

The Empirical Price Duration Distribution and Monetary Non-Neutrality

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Abstract

Allowing for price adjustment probabilities that vary with the number of periods since an item last adjusted ('duration-dependence') provides a significantly better fit of observed price spells in CPI and grocery store micro data than the Calvo model, even if the latter is extended to incorporate item-specific adjustment probabilities. Furthermore, extending the Calvo model to match both duration-dependence and cross-item heterogeneity, as observed in the micro data, leads to an increase of 100-230% in monetary non-neutrality, even with no strategic-complementarities. As much as half of this increase is driven by duration-dependent adjustment probabilities.

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1 Introduction

Time-dependent pricing models such as Calvo [1983] are widely used in policy analysis due to their analytical tractability. However, these models impose strong restrictions on how the probability of adjustment depends on the current duration of a price spell. In the Calvo model, the probability of adjustment is constant while in a Taylor model, the probability of adjustment is zero over some interval and then jumps to one. In this paper I show that in a generalized time-dependent model, that imposes no restrictions on the probability of adjustment, aggregate impulse response functions can be completely characterized by the distribution of the number of periods since prices last adjusted (the *price duration distribution*).

Using CPI micro data as well as Dominick's grocery scanner data, I calculate the empirical price duration distribution, explicitly accounting for the presence of censored data, and provide evidence that it departs

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significantly from that implied by the Calvo model, even if the Calvo model is extended to allow for cross-item¹ heterogeneity in the probability of adjustment as in Carvalho [2006] or Alvarez and Burriel [2010]. Calibrating the generalized time-dependent model to match the empirical duration distribution increases monetary non-neutrality by 100-230% relative to the Calvo model. Between 33-52% of this increase is driven by duration-dependent probabilities of adjustment for individual items while the remainder is explained by fixed cross-item heterogeneity of the type considered by Alvarez and Burriel [2010]. Thus, while Alvarez and Burriel [2010] emphasizes the fact that broad sector level Calvo heterogeneity is not enough to match aggregate dynamics, my analysis shows that even their alternative: item-level Calvo heterogeneity, still generates large departures from the micro data.

I develop a new empirical strategy that takes account of heterogeneity across items, and I show that price adjustment hazards are decreasing at short horizons². If items face a constant probability of adjustment, then the probability of adjustment if the price changed last period should be equal to the probability of adjustment if the price did not change last period. That is, if I_t is an indicator for a price change at time t , then under a Calvo model $P(I_t = 1|I_{t-1} = 1) = P(I_t = 1|I_{t-1} = 0)$. Under a multi-sector Calvo model, these probabilities can differ across items but the two probabilities must be equal for each item. Figures 1 and 2 display scatter plots of these two item-level probabilities for CPI and Dominick's data, respectively. From these figures, it is clear that the probability of adjusting after one period is not the same as the probability of adjusting conditional on not adjusting last period. Under a Calvo model, these scatter plots should just be a single point, as all items change prices with a common, constant probability. Under a Calvo model with cross-item heterogeneity, these points should all lie on the black 45 degree line as the probabilities should be equal for each item. In fact, the majority of the observations lie above the 45 degree line: items are more likely to adjust in the period after adjustment than if they did not just adjust.

These figures are striking evidence in support of non-constant probabilities of adjustment, and since they are computed at the item level, they are not subject to concerns of survivor bias. While these figures show that the probability of adjustment is not constant, on their own, they do not provide a great deal of information about the overall duration distribution. Since I show that the overall duration distribution is what matters for monetary non-neutrality in a generalized time-dependent model, I next estimate the empirical du-

¹Henceforth, I use item to refer to fully disaggregated price observations, e.g. 2 liter coke at a particular Chicago outlet. Carvalho [2006] considers cross-sector heterogeneity while Alvarez and Burriel [2010] extends the model to cross-item heterogeneity.

²Differences across items in the average frequency of adjustment is well established. Bils and Klenow [2004] and Nakamura and Steinsson [2008] show that some items change prices nearly every month while others almost never change prices. Existing evidence on whether the probability of price adjustment changes with duration is more mixed. Nakamura and Steinsson [2008] find evidence that the probability of adjustment falls with duration in many product categories, while Klenow and Kryvtsov [2008] argue that the overall probability of adjustment is constant. To my knowledge, I am the first to show that there is clear heterogeneity at the item level between the first and later months. Previous work also does not make the direct link to monetary non-neutrality implied by time-dependent models and does not quantify the relative importance of duration-dependence and cross-item heterogeneity.

ration distribution. By comparing this empirical duration distribution to the empirical duration distribution predicted by a Calvo model as well as that predicted by a Calvo model with cross-item heterogeneity, I can decompose departures from the Calvo distribution into those driven by cross-item heterogeneity and those driven by duration-dependent probabilities of adjustment. Overall, I find that in the CPI data, matching the empirical duration distribution doubles monetary non-neutrality relative to that predicted by a Calvo model with no heterogeneity. At least one-third of this increase is driven by duration-dependence. In Dominick's data, I find that matching the empirical distribution more than triples monetary non-neutrality and that just over half of this increase is driven by duration-dependence.

Although the absolute importance of duration-dependence is quite large in both data sets, the relative importance is greater in Dominick's data. I explore what drives this difference and conclude that sampling frequency plays a major role. Dominick's data is sampled weekly, while the CPI data is only sampled monthly. When the true probability of adjustment is non-constant, sampling at lower frequencies changes the average frequency of adjustment, and under Calvo pricing, a change in the average frequency of adjustment alters the entire duration distribution. I present evidence that for the CPI this bias can be quite substantial and that, after imposing a rough correction, duration-dependence explains half rather than one-third of the increase in monetary non-neutrality relative to the Calvo model.

The importance of duration-dependence is robust to theoretical extensions to my benchmark generalized time-dependent model³. For simplicity the benchmark model assumes that money growth follows a random walk and that there are no strategic-complementarities. I enrich the generalized time-dependent model to allow for these features and show that they enhance the importance of duration-dependence.

The remainder of the paper proceeds as follows: Section 2 presents the benchmark generalized time-dependent model. Section 3 discusses the data and presents empirical results. Section 4 describes extensions to the benchmark model. Section 5 discusses potential explanations that could endogenously generate duration-dependence in the probability of adjustment, and Section 6 concludes.

2 Generalized Time-Dependent Pricing Model

I now describe the benchmark generalization of the Calvo model with no restrictions on the shape of the duration hazard of price adjustment. This model is similar to that in Wolman [1999], but I do not impose the restriction that the probability of price adjustment be non-decreasing with duration, and I solve the model in a manner that allows it to be taken directly to the micro data.

³Again, the generalized time-dependent model is essentially a Calvo model that imposes no ex ante restrictions on the relationship between the duration of a price spell and the probability of adjustment.

Assume that each period a generic firm in sector j , whose price has lasted for k periods, adjusts its price with probability $1 - \alpha_{j,k}$. As in the standard Calvo model, adjustment draws are independent across firms, and firms that do not adjust prices keep their previous price. Note that this nests the standard Calvo model when $\alpha_{j,k} \equiv \alpha$ for all j, k . It nests the Carvalho [2006] model (henceforth multi-sector Calvo model) when $\alpha_{j,k} \equiv \alpha_j$ for all k , and it nests the recent work by Alvarez and Burriel [2010] when each item is identified with a separate sector j , with constant adjustment probability. For this generalized time-dependent model, we get the following proposition:

Proposition 1 *Assume that each firm in sector j , whose price has lasted for k periods, adjusts its price with probability $1 - \alpha_{j,k}$. Further, assume that nominal output follows a random walk and that there are no strategic complementarities. Let ω_k be the total population weight with current duration k . Then the impulse response function of the price level to a nominal output shock ε_t is given by*

$$\frac{\partial \log P_{t+k}}{\partial \varepsilon_t} = \sum_0^k \omega_k,$$

so that the impulse response to the output gap is given by

$$\frac{\partial \log \hat{Y}_{t+k}}{\partial \varepsilon_t} = 1 - \sum_0^k \omega_k,$$

and the total impulse response function to the output gap is $\sum_{k=0}^{\infty} k\omega_k$.

