

Modeling Variation in Intensive Longitudinal Data

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Abstract

Ecological Momentary Assessment (EMA) and/or Experience Sampling (ESM) methods are increasingly used in many research areas to study subjective experiences within changing environmental contexts. In these studies, many observations are often obtained for each subject, resulting in what is sometimes termed intensive longitudinal data. As a result, one can characterize not only a subject's mean, but also their variance, and specify models for both. In this chapter, we focus on an adolescent study using EMA where interest is on characterizing changes in mood variation. We describe how covariates can influence the mood variances, and also allow for a subject-level random effect to the within-subject variance specification. This permits subjects to have influence on the mean, or location, and variability, or (square of the) scale, of their mood responses. These mixed-effects location scale models have useful applications in many research areas where interest centers on the joint modeling of the mean and variance structure.

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Introduction

Modern data collection procedures, such as ecological momentary assessments (EMA) (Shiffman, Stone, & Hufford, 2008; Smyth & Stone, 2003; Stone & Shiffman, 1994), experience sampling (de Vries, 1992; Feldman Barrett & Barrett, 2001; Scollon, Kim-Prieto, & Diener, 2003), and diary methods (Bolger, Davis, & Rafaeli, 2003), have been developed to record the momentary events and experiences of subjects in daily life. These procedures yield relatively large numbers of subjects and observations per subject, and data from these designs are sometimes referred to as intensive longitudinal data (Walls & Schafer, 2006). Such designs follow the “bursts of measurement” approach described by Nesselroade and McCollam (2000), who called for such an approach in order to assess intra-individual variability. In this approach, a large number of measurements are obtained over a relatively short time span (e.g., a week). As noted by Nesselroade and McCollam (2000), this increases the research burden in several ways; however, it is important for studying intra-individual variation and to explain why subjects differ in variability rather than solely in mean level (Bolger et al., 2003). In this chapter, we describe data from an EMA study of adolescents, where interest was on determinants of the variation in the adolescents’ moods.

In mental health research, EMA procedures have been used in studying pediatric affective disorders (Axelson et al., 2003), eating disorders (Boseck et al., 2007; le Grange, Gorin, Dymek, & Stone, 2002), drug abuse (Epstein et al., 2009), schizophrenia (Granholtm, Loh, & Swendsen, 2008; Kimhy et al., 2006), borderline personality disorder (Trull et al., 2008), stress and anxiety (de Vries, Caes, & Delespaul, 2001; Yoshiuchi, Yamamoto, & Akabayashi, 2008), and sexual abuse (Simonich et al., 2004). Similarly, in smoking research, EMA studies include those studying relapse in people who are quitting smoking (Shiffman, 2005), relapse

among adolescent smokers (Gwaltney, Bartolomei, Colby, & Kahler, 2008), examining the urge to smoke (O'Connell et al., 1998), and our own EMA studies on adolescents (Mermelstein, Hedeker, Flay, & Shiffman, 2002, 2007). Recently, a number of review articles on EMA studies have been published in several diverse research areas that indicate the wide range of studies using EMA methods (aan het Rot, Hogenelst, & Schoevers, 2012; Armeij, Schatten, Haradhvala, & Miller, 2015; Heron, Everhart, McHale, & Smyth, 2017; Liao, Skelton, Dunton, & Bruening, 2016; May, Junghaenel, Ono, Stone, & Schneider, 2018; Rodriguez-Blanco, Carballo, & Baca-Garcia, 2018; Serre, Fatseas, Swendsen, & Auriacombe, 2015; Walz, Nauta, & aan het Rot, 2014; Wen, Schneider, Stone, & Spruijt-Metz, 2017).

Data from EMA studies are inherently multilevel with, for example, (level-1) observations nested within (level-2) subjects. Thus, linear mixed models (LMMs, aka multilevel or hierarchical linear models) are often used for EMA data analysis, and several books and/or book chapters describe mixed model analysis of EMA data (Bolger & Laurenceau, 2013; Schwartz & Stone, 2007; Walls & Schafer, 2006). A basic characteristic of these models is the inclusion of random subject effects into regression models in order to account for the influence of subjects on their repeated observations. The variance of these random effects indicates the degree of variation that exists in the population of subjects, or the between-subjects variance. Analogously, the error variance characterizes how much variation exists within a subject, or the within-subjects variance. These variances are usually treated as being homogeneous across subject groups or levels of covariates.

