our model abstracts from. We thus sample at a frequency where these adjustment mechanisms play out fully. The intermediary equity measure is the sum across the commercial bank and broker/dealer sectors (SIC codes 6000-6299) of their stock price times the number of shares from the CRSP database. The consumption and investment data are from NIPA. Consumption is non-housing services and nondurable goods. Investment is business investment in software, equipment, structures, and residential investment. Land price data is from the Lincoln Institute (<http://www.lincolnst.edu/subcenters/land-values/price-and-quantity.asp>), where we use *LAND-PI* series based on Case-Shiller-Weiss. These measures are expressed in per-capita terms and adjusted for inflation using the GDP deflator. The credit risk spread is drawn from Gilchrist and Zakrajsek (2012). There is a large literature showing that credit spreads (e.g., the commercial paper to Treasury bill spread) are a leading indicator for economic activity (e.g., Philippon (2010)). Credit spreads have two components: expected default and an economic risk premium that lenders charge for bearing default risk. In an important recent paper, Gilchrist and Zakrajsek (2012) show that the spread’s forecasting power stems primarily from variation in the risk premium component (the “excess bond premium”). The authors also show that the risk premium is closely related to measures of financial intermediary health. Our model has predictions for the link between intermediary equity and the risk premium demanded by intermediaries, while being silent on default.¹⁴ We convert the Gilchrist and Zakrajsek’s risk premium into a Sharpe ratio by scaling by the risk of bond returns, as the Sharpe ratio is the natural measure of risk-bearing capacity in our model.¹⁵ The Sharpe ratio is labeled EB in the table.

¹⁴There is no default in the equilibrium of the model. Of course, one can easily price a defaultable corporate bond given the intermediary pricing kernel, where default is chosen to match observables such as the correlation with output. We do not view having default in the equilibrium of the model as a drawback of our approach.

¹⁵ Suppose that the yield on a corporate bond is y^c , the yield on the riskless bond is y^r and the default rate on the bond is $E[d]$. The expected return on the bond is $y^c - y^r - E[d]$, which is the counterpart to the excess bond premium of Gilchrist and Zakrajsek (2012). To compute the Sharpe ratio on this investment, we need to divide by the riskiness of the corporate bond investment. Plausibly, the risk is proportional to $E[d]$ (for example, if default is modeled as the realization of Poisson process, this approximation is exact). Thus the ratio $\frac{y^c - y^r - E[d]}{E[d]}$ is proportional to the Sharpe ratio on the investment, and this is how we construct the Sharpe ratio.

6.2 Conditional Moments

Table 2 presents covariances depending on whether or not the intermediary sector is in a “distress” period. Table 3 lists the distress classification. Ideally, we would like to split the data based on observations of e_t , which measures the equity capacity of the intermediary sector. However, e_t is not directly observable in data. Instead, the model suggests that there is a one-to-one link between the Sharpe ratio and e_t . Thus, we consider as distress periods the highest one-third of realizations of the EB Sharpe ratio, but requiring that the distress or non-distress periods span at least two contiguous quarters. In choosing the distress/non-distress classification, we face the tradeoff that if we raise cutoff to define distress (say, worst 10% of observations as opposed to 33%), then the data is more reflective of the crisis effects suggested by the model but we have too little data on which to compute meaningful statistics. After experimenting with the data, we have settled on the one-third/two-thirds split.

The distress periods roughly correspond to NBER recession dates, with some exceptions. We classify distress periods in 1985Q4-1987Q3, 1988Q4-1990Q1, and 1992Q3-1993Q2. The NBER recession over this period is from 1990 to 1991. The S&L crisis and falling real estate prices in the late 80s put pressure on banks which appear to result in a high EB and hence leads us to classify these other periods as distress. We also classify further distress periods in 2010 and 2011-2013, corresponding to the intensifying of the European financial crisis. The NBER recession ends in June 2009.

Table 2 shows that there is an asymmetry in the covariances across the distress and non-distress periods, qualitatively consistent with the model. There is almost no relation between equity and the other variables in the non-distress periods, while the variables are closely related in the distress periods. Volatilities are much higher in the distress periods than the non-distress periods. Table 2 also presents results for alternative classifications of the distress periods. All of the classifications display the pattern of asymmetry so that our results are not driven by an arbitrary classification of distress. The only column that looks different is the last one where we drop the recent crisis. For this case, most of the covariances in the distress period drop substantially, as one would expect. Nevertheless, the asymmetry across distress and non-distress periods is still evident.