Proof. Let $G_{j,k}$ be the probability that a generic spell in sector j lasts at least k periods. Then, using the notation of Woodford [2003], each firm in sector j solves

$$E_t \left[\sum_{k \geq 0} G_{j,k} Q_{t,t+k} \prod_1 (p_{j,t}^*, P_{t+k}, P_{j,t+k}; Y_{t+k}; \zeta_{t+k}) \right] = 0.$$

Log-linearizing around the steady-state and assuming no strategic complementarities gives that a firm's reset price $p_{j,t}^*$ solves

$$\left[\sum_{k \geq 0} G_{j,k} \beta^k E_t \left[\log p_{j,t}^* - \log P_{t+k} - \log \hat{Y}_{t+k} - \log \hat{Y}_{t+k}^n \right] \right] = 0.$$

Rearranging we get that

$$\sum_{k \geq 0} G_{j,k} \beta^k \log p_{j,t}^* = \sum_{k \geq 0} G_{j,k} \beta^k E_t \log \mathcal{Y}_{t+k} + \text{Terms Independent of Policy}.$$

Finally, using the assumption that nominal output follows a random walk so that $E_t \log \mathcal{Y}_{t+k} = \log \mathcal{Y}_t$ and

ignoring terms that do not depend on monetary policy we get that

$$\log p_{j,t}^* = \log \mathcal{Y}_t. \quad (1)$$

Now note that the price level is given by

$$\begin{aligned} P_t^{1-\theta} &= \int f(j) P_{j,k}^{1-\theta} dj \\ &= \int f(j) \left[\sum_{k \geq 0} w_{j,k} p_{j,t-k}^* \right]^{1-\theta} dj \end{aligned}$$

where $w_{j,k}$ is the fraction of firms in sector j that last adjusted k periods ago and $f(j)$ is sector j 's population weight. Log-linearizing implies that

$$\log P_t = \int f(j) \sum_{k \geq 0} w_{j,k} \log p_{j,t-k}^* dj.$$

Substituting for the optimal reset price (1) gives:

$$\log P_t = \sum_{k \geq 0} \omega_k \log \mathcal{Y}_{t-k}$$

where I introduce the notation $\omega_k \equiv \int f(j) w_{j,k} dj$ so that ω_k is the total population weight with duration k .

Then

$$\begin{aligned} \pi_t &= \log P_t - \log P_{t-1} \\ &= \sum_{k \geq 0} \omega_k \varepsilon_{t-k}. \end{aligned}$$

Thus, the impulse response function of inflation to ε_t is given by

$$\frac{\partial \pi_{t+k}}{\partial \varepsilon_t} = \omega_k.$$

Similarly,

$$\begin{aligned} \frac{\partial \log P_{t+k}}{\partial \varepsilon_t} &= \sum_0^k \omega_k. \\ \frac{\partial \log \hat{Y}_{t+k}}{\partial \varepsilon_t} &= 1 - \sum_0^k \omega_k. \end{aligned}$$

This implies that the cumulative output gap impulse response to a nominal output shock is given by

$$\widehat{Y}_{tot} = \sum_{k=0}^{\infty} k\omega_k.$$

■

I take the cumulative impulse response, \widehat{Y}_{tot} , as my measure of monetary non-neutrality in what follows, but other reasonable measures give similar results. In general, the duration distribution can take almost any shape, yet standard time-dependent pricing models imply extremely strong restrictions on the shape of this distribution. Under a Calvo model, each item adjusts price with constant probability $1 - \alpha$ so that $\omega_k = (1 - \alpha)\alpha^k$ and $\widehat{Y}_{tot} = \frac{\alpha}{1-\alpha}$. Under the multi-sector Calvo model, $\widehat{Y}_{tot} = \int f(j)\frac{\alpha}{1-\alpha}dj$. With Taylor pricing, the duration distribution is uniformly distributed with weight equal to the frequency of adjustment. Thus, in these standard pricing models, knowing the frequency of adjustment completely determines \widehat{Y}_{tot} . However, in general, knowing the frequency of adjustment provides very little information about \widehat{Y}_{tot} . Given an average frequency of adjustment, $1 - \alpha$, it is straightforward to show that all we can tell in general is that $\widehat{Y}_{tot}(\alpha) \in \left[\frac{\alpha}{2(1-\alpha)}, \infty\right)$ ⁴.

My empirical strategy is thus twofold. First, I use micro data to calculate the empirical duration distribution and compare this to that predicted by the Calvo model with the same frequency of adjustment. Second, departures from the Calvo model can come from two sources: cross-item heterogeneity in the Calvo frequency of adjustment and duration-dependence in the probability of adjustment. The multi-sector Calvo model captures the first source of departure but does not allow for duration-dependence in the probability of adjustment. The empirical duration distribution places no restrictions on the shape of each item's adjustment hazard and thus allows for both cross-item heterogeneity and duration-dependence. By calculating the duration distribution under the multi-sector Calvo model, I can see how far cross-item heterogeneity can go towards matching the empirical duration distribution⁵. Any remaining differences between the duration distribution under the multi-sector Calvo model and the empirical duration distribution must be driven by duration-dependence. In this manner, I can assess the relative importance of cross-item heterogeneity and duration-dependence for departures from the duration distribution predicted by Calvo.

⁴Formally, we solve: \sup (or \inf) $\sum k\omega_k$ s.t. $\omega_0 = 1 - \alpha$; $\omega_{k+1} \leq \omega_k$; $\sum \omega_k = 1$. The \sup occurs when $\omega_0 = 1 - \alpha$ and $\lim_{k \rightarrow \infty} \omega_k = \alpha$ which implies monetary non-neutrality becomes infinite. The \inf occurs when as much weighted is shifted forward as possible, subject to the constraints. This occurs under Taylor pricing, which implies monetary non-neutrality one half of Calvo.

⁵Effectively, the multi-sector Calvo model is picking up how much of the departure from a constant adjustment hazard can be explained by survivor bias.

3 The Empirical Duration Distribution

3.1 Data and Price Filter

The restricted access *CPI research database* collected by the Bureau of Labor Statistics (BLS) contains the individual prices for the thousands of non-shelter items underlying the CPI. Prices are collected monthly in New York, Los Angeles and Chicago, and I restrict my analysis of CPI data to these observations. The database contains thousands of individual "quote-lines" with price observations for many months. Quote-lines are the greatest level of disaggregation possible and correspond to an individual item at a particular outlet. An example of a quote-line collected in the research database is 2-liter coke at a particular Chicago outlet. These quote-lines are then classified into various product categories called "Entry Level Items" or ELIs. The ELIs can then be grouped into several levels of more aggregated product categories finishing with eight major expenditure groups: Apparel, Education and Communication, Food, Other Goods and Services, Housing, Medical Care, Recreation, and Transportation. For more details on the structure of the database see Nakamura and Steinsson [2008].

In addition to CPI micro data, I also perform my empirical analysis on high-frequency grocery store scanner data. From September 1989 to May 1997, Dominick's Finer Foods and the Chicago Booth School of Business partnered to collect weekly store-level data on a wide variety of products sold in the grocery chain.⁶ Weekly price observations were collected for more than 9,000 UPCs in 29 different product categories throughout 86 stores in the 100-store Dominick's chain. An example of a UPC product is "Sargento Shredded Cheddar, 16 oz." within the product category "Cheeses."

For notational consistency, I will refer to quote-lines, UPCs, and price-setters in the theoretical models as "items". Thus, in all cases, an item price series refers to the most disaggregated data series.

In constructing price duration data, I must take a stand on which price changes to use. The products in both data sets exhibit frequent sales with less frequent regular price changes, and there is not a clear consensus on which price changes matter for aggregate non-neutrality. As a conservative benchmark, I focus mainly on regular price changes. This benchmark is relatively conservative because by definition temporary price changes tend to be clustered together in time, with an increase quickly following a decrease or vice versa. For this reason, duration-dependence is even more important when I use alternative measures of price changes that include temporary changes. In the benchmark, following Nakamura and Steinsson [2008], I filter out v-shaped sales to construct a regular price series⁷.

Finally, there is one additional concern that arises in the CPI database and not in the Dominick's data:

⁶The data set is available at <http://research.chicagogsb.edu/marketing/databases/dominicks/index.aspx>

⁷Constructing a regular price using the method suggested in Kehoe and Midrigan [2008] does not qualitatively affect the conclusions.

product substitution. When a product disappears from shelves, the BLS replaces it with the most similar available product. In addition, products are also substituted so that the prices tracked in the database roughly reflect actual expenditure by consumers. Forced product substitution, which occurs when a product that has disappeared is replaced with a new product, is flagged in the database. Following Klenow and Kryvtsov [2008] (and some specifications of Nakamura and Steinsson [2008]) I merge quote-lines following product substitutions into a single quote-line. If the price changes at the time of product substitution then this appears as a price change within the quote-line while if the price of the old and new product are identical then I do not count this as a price change⁸.

3.2 The Duration Distribution of Price Spells

Are prices more likely to change if they have changed recently than if they have not changed in some time? The price adjustment duration hazard captures how the probability of price adjustment changes with the duration of a price spell. The duration hazard of a price change at time t is the probability that the price will change after exactly t periods given that it has survived for t periods. More formally, if T is a random variable that denotes the length of a price spell, then the hazard function is given by $h(t) = P(T = t | T \geq t)$. It is well known that hazard functions estimated off of pooled samples exhibit downward survivor bias. For example, see Kiefer [1988]. I bypass this problem in two ways: First, for each product I explore whether it is more likely to change in the period immediately following a price change than at any later time. Then, more importantly, I directly compute the duration weights observed in the data and compare these to duration weights predicted by both the Calvo and multi-sector Calvo models.

