Online Appendix for House Prices and Consumer Spending

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A.4 Description of Computational Procedures

In this appendix, we describe the solution to the baseline model and its extensions. The house-hold state vector is $\mathbf{s} \equiv (A, H, z, P, j)$, and the model is solved by backward induction from the final period of life. When working, households solve:

$$V(\mathbf{s}) = \max\left\{V^{adjust}(\mathbf{s}), V^{noadjust}(\mathbf{s}), V^{rent}(\mathbf{s})\right\}.$$

The three value functions, for adjusters, non-adjusters, and renters, are given by

$$\begin{aligned} V^{adjust}(\mathbf{s}) &= \max_{C,A',H'} U\left(C, H'\right) + \beta E\left[V(\mathbf{s}')|z\right] \\ s.t. \quad & A' + PH' + C = (1+r)A + Y(z) + (1-F)(1-\delta)PH, \\ & A' \geq -(1-\theta)\frac{1-\delta}{1+r}PH', \quad \mathbf{s}' = (A', H', z', P', j+1), \end{aligned}$$

$$\begin{aligned} V^{noadjust}(\mathbf{s}) &= \max_{C,A'} U\left(C, H\right) + \beta E\left[V(\mathbf{s}')|z\right] \\ s.t. \quad & A' + C = (1+r)A + Y(z) - \delta PH, \\ & A' \geq -(1-\theta)\frac{1-\delta}{1+r}PH, \quad \mathbf{s}' = (A', H, z', P', j+1), \end{aligned}$$

$$\begin{aligned} V^{rent}(\mathbf{s}) &= \max_{C,A',\tilde{H}} U\left(C, \tilde{H}\right) + \beta E\left[V(\mathbf{s}')|z\right] \\ s.t. \quad & A' + C + \phi P\tilde{H} = (1+r)A + Y(z) + (1-F)(1-\delta)PH, \\ & A' \geq 0, \quad \mathbf{s}' = (A', 0, z', P', j+1). \end{aligned}$$

The problem for a retired household is identical except that social security benefits replace labor earnings. At the age of retirement households also receive an additional lump sump transfer to match the level of retirement wealth which is now liquid, as described in the text. At the time of death households' continuation value is given by the bequest motive in the text.

To solve the model numerically, we proceed as follows. First, note that the presence of random walk house prices with i.i.d. changes x_t, CRRA preferences and a constant price-rent ratio allows us to combine the separate states P and H into a single state $\hat{H} \equiv PH$ and instead solve the equivalent recursive problem:

$$V^{adjust}(\mathbf{s}) = \max_{C,A',\hat{H}'} U\left(C, \hat{H}'\right) + \beta E\left[x'^{-(1-\sigma)(1-\alpha)} V(\mathbf{s}')|z\right]$$
$$s.t. \quad A' + \hat{H}' + C = (1+r)A + Y(z) + (1-F)(1-\delta)\hat{H},$$
$$A' \geq -(1-\theta)\frac{1-\delta}{1+r}\hat{H}', \quad \mathbf{s}' = (A', \hat{H}', z', j+1),$$

$$V^{noadjust}(\mathbf{s}) = \max_{C,A'} U\left(C, \hat{H}\right) + \beta E\left[x'^{-(1-\sigma)(1-\alpha)} V(\mathbf{s}')|z\right]$$
$$s.t. \quad A' + C = (1+r)A + Y(z) - \delta\hat{H},$$
$$A' \geq -(1-\theta)\frac{1-\delta}{1+r}\hat{H}, \quad \mathbf{s}' = (A', \hat{H}', z', j+1),$$

$$V^{rent}(\mathbf{s}) = \max_{C,A',\tilde{H}} U\left(C, \tilde{H}\right) + \beta E\left[x'^{-(1-\sigma)(1-\alpha)} V(\mathbf{s}')|z\right]$$
$$s.t. \quad A' + C + \phi\tilde{H} = (1+r)A + Y(z) + (1-F)(1-\delta)\hat{H},$$
$$A' \geq 0, \quad \mathbf{s}' = (A', 0, z', j+1).$$

Note that $P' = x'P$ so that the x' in the expectation integrates over the possible realizations of house price growth from today to tomorrow. Given the above assumptions, this enters the household problem equivalently to i.i.d. discount rate shocks and enters the problem only in its role in evaluating expected continuation values. It does not enter as a current state since shocks are i.i.d. and previous values of the shock are fully reflected in the state $\hat{H}$.