Table 2: Covariances in Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB), using data from 1975Q1 to 2015Q4. Suppose quarter t is classified as a distress quarter. We compute growth rates as annual changes in log value from $t - 2$ to $t + 2$. The Sharpe ratio is the value at t . The first column is using the distress classification of Table 3. The second uses NBER recession dates, from Table 3. The third uses these recession dates, plus two adjoining quarters at the start and end of the recession. The last is based on the expanded NBER recession dates but drops the period after 2007Q2.

| | EB | NBER Recession | NBER+/-2Qs | NBER+, Drop Crisis |
|-------------------------------|-------|----------------|------------|--------------------|
| Panel A: Distress Periods | | | | |
| vol(Eq) | 25.73 | 28.72 | 27.14 | 22.11 |
| vol(I) | 7.71 | 7.24 | 6.93 | 4.70 |
| vol(C) | 1.72 | 1.79 | 1.83 | 1.37 |
| vol(PL) | 15.44 | 15.11 | 10.51 | 8.10 |
| vol(EB) | 65.66 | 107.16 | 85.04 | 36.23 |
| cov(Eq, I) | 1.02 | 1.10 | 0.60 | 0.20 |
| cov(Eq, C) | 0.20 | 0.10 | 0.07 | -0.04 |
| cov(Eq, PL) | 2.38 | 3.12 | 1.88 | 0.11 |
| cov(Eq, EB) | -8.50 | -19.03 | -11.32 | 1.66 |
| Panel B: Non-distress Periods | | | | |
| vol(Eq) | 20.54 | 19.42 | 18.90 | 19.15 |
| vol(I) | 5.79 | 5.92 | 4.75 | 4.99 |
| vol(C) | 1.24 | 1.29 | 1.09 | 0.91 |
| vol(PL) | 9.45 | 10.51 | 10.26 | 8.63 |
| vol(EB) | 16.56 | 29.95 | 29.33 | 30.95 |
| cov(Eq, I) | -0.07 | -0.06 | -0.18 | -0.14 |
| cov(Eq, C) | -0.01 | 0.01 | 0.00 | -0.01 |
| cov(Eq, PL) | -0.43 | -0.23 | -0.31 | -0.59 |
| cov(Eq, EB) | 0.60 | 0.19 | 0.02 | 0.54 |

6.3 Simulated Conditional Moments

We compare the results from simulating the model to the data presented in Table 2. When simulating the model we follow the one-third/two-thirds procedure as when computing moments in historical data and label distress as the worst one-third of the sample realizations. Importantly therefore our definitions are consistent and comparable across both model and data. From Figure 4, points on the x -axis where $e < e_{distress} = 0.657$ are classified as distressed.

We simulate the model, quarterly, for 2000 years. To minimize the impact of the initial condition, we first simulate the economy for 2000 years, and then record data from the economy for the next 2000 years. We then

Table 3: Distress Classification

| Distress Periods | NBER Recessions |
|------------------|-----------------|
| 1975Q1 - 1975Q4 | 11/73 - 3/75 |
| 1982Q3 - 1982Q4 | 7/81 - 11/82 |
| 1986Q1 - 1987Q1 | |
| 1989Q1 - 1990Q1 | |
| | 7/90 - 3/91 |
| 1992Q3 - 1993Q1 | |
| 2000Q1 - 2003Q1 | 3/01 - 11/01 |
| 2007Q4 - 2009Q3 | 12/07 - 6/09 |
| 2010Q2 - 2010Q4 | |
| 2011Q3 - 2013Q1 | |

compute sample moments and the probability of distress region accordingly. We run the simulation 5000 times and report the sample average.

Table 4 provides numbers from the data and the simulation. When reading these numbers it is important to keep in mind that our calibration predominantly targets on non-distress periods, as opposed to the asymmetry across distress and non-distress periods. Thus one criterion for the success of our work is whether the non-linearity imposed by the theoretical structural of the model can match the asymmetry in the data.

In the data, the covariance between equity and investment is 1.02% in distress and -0.07% in non-distress. In the simulation, these numbers are 1.12% and 0.30%. The model also comes close to matching the asymmetry in land price volatility and covariance with land prices and equity. In the data, the volatility numbers are 15.44% and 9.45%; while the corresponding land price volatilities from the model are 15.82% and 8.23%. The land-equity covariances in the data are 2.38% and -0.43% ; while in the model, they are 3.00% and 0.47%. The model also generates substantial asymmetry in the Sharpe ratio, although not as high as in the data.

The model misses substantially in a few dimensions. Most importantly, while not immediately apparent from the table, the volatility of output (consumption plus investment) is low at 3.9%. Over a short period of time, output is equal to AK and thus the volatility of output is driven by the exogenous volatility in K , which is 3%. This creates the following problem: negative shocks reduce investment through the financial intermediation channel, but for given output, consumption has to rise. We can see this effect when comparing the covariance of equity and consumption to the covariance of equity and investment. The investment covariance is positive, while the consumption covariance is negative. The second problem created is that investment volatility is

uniformly too low while consumption volatility is nearer the data. It is clear that more work needs to be done in order to better match both investment and consumption dynamics. For example, introducing endogenous labor supply can lead the endogenous output volatility to differ significantly from σ .

6.4 Comparative Statics

The last five columns in the table consider variations where in different ways we change the volatility of the economy. In each of these variations, we vary η to ensure that the probability of being constrained remains at 3%.

The variation with σ raised to 4% from 3% increases the volatility of most variables considerably. The increase is larger in the distress period than the non-distress period which should be expected given the non-linearity in the model. The the volatility of investment rises substantially more than the volatility of consumption. This comparison makes clear that the main effect of the constraint we have introduced is on investment. Increasing σ raises the effects of the non-linear constraint and particularly affects investment.

