

## Outline

### Truth, Subderivations and the Liar

Why Should I Care about the Liar Sentence?

Uses of the Truth Concept - (i) Disquotation.

Uses of the Truth Concept - (ii) Challenging Assumptions.

Formalization of the Main Intuition.

Main Technical Results.

1. Why Should I Care about the Liar Sentence?
2. Uses of the Truth Concept - (i) Disquotation.
3. Uses of the Truth Concept - (ii) Challenging Assumptions.
4. Formalization of the Main Intuition.
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## Truth, Subderivations and the Liar

### Truth, Subderivations and the Liar

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## The Liar Sentence.

### Truth, Subderivations and the Liar

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- Let  $L$  be the sentence:  
This sentence is false.
- This sentence causes trouble.  
*If it is true, then it is false. So it can't be true. Thus, it is false.*  
*If it is false, then it is true. So it can't be false. Thus, it is true.*  
*Thus,  $L$  is true and false.*
- But no sentence can be true and false. So we have a problem.
- Is there an obvious way out? And if not, who cares? Is this a puzzle that should keep us up at night?

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## Why Should I Care about the Liar Sentence?

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## The Collapse of Logic.

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- Using the principles of logic alone, we have argued that  $L$  is both true and false; i.e.,  $L \& \neg L$ .
- (Aristotle) In classical logic, *anything* follows from a contradiction.
- To see why  $\neg L$  is true. So

$L$  or Santa Claus exists

is true.

- But  $L$  is also false. And from

$L$  or Santa Claus exists

and

$L$  is false,

it follows that Santa Claus exists.

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- So using only the principles of logic, we can prove *anything*.
- But the whole point of logic was to provide tools for distinguishing things that we are entitled to assert or infer from things we aren't entitled to assert or infer.
- So unless something we have said is wrong, the whole project of logic collapses.

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## A Quick Solution? (i)

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- Our reasoning presupposes that the Liar sentence is true or false:  
*If it is true, then it is false. So it can't be true. Thus, it is false.*
- But perhaps some sentences (such as  $L$ ) are neither true nor false.

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- **Difficulty:** So we are rejecting the claim

$L$  or  $\neg L$ .

- Presumably this means that we are endorsing

$\neg(L$  or  $\neg L)$ .

- But in classical logic, this is equivalent to

$\neg L \& \neg \neg L$ .

- And so we still have a contradiction.
- So if we want to take this approach, we have to revise classical logic quite deeply. This requires care.

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## A Quick Solution? (ii)

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- Is the Liar Sentence really a grammatical sentence? If not, we don't have to worry about it.

- **Response 1:** There are versions of the Liar that are not *manifestly* self-referential. For example:

The topmost sentence written on the blackboard in my office is false.

- Does whether a sentence is grammatical really depend on where it is or isn't written?

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## A Quick Solution? (ii)

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- **Response 2:** One can explicitly construct mathematical analogs of the Liar.

- Take the first-order language of arithmetic  $(\{0, ', +, \times\})$  and add a one-place predicate  $T(\cdot)$ .

- Then one can construct a sentence  $L$  in this language such that

$$PA \vdash L \leftrightarrow \neg T(\ulcorner L \urcorner)$$

- On what grounds could we say that  $L$  is ungrammatical?

- Given the difficulty of these challenges, we follow the modern tradition that takes grammatical Liar sentences to really exist.

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- It follows that some part of logic (as it is traditionally understood) or the way in which we understand the concept of truth must be revised.

- Here, there is no consensus.

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## Uses of the Truth Concept - (i) Disquotation.

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- What is the utility of the concept of truth?
- (Tarski, Quine) We use the concept of truth to endorse sentences / endorse descriptions of sentences.

- One use of the truth concept - 'disquotational'

'Grass is green' is true.

- Another use - to endorse a sentence under a description:

'What John said was probably true.'

(Note: maybe the speaker doesn't even know what John said)

$(\exists s) (\text{John-just-said}(s) \ \& \ \text{Probably}(\text{True}(s)))$ .

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- In order for the concept of truth to be usable in this way, we need to endorse something like the T-schema

for any proposition  $p$ ,  $T(\ulcorner p \urcorner) \leftrightarrow p$ .

- This means we must endorse

$T(\ulcorner L \urcorner) \leftrightarrow L$ .

which gets us into trouble.

- I would like to argue that the concept of truth is used in other ways, and that realizing this offers us another way out.

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- I want to point out a different use of the truth concept in *challenging assumptions*.