As will be seen shortly, the most difficult part of estimating the empirical duration distribution is the presence of censored spells. Computing the empirical duration distribution requires estimating the length of censored spells. In contrast, the procedure first mentioned in the introduction is subject to neither concerns with censoring nor concerns with survivor bias. For each item in the data set, I compute the probability that the item changes price in period t given that it changed in period $t - 1$ and the probability that it changes price in period t given that it did not change in period $t - 1$. That is if I_t is an indicator that takes value 1 when a price change occurs, I compute $P(I_t = 1 | I_{t-1} = 1)$ and compare it to $P(I_t = 1 | I_{t-1} = 0)$. These probabilities are computed separately for each item, so problems with heterogeneity are eliminated.

Again, figures 1 and 2 display scatter plots of these two item-level probabilities for CPI and Dominick's data, respectively. From these figures, it is clear that there are differences in the probability of adjustment between the first and later periods after adjustment. Under the Calvo model, these scatter plots should just

⁸My results are not sensitive to other treatments of product substitution including treating product substitutions as separate quote-lines and treating all product substitutions as price changes.

be single points, while under a multi-sector Calvo model, they should all lie on the 45 degree line shown in black. In fact, the majority of the observations lie above the 45 degree line: items are more likely to adjust in the period after adjustment than if they did not just adjust. This feature is particularly pronounced for the Dominick’s data.

While duration-dependence is clearly evident when all durations are stratified into only two different duration groups, it is still possible that, overall, duration-dependence has little quantitative importance. Furthermore, even in this sample case, the importance of duration-dependence clearly varies across items. Some items are more likely to change in the period following adjustment while others are less likely to change. As shown in Section 2, only by computing the overall duration distribution can we assess the importance of duration-dependence for the aggregate response to monetary shocks.

In principle, calculating the empirical duration distribution nonparametrically⁹ is straightforward given long enough price observations: for each item¹⁰ j , the weight on duration k is given by

$$w_{j,k} = \frac{\text{\#observations of current duration } k \text{ for item } j}{\text{total number observations for item } j}.$$

The overall duration distribution is then given by $\omega_k \equiv \int f(j)w_{j,k}dj$ where $f(j)$ is the expenditure weight on item j . Similarly, the duration distribution under the Calvo model is given by $\omega_k = (1 - \alpha)\alpha^{k-1}$ where $1 - \alpha = \int f(j)w_{j,0}dj$. In the multi-sector Calvo model, the duration distribution for each item is given by $w_{j,k} = w_{j,0}(1 - w_{j,0})^{k-1}$ and the overall distribution is again given by $\int f(j)w_{j,k}dj$.¹¹ The major complication that arises in computing the empirical duration distribution is the presence of censored spells. Because prices are only sampled during a finite window, even if a spell actually lasts for an extremely long time, no price spells longer than the sampling window will be recorded. How should these censored spells be treated when calculating the price duration distribution in the data? One extreme solution is to simply exclude censored spells from the data and to only calculate the distribution of price durations over spells with certain duration. This is the approach taken by Klenow and Kryvtsov [2008] when calculating their pooled duration hazard. However, excluding censored spells, particularly in the CPI data set, is problematic for three reasons:

- 1) In the CPI data, nearly 50% of all price observations belong to censored spells so that excluding censored spells means excluding a large fraction of the data¹².

⁹A number of recent papers have estimated hazards (which have a one-to-one relationship with the duration distribution) for both BLS and Dominick’s data but they require assumptions on the general functional form of the hazard function.

¹⁰This calculation assumes that sectors correspond to individual items. This is essentially attributing as much weight as possible to cross-item heterogeneity and as little to duration-dependence as possible. Less conservative procedures, that assume more aggregated sectors, imply greater importance for duration-dependence.

¹¹Note that $w_{j,0}$ is item j ’s average frequency of adjustment.

¹²Censored observations account for a greater percentage of all observations towards the beginning and end of the sample window, but the percent of observations belonging to a censored spell in any given month is still quite large.

2) More importantly, censored spells do not look like uncensored spells. The average monthly frequency of adjustment before excluding censored spells is equal to 18.6%. Once censored spells are excluded, the average frequency of adjustment rises to 29%. Equivalently, the average spell duration falls from 4.9 months to 2.9 months when censored spells are excluded. Thus, censored spells are much longer than uncensored spells on average, even if the time of censoring is counted as a price change. If prices are assumed to continue after the time of censoring then the difference is even more extreme.

3) Under a Calvo model of price adjustment, price durations can be arbitrarily long with positive probability. With sample censoring, price durations can be no longer than the sample window even if the price spell in reality lasts much longer than the sample window. Thus, in order to make a fair comparison between the predictions of a Calvo model with a constant probability of adjustment and the distribution implied by the data, one should allow for the possibility that price spells continue to last beyond the sample window.

For these reasons, instead of excluding censored spells¹³, I instead include them and estimate their length. This requires strong assumptions, but nevertheless, imputing censored spells using a conservative procedure should provide a better picture of the actual distribution of price durations than would be obtained by excluding all of the relatively long, censored spells in the data set. In the benchmark analysis, I calculate the average frequency of adjustment for each item and then assume that censored spells end with this probability¹⁴. Thus, I essentially impose a Calvo model on censored spells. This will tend to understate the importance of duration-dependence to the extent that the true, unobserved hazard is downward sloping, as is suggested by hazards estimated on uncensored spells¹⁵.

3.3 Empirical Results

Once censored spells are imputed it is straightforward to compute the empirical duration distribution, the distribution under the Calvo model, and the distribution under the multi-sector Calvo model. Figures 3 and 4 plot the three duration distributions for CPI and Dominick's data, respectively. In each data set, more of the weight under the empirical duration distribution is distributed at long price durations than even

¹³Left censored spells are still excluded, but represent a smaller fraction of the sample. Results are not sensitive to their inclusion under a similar procedure to estimate their length.

¹⁴The validity of this procedure requires that censoring is at random. In the CPI database, this is likely to be the case, as product substitutions are already explicitly merged in my price filtering procedure. The majority of censored spells then arise because the BLS periodically rotates out items in a time-dependent manner. This procedure is independent of the price behavior of the item. Furthermore, observed prices do not systematically differ from average at the time of censoring. Censoring in general appears to be less important in the Dominick's data. Only a relatively small fraction of price observations belong to censored spells. Furthermore, sales do not differ systematically from average at the time of censoring, suggesting that censoring is not driven by e.g. the removal of items with weak sales.

¹⁵An even more conservative procedure assumes that prices change at the time of censoring. The importance of duration-dependence survives in this case although its effect is dampened.

under the multi-sector Calvo model¹⁶. Under the generalized time-dependent model, the cumulative output impulse response is given by $\widehat{Y}_{tot} = \sum_{k=0}^{\infty} k\omega_k$ so that long durations matter disproportionately for monetary non-neutrality. Thus, differences at long durations are substantially more important than differences at short durations. Figures 5 and 6 plot each duration’s contribution to monetary non-neutrality, $k\omega_k$. In these plots it is clear that differences between the empirical duration distribution and the multi-sector Calvo model are magnified at long durations. Further, the spike in the adjustment hazard documented by Klenow and Kryvtsov [2008] shows up clearly in the contribution to monetary non-neutrality from that period forward, but it is not of huge global importance.

Table 1 reports \widehat{Y}_{tot} for the two different data sets under the three different time-dependent models. In the CPI, moving from Calvo to the generalized time-dependent model, calibrated to match the empirical duration distribution, increases monetary non-neutrality by just over 100%. Moving between the two models with Dominick’s data increases non-neutrality by 230%. In the CPI, 32% of this increase is driven by duration-dependence. That is, moving from the Calvo model to the multi-sector Calvo model increases monetary non-neutrality by 70%, and the remainder of the 102% increase comes from relaxing the assumption of constant adjustment probabilities at the item level. In the Dominick’s data, 54% of the increase is driven by duration-dependence. Thus, even allowing for item-specific adjustment probabilities as in Alvarez and Burriel [2010] leaves a very large fraction of the empirical duration distribution unexplained.