In EMA studies, it is common to have up to thirty or forty observations per subject, and this allows greater modeling opportunities than what conventional LMMs allow. In particular, one very promising extended approach is the modeling of both between-subject (BS) and within-subject (WS) variances as a function of covariates, in addition to their effect

on overall mean levels. For example, if a person's mood is the outcome, then one can consider the effect of covariates on their mood level (e.g., how happy/sad they are on average), how similar/different they are in their mood levels to others (e.g., how homogeneous/heterogeneous are particular groups of subjects), as well as on their own variation in mood (e.g., how consistent/erratic their mood is).

Momentary mood may be influenced by both stable trait factors and situational or momentary influences and contexts. A persistent debate among researchers interested in personality and psychopathology, for example, has been whether mood variability is a more stable trait or a more situationally specific state; parsing out the between-subject and within-subject variances helps to better address this research question. Of interest to researchers has been whether mood variability is related to a host of standard personality traits (e.g., introversion, extraversion; e.g., Hepburn and Eysenck (1989)) or how much it may be influenced by being with others, such as contagion effects on mood (e.g., Neumann and Strack (2000)). Examining whether these influences on mood are more personality (e.g., extraversion) or situational (e.g., influenced by others) becomes possible by examining the effects of specific covariates on the BS and WS variances.

Expanding on the work of Cleveland, Denby, and Liu (2000), Hedeker, Mermelstein, and Demirtas (2008) describe an extended LMM for variance modeling of EMA data, dubbed the mixed-effects location scale (MELS) model. Like all LMMs, this model allows covariates and a random subject effect to influence the mean response of a subject. However, this model also includes a log-linear structure for both the WS and BS variance, allowing covariates to influence both sources of variation. Finally, a random subject effect is included in the WS variance specification. This permits the WS variance to vary at the subject level, above and beyond the influence of covariates on this variance. In this chapter we more fully describe the MELS model, and show how it can be used to model changes in mood levels and mood

variation as a function of covariates. We will also describe some of the software programs that can be used to estimate the parameters of the MELS model towards the end of the chapter. A word of caution about the notation used in this chapter should be made. Because of the modeling of the variances, the notation of the MELS model is perhaps more involved than standard multilevel and/or hierarchical linear models (HLMs). For example, it is customary to use τ s to represent variances in HLMs, however in what follows τ s will be used to denote fixed-effects in the WS variance submodel. Similarly, we will use α s to represent fixed-effects in the BS variance submodel. We apologize for any confusion that our choices for notation creates.

MELS Model

Consider the following mixed-effects regression model (aka hierarchical linear or multilevel model) for the measurement y of subject i ($i = 1, 2, \dots, N$ subjects) on occasion j ($j = 1, 2, \dots, n_i$ occasions):

$$y_{ij} = \mathbf{x}'_{ij} \boldsymbol{\beta} + \nu_i + \varepsilon_{ij}, \quad (1)$$

where \mathbf{x}_{ij} is the $p \times 1$ vector of regressors (typically including a “1” for the intercept as the first element) and $\boldsymbol{\beta}$ is the corresponding $p \times 1$ vector of regression coefficients. The regressors can either be at the subject level, vary across occasions, or be interactions of subject-level and occasion-level variables. In the multilevel terminology, subjects are at level 2, while the repeated observations are at level 1. Thus, the level-2 random subject effect ν_i indicates the influence of individual i on his/her repeated level-1 measurements. The population distribution of these random effects is assumed to be a normal distribution with zero mean and variance σ_ν^2 . The errors ε_{ij} are also assumed to be normally distributed in the population with zero mean and variance, σ_ε^2 and independent of the random effects. Here,

σ_v^2 represents the BS variance and σ_ε^2 is the WS variance.

For simplicity, consider the null model with no covariates, namely, $y_{ij} = \beta_0 + \nu_i + \varepsilon_{ij}$. Here, ν_i represents a subject's mean deviation from the population intercept β_0 , the latter representing the population mean of the outcome variable in this model with no covariates. A subject's mean is therefore $\beta_0 + \nu_i$. If subjects are very similar to each other, then $\nu_i \approx 0$ and σ_v^2 will approach 0. Conversely, as subjects differ, $\nu_i \neq 0$ and σ_v^2 will increase from 0. Thus, the magnitude of the BS variance σ_v^2 indicates how different subjects are from each other in terms of their means. We refer to this as the degree of homogeneity/heterogeneity across subjects.