The variation with $\phi = 0$ is interesting in that it reveals the workings of the model. When $\phi = 0$, land drops out of the model. From Figure 4 note that land price volatility rises in the constrained region while the volatility of q remains roughly constant. Thus, when land is removed from the economy, the volatility of intermediary equity in the distress region falls from 21.68% to 9.92%. The intermediary pricing kernel is far less volatile which in turn greatly reduces the non-linearity in the model. Recall in the baseline model with land, reduced demand for assets in the constrained region causes land prices to fall sharply as land is in fixed supply, while physical capital is subject to adjustment costs so that reduced demand both reduces quantity and price. This distinction is what drives the high volatility of land relative to physical capital in our baseline. Eliminating land thereby dramatically reduces the non-linear effects produced by the model.

The next column presents a variation where we change γ to 2.3. We are effectively increasing the risk aversion of bankers in this case which increases the Sharpe ratio and asset price volatility. The absolute effect is larger in the constrained region because increasing γ scales up the risk premium. Indeed, we see that this variation matches the asymmetry in the Sharpe ratio quite well, although it overstates the covariances between equity and other variables.

The last column in the table considers a variation with a lower λ . Reducing the leverage of intermediaries in

the unconstrained region reduces the amount of risk borne by intermediary equity and thus reduces risk premia and the intermediation effects of our model. The effects are more pronounced in the distress region than the non-distress region. Thus, this variation is qualitatively similar to the effect of reducing γ .

Finally, we present a table in the appendix with results from the model and data, using a distress classification of 10% of observations and 20% of observations. Both data and model display asymmetry across distress and non-distress periods, consistent with our main results. If we focus on the covariance between equity and investment, the data show an increasing covariance in the distress region as we focus on successively worse classifications of the distress episode. The model is able to match this increasing covariance. Overall, it is evident that our model/data matching exercise is not driven by the choice of the distress cutoff.

7 Systemic Risk

We now turn our attention to quantifying systemic risk, which we define as the probability that the economy can transit into a state where the capital constraint binds. Figure 7 plots the stock market value of intermediary equity and the EB-Sharpe ratio from 2006 to 2010. The financial crisis is evident as the spike in EB and fall in equity, reaching a climax in the fall of 2008. The equity and EB variables show some sign of stress beginning in early 2007, but the movements are small in this period compared to the crisis. That is, the financial crisis is a non-linear phenomenon, which is exactly what our model attempts to capture.

We are interested in using our model to say something about the likelihood of the impending crisis in mid-2007, at a date before the financial crisis. Figure 7 hints at the challenges presented by this exercise. Both the EB and the equity variables in early 2007 show little sign of the crisis that followed (the VIX, which is not plotted, follows a similar pattern as the EB). A purely statistical exercise which uses this data to forecast the crisis will have little chance of predicting the crisis. The first part of the section formalizes this observation within our model, showing that even with our non-linear model, the likelihood of the crisis given 2007 initial conditions is low. This is a negative result, but should not be surprising: the initial conditions are chosen to be consistent with the early 2007 EB measure, and given that this measure shows little stress in the data, our rational expectations model produces a low likelihood of the crisis. A lesson from our analysis is that it is not possible to construct a model in which spreads are low ex-ante, as in the data, and yet the probability of a crisis is high.

The second part of the section shows how our model can be used to understand systemic risk. The utility of our structural model is that we can compute these probabilities based on alternative scenarios, as under a stress test. That is, the model helps us understand the type of information that agents did not know ex-ante but was important in subsequently leading to a crisis. The model is also useful in identifying the types of stress scenarios that most significantly threaten financial stability.

7.1 Simulation of 2007-2009 Crisis

We first use our model to attempt to replicate the crisis of 2007-2009, as reflected in Figure 2. To do so, we need to pick an initial condition in terms of e and a sequence of shocks that can reflect events in 2007-2009.

The choice of initial condition is important for the exercise that follows. The economy transits into the distress region at some point between 2007Q3 and 2007Q4. Since the 33% threshold in our model simulation is a value of $e_{distress} = 0.657$, we assume that the economy in 2007Q3 is at $e = 0.657$. Note we are using information from the EB to pin down the initial condition. In principle more data (VIX, equity, etc.) can also be used to set the initial condition. Because all of these variables suggest little stress in early 2007, our initial condition is unlikely to change based on these additional datapoints.

Starting from the $e = 0.657$ state, we impose a sequence of exogenous quarterly shocks, $\sigma(Z_{t+0.25} - Z_t)$ to the capital dynamics equation (2). The shocks are chosen so that the model implied intermediary equity matches the data counterpart during 2007Q3 to 2009Q4 in Figure 2. These shocks are in units of percentage change in capital. From 2007Q3 to 2009Q4, the shocks are $(-5.0\%, -1.5\%, -1.5\%, -0.9\%, -2.2\%, -2.6\%, -2.5\%, -0.7\%, -0.7\%)$ which totals about -16.3% (geometric sum). We compute the values of all endogenous variables, intermediary equity, land prices, investment, and the Sharpe ratio, after each shock.