Imagine a prosecutor arguing:

*Suppose John was in the park at 5pm.*

*Then he would have had access to a weapon,*

*He would have had ample opportunity to kill the victim,*

*...*

*He should be convicted.*

John's lawyer retorts:

*But suppose it's not true that John was in the park at 5pm ...*

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## Uses of the Truth Concept - (ii) Challenging Assumptions.

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Uses of the Truth Concept - (i) Disquotation.

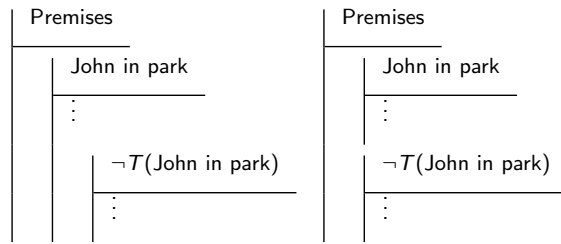
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But suppose it's not true that John was in the park at 5pm ...

- This could be read in two ways:



- Surely the second reading is correct.

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## Uses of the Truth Concept - (ii) Challenging Assumptions.

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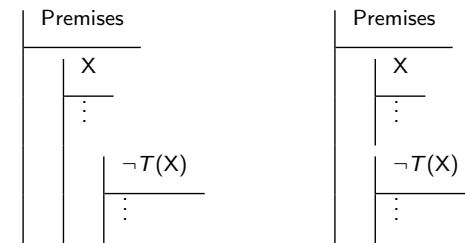
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**General Claim 1:** If  $X$  is supposed, then  $\neg T(X)$  is supposed, this *must* be read in the second way shown below rather than the first way shown below.



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- To make a supposition of the form  $\neg T(X)$  is to *challenge / distance oneself* from any assumption to the contrary.
- This act of challenging cannot be done *within* such any subderivation governed by assumption to the contrary, but must be done *outside* any such subderivation.
- In this way, the truth predicate plays a role in managing the movement between the various subderivations, sub-sub-derivations, etc., in an argument.
- I take this to be part of the *grammar* of the truth predicate.
- Caveat This doesn't seem like an inviolable rule of grammar - how damaging that is for the subsequent claims needs to be assessed.

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- Let us push this idea a little further. Consider the argument:

Suppose John was in the park at 5pm.

...

*He should be convicted.*

John's lawyer retorts:

*But suppose it's not true that John was anywhere near the park all day ...*

- Again, this should be seen as John's lawyer *challenging* the prosecutor's assumption and starting a new subderivation.

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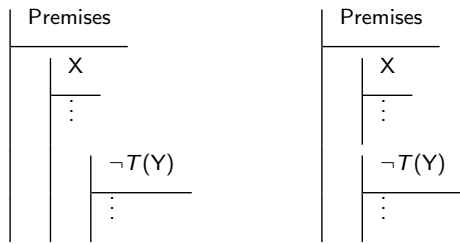
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**General Claim 2:** If  $X$  is supposed, then  $\neg T(Y)$  is supposed, then if  $X$  is incompatible with  $\neg Y$  this *must* be read as a new subderivation, rather than a new sub-sub derivation.



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- Let us push this idea even further. Consider the argument:

Suppose John was in the park at 5pm.

...

*He should be convicted.*

John's lawyer retorts:

*But suppose John hadn't received the invitation in question*

...

*Then it wouldn't have been true that John was anywhere near the park that day*

- The structure here is a little more complex - I maintain that this too should be seen as John's lawyer *challenging* the prosecutor's assumption in a new subderivation.

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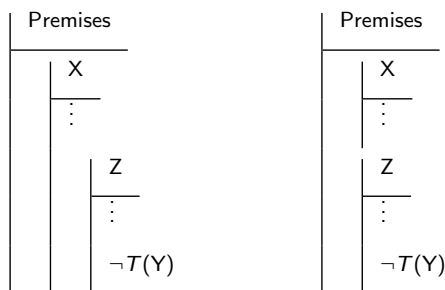
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- Structure:** where  $X$  is incompatible with  $\neg Y$ , we have



- Intuition:** even *deriving*  $\neg T(Y)$  (rather than just assuming it) amounts to a rejection of the subderivation governed by  $X$ .

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- General Claim 3:** If  $X$  is supposed, then some  $Z$  is supposed and  $\neg T(Y)$  derived, then if  $X$  is incompatible with  $\neg Y$  this *must* be read as a new subderivation, rather than a new sub-sub derivation.