These quantitative effects are quite large. For another perspective on the magnitude of these changes, I perform the exercise proposed by Carvalho [2006]. I ask, what frequency of adjustment in the Calvo model best approximates the impulse response function in the multi-sector Calvo model and in the unrestricted time-dependent model. For each Calvo frequency of adjustment, I compute the output impulse response function and compare this to the impulse response function in the multi-sector Calvo and generalized time-dependent models and find the frequency that minimizes the mean squared error. Table 2 reports these numbers. In both data sets, the required frequency of adjustment roughly halves when moving from the Calvo model to the unrestricted, generalized time-dependent model¹⁷. Again, the effects of duration-dependence are large in both data sets, but the relative importance is greater in the Dominick’s data.

Many plausible explanations exist for the different relative importance of duration-dependence in the CPI and Dominick’s data. Perhaps the most intuitive explanation for differences in the relative importance of duration-dependence between the two data sets is differences in sample representativeness. The Dominick’s data set only covers a set of 29 product categories sold in grocery stores while the CPI research database covers

¹⁶Bootstrapped standard errors are calculated but not reported, for clarity. The error bands are extremely small since the data sets contain millions of observations, so differences between the three different series are statistically significant.

¹⁷The baseline frequency of adjustment is somewhat higher than the mean frequency numbers reported by Nakamura and Steinsson [2008]. This is because items with no price changes must be excluded and because left-censored spells, which are on average longer than uncensored spells are also excluded.

prices that make up approximately 70% of U.S. expenditure. If there is heterogeneity in duration-dependence across items, as figures 1 and 2 suggest, then perhaps food items just exhibit greater duration-dependence than other product categories. This can be at least partially investigated by redoing the CPI analysis using only food expenditures data. These items should be similar in nature to those sold in the Dominick's grocery store data, so if they exhibit duration-dependence in the CPI that is similar in magnitude to that observed in the Dominick's data, then this is potentially the primary explanation for differences between the two data sets. However, redoing the CPI analysis exclusively on food expenditures, I find that the generalized time-dependent model generates 22% more monetary non-neutrality than the multi-sector Calvo model, while for the CPI as a whole, this difference is 19%. Thus, a lack of representativeness of Dominick's data is unlikely to be a significant part of the explanation for the differences between the two data sets. While duration-dependence does appear to be of greater relative importance for food expenditures than for all items in the CPI, the difference is not nearly large enough to explain the whole discrepancy between the two data sets.

Another important difference between the Dominick's and CPI data is sampling frequency. Dominick's scanner data is recorded weekly while the CPI database only records prices once each month. If the probability of adjustment does not depend on duration then this is not problematic, but when there is duration-dependence, sampling at lower frequencies can potentially greatly bias the calculation of the duration distribution for Calvo models. If the probability of adjustment is decreasing in duration, then sampling at lower frequencies means that many spells of short duration are missed so that the estimated frequency of adjustment falls. Under the Calvo model, a change in the frequency of adjustment alters the entire shape of the duration distribution. Thus, missing a small number of short spells due to lower frequency sampling can greatly increase the implied long duration weight and the implied monetary non-neutrality under the Calvo model. This bias is also stronger at low frequencies of adjustment than at high frequencies of adjustment, so by Jensen's inequality the absolute importance of sampling frequency bias is even greater under the multi-sector Calvo model. In contrast, the unrestricted duration distribution need not have a strong link between the average frequency of adjustment and the overall duration distribution. Missing a few short spells when calculating the empirical duration distribution does not change the entire shape of the distribution: the unrestricted duration distribution is much more robust to sampling frequency than is the Calvo model.

This bias can be estimated quantitatively using the weekly Dominick's data. By artificially sampling the weekly Dominick's data at monthly frequencies and recalculating the duration distribution, the effects of sampling frequency on the implied duration distribution can be estimated. Since the underlying data is the same, any differences between the duration distribution under weekly sampling and that under monthly

sampling is driven by the sampling frequency. Artificially sampling the Dominick’s data monthly leads to a 15% increase in monetary non-neutrality under the empirical distribution while it leads to a 40% increase under the multi-sector Calvo model. The increase in monetary non-neutrality from weekly sampling in a standard Calvo model is nearly 60%. Together, this implies that nearly 50% of the empirical duration distribution’s departures from a benchmark Calvo model are explained by duration-dependence in the CPI, if sampling frequency bias in the CPI is similar to that in the Dominick’s data.

This analysis suggests that sampling frequency likely explains most of the difference between the CPI and Dominick’s data. After the somewhat ad hoc correction for sampling frequency bias, duration-dependence likely accounts for around half of departures from the Calvo model in both the CPI and in Dominick’s data.

4 Extensions to the Benchmark Model

The generalized time-dependent model described thus far assumed that nominal output follows a random walk and that there are no strategic-complementarities. In this section I relax both of these assumptions and show that they increase the importance of duration-dependence for monetary non-neutrality. First, I extend the baseline model to allow for more empirically realistic autocorrelation in the growth rate of nominal output. That is, I assume that nominal output follows an ARIMA(1,1,0) process:

$$\Delta \log \mathcal{Y}_t = \rho \Delta \log \mathcal{Y}_{t-1} + \epsilon_t.$$

In this case, inflation and output dynamics no longer depend on the duration distribution alone but instead depend on the duration distribution within each of the j sectors in the economy. The following proposition extends Proposition 1 to the case of autocorrelated nominal output shocks:

Proposition 2 *Assume that each firm in sector j , whose price has lasted for k periods, adjusts its price with probability $1 - \alpha_{j,k}$. Further, assume that nominal output follows an ARIMA(1,1,0) and that there are no strategic complementarities. Let $\omega_{j,k}$ be the population weight with duration k in sector j and let $f(j)$ be sector j ’s weight in the overall economy. Then the impulse response function of the price level to a nominal output shock ϵ_t is given by*

$$\frac{\partial E_t \log P_{t+n}}{\partial \epsilon_t} = \frac{1}{1-\rho} \sum_{k=0}^n \omega_k - \int f(j) \frac{1}{1-\rho} \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \rho^k}{\sum_{k=0}^{\infty} w_{j,k} \beta^k} \sum_{l=0}^n w_{j,l} \rho^{n+1-l} dj,$$

so that the impulse response to the output gap is given by

$$\frac{\partial E_t \log \widehat{Y}_{t+n}}{\partial \epsilon_t} = \frac{1 - \rho^k}{1 - \rho} - \frac{1}{1 - \rho} \sum_{k=0}^n \omega_k + \int f(j) \frac{1}{1 - \rho} \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \rho^k}{\sum_{k=0}^{\infty} w_{j,k} \beta^k} \sum_{l=0}^n w_{j,l} \rho^{n+1-l} dj.$$

Proof. See Appendix. ■

Once the random walk assumption for nominal output is relaxed, the duration distribution within sectors rather than within the entire economy now matters for aggregate dynamics. Thus, to empirically assess the degree of monetary non-neutrality under different assumptions for the duration hazard, I must take a stronger stand on what a sector in the economy is. The results are robust to a wide variety of different assumptions, but I present results under sectors corresponding to the 29 product categories in the Dominick’s data. These categories are defined more narrowly than the categories in the CPI and are thus less likely to have concerns of survivor bias, so for brevity I only report results for Dominick’s data. However, results are similar for the CPI when the CPI is divided into 272 ELI sectors. Results are also similar for both data sets when these initial sectors are further subdivided into high and low frequency adjusters.

To calibrate the model with autocorrelated nominal output growth, I assume, following Midrigan [2008], that $\rho = .61$ at the monthly frequency and $\rho = .9$ at the weekly frequency. I set the discount factor to 0.96 annually. Figure 7 displays the output impulse response to a $(1 - \rho)\%$ shock to the nominal output growth rate¹⁸. Allowing for autocorrelation clearly amplifies the difference between the multi-sector Calvo model and the generalized time-dependent model. This is because the effect of the initial shock builds up over time so that price spells with long duration become even more important than when nominal output follows a random walk. Since the empirical duration distribution has greater long duration weight than the duration distribution under the multi-sector Calvo model, the effects of money shocks are amplified.

I next explore how adding strategic complementarities to the benchmark model influences the aggregate dynamics. Once strategic-complementarities are added, there is no longer a closed form solution for the generalized time-dependent model. However, under the assumption that lagged nominal inflation shocks no longer affect inflation after a finite number of periods, the model can be solved numerically. The appendix discusses the solution in detail.

As in Carvalho [2006], I find that strategic complementarities increase the output response to nominal shocks in the generalized time-dependent model, the multi-sector Calvo model and the Calvo model, but figure 8 shows that the increase is greatest for the generalized time-dependent model. This is because the *strategic-interaction* effect that Carvalho [2006] discusses applies whenever there are some firms that do not change prices for long periods of time, whether the reason for their inaction is fixed heterogeneity

¹⁸This implies that the cumulative effect of the shock on nominal output is 1%.

across items or an adjustment hazard that decreases with duration. Since the empirical distribution has a greater fraction of firms that have not adjusted prices in a very long time than under Calvo models, the strategic-interaction effect is amplified. Since there are more long price spells, strategic-complementarities imply that items that change prices change by smaller amounts, and so inflation responds more sluggishly and the real output response to nominal shocks is magnified.