By matching the intermediary equity data, our model focuses on shocks in the world that most directly affect the intermediary sector. Note also that a given shock, say the first one with -5.0% magnitude, reflects not only losses by banks for 2007Q3, but also losses anticipated by investors over the future; the latter is impounded in the current market value of equity. That is, as the world is evolving over 2007Q3 to 2009Q4, investors receive information that cause them to anticipate losses to the intermediary sector which then immediately reduces the market value of the intermediary sector. We pick the σdZ shock in a given quarter to match the reduction in the market value of the intermediary equity over that quarter.

Figure 3 plots the values of the endogenous variables from the model simulation at each quarter (all variables are normalized to one in 2007Q2). The exogenous shocks total 16.3% while intermediary equity and land prices fall by 63% and 47%, respectively, in the trough of the model. Thus, there is clearly an amplification of shocks. The equity capital constraint comes to bind after the first four shocks, totaling -8.9% , and corresponding to 2008Q3. The Sharpe ratio rises dramatically after 2008Q3. Also, note that from that point on, the shocks are smaller but the responses of endogenous variables are larger, reflecting the non-linearity of the model.

Figure 3 should be compared to Figure 2. It is apparent that the model can replicate important features of the crisis with a sequence of shocks that plausibly reflects current and anticipated losses on bank mortgage investments. In addition, the analysis shows that focusing only on shocks to intermediary equity results in an equilibrium that matches the behavior of aggregate investment, the Sharpe ratio, and land prices. This result suggests that an intermediary-capital-based mechanism, as in our model, can be a successful explanation for the macroeconomic patterns from 2007 to 2009.

7.2 Probability of Systemic Crisis and a Leverage Counterfactual

We use our model to compute the probability of falling into a systemic crisis. Consider the sequence of shocks as in Figure 3 that leads the capital constraint to bind in 2008Q1. We ask, what is the probability of the capital constraint binding any time over the next T years, given the initial condition of being on the distress boundary ($e_{distress} = 0.657$). These probabilities are 3% for 1 year, 16% for 2 years, and 44% for 5 years. This result confirms the intuition offered earlier. Since our initial condition is based on financial market measures that showed little sign of stress prior to the summer of 2007, our model does not flash red that a crisis will follow. That is, without the benefit of hindsight, in both the model and data the probability of the 2007-2009 crisis is not high.

As many observers have pointed out, it is clear with the benefit of hindsight that there was a great deal of leverage “hidden” in the system. For example, many were unaware of the size of the structured investment vehicles (SIVs) that commercial banks had sponsored and the extent to which these assets were a call on bank’s liquidity and capital. As Acharya, Schnabl and Suarez (2013) have documented, much of the assets in SIVs came back onto bank balance sheets causing their leverage to rise. Likewise, hedge funds and broker/dealers were carrying high leverage in the repo market, but this was not apparent to observers given the opacity of

the repo market. As Gorton and Metrick (2012) have argued, this high leverage was a significant factor in the crisis. However, in early 2007, this high leverage was “hidden” in financial markets, and is perhaps one reason why financial market indicators did not signal a crisis.

We consider a counterfactual to see how accounting for the hidden leverage in the system may change the probability of the crisis. The experiment we lay out should be thought of as stress test that asks how much higher the crisis probability would be if a regulator had known that the leverage was higher than widely understood. In our baseline calibration, financial sector leverage is 3.77. Recall that the return on equity produced by an intermediary is,

$$d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1 - \alpha_t^k - \alpha_t^h)r_t dt.$$

where,

$$\alpha_t^k = \frac{1}{1 - \lambda} \frac{q_t K_t}{W_t} \quad \text{and,} \quad \alpha_t^h = \frac{1}{1 - \lambda} \frac{P_t}{W_t},$$

when the capital constraint does not bind. The leverage parameter, λ , enters by affecting the α s and thus the exposure of intermediary equity to returns on housing and capital.

We recompute financial sector leverage in the data in 2007 accounting for two other types of leverage. We assume that the financial sector carries an additional \$1.2 trillion of assets with zero capital. This is based on the amount of SIVs pre-crisis and the results from Acharya, Schnabl and Suarez (2013) that these structures succeeded in evading all capital requirements. We also assume that the financial sector carries \$1 trillion of repo assets at a 2% haircut (capital requirement). These numbers are based on data on the repo market from Krishnamurthy, Nagel and Orlov (2013). These computations result in an increase in financial sector leverage from 3.77 to 4.10.¹⁶ Translating this into our calibration, we replace $\lambda = 0.75$ with $\hat{\lambda} = 0.772$ in the expressions above. This increases the leverage of the intermediaries, causing the α s to rise.

We assume, however, that this increase in leverage is “hidden,” in the sense that agents continue to make decisions as if $\lambda = 0.75$ so that the equilibrium decision rules, prices, and returns correspond to the baseline calibration. But when returns are realized, the hidden leverage leads to a larger-than-expected effect (i.e. $\hat{\lambda} = 0.772$) on the return to intermediary equity. Thus our experiment is trying to hold fixed agents decisions

¹⁶Here are the details. In our baseline, we choose $\lambda = 0.75$ based on total intermediary assets of \$23.9tn and debt of \$17.6tn. In the variation which consolidates SIVs and repo, assets rise to \$25.1tn and debt rises to \$19.8tn.

rules and equilibrium prices and returns, and only allowing these returns to have a levered effect on the dynamics of intermediary equity. With the higher leverage, one can expect that shocks will be amplified and thus the crisis state will be more likely. We compute exactly how much more likely by simulating the model. The appendix describes the simulation procedure in detail.