Analogously, ε_{ij} is subject i 's error at time j , which represent deviations from their mean. If the observations from all subjects are all close to their means, $\varepsilon_{ij} \approx 0$ and σ_ε^2 will approach 0. Alternatively, as the observations from subjects deviate from their means, $\varepsilon_{ij} \neq 0$ and σ_ε^2 will increase from 0. The magnitude of the WS variance σ_ε^2 indicates how data vary within subjects, which we refer to as the degree of consistency/erraticism within subjects.

To allow covariates (*i.e.*, regressors) to influence the BS and WS variances, we can utilize a log-linear representation, as has been described in the context of heteroscedastic (fixed-effects) regression models (Aitkin, 1987; Harvey, 1976), namely,

$$\sigma_{\nu_{ij}}^2 = \exp(\mathbf{u}'_{ij}\boldsymbol{\alpha}), \quad (2)$$

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\boldsymbol{\omega}'_{ij}\boldsymbol{\tau}). \quad (3)$$

The variances are subscripted by i and j to indicate that their values change depending on the values of the covariates \mathbf{u}_{ij} and $\boldsymbol{\omega}_{ij}$ (and their coefficients). Both \mathbf{u}_{ij} and $\boldsymbol{\omega}_{ij}$ would

usually include a (first) column of ones for the reference BS and WS variances (α_0 and τ_0), respectively. Thus, the BS variance equals $\exp \alpha_0$ when the covariates \mathbf{u}_{ij} equal 0, and is increased or decreased as a function of these covariates and their coefficients $\boldsymbol{\alpha}$. Specifically, for a particular covariate u^* , if $\alpha^* > 0$, then the BS variance increases as u^* increases (and vice versa if $\alpha^* < 0$). Note that the exponential function ensures a positive multiplicative factor for any finite value of α , and so the resulting variance is guaranteed to be positive. The WS variance is modeled in the same way. The coefficients in $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ indicate the degree of influence on the BS and WS variances, respectively, and the ordinary random intercept model is obtained as a special case if $\boldsymbol{\alpha} = \boldsymbol{\tau} = 0$ for all covariates in \mathbf{u}_{ij} and $\boldsymbol{\omega}_{ij}$ (*i.e.*, excluding the reference variances α_0 and τ_0).

We can further allow the WS variance to vary across subjects, above and beyond the contribution of covariates, namely,

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\boldsymbol{\omega}'_i \boldsymbol{\tau} + \omega_i), \quad (4)$$

where the random subject (scale) effects ω_i are distributed in the population of subjects with mean 0 and variance σ_{ω}^2 . The idea for this is akin to the inclusion of the random (location) effect in Equation (1), namely, covariates do not account for all of the reasons that subjects differ from each other. In this model, v_i is a random effect which characterizes a subject's mean, or location, and ω_i is a random effect which characterizes a subject's variance, or (square of the) scale. These two random effects are correlated with covariance parameter $\sigma_{v\omega}$, which indicates the degree to which the random location and scale effects are associated with each other.

A Better Understanding of the MELS Model Parameters

Here, we try to provide a more concrete description of the parameters of the mean and variance submodels with some simple illustrations. Consider, first, the modeling of the BS variance in Equation (2). It might seem odd that the BS variance can change depending on a time-varying variable, say u_{ij} . In fact, in Hedeker et al. (2008) we did not consider this possibility. However, this is clearly possible as we will now explain. Suppose that at each prompt a subject indicates whether they are alone or with others. This variable, denoted as Others, is therefore a time-varying (level-1) variable which could be coded 0 when a subject is alone and 1 if the subject is with others for that prompt. Suppose that the BS variance is decreased when subjects are with others. In other words, subjects are more heterogeneous in terms of their mood responses when those mood responses are obtained while they are alone, and less heterogeneous when their mood responses are obtained while they are with others. Figure 1 depicts such a situation.