Thus, while the benchmark generalized time-dependent model makes many assumptions for simplicity, these assumptions are not central to the result that duration-dependence is quantitatively important for monetary non-neutrality. Autocorrelated money growth shocks and strategic-complementarities both lead to increases in the importance of duration-dependence for aggregate dynamics.

5 Endogenous Duration-Dependence

The empirical results in this paper document that duration-dependence is present in the data. However, in my theoretical results, I analyze the implications of exogenously matching this duration-dependence, yet I make no attempt to explain why there is duration-dependence in the data.

Investigating mechanisms that could endogeneously generate the empirical duration distribution is thus an interesting area for future research. It is well known that different pricing mechanisms can have vastly different implications for monetary non-neutrality. Models of state-dependent pricing tend to generate little monetary non-neutrality while models with time-dependent pricing generate substantially greater degrees of non-neutrality.

There are a number of possible explanations that could give rise to a probability of adjustment that decreases with the duration of a price spell. These explanations fall largely under two classes: 1) Time varying costs of price adjustment and 2) Time varying values of price adjustment. Under the first heading, customer attachment models as well as models with costly information acquisition can make the cost of an adjustment increase with the time since last adjustment. In a detailed study of a single large firm's pricing decisions Zbaracki et al. [2004] find that information acquisition and customer negotiation costs are among the most significant sources of price rigidity. If customers become attached to a given price and this attachment grows with time, then the customer costs of price changes may also grow with time. Vavra [2009] builds such a customer attachment model and shows that it can generate a downward sloping duration hazard of price adjustment. Nakamura and Steinsson [2005] build a similar customer attachment model to explain temporary sales.

Similarly, with costly information acquisition, it may be less costly to change a price if it has recently changed. Once information has been gathered, price-setters may have better information about market

conditions or may be devoting more of their energy to pricing decisions. If this is the case then it may be less costly to change the price a second time following an initial price change. In recent work, Woodford [2008] builds a pricing model with constrained information. While his model does not generate a downward sloping adjustment hazard because he assumes that price-setters are perfectly informed for exactly one-period after holding a price review, his model could be extended to allow for multiple-periods of review and lagged information on market conditions following a price review and would then likely generate a downward sloping hazard.

Those explanations would generate a probability of adjustment that falls with price duration because the cost of adjustment rises with price duration. Alternatively, the value of adjustment could fall with price duration while firms face a constant cost of adjustment. Again customer-firm relationships could potentially generate these implications. If customers get used to a given price and their demand becomes less elastic with the duration of a price spell, then the elasticity of demand could rise sharply following a price change. With a higher elasticity of demand it is more important for firms to get the price correct. This could induce them to change prices rapidly for some period of time after the initial change, until a sequence of idiosyncratic shocks left them with no desire to change prices for several periods, at which point the elasticity of demand and the probability of adjustment would again fall. Such an explanation would be endogenous to an individual firm's pricing decision; if it changes its price it faces more elastic demand. Similar explanations could hold at the aggregate level. Bloom [2009] argues that time-varying uncertainty has a significant impact on firm's investment decisions. Time-varying uncertainty could easily lead to periods in which firms wish to change prices often and other periods when they rarely want to change prices. This would imply a downward sloping duration hazard of adjustment so that price changes would be clustered in time within a given firm.

These explanations are by no means exhaustive or mutually exclusive, and I leave their exploration to future work. Homescan data as well as sales data in the Dominick's data set could potentially be useful for differentiating various motives for duration-dependent probabilities of adjustment.

6 Conclusion

In this paper, I use price micro data to analyze the importance of duration-dependent probabilities of adjustment for the empirical duration distribution of price spells. I show that in a generalized time-dependent pricing model, the duration distribution completely characterizes aggregate dynamics. I then show that there is clear duration-dependence at the item level with substantial differences between the first period after adjustment and later periods. Finally, I provide evidence that duration-dependence is important

for the overall shape of the duration distribution.

Moving from the duration distribution generated by a Calvo model to the duration distribution measured in the data increases monetary non-neutrality in the generalized time-dependent model by 100-230%, and as much as half of this increase is the result of duration-dependent adjustment probabilities rather than cross-item heterogeneity in the Calvo frequency of adjustment. The contribution of duration-dependence to monetary non-neutrality is greater in the Dominick's data than under the CPI data, but I present evidence that sampling frequency bias in the CPI likely accounts for a significant portion of this difference. After a rough correction for this sampling frequency bias, I find that roughly one half of departures from the Calvo model are explained by duration-dependence in both the CPI and Dominick's data.

While the benchmark analysis makes several simplifying assumptions, I also provide evidence that autocorrelated nominal output growth shocks and strategic-complementarities amplify the importance of duration-dependence.

Time-dependent pricing models are popular in policy analysis, but they are missing a first order feature of the data. Matching empirical duration-dependence is quantitatively important in these models. If Dominick's data is used as a benchmark, then duration-dependence is even more important for monetary non-neutrality than is the inclusion or exclusion of temporary price changes, a topic that has received much recent attention¹⁹.

The fact that matching the empirical duration distribution generates large amounts of monetary non-neutrality even without strategic complementarities may help reconcile research that estimates large and persistent responses to nominal output shocks at the aggregate level with the recent work of Bils et al. [2009], that finds strong strategic complementarities are inconsistent with the behavior of their measure of "reset price inflation".

The results in this paper suggest that more attention should be devoted to duration-dependence as it likely drives a large fraction, if not the majority, of departures from the Calvo model and potentially has very large quantitative effects on monetary non-neutrality.

¹⁹Keohoe and Midrigan [2008] Nakamura and Steinsson [2008]

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7 Appendix

7.1 Autocorrelated Money Growth Shocks

Now assume that instead of following a random walk, nominal output follows an ARIMA(1,1,0) process:

$$\Delta \log \mathcal{Y}_t = \rho \Delta \log \mathcal{Y}_{t-1} + \epsilon_t.$$

Ignoring terms independent of policy, the above algebra again implies that

$$\log p_{j,t}^* = \frac{\sum_{k \geq 0} G_{j,k} \beta^k E_t \log \mathcal{Y}_{t+k}}{\sum_{i \geq 0} G_{j,k} \beta^i}$$

but now $E_t \log \mathcal{Y}_{t+k}$ is no longer a constant, since

$$\frac{\partial E_t \log \mathcal{Y}_{t+k}}{\partial \epsilon_t} = \frac{(1 - \rho^{k+1})}{(1 - \rho)}.$$

For ease of notation, we can use that

$$G_{j,k} = \frac{w_{j,k}}{w_{j,0}}$$

to rewrite the reset price in terms of the distribution of firms across times since last adjustment:

$$\log p_{j,t}^* = \frac{\sum_{k \geq 0} w_{j,k} \beta^k E_t \log \mathcal{Y}_{t+k}}{\sum_{k \geq 0} w_{j,k} \beta^k}. \quad (2)$$

The price level will again be given by

$$\log P_t = \int f(j) \sum_{i \geq 0} w_{j,i} \log p_{j,t-i}^* dj. \quad (3)$$

Taking lags of (2) we get that

$$\log p_{j,t-i}^* = \frac{\sum_{k \geq 0} w_{j,k} \beta^k E_{t-i} \log \mathcal{Y}_{t-i+k}}{\sum_{k \geq 0} w_{j,k} \beta^k}$$

so that substituting into (3) gives

$$\log P_t = \int f(j) \sum_{i \geq 0} w_{j,i} \frac{\sum_{k \geq 0} w_{j,k} \beta^k E_{t-i} \log \mathcal{Y}_{t-i+k}}{\sum_{i \geq 0} w_{j,k} \beta^k} dj$$