The probability of the crisis over the next year rises from 3% to 10%, while for two years it rises from 16% to 30%, and for five years it rises from 44% to 57%. To better understand the role of “hidden leverage,” we consider the benchmark case in which our agents do understand that the financial sector is with a higher $\lambda = 0.772$. In this benchmark case, rational bankers scale back their positions, which almost completely undoes the prospective risk: the implied crisis probability only increases slightly to 4% (from 3%) for one year, 19% (from 16%) for two years, and 46% (from 44%) for five years. This computation quantifies how important a factor hidden leverage was in contributing the crisis. The exercise also shows how stressing the financial system, because of the non-linearity in the model, can have a large impact on crisis probabilities.

7.3 Stress Tests

The hidden leverage exercise is an example of a stress test. The fact that financial market indicators offered a poor signal of the crisis has led regulators to emphasize stress testing as a tool to uncover vulnerabilities in the financial system. Current regulatory stress tests are exercises which measure the impact of a stress event on the balance sheet of a bank. The typical stress test maps a scenario into a loss to equity holders. For example, a stress-test may assess how much equity capital a given bank will lose in the event that loss rates on mortgage loans double. Our model offers two ways in which to improve the stress test methodology. First, we have learned from the 2007/08 crisis that banks with shrinking capital base will be more reluctant to extend new housing related loans, which may further exacerbate the losses on existing mortgage assets. This further hurts bank equity capital, and so on (this point was identified by Brunnermeier, Gorton and Krishnamurthy (2011)). That is these exercises miss the general equilibrium feedback effect of the stress on aggregate bank balance sheets to the real sector and back to bank balance sheets. Our general equilibrium model allows us to compute the fixed point of this feedback mechanism. Second, our model allows us to translate the result of the stress test into likely macroeconomic outcomes and particularly the probability of a crisis. This is likely a more useful metric for evaluating financial stability than a loss to bank capital.

The following thought experiment illustrates how the partial equilibrium approach ignoring the feedback amplification effect could give misleading answers. Suppose that we are at 2007Q3 with $e = e_{distress} = 0.657$, and the hypothetical stress scenario is a -30% loss on the bank equity. What is the underlying fundamental shock? Since the leverage in our model is about 4, the answer from a partial equilibrium perspective is a shock of $\sigma dZ_t = -30\%/4 = -7.5\%$. However, at $e = 0.657$, once feeding this -7.5% shock into our model over a quarter, we find that the full feedback effect in general equilibrium leads the return on equity to be -60% , which is far greater than the initial assumed shock of -30% .

Our model is useful to help determine the size of the dZ_t shock that generates a given equity loss. In practice, the stress test scenarios considered by the Federal Reserve were over six quarters rather than over one quarter. Hence, starting at $e = 0.657$, we consider feeding in negative shocks equally over six quarters, so that the resulting 6-quarter return on equity matches a particular stress test scenario, as shown in the left-hand-side column of Table 5. We report the geometric sum of six-quarter fundamental dZ_t shocks in the middle column. The right-hand-side column in Table 5 gives the probability of a crisis within the next 2 years after experiencing those six-quarter losses. We find that if the bank equity suffers a loss of -5% , the probability of a crisis within the next 2 years rises modestly to 19.2% . But a -25% loss on bank equity pushes the economy into the crisis state, and this is why with probability 100% there will be a crisis in the next 2 years.

We conclude this section by stressing that our analysis in this section should be viewed as illustrative. We can consider other shocks, scenarios, and initial conditions. The model can be used by mapping these scenarios into the dynamics of the state variable e_t which is the key to understanding crisis probabilities.

8 Conclusion

We presented a fully stochastic model of a systemic crisis in which the main friction is an equity capital constraint on the intermediary sector. We first showed that the model offers a good quantitative representation of the U.S. economy. In particular, the model is able to replicate behavior in non-distress periods, distress periods, and extreme systemic crisis, quantitatively matching the nonlinearities that distinguish patterns across these states. We then used the model to evaluate and quantify systemic risk, defined as the probability of reaching a state where capital constraints bind across the financial sector. We showed how the model can be used to evaluate the macroeconomic impact of a stress scenario on the systemic risk probability.