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There is also a dual set of intuitions about  $\neg X$  being supposed, and then  $T(Y)$  being supposed. So consider the following dialogue:

- The lawyer argues:

Suppose John wasn't anywhere near the park at the time.

...

*He shouldn't be convicted.*

The prosecutor retorts:

*But suppose it's true that John was in the park at 5pm ...*

- This should be seen as the prosecutor *challenging* the lawyer's assumption and starting a new subderivation.

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**General Claim 4:** If  $\neg X$  is supposed, then some  $Z$  is supposed and  $T(Y)$  derived, then if  $\neg X$  is incompatible with  $Y$  this *must* be read as a new subderivation, rather than a new sub-sub derivation.

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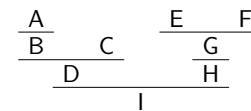
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- In order to formalize our main idea and see what its consequences are, we use a different style of natural deduction, in which proofs look like *trees*:



- The conclusion (in this case,  $I$ ), is at the bottom of the tree, and the leaves (in this case,  $A$ ,  $C$ ,  $E$  and  $F$ ), are either premises or assumptions to be discharged.
- Each sentence on the tree follows from the sentence(s) immediately above it by one of the basic rules of inference.

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- The basic logical symbols are  $\perp$ ,  $\supset$ ,  $\&$ ,  $T$  and  $\forall$ .

The negation symbol  $\neg A$  is defined as  $A \supset \perp$ ,  
 Disjunction  $A \vee B$  is defined as  $(\neg A) \supset B$ ,  
 Existential quantification  $(\exists x)(\dots)$  is defined as  $\neg(\forall x)\neg(\dots)$ .

- There are *standard rules of inference*: e.g.,

$$\frac{A \ \& \ B}{A} \qquad \frac{A \ \& \ B}{B} \qquad \frac{A \quad B}{A \ \& \ B}$$

$$\frac{A \supset B \quad A}{B} \qquad \frac{A}{T(A)} \qquad \frac{T(A)}{A}$$

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- And there are *assumption discharging rules of inference*: e.g.,

$$\frac{[A] \dots B}{A \supset B} \qquad \frac{[\neg A] \dots \perp}{A}$$

- We also have standard quantifier rules.

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- Example: a proof of  $\neg(X \supset Y)$  from  $X$  and  $\neg Y$ :

$$\frac{\neg Y \quad \frac{X \quad [X \supset Y]}{Y}}{\perp}}{\neg(X \supset Y)}$$

- This style of proof separates an argument nicely into independent 'threads' of argumentation.

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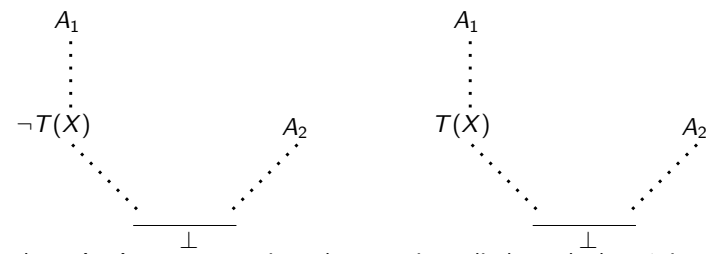
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- In accordance with the intuitions developed in the previous section, we prohibit the patterns:



where  $A_1, A_2$  are assumptions that remain undischarged when  $\perp$  is derived.

- We call this *restricted truth logic*.



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- Consider the following proof of  $L$  from premises  $L \supset \neg T(L)$  and  $L \supset \neg T(L)$  (Note: this can be used as part of a proof of  $\perp$  from the same premises):

$$\frac{\frac{[T(L)]}{L} \quad L \supset \neg T(L)}{\neg T(L)} \quad [T(L)]}{\frac{\perp}{\neg T(L)}} \quad \frac{}{L} \quad \frac{}{\neg T(L) \supset L}$$

- This is not a proof in restricted truth logic.

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- In fact, we have:

**Theorem 1:** There is no proof of  $\perp$  from premises  $L \supset \neg T(L)$  and  $L \supset \neg T(L)$  in restricted truth logic.

- Thus, restricted truth logic stops the 'collapse of logic' in the presence of liar sentences.

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- Next, we consider the fact that adding a truth predicate to the theory of arithmetic in a certain way creates a proof of  $\perp$ .
- Specifically, begin with the usual axioms of  $PA$ , add a single unary predicate  $T$  to the language, and allow the rules of inference:

$$\frac{\sigma(\vec{x})}{T(\ulcorner \sigma(\vec{x}) \urcorner)} \quad \frac{T(\ulcorner \sigma(\vec{x}) \urcorner)}{\sigma(\vec{x})}$$

Call the resulting system  $PA_T$ .