In Figure 1, the “alone” responses are grouped together on the left-hand side and the “with others” responses are on the right-hand side. Consider the solid lines first, which represent the mean and the effect on the mean attributable to being with others. Notice that the mean level is lower for alone (approximately 0) than with others (approximately 1). Thus, on average, being with others raises the mean level of mood by 1 unit. To represent subject heterogeneity, the figure presents the hypothetical data of two subjects, one below and one above the mean levels. These subject lines are depicted as dotted lines, and, as mentioned, their alone/with others responses are separated and grouped together on the left/right-hand side of the figure. For simplicity, only two subjects are presented, but for a real dataset there would be as many subject lines as there are subjects in the sample. Also, for each subject, the dots represent the prompt-level (level-1) responses. In Figure 1, each subject has many responses when alone and many responses with others, however in a real dataset the numbers of observations could vary considerably, both across subjects and the alone/with others

situations. Each subject's dotted lines represent their mean mood level when alone and with others, respectively. Thus, the subject in the upper part of the figure (above the mean) has higher relative mood levels than the subject in the lower part of the figure (below the mean), both when alone and with others. Notice that the distance between the dotted lines of the two subjects is greater for the alone responses (a difference of about 4 units) than when the two subjects are with others (a difference of about 2.5 units). In other words, subjects are more similar to each other in their mood responses when those mood responses are obtained with others, relative to when they are alone. Or, the level of subject heterogeneity varies across the different values of the Others variable. This is precisely how a level-1 variable (Others) can influence the level-2 BS variance.

In terms of a model of the BS variance, we might posit

$$\sigma_{v_{ij}}^2 = \exp(\alpha_0 + \alpha_1 \text{Others}_{ij}), \quad (5)$$

and according to our figure, $\alpha_1 < 0$, since the BS variance is reduced for the “with others” responses (“with others” responses coded as 1, relative to the “alone” responses coded as 0). Notice that even though the α coefficients can be negative, the exponential function ensures that the resulting variance is a positive value. Also, for simplicity, here we simply have a single (level-1) covariate in our BS variance model, but more generally the model could include level-2 variables (e.g., a subject's gender), other level-1 variables (e.g., day of the prompt), and cross-level interactions.

As depicted in Figure 1, the WS (level-1) variance σ_e^2 is constant. Thus, the dispersion of the dots around the subjects' mean levels is the same across subjects and both levels of the Others variable. This implies that subjects are equally consistent in their mood reports, and likewise that the mood responses obtained when alone or with others are also equally consistent. Suppose, however, that the mood responses obtained when alone vs with others

are not equally consistent. This would imply that the level-1 WS variance could be influenced by covariates, say,

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\tau_0 + \tau_1 \text{Others}_{ij}), \quad (6)$$

in a similar manner as the BS variance model. More generally, the WS variance could be influenced by covariates at level-1, level-2, or cross-level interactions. Additionally, subjects themselves could vary in their response consistency/erraticism, over and above the effects of covariates. Certainly, in terms of mood, there is likely to be a large unique subject component to the consistency/erraticism of their responses. To extend the WS variance model, consider

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\tau_0 + \tau_1 \text{Others}_{ij} + \omega_i) \quad (7)$$

where ω_i is the random effect of subject i on the WS (level-1) variance. More consistent subjects would have negative values of ω_i , while more erratic subjects would have positive ω_i values. Similar to the random subject effects ν_i on the mean (or location), these random subject effects ω_i on variance (or scale) are assumed to be normally distributed in the population of subjects with mean 0 and variance σ_ω^2 .

Figure 1, with constant error variance, would assume that $\tau_1 = \sigma_\omega^2 = 0$. Namely, there is no effect of Others on the dispersion of points, and all subjects have the same degree of dispersion (*i.e.*, no subject heterogeneity in the WS variance). Alternatively, Figure 2 illustrates the effects of Others and subjects on the dispersion of points (*i.e.*, the WS variance). Notice that the subject in the upper part of the plot has more dispersed points than the subject in the lower part of the plot, for both “alone” and “with others” observations. Thus, the top subject is more erratic and would have a larger value of ω_i than the bottom subject. Also, for each subject, the dispersion of points is greater when subjects are alone than

with others, which would imply that $\tau_1 < 0$. To summarize the effects of the level-1 Others variable that is illustrated in Figure 2: it raises the mean level ($\beta_1 > 0$), while reducing both the level-1 and level-2 variances ($\alpha_1 < 0$ and $\tau_1 < 0$, respectively). Thus, when subjects are with others, their mood levels are elevated, more similar to each other, and more consistent.

A final parameter in the MELS model is the association or covariance between the random location and scale parameters $\sigma_{v\omega}$. Based on Figure 2, the subject in the top part has both a higher mean level and a greater dispersion of points, than the subject in the lower part of the figure. This would imply a positive covariation between these two random effects, or namely, $\sigma_{v\omega} > 0$.