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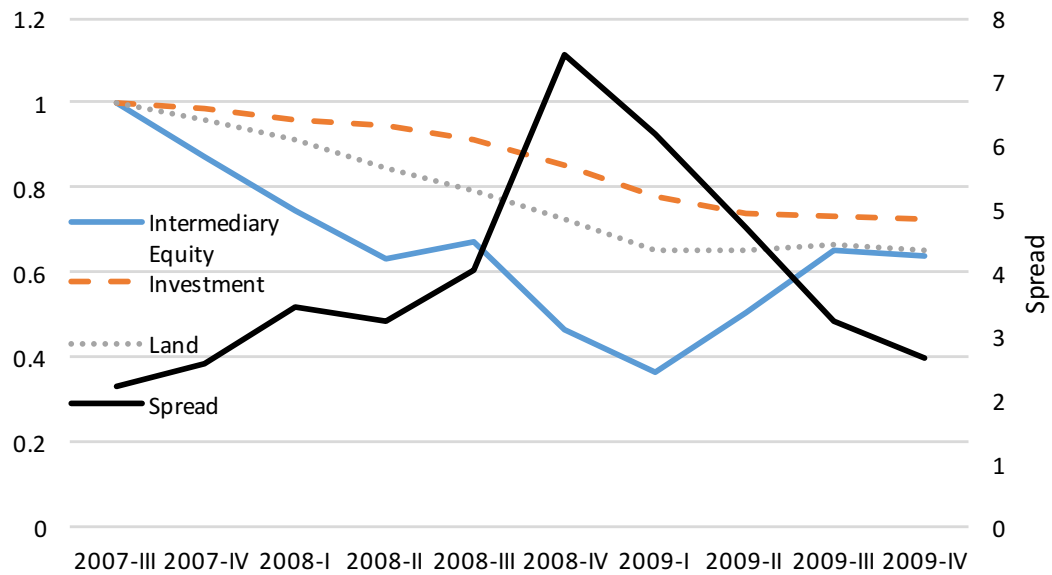


Figure 2: Data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Excess bond premium (labeled spread) is on right-axis. Variables are scaled by their initial values in 2007Q2.

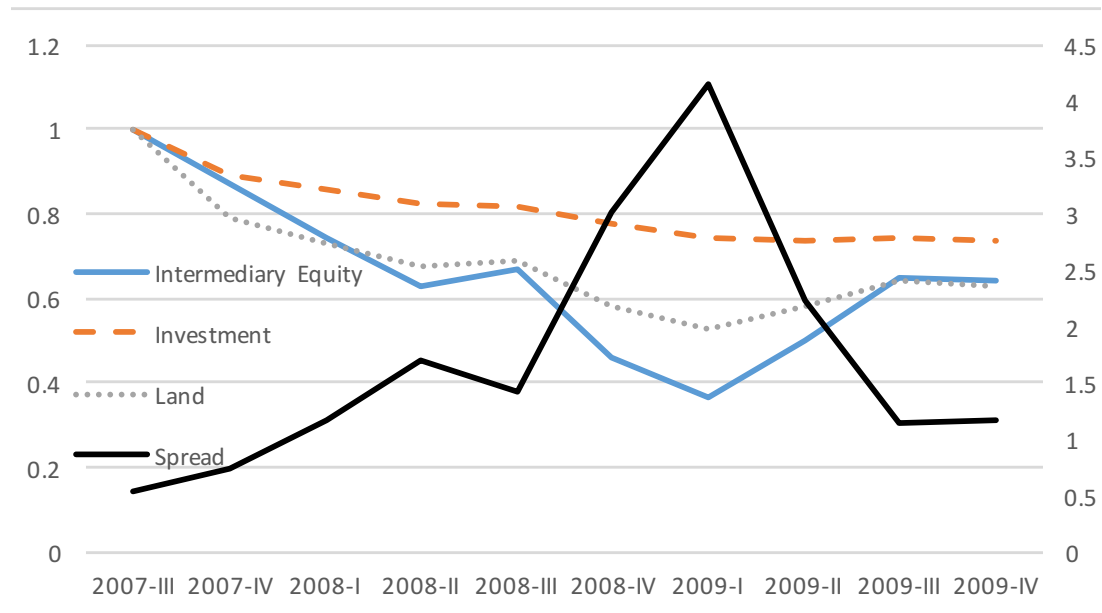


Figure 3: Model simulation matching data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Sharpe ratio is on right-axis. Variables are scaled by their initial values in 2007Q2.

Table 4: Model Simulation and Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB). Growth rates are computed as annual changes in log value from t to $t + 1$. The Sharpe ratio is the value at $t + 1$. The column labeled data are the statistics for the period 1975Q1 to 2015Q4. The Sharpe ratio is constructed from the excess bond premium, and other variables are standard and defined in the text. The next four columns are from the model, reflecting different parameter choices. Numbers are presented conditional on being in the distress period or non-distress period. For the data, the classification of the periods follows Table 3. For the model simulation, the distress period is defined as the 33% worst realizations of the Sharpe ratio.