- Because liar sentences can be constructed in this system,  $PA_T \vdash \perp$ .

**Theorem 2:** In restricted truth logic,  $PA_T \not\vdash \perp$ .

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- Do we lose something important in adopting restricted truth logic?
- To address this we talk about *semantics*.
- Suppose we have a first order model  $\mathcal{M}$ , with a domain  $M = \{o_1, o_2, \dots\}$ , and predicates  $P_1, P_2, \dots$ .
- The for a sentence  $\sigma$  of the language, we say  $\mathcal{M} \models \sigma$  iff there is a proof of  $\sigma$  using the true atomic sentences / negated atomic sentences of  $\mathcal{M}$  as premises, where we allow the infinitary  $\forall$  rule:

$$\frac{\varphi(o_1) \quad \varphi(o_2) \quad \varphi(o_3) \quad \dots}{\forall x \varphi(x)}$$

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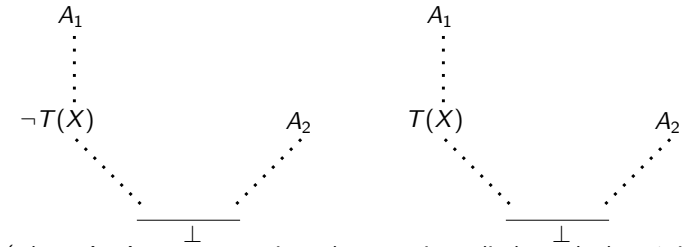
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- **Generalization:** If we add an operator  $T$  to the language, then for a sentence  $\sigma$  of the language we say  $\mathcal{M} \models_{tr} \sigma$  iff there is a proof of  $\sigma$  from the true atomic sentences / negated atomic sentences of  $\mathcal{M}$ , where we allow the infinitary  $\forall$  rule, the  $T$  rules, and we prohibit the patterns:



(where  $A_1, A_2$  are assumptions that remain undischarged when  $\perp$  is derived.)

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## Main Technical Results.

Truth,  
Subderivations  
and the Liar

Why Should I  
Care about the  
Liar Sentence?

Uses of the  
Truth Concept -  
(i) Disquotat.

Uses of the  
Truth Concept -  
(ii) Challenging  
Assumptions.

Formalization of  
the Main  
Intuition.

Main Technical  
Results.

- In 'Outline of a theory of Truth' Kripke constructs different satisfaction relations ( $\models$ ) for languages containing truth predicates that meet minimal standards of adequacy.
- One of these, the *minimal fixed point*  $\models_{min}$ , is the 'smallest' reasonable interpretation of the  $\models$  relation in the presence of a truth predicate.
- Theorem (sort of) 3:** If  $\mathcal{M} \models_{min} \sigma$ , then  $\mathcal{M} \models_{tr} \sigma$ .
- Theorem (sort of) 4:** For any  $\mathcal{M}$ ,  $\mathcal{M} \not\models_{tr} \perp$ .
- So  $\models_{tr}$  captures everything in Kripke's minimal fixed point.
- This is some sort of (limited) argument that we lose nothing crucial in truth restricted logic.

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- Still, we must lose *something* in truth restricted logic.
- As with many systems that deny the law of the excluded middle, the following are not true:

$$\begin{aligned} &(\forall \sigma) T(\ulcorner \neg \neg \sigma \supset \sigma \urcorner), \\ &(\forall \sigma) T(\ulcorner \sigma \supset \neg \neg \sigma \urcorner), \\ &(\forall \sigma) T(\ulcorner \neg T(\sigma) \supset T(\neg \sigma) \urcorner), \end{aligned}$$

- In addition, schemas such as:

$$\begin{aligned} &(A \supset B) \supset (\neg B \supset \neg A), \\ &(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C)), \end{aligned}$$

are not provable in full generality.

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## Main Technical Results.

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Results.**

**Main Claim.** I claim that our intuitive reasoning is done in something like restricted truth logic, which is why logic does not collapse in the face of the usual paradoxes that involve the truth predicate.

This involves giving up some of the principles of classical logic - to what extent these sacrifices are acceptable requires more thorough thought.

A more detailed comparison with alternatives (e.g., Hartry Field's system) needs to be performed, though an advantage of restricted truth logic is that it has a philosophic / linguistic motivation that Field's system lacks.

There are various directions in which the technical results given can be expanded and explored further.