Further Interpretation of Variance Effects

Suppose now that the BS variance submodel includes a subject-level (level-2) variable Male_i (coded 0 for females and 1 for males), in addition to the time-varying (level-1) variable Others_{ij} (coded 0 for alone and 1 for with others).

$$\sigma_{v_{ij}}^2 = \exp(\alpha_0 + \alpha_1 \text{Male}_i + \alpha_2 \text{Others}_{ij}) \quad (8)$$

Holding the variable Others constant, the BS variance equals $\exp(\alpha_0)$ for females, and $\exp(\alpha_0 + \alpha_1)$ for males. The ratio of the BS variances (male to female) equals $\exp(\alpha_0 + \alpha_1) / \exp(\alpha_0) = \exp(\alpha_1)$. Thus, exponentiating the slope parameters in the BS variance model yields variance ratio interpretations: the ratio of the BS variances for a unit change in the regressor. This is akin to the interpretation in a Poisson regression model (see Chapter 8 for example), which also has a log link function, in which the exponentiated slopes have rate ratio interpretations. Similarly, the exponentiated slopes in the WS variance submodel represent ratios of the WS variances for a unit change of the regressor. Thus, for the variance

submodels, reporting of the variance ratios ($\exp \alpha$ for BS variance, and $\exp \tau$ for WS variance), along with 95% confidence intervals, can be useful. Notice that a variance ratio of 1 would imply no effect of the variable on the variance, so if the 95% confidence interval includes 1 then the variable does not have a statistically significant effect at the 0.05 level.

Coding of Regressors

The mean, BS variance, and WS variance submodels are all regression models, and so the coding of the regressors is an important aspect to consider. For example, in the BS variance submodel above, if both regressors Male and Others are dummy-coded variables (0 or 1), then α_0 represents the BS variance when females are alone (*i.e.*, when both regressors equal 0). Alternatively, if both variables were coded 1 or 2, then the slopes would be the same, but the intercept α_0 would represent the BS variance when both variables equal 0 (which does not occur).

For continuous regressors, the coding of the variables is perhaps of greater consideration. Suppose that instead of a subject's sex, we included their age in the model:

$$\sigma_{v_{ij}}^2 = \exp(\alpha_0 + \alpha_1 \text{Age}_i + \alpha_2 \text{Others}_{ij}). \quad (9)$$

In this case, α_0 would represent the BS variance when a subject aged 0 is alone. Such an interpretation is often an extrapolation outside of the data range (unless, perhaps, one was studying infants). Thus, for a continuous variable like Age it often makes sense to subtract off the lowest level in the sample, or to center the variable around its mean. This is especially important if one includes interactions. For example, suppose we included an interaction of age by being with others:

$$\sigma_{v_{ij}}^2 = \exp(\alpha_0 + \alpha_1 \text{Age}_i + \alpha_2 \text{Others}_{ij} + \alpha_3 \text{Age}_i \times \text{Others}_{ij}). \quad (10)$$

Here, with the interaction in the model, the main effect of Others (α_2) represents the effect of being with others versus being alone for subjects of zero age. Again, in most studies this would be an extrapolation away from the data range, and essentially uninterpretable.

Subtracting off the sample's low value or its mean from Age_i greatly helps here, so that α_2 would represent the effect of being with others versus being alone for subjects at the lowest or mean age level. Also, the scaling of continuous regressors is worth considering. For example, one might want to express the age variable in deciles, say $\text{Age}_i/10$, so that the coefficients associated with age pertain to a 10-unit change in age (rather than a unit change in age). These considerations can lead to more interpretable results, as well as ease the computational complexity inherent in estimation of the model parameters.

Between-Subject and Within-Subject Effects

As is well-known in the multilevel literature, the effects of time-varying (level-1) variables can be decomposed into BS and WS effects (Hedeker & Gibbons, 2006; Neuhaus & Kalbfleisch, 1998; van de Pol & Wright, 2009). For example, suppose we are considering the following mean submodel for our modeling of mood (y_{ij}):

$$y_{ij} = \beta_0 + \beta_1 \text{Age}_i + \beta_2 \text{Others}_{ij} + \nu_i + \varepsilon_{ij}. \quad (11)$$