| | Data | Baseline | $\sigma = 4\%$ | $\phi = 0$ | $\gamma = 2.3$ | $\lambda = 0.6$ |
|-------------------------------|-------|----------|----------------|------------|----------------|-----------------|
| Panel A: Distress Periods | | | | | | |
| vol(Eq) | 25.73 | 21.68 | 25.12 | 9.92 | 25.62 | 6.78 |
| vol(I) | 7.71 | 6.95 | 23.36 | 3.35 | 8.72 | 3.54 |
| vol(C) | 1.72 | 4.46 | 6.17 | 2.31 | 8.04 | 1.62 |
| vol(PL) | 15.44 | 15.82 | 17.68 | | 19.03 | 5.74 |
| vol(EB) | 65.66 | 34.56 | 45.37 | 6.77 | 67.34 | 6.85 |
| cov(Eq, I) | 1.02 | 1.12 | 4.87 | 0.18 | 1.95 | 0.20 |
| cov(Eq, C) | 0.20 | -0.82 | -1.14 | -0.05 | -1.72 | -0.05 |
| cov(Eq, PL) | 2.38 | 3.00 | 3.86 | | 4.61 | 0.36 |
| cov(Eq, EB) | -8.50 | -8.77 | -14.24 | -0.50 | -12.63 | -0.26 |
| Panel B: Non-distress Periods | | | | | | |
| vol(Eq) | 20.54 | 5.71 | 6.59 | 3.00 | 7.08 | 3.70 |
| vol(I) | 5.79 | 5.23 | 12.74 | 3.01 | 5.71 | 3.15 |
| vol(C) | 1.24 | 1.66 | 3.68 | 2.92 | 3.12 | 2.03 |
| vol(PL) | 9.45 | 8.23 | 9.18 | | 8.91 | 4.11 |
| vol(EB) | 16.56 | 5.62 | 7.95 | 0.04 | 20.17 | 0.57 |
| cov(Eq, I) | -0.07 | 0.30 | 0.83 | 0.09 | 0.37 | 0.12 |
| cov(Eq, C) | -0.01 | -0.08 | -0.15 | 0.09 | -0.13 | 0.07 |
| cov(Eq, PL) | -0.43 | 0.47 | 0.60 | | 0.59 | 0.15 |
| cov(Eq, EB) | 0.60 | -0.28 | -0.54 | 0.00 | -1.15 | -0.01 |

Table 5: Probability of Crisis and Return on Equity

The table gives the probability of a crisis within the next 2 years for a number of different six-quarter scenarios. The scenarios are chosen based on feeding in shocks over six quarters to match a given return on equity (left hand column).

| Return on Equity | 6 QTR Shocks | Prob(Crisis within 2 years) |
|------------------|--------------|-----------------------------|
| -2% | -0.9% | 11.0 % |
| -5% | -2.3% | 19.2% |
| -10% | -3.7% | 32.0% |
| -15% | -5.7% | 59.9% |
| -25% | -7.0% | 100.0% |



Figure 4: Price and policy functions for baseline parameters given in Table 1. The x -axis is the scaled intermediary capital state variable, e . Note that the range for the x -axis in bottom right-hand panel is smaller than the other graphs, to make it possible to see e_{crisis} and $e_{distress}$.