In order to rectangularize the choice set and simplify the computational problems imposed by the endogenous liquidity constraint, we follow Díaz and Luengo-Prado (2010) and reformulate our problem in terms of voluntary equity, defined as

$$Q \equiv A + (1-\theta)\frac{1-\delta}{1+r}\hat{H}.$$

After substituting the budget constraint into the utility function to eliminate C as a choice variable, the value function can then be rewritten in terms of the two non-negative state variables Q and $\hat{H}$. Note that A' and H' are chosen prior to next period shocks to house prices. Thus shocks to house prices imply that realized Q' and $\hat{H}'$ will become stochastic variables which differ from the value chosen by households today. Namely, given a chosen pair $Q', \hat{H}'$, the realized value of $\hat{H}$ next period will be $x'\hat{H}$ and the realized value of Q next period will be $Q' + (1-\theta)\frac{1-\delta}{1+r}(x'-1)\hat{H}'$, where ΔP is the house price shock. This implies that although households in our baseline model are constrained to always choose $Q' \geq 0$, realized voluntary equity can be negative, for a large enough negative house price shock. To account for this, we solve the model for states that include negative voluntary equity even though households are constrained to choose non-negative values for this variable.[60]

We discretize the problem so it can be solved on the computer by first discretizing z and x' using the algorithm of Tauchen (1986). We use 13 grid points for z and 5 grid points for x'. We then approximate the functions $V_j^{adjust}, V_j^{noadjust}$, and V_j^{rent} as mulitlinear functions in the endogenous states. In our benchmark calculation, we use 120 knot points for Q (we space these points more closely together near the constraint) and 40 knot points for $\hat{H}$. The presence of fixed adjustment costs on housing together with the borrowing constraint make the household policy function highly non-linear. For this reason, we follow Berger and Vavra (2015) and compute optimal policies for a given state-vector using a Nelder-Meade algorithm initialized from 3 different starting values, to reduce the problem of finding local maxima. The value of adjusting, not adjusting and renting are then compared to generate the overall policy function. We proceed via backward induction from the final period of life.

To simulate the model, we initialize cohorts to match the values of the SCF for age 22-27 year old households. First, we randomly split the sample into two groups to match the fraction of homeowners and renters. Then within each group, we split the sample into 4 income bins and assign the median value of housing and liquid assets from the SCF in that same income bin. (By definition, the value of housing for the renter groups is always zero). The model is simulated with 100,000 households and house price impulse responses are computed for each cohort.

In section 5.1 we introduce long-term debt. In this version of the model, non-adjusting households can increase voluntary equity with no cost, as in the baseline model, but households who want to decrease voluntary equity when $a < 0$ must pay a fixed cost proportional to the value of their house to do so. We also assume that when households neither refinance or move, they need not satisfy the collateral constraint on new debt, but they must pay off some fraction of their existing debt: In particular, when refinancing, households face the constraint:

$$-A' \leq (1-\theta)\frac{1-\delta}{1+r}PH',$$

but when not adjusting, they instead face the constraint:

$$A' \geq \begin{cases} \chi A, & \text{if } A < 0 \\ 0, & \text{if } A \geq 0 \end{cases}$$

[60]Shocks to house prices in the model are not large enough to ever reach a situation where realized Q is so negative that households would be unable to choose $Q' \geq 0$ without having negative consumption.

Given this constraint, we then solve $V(\mathbf{s}) = \max\left\{V^{adjust}(\mathbf{s}), V^{noadjust}(\mathbf{s}), V^{refi}(\mathbf{s}), V^{rent}(\mathbf{s})\right\}$ where $V^{noadjust}(\mathbf{s})$ now includes the above constraint, and $V^{refi}(\mathbf{s})$ is identical to $V^{noadjust}(\mathbf{s})$ in the baseline problem but with budget constraint $A' + C = (1+r)A + Y(z) - \delta PH - F^{refi}PH$, where F^{refi} is calibrated to match targets described in the text.

We also explore three extensions which do not allow us to solve the problem using $\hat{H}$ instead of separate states P and H. In particular, moving from Cobb-Douglas to CES preferences, eliminating the constant price-rent ratio, or eliminating the random walk and working with AR shocks no longer allows us to make this substitution. In this case, we now include P explicitly as a state variable, which we allow to take on 25 evenly spaced valued from -25 σ_P to 25 σ_P. That is, each node in the price grid is two standard deviations apart, and the overall price grid is wide enough that no households hit the boundary during their finite life. Re-solving the model with a finer grid delivers similar results but substantially slows computations. In order to avoid interpolating between price grid points, we set $\mu = 0$ in these extensions but recalibrate our baseline model to match the same moments with $\mu = 0$. Since adding an additional state-variable substantially slows the problem, we reduce the grid for Q to 110 points and the grid for H to 36 points. Solving the problem with this additional state nevertheless is substantially slower, so we do not recalibrate the model in these extensions. However, these models still hit the original targets reasonably well.