Suppose that we obtain a positive effect for Others_{ij} (*i.e.*, $\hat{\beta}_2 > 0$). Does this indicate that (a) mood is elevated for a subject when they are with others, or does it indicate that (b) the average mood is elevated for subjects that tend to be with others to a greater extent? This latter interpretation (b) is the BS effect, namely the association between a subject's average mood (\bar{y}_i) and their average of being with others ($\overline{\text{Others}_i}$). In this case of a dummy variable, this average would be the proportion of prompts that a subject reports being with others. The former interpretation (a) is the WS effect, or the relationship for a given subject momentary

mood ($y_{ij} - \bar{y}_i$) and being with others ($\text{Others}_{ij} - \overline{\text{Others}_i}$), relative to their averages on both variables. In Equation (11) above, it is assumed that the BS and WS effects (of being with others) are equal. This may not be the case, and in EMA studies it is often the momentary or WS effect that is of greatest interest. To allow for separate BS and WS effects, we simply augment the model in the following way:

$$y_{ij} = \beta_0 + \beta_1 \text{Age}_i + \beta_2 \overline{\text{Others}_i} + \beta_3 (\text{Others}_{ij} - \overline{\text{Others}_i}) + \nu_i + \varepsilon_{ij}. \quad (12)$$

Here, β_2 represents the BS effect and β_3 is the WS effect of the time-varying Others_{ij} variable. Because the model of Equation (11) is nested within the model of Equation (12), under the assumption that the BW and WS effects are equal, one can use a likelihood-ratio test to assess this assumption. However, because of the primary interest in the WS effect in EMA studies, the model of Equation (12) would generally be preferred, regardless of the results of such a test.

This decomposition can also be applied to the variance submodels, for example:

$$\sigma_{\nu_{ij}}^2 = \exp(\alpha_0 + \alpha_1 \text{Age}_i + \alpha_2 \overline{\text{Others}_i} + \alpha_3 (\text{Others}_{ij} - \overline{\text{Others}_i})), \quad (13)$$

for the BS variance. Here, the BS effect α_2 characterizes heterogeneity/homogeneity for subjects that are more/less with others. The WS effect α_3 is what is depicted in Figure 2, namely the degree to which subject heterogeneity (dispersion of the dotted lines) is affected when a subject is with others, relative to being alone. Similarly, the WS variance submodel can include WS and BS effects of time-varying (level-1) variables:

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\tau_0 + \tau_1 \text{Age}_i + \tau_2 \overline{\text{Others}_i} + \tau_3 (\text{Others}_{ij} - \overline{\text{Others}_i}) + \omega_i). \quad (14)$$

The BS effect τ_2 represents the degree of consistency/erraticism comparing subjects with different levels of being with others, while the WS effect τ_3 reflects consistency/erraticism

differences when a subject is with others, relative to being alone. Again, the WS effect is depicted in Figure 2, in which subjects display more consistency in their responses when they are with others.

Variance-Covariance and Standardization of the Random Effects

The random location ν_i and scale ω_i effects follow a bivariate normal distribution (in the population of subjects) with means equal to 0 and variance covariance matrix Σ given by:

$$\Sigma = \begin{bmatrix} \sigma_\nu^2 & \sigma_{\nu\omega} \\ \sigma_{\nu\omega} & \sigma_\omega^2 \end{bmatrix}. \quad (15)$$

Here, σ_ν^2 is the variance of the location effects, σ_ω^2 is the variance of the scale effects, and $\sigma_{\nu\omega}$ is the covariance of the two. For estimation of the model parameters, it is beneficial to standardize the random effects (*i.e.*, as standard normals). The reason for this is that the maximum likelihood solution requires integration over the bivariate normal distribution of the random effects, and this is facilitated if the random effects are always in standardized form, rather than taking on a different unstandardized form for each dataset. To standardize the random effects, we can use the Cholesky factorization (Bock, 1975), or matrix square-root, of the variance covariance matrix, namely $\Sigma = SS'$, where S is the lower triangular Cholesky factor (or matrix square-root).

$$\begin{bmatrix} \nu_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} s_{1ij} & 0 \\ s_{2ij} & s_{3ij} \end{bmatrix} \begin{bmatrix} \theta_{1i} \\ \theta_{2i} \end{bmatrix} = \begin{bmatrix} \sigma_{\nu_{ij}} & 0 \\ \sigma_{\nu\omega} / \sigma_{\nu_{ij}} & \sqrt{\sigma_\omega^2 - \sigma_{\nu\omega}^2 / \sigma_{\nu_{ij}}^2} \end{bmatrix} \begin{bmatrix} \theta_{1i} \\ \theta_{2i} \end{bmatrix} \quad (16)$$