Then, noting that all expectations before time t don't depend on ε_t we get that

$$\begin{aligned}
\frac{\partial E_t \log P_t}{\partial \varepsilon_t} &= \frac{\partial}{\partial \varepsilon_t} E_t \int f(j) \sum_{i \geq 0} w_{j,i} \frac{\sum_{k \geq 0} w_{j,k} \beta^k E_{t-i} \log \mathcal{Y}_{t-i+k}}{\sum_{i \geq 0} w_{j,k} \beta^k} dj \\
&= \frac{\partial}{\partial \varepsilon_t} E_t \int f(j) w_{j,0} \frac{\sum_{k \geq 0} w_{j,k} \beta^k E_t \log \mathcal{Y}_{t+k}}{\sum_{k \geq 0} w_{j,k} \beta^k} dj \\
&= \int f(j) w_{j,0} \frac{\sum_{k \geq 0} w_{j,k} \beta^k \frac{(1-\rho^{k+1})}{(1-\rho)}}{\sum_{k \geq 0} w_{j,k} \beta^k} dj
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial E_t \log P_{t+1}}{\partial \varepsilon_t} &= \int f(j) \left[w_{j,1} \frac{\partial E_t \sum_{k=0}^{\infty} w_{j,k} \beta^k \log \mathcal{Y}_{t+k}}{\partial \varepsilon_t \sum_{k=0}^{\infty} w_k \beta^k} + w_{j,0} \frac{\partial E_t E_{t+1} \sum_{k=0}^{\infty} w_{j,k} \beta^k \log \mathcal{Y}_{t+1+k}}{\partial \varepsilon_t \sum_{k=0}^{\infty} w_k \beta^k} \right] dj \\
&= \int f(j) \left[w_{j,1} \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \frac{(1-\rho^{k+1})}{(1-\rho)}}{\sum_{k=0}^{\infty} w_k \beta^k} + w_{j,0} \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \frac{(1-\rho^{k+2})}{(1-\rho)}}{\sum_{k=0}^{\infty} w_k \beta^k} \right] dj \\
&= \int f(j) \frac{1}{1-\rho} \left[w_{j,1} + w_{j,0} - [w_{j,1} \rho + w_{j,0} \rho^2] \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \rho^k}{\sum_{k=0}^{\infty} w_{j,k} \beta^k} \right] dj
\end{aligned}$$

and in general

$$\begin{aligned}
\frac{\partial E_t \log P_{t+n}}{\partial \varepsilon_t} &= \int f(j) \left[\frac{1}{1-\rho} \sum_{k=0}^n w_{j,k} - \frac{1}{1-\rho} \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \rho^k}{\sum_{k=0}^{\infty} w_{j,k} \beta^k} \sum_{l=0}^n w_{j,l} \rho^{n+1-l} \right] dj \\
&= \frac{1}{1-\rho} \sum_{k=0}^n \omega_k - \int f(j) \frac{1}{1-\rho} \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \rho^k}{\sum_{k=0}^{\infty} w_{j,k} \beta^k} \sum_{l=0}^n w_{j,l} \rho^{n+1-l} dj.
\end{aligned}$$

Then

$$\frac{\partial E_t \log \widehat{Y}_{t+n}}{\partial \varepsilon_t} = \frac{1-\rho^{n+1}}{1-\rho} - \frac{1}{1-\rho} \sum_{k=0}^n \omega_k + \int f(j) \frac{1}{1-\rho} \frac{\sum_{k=0}^{\infty} w_{j,k} \beta^k \rho^k}{\sum_{k=0}^{\infty} w_{j,k} \beta^k} \sum_{l=0}^n w_{j,l} \rho^{n+1-l} dj.$$

>From this expression, it is clear the the impulse response function no longer depends solely on the overall duration distribution in the economy but instead depends on the duration distribution within the j sectors. If I impose all sectors to have identical duration weights so that I allow for heterogeneity across time but not across items (in contrast Carvalho [2006] allows for heterogeneity across items but not across time), then the expression reduces to

$$\frac{\partial E_t \log \widehat{Y}_{t+n}}{\partial \epsilon_t} = \frac{1 - \rho^k}{1 - \rho} - \frac{1}{1 - \rho} \sum_{k=0}^n \omega_k + \frac{1}{1 - \rho} \frac{\sum_{k=0}^{\infty} \omega_k \beta^k \rho^k}{\sum_{k=0}^{\infty} \omega_k \beta^k} \sum_{l=0}^n \omega_l \rho^{n+1-l} dj.$$

7.2 Strategic Complementarities

Let each firm in sector j , with price duration k , adjust its price with probability $1 - \alpha_{j,k}$. Assume that nominal output follows a random walk and assume that there are strategic complementarities $\zeta < 1$ where firm j 's reset price solves

$$\sum_{k=0}^{\infty} \frac{\omega_{j,k}}{\omega_{j,0}} \beta^k E_t \left[\log p_{j,t}^* - \log P_{t+k} - \zeta \left(\log \widehat{Y}_{t+k} - \log \widehat{Y}_{t+k}^n \right) \right] = 0.$$
 Further, assume that lagged nominal inflation shocks no longer affect inflation after a finite number of periods, M , so that $\varepsilon_{t-m} = 0$ for $m > M$ where ε_{t-m} is the innovation to nominal output at time $t - m$. Then the impulse response to the price level $\frac{\partial \log P_{t+k}}{\partial \varepsilon_t} = \sum_{i=0}^k \psi_i$ where ψ_i solve a finite system of $M + 1$ equations in the lagged nominal output innovations and $\psi_k = 0$ for $k > M$.

With strategic complementarities, the firm now resets prices to solve

$$\sum_{k=0}^{\infty} \frac{\omega_{j,k}}{\omega_{j,0}} \beta^k E_t \left[\log p_{j,t}^* - \log P_{t+k} - \zeta \left(\log \widehat{Y}_{t+k} - \log \widehat{Y}_{t+k}^n \right) \right] = 0$$

This implies that the reset price is equal to

$$\log p_{j,t}^* = (1 - \zeta) E_t \frac{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k [\log P_{t+k}]}{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k} + \zeta \log \mathcal{Y}_t + TIP \quad (4)$$

and since the price level is again equal to

$$\log P_t = \int f(j) \sum_{i \geq 0} w_{j,i} \log p_{j,t-i}^* dj$$

we get that

$$\log P_t - \log P_{t-1} = \int f(j) \sum_{i=0}^{\infty} w_{j,i} [\log p_{j,t-i}^* - \log p_{j,t-1-i}^*] dj. \quad (5)$$

Then subtracting (4) from itself lagged one period gives

$$\log p_{j,t-i}^* - \log p_{j,t-1-i}^* = (1 - \zeta) \frac{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k (E_{t-i} \log P_{t-i+k} - E_{t-i-1} \log P_{t-i-1+k})}{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k} + \zeta [\log \mathcal{Y}_{t-i} - \log \mathcal{Y}_{t-i-1}]$$

Substituting this into (5) gives that

$$\begin{aligned}
\log P_t - \log P_{t-1} &= \int f(j) \sum_{i=0}^{\infty} \omega_{j,i} [\log p_{j,t-i}^* - \log p_{j,t-1-i}^*] dj \\
&= \int f(j) \sum_{i=0}^{\infty} \omega_{j,i} \left[(1-\zeta) \frac{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k (E_{t-i} \log P_{t-i+k} - E_{t-i-1} \log P_{t-i-1+k})}{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k} \right. \\
&\quad \left. + \zeta [\log \mathcal{Y}_{t-i} - \log \mathcal{Y}_{t-i-1}] \right] dj \tag{6}
\end{aligned}$$

Now conjecture that

$$\log P_t = \sum_{l=0}^{\infty} \psi_l \log \mathcal{Y}_{t-l},$$

and we look for a solution of this form. Now, assuming for simplicity that nominal output follows a random walk, we get:

$$\begin{aligned}
&\sum_{k=0}^{\infty} \omega_{j,k} \beta^k (E_{t-i} \log P_{t-i+k} - E_{t-i-1} \log P_{t-i-1+k}) \\
&= \sum_{k=0}^{\infty} \omega_{j,k} \beta^k \left(E_{t-i} \sum_{l=0}^{\infty} \psi_l \log \mathcal{Y}_{t-i+k-l} - E_{t-i-1} \sum_{l=0}^{\infty} \psi_l \log \mathcal{Y}_{t-i-1+k-l} \right) \\
&= \sum_{k=0}^{\infty} \omega_{j,k} \beta^k \left(\sum_{l=0}^k \psi_l \log \mathcal{Y}_{t-i} + \sum_{l=k+1}^{\infty} \psi_l \log \mathcal{Y}_{t-i+k-l} - \sum_{l=0}^k \psi_l \log \mathcal{Y}_{t-i-1} - \sum_{l=k+1}^{\infty} \psi_l \log \mathcal{Y}_{t-i-1+k-l} \right)
\end{aligned}$$