A.5 Extension: CES Preferences

In this section, we extend the analytical analysis of the frictionless model in 3 to CES preferences for consumption and housing. This complements the numerical analysis in 5.3, which explores the effects of CES preferences in our baseline model.

In the body of the paper, we assume Cobb-Douglas utility—i.e., elasticity of substitution equal to 1—and use that assumption to derive Proposition 1. Here we show that the proposition can be extended to the case of CES preferences if we make the additional assumption of $\theta = 0$, that is, if we consider a very loose collateral requirement. In that case, our sufficient statistic formula extends naturally by adding a new term that is positive in the case of elasticity of substitution bigger than 1 and negative in the opposite case. Under plausible parametrizations the magnitude of the elasticity remains large and the Cobb-Douglas based formula implies results similar to the exact CES formula.

Let the utility function be:

$$U(C_{it}, H_{it}) = \frac{1}{1-\sigma}\left(\alpha C_{it}^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha) H_{it}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon(1-\sigma)}{\epsilon-1}}$$

where ϵ is the intra-temporal elasticity of substitution between non-durable consumption and housing services. For simplicity we focus an environment with constant house prices and analyze the response to a permanent, unexpected price change, but results extend naturally to the stochastic case.

Proposition 2 *Consider the model with CES preferences, liquid housing wealth, and $\theta = 0$. The individual response of non-durable consumption to an unexpected, permanent, proportional change in house prices dP/P is*

$$(\epsilon - 1) \frac{\frac{r+\delta}{1+r} PH_{it}}{C_{it} + \frac{r+\delta}{1+r} PH_{it}} C_{it} + MPC_{it} \cdot (1-\delta) PH_{it-1}.$$

Proof. With constant prices, the user cost of housing (or implicit rental rate) is $\frac{r+\delta}{1+r}P$. So we can define total spending on non-durables and housing services

$$X_{it} \equiv C_{it} + \frac{r+\delta}{1+r} PH_{it},$$

and the price index

$$P^X \equiv \left[\alpha^\epsilon + (1-\alpha)^\epsilon \left(\frac{r+\delta}{1+r} P \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$

The household's optimization problem can then be decomposed into an intertemporal optimization problem, characterized by the Bellman equation

$$V(W, s) = \max \frac{1}{1-\sigma} \left(\frac{X}{P^X} \right)^{1-\sigma} + \beta E \left[V(W', s') \right],$$

subject to

$$W' = (1+r) \left[W + Y(s) - X \right] \geq 0,$$

and an intratemporal utility maximization problem. The solution to the intertemporal problem is independent of P^X as it only appears as a multiplicative constant in the objective function. So the policy $X(W, s)$ is independent of P. The solution to the intratemporal problem gives

$$C = \frac{\alpha^\epsilon}{\alpha^\epsilon + (1-\alpha)^\epsilon (\frac{r+\delta}{1+r} P)^{1-\epsilon}} X, \quad H = \frac{(1-\alpha)^\epsilon (\frac{r+\delta}{1+r} P)^{-\epsilon}}{\alpha^\epsilon + (1-\alpha)^\epsilon (\frac{r+\delta}{1+r} P)^{1-\epsilon}} X.$$

The response of C to P conditional on X is then

$$\frac{\partial C}{\partial P} = (\epsilon - 1) \frac{r+\delta}{1+r} \frac{H}{X} C.$$

Combining this effect with the effect on X through W, yields the desired result. ■

An elasticity of substitution different from one implies that there is an additional term, proportional to the implicit share of housing services in the total consumption basket. In the model, this share is tightly linked to the ratio of house values to consumption. For example, take an agent with housing-to-consumption ratio 3.5—which is the roughly the average for agents in the 40s bin. With $r = 2.4\%$ and $\delta = 2.2\%$ this implies a share of housing services to total spending equal to 0.13. For such an agent if $\epsilon = 1.1$, the additional term is equal to $0.1 \times 0.13 = 0.013$. If the same agent has an MPC of 0.1 the magnitude of the baseline sufficient statistic is $0.1 \times 3.5 = 0.35$, so the additional term plays a minor role quantitatively.