Here, θ_{1i} and θ_{2i} are the standardized location and scale random effects, respectively, with means of 0 and variances of 1. In this representation, the Cholesky elements s_{1ij} , s_{2ij} , and s_{3ij} would be estimated, and subscripts i and j are included on these elements because the BS

variance $\sigma_{v_{ij}}^2$ can vary across subjects and/or occasions. The model can now be written as

$$y_{ij} = \mathbf{x}'_{ij} \boldsymbol{\beta} + s_{1ij} \theta_{1i} + \varepsilon_{ij} \quad (17)$$

where $s_{1ij} = \sigma_{v_{ij}} = \sqrt{\exp(\mathbf{u}'_{ij} \boldsymbol{\alpha})}$, and the errors ε_{ij} have variance given by

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\boldsymbol{\omega}'_{ij} \boldsymbol{\tau} + s_{2ij} \theta_{1i} + s_{3ij} \theta_{2i}). \quad (18)$$

In this representation, the covariance of the random effects ($\sigma_{v\omega}$) is obtained as the product of the Cholesky elements $s_{1ij} \times s_{2ij}$, and the variance of the random scale (σ_{ω}^2) equals

$$s_{2ij}^2 + s_{3ij}^2.$$

Alternative formulation for association of location and scale

Suppose that instead of allowing the location and scale random effects to be correlated, we assume that they are independent (*i.e.*, $\sigma_{v\omega} = 0$, and therefore $s_{2ij} = 0$), but that the location random effect θ_{1i} explicitly influences the WS variance. In this case, the WS variance could be expressed as

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\boldsymbol{\omega}'_{ij} \boldsymbol{\tau} + \tau_l \theta_{1i} + \sigma_{\omega} \theta_{2i}). \quad (19)$$

where the regression coefficient τ_l represents the (linear) influence of the location random effect θ_{1i} on the (log of the) WS variance. These two models of the WS variance are essentially the same, although in Equation (18) the parameter s_{2ij} is indicative of the covariance between the random location and scale effects, whereas in Equation (19) the parameter τ_l represents the effect of the random location effect on the WS variance. Also, the Cholesky element s_{3ij} in Equation (18) is replaced by the (simpler) square root of the random scale σ_{ω} in Equation (19). We have merely shifted from a correlation-like association

between the mean and variance to a regression setting in which the mean influences the variance.

Although equivalent in the present case, the latter representation can be more easily generalized to represent various forms of the relationship between the random location effect and the WS variance. For example, one can easily extend the model to allow for a quadratic relationship, namely,

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\boldsymbol{\omega}'_{ij}\boldsymbol{\tau} + \tau_l\theta_{1i} + \tau_q\theta_{1i}^2 + \sigma_{\omega}\theta_{2i}). \quad (20)$$

Here, τ_l and τ_q represent the linear and quadratic effect, respectively, of the random location effect θ_{1i} on the (log of the) WS variance. A quadratic relationship between the mean and variance would seem to be useful for rating scale data with ceiling and/or floor effects, where subjects that have mean levels at either the maximum or minimum value of the rating scale also have near-zero variance. For example, if the rating scale goes from 1 to 10, then any subject with a mean level near either 1 or 10 would almost certainly have a very small variance, giving rise to the potential for a quadratic relationship between the mean and variance. In this regard, the program MIXREGLS (Hedeker & Nordgren, 2013) allows for three possibilities: (1) no association ($\tau_l = \tau_q = 0$); (2) linear association only ($\tau_l \neq 0, \tau_q = 0$); and (3) linear and quadratic association ($\tau_l \neq 0, \tau_q \neq 0$).

Intraclass correlation

The expectation of y_{ij} , $E(y_{ij})$, is simply $\mathbf{x}'_{ij}\boldsymbol{\beta}$, as in an ordinary multiple regression model. Given that the MELS model allows extensive modeling of the BS and WS variances, the variance of y_{ij} is a bit more involved. For example, for the situation in which the random location θ_{1i} has a quadratic effect on the WS variance (*i.e.*, $\tau_q \neq 0$), the variance of y_{ij} varies as a function of the random location effect. However, for the simpler situation in which