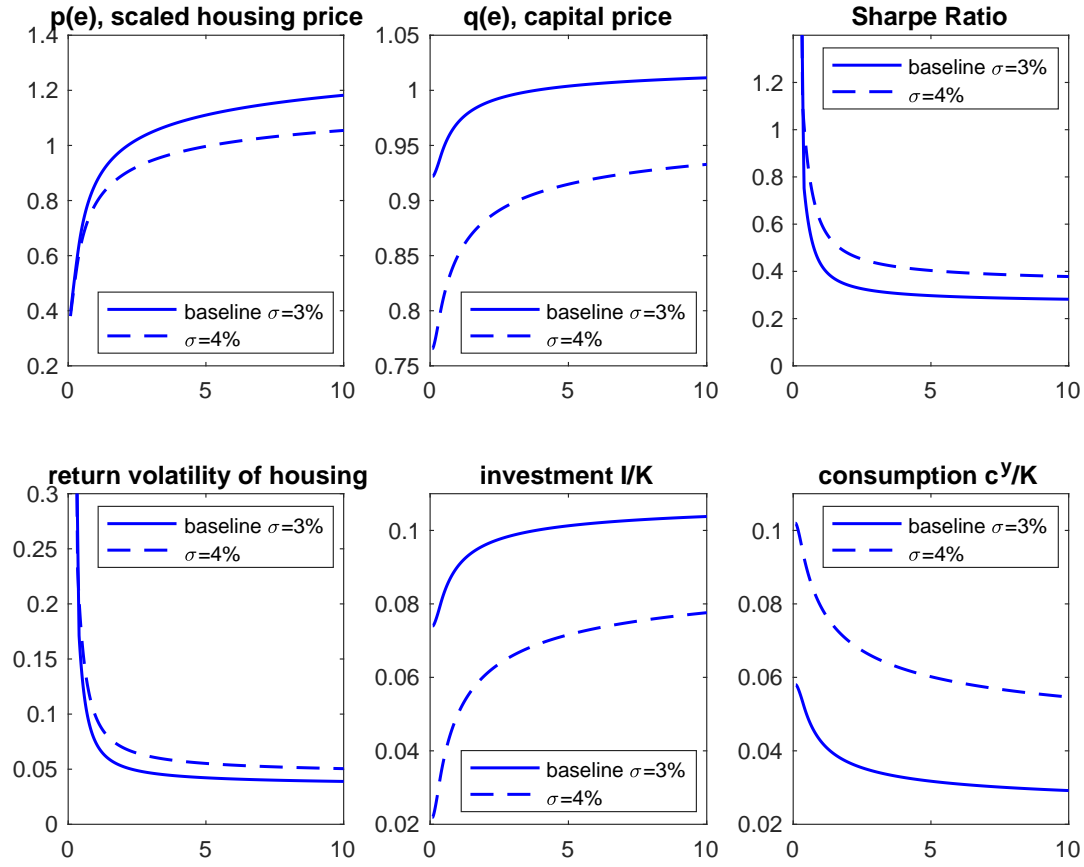


Figure 5: Price and policy functions for $\sigma = 4\%$ case; other baseline parameters are given in Table 1. The x -axis is the scaled intermediary capital state variable, e .

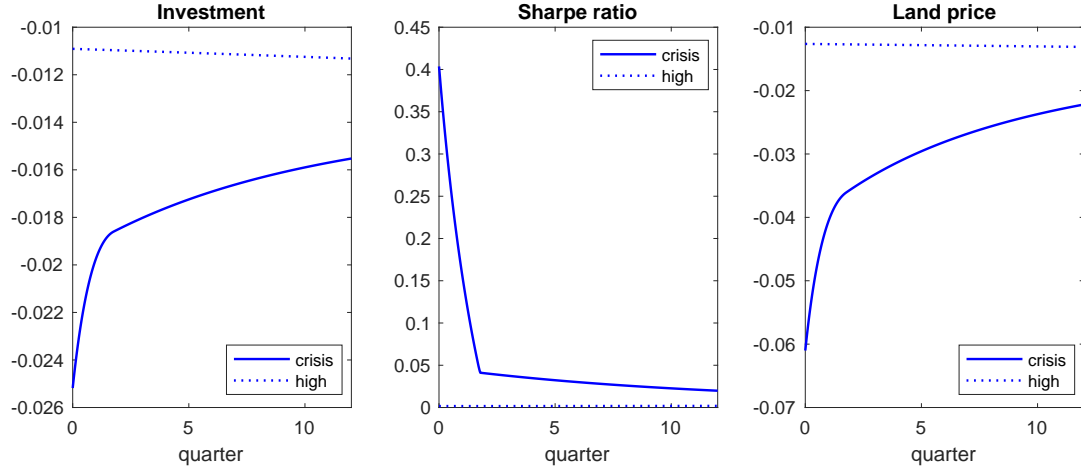


Figure 6: Effect of a $dZ_t = -1\%$ shock on investment, Sharpe ratio, and land prices, conditional on crisis, $e = e_{crisis}$, (solid line) and high e states (dashed line). We trace out the impulse response to this shock, as described in the text, over 15 quarters.

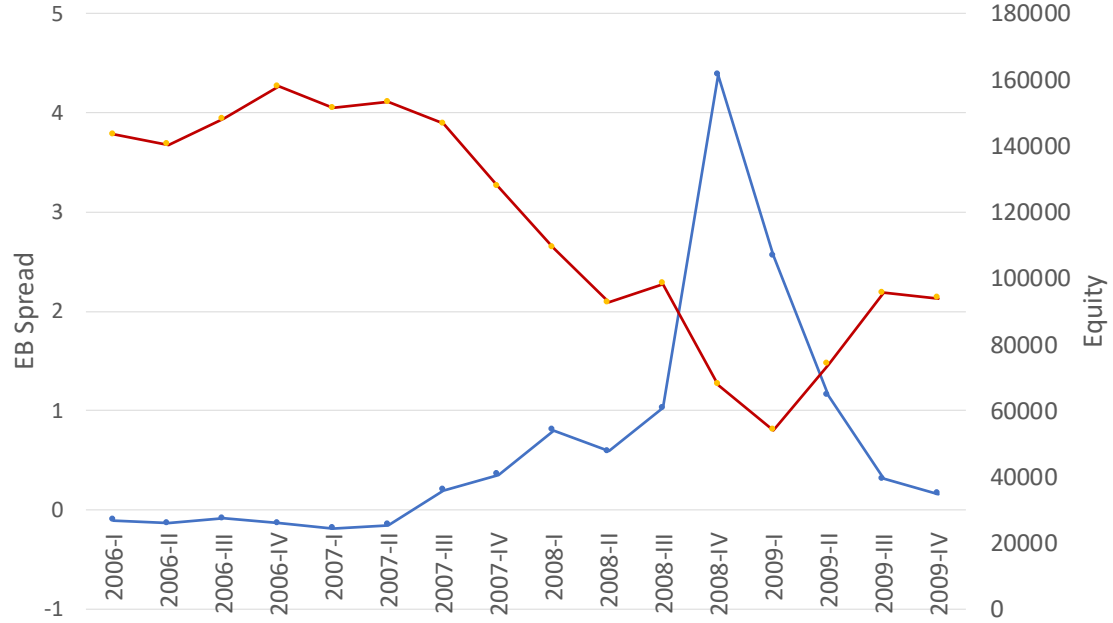


Figure 7: EB and Intermediary Equity are graphed from 2006 to 2010.