Substituting into (6)

gives that

$$\begin{aligned}
&\log P_t - \log P_{t-1} \\
&= \int f(j) \sum_{i=0}^{\infty} \omega_{j,i} \left[(1-\zeta) \frac{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k (E_{t-i} \log P_{t-i+k} - E_{t-i-1} \log P_{t-i-1+k})}{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k} + \zeta [\log \mathcal{Y}_{t-i} - \log \mathcal{Y}_{t-i-1}] \right] dj \\
&= \int f(j) \sum_{i=0}^{\infty} \omega_{j,i} \left[\frac{(1-\zeta) \sum_{k=0}^{\infty} \omega_{j,k} \beta^k \left(\sum_{l=0}^k \psi_l \log \mathcal{Y}_{t-i} + \sum_{l=k+1}^{\infty} \psi_l \log \mathcal{Y}_{t-i+k-l} - \sum_{l=0}^k \psi_l \log \mathcal{Y}_{t-i-1} - \sum_{l=k+1}^{\infty} \psi_l \log \mathcal{Y}_{t-i-1+k-l} \right)}{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k} + \zeta [\log \mathcal{Y}_{t-i} - \log \mathcal{Y}_{t-i-1}] \right] dj \\
&= \int f(j) \sum_{i=0}^{\infty} \omega_{j,i} \left[(1-\zeta) \frac{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k \left(\varepsilon_{t-i} \sum_{l=0}^k \psi_l + \sum_{l=k+1}^{\infty} \psi_l \varepsilon_{t-i+k-l} \right)}{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k} + \zeta \varepsilon_{t-l} \right] dj
\end{aligned}$$

Then since

$$\begin{aligned}\log P_t - \log P_{t-1} &= \sum_{l=0}^{\infty} \psi_l [\log \mathcal{Y}_{t-l} - \log \mathcal{Y}_{t-1-l}] \\ &= \sum_{l=0}^{\infty} \psi_l \varepsilon_{t-l}\end{aligned}$$

we have

$$\sum_{l=0}^{\infty} \psi_l \varepsilon_{t-l} = \int f(j) \sum_{i=0}^{\infty} \omega_{j,i} \left[(1-\zeta) \frac{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k \left(\varepsilon_{t-i} \sum_{l=0}^k \psi_l + \sum_{l=k+1}^{\infty} \psi_l \varepsilon_{t-i+k-l} \right)}{\sum_{k=0}^{\infty} \omega_{j,k} \beta^k} + \zeta \varepsilon_{t-l} \right] dj$$

Now assume that $\omega_k = 0$ for $k > M$ and that $\varepsilon_{t-i} = 0$ for $i > M$ so that current inflation depends on only a finite lag process of nominal spending shocks. The system then reduces to

$$\sum_{l=0}^M \psi_l \varepsilon_{t-l} = \int f(j) \sum_{i=0}^M \omega_{j,i} \left[(1-\zeta) \frac{\sum_{k=0}^M \omega_{j,k} \beta^k \left(\varepsilon_{t-i} \sum_{l=0}^k \psi_l + \sum_{l=k+1}^{M+k-i} \psi_l \varepsilon_{t-i+k-l} \right)}{\sum_{k=0}^M \omega_{j,k} \beta^k} + \zeta \varepsilon_{t-l} \right] dj$$

Grouping coefficients on ε_{t-i} 's we get the following system of equations:

$$\begin{aligned}\varepsilon_t &: \psi_0 = \int f(j) \left[\omega_{j,0} (1-\zeta) \frac{\sum_{k=0}^M \omega_{j,k} \beta^k \sum_{l=0}^k \psi_l}{\sum_{k=0}^M \omega_{j,k} \beta^k} + \omega_{j,0} \zeta \right] dj \\ \varepsilon_{t-1} &: \psi_1 = \int f(j) \left[\omega_{j,1} (1-\zeta) \frac{\sum_{k=0}^M \omega_{j,k} \beta^k \left(\sum_{l=0}^k \psi_l \right)}{\sum_{k=0}^M \omega_{j,k} \beta^k} + \omega_{j,1} \zeta + \omega_{j,0} (1-\zeta) \frac{\sum_{k=0}^{M-1} \omega_{j,k} \beta^k \psi_{k+1}}{\sum_{k=0}^M \omega_{j,k} \beta^k} \right] dj \\ &\dots \\ \varepsilon_{t-M} &: \psi_M = \int f(j) \left[\omega_{j,M} (1-\zeta) \frac{\sum_{k=0}^M \omega_{j,k} \beta^k \left(\sum_{l=0}^k \psi_l \right)}{\sum_{k=0}^M \omega_{j,k} \beta^k} + \omega_{j,M} \zeta + \omega_{j,M-1} (1-\zeta) \frac{\sum_{k=0}^{M-1} \omega_{j,k} \beta^k \psi_{k+1}}{\sum_{k=0}^M \omega_{j,k} \beta^k} \right. \\ &\quad \left. + \omega_{j,M-2} (1-\zeta) \frac{\sum_{k=0}^{M-2} \omega_{j,k} \beta^k \psi_{k+2}}{\sum_{k=0}^M \omega_{j,k} \beta^k} + \dots + \omega_{j,1} (1-\zeta) \frac{\omega_{j,0} \psi_{M-1} + \omega_{j,1} \beta \psi_M}{\sum_{k=0}^M \omega_{j,k} \beta^k} + \omega_{j,0} (1-\zeta) \frac{\omega_{j,0} \psi_M}{\sum_{k=0}^M \omega_{j,k} \beta^k} \right] dj,\end{aligned}$$

which, given $f(j)$ is a finite system of $M+1$ equations in $M+1$ variables. Although not particularly attractive, this system can be solved numerically for large M to compute the implied impulse response to

the price level:

$$\frac{\partial \log P_{t+k}}{\partial \varepsilon_t} = \sum_{i=0}^k \psi_i$$

and the impulse response to the output gap:

$$\frac{\partial \log \widehat{Y}_{t+k}}{\partial \varepsilon_t} = 1 - \sum_{i=0}^k \psi_i$$

Table 1		
Cumulative Percent Increase in Monthly Output Gap from 1% Increase in Nominal Output		
	CPI Data	Dominick's Data
Empirical Model	4.77%	2.11%
Multi-Sector Calvo	4.01%	1.32%
Calvo	2.36%	.64%

Table 2		
Average Frequency of Adjustment in Calvo Model Required to Match Multi-Sector Calvo and Empirical Model		
	CPI Data	Dominick's Data ²⁰
To Match Empirical Model	16.6%	13.5%
To Match Multi-Sector Calvo	19.1%	19.7%
Calvo (Baseline)	29.0%	28.1%

²⁰CPI frequencies are monthly and Dominick's frequencies are weekly

Figure 1: Item-Level Relationship in CPI Between Adjustment Probability After One Period and Adjustment at All Other Times

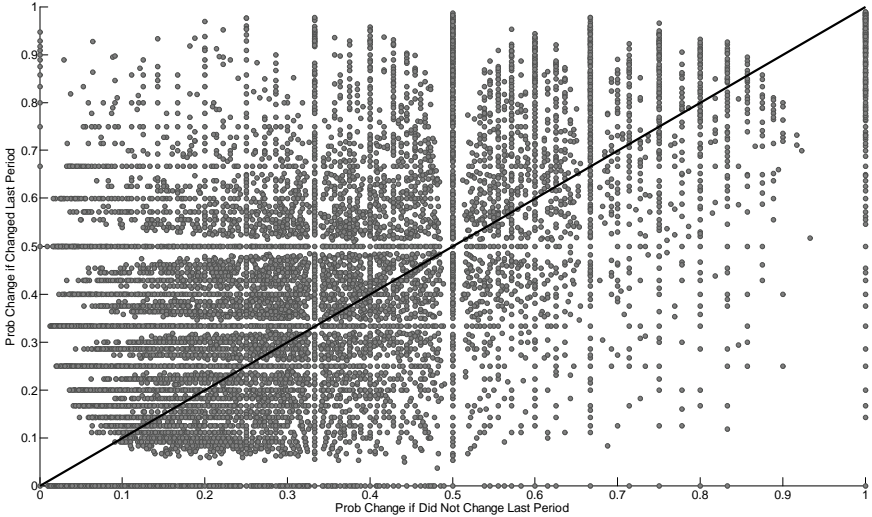


Figure 2: Item-Level Relationship in Dominick's Between Adjustment Probability After One Period and Adjustment at All Other Times

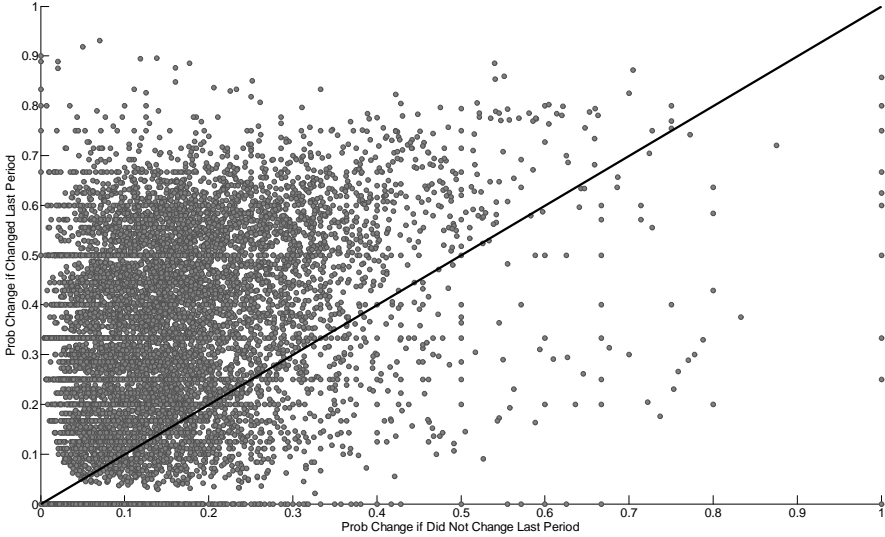


Figure 3: Duration Distribution for CPI Data

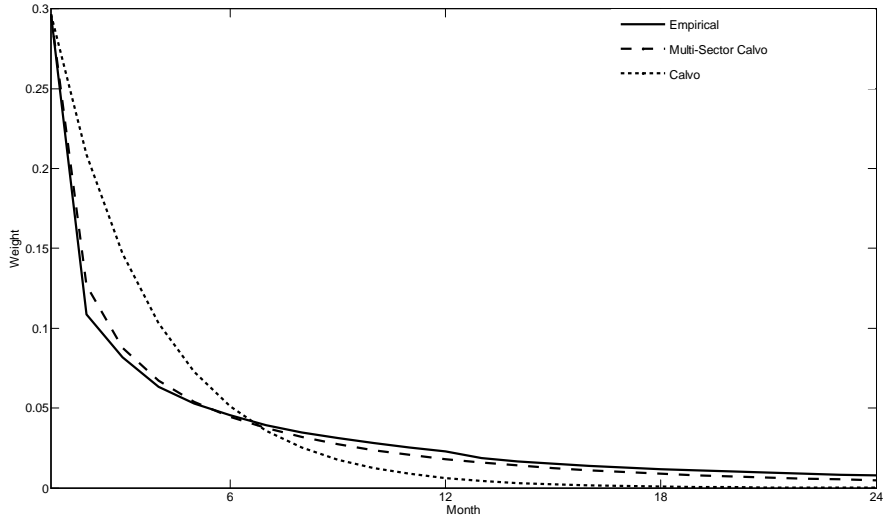


Figure 4: Duration Distribution for Dominick's Data

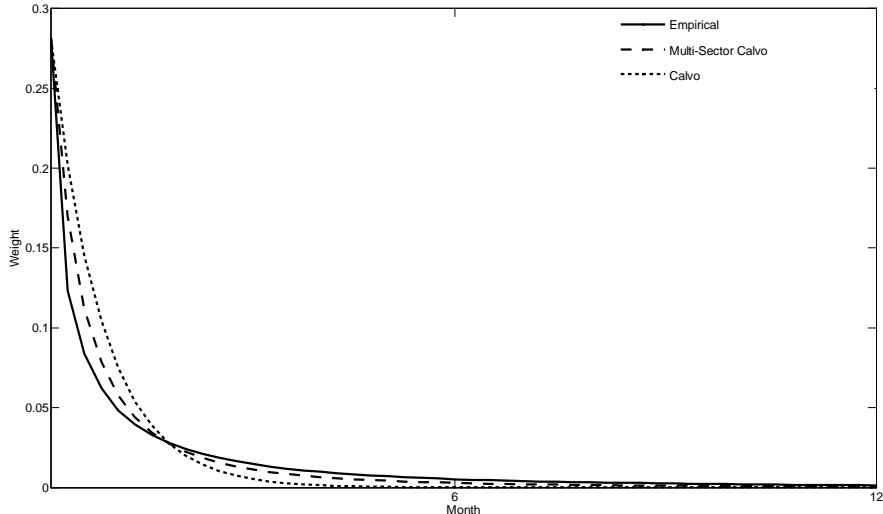


Figure 5: Each Duration's Contribution to Total Monetary Non-Neutrality. CPI

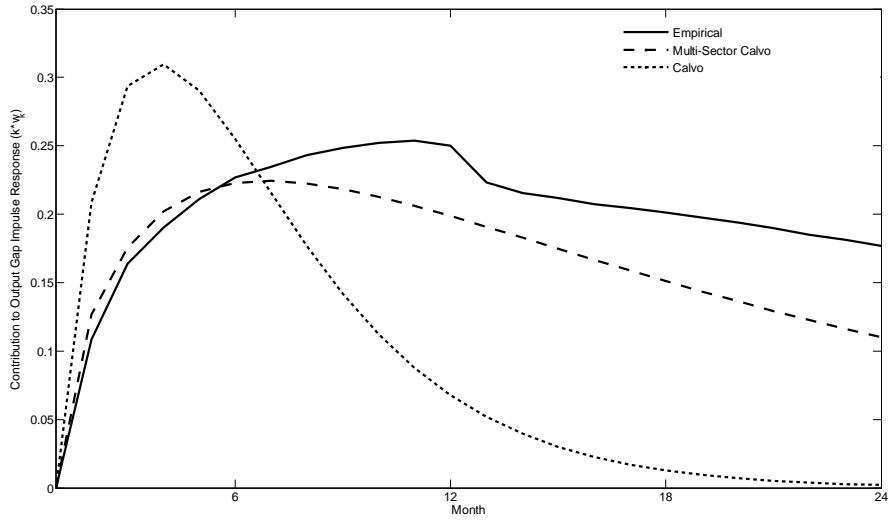


Figure 6: Each Duration's Contribution to Total Monetary Non-Neutrality. Dominick's

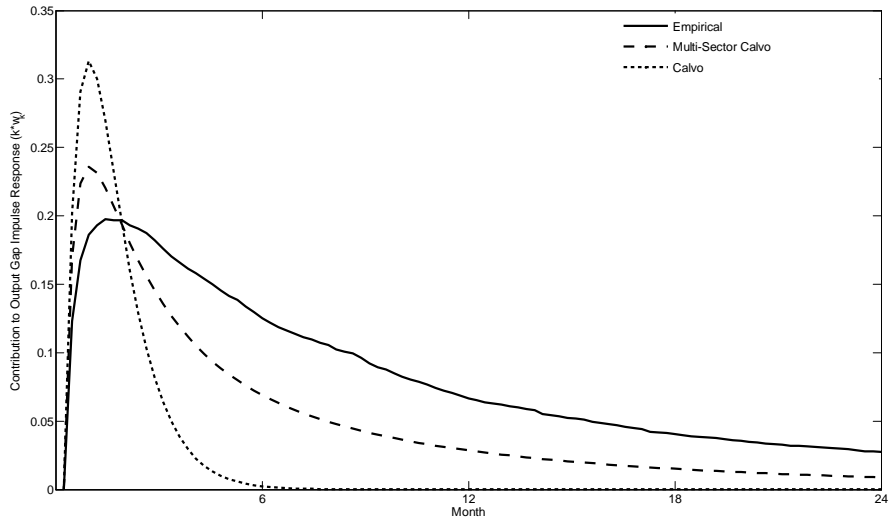


Figure 7: Output Gap Impulse Response to an Increase in the Nominal Output Growth Rate. Dominick's Data

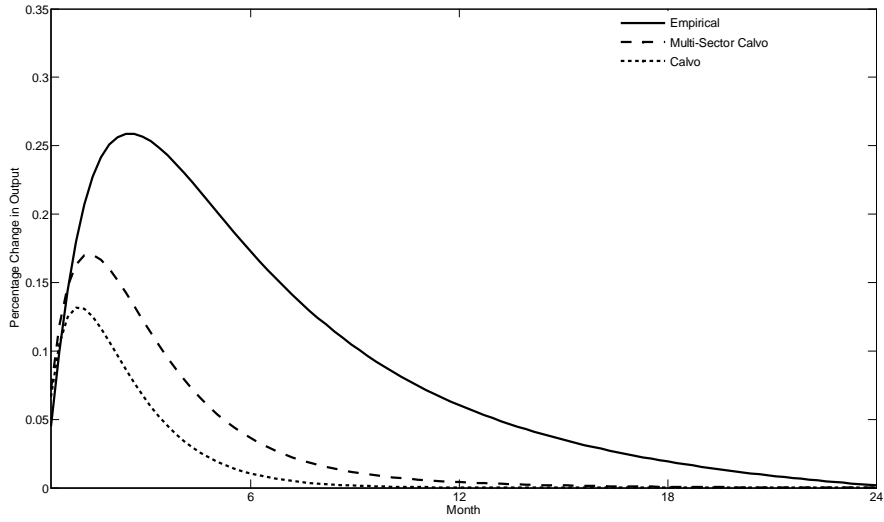


Figure 8: Effects of Strategic-Complementarities on Output Gap Impulse Response. Dominick's Data