$\tau_q = 0$ (i.e., the random location θ_{li} has only a linear effect on the WS variance), the variance of y_{ij} is given by

$$V(y_{ij}) = \exp(\mathbf{u}'_{ij}\boldsymbol{\alpha}) + \exp(\boldsymbol{\omega}'_{ij}\boldsymbol{\tau} + \frac{1}{2}[\tau_l^2 + \sigma_\omega^2]). \quad (21)$$

This is the sum of the contributions of the BS and WS variance submodels. Within the latter, the factor is $\frac{1}{2}[\tau_l^2 + \sigma_\omega^2]$ based on the expectation of log-normally distributed variables (Skrondal & Rabe-Hesketh, 2004). The covariance for any two observations j and j' that are nested within the same subject i (e.g., two different observations made on the same subject) equals

$$C(y_{ij}, y_{ij'}) = \sigma_{v_{ij}}^2 = \exp(\mathbf{u}'_{ij}\boldsymbol{\alpha}) \text{ for } j \neq j'. \quad (22)$$

As in an ordinary random-intercept multilevel model, this is simply the BS variance.

However, here, because the BS variance is modeled in terms of covariates it can vary across values of these covariates \mathbf{u}_{ij} . Expressed as a correlation, this yields the intraclass correlation (ICC), denoted as r_{ij} ,

$$r_{ij} = \frac{\exp(\mathbf{u}'_{ij}\boldsymbol{\alpha})}{\exp(\mathbf{u}'_{ij}\boldsymbol{\alpha}) + \exp(\boldsymbol{\omega}'_{ij}\boldsymbol{\tau} + \frac{1}{2}[\tau_l^2 + \sigma_\omega^2])} \quad (23)$$

Note that the ICC, which is equal to the BS variance divided by the sum of the BS and WS variance, represents the proportion of total *unexplained* variation that is at the subject level.

Here, the word *unexplained* refers to the residual variation of the dependent variable not explained by the mean submodel covariates \mathbf{x} . In the MELS model, the ICC can be obtained for specific values of the covariates \mathbf{u}_{ij} and $\boldsymbol{\omega}_{ij}$, which can include both time-invariant and time-varying covariates. This is why r carries both i and j subscripts in Equation (23).

Thus, another use of the MELS model can be to examine the degree to which the ICC varies across particular covariate values, or sets of covariate values.

Example

Data for the analyses reported here come from a longitudinal, natural history study of adolescent smoking (Mermelstein et al., 2002). Students included in the study were either in 8th or 10th grade at baseline, and self-reported on a screening questionnaire 6-8 weeks prior to baseline that they either had never smoked, but indicated a probability of future smoking, or had smoked in the past 90 days, but had not smoked more than 100 cigarettes in their lifetime. Written parental consent and student assent were required for participation. The data collection modalities included self-report questionnaires, a week-long time/event sampling method via palmtop computers (EMA), and in-depth interviews. Data for the current analyses came from the EMA portion. Adolescents carried the hand held computers with them at all times during the 7 consecutive day data collection period and were trained to both respond to random prompts from the computers and to event record (initiate a data collection interview) smoking episodes. Questions included ones about place, activity, companionship, mood, and other subjective items. The hand-held computers date and time-stamped each entry. For the analyses reported, we treated the responses obtained from the random prompts at baseline. In all, there were 17,402 random prompts obtained from 510 students with an approximate average of 34 prompts per student (range = 3 to 58).

The dependent variable considered is a measure of the subject's negative affect (NegAff) at each random prompt. This measure consists of the average of several individual mood items that were identified via factor analysis. Each item was rated from 1 to 10 with higher values indicating higher levels of negative mood. Over all prompts, and ignoring the clustering of the data, the marginal mean of NegAff was 2.41 (sd=1.49). Of interest is the

degree of heterogeneity in this mood measure in terms of both WS and BS variation. To get a sense of this, Figures 3-4 provide histograms of the subject-level means and variances, calculated for each subject based on their negative affect responses. The variances are expressed on the natural log scale to reflect the metric in which they will be modeled. Notice that both means and variances vary rather considerably across subjects. Modeling of the BS variance will attempt to relate covariates to the variability in the distribution of the subject means of negative affect depicted in Figure 3. In other words, what might be related to the homogeneity/heterogeneity in subject mean levels of negative affect. Similarly, modeling of the WS variance will examine if there are covariates that are related to the variability levels in the distribution of the subject variances of negative affect depicted in Figure 4. This might help us better understand the factors that explain why subjects are more/less consistent/erratic in negative affect.

To begin, we will estimate a null model with no covariates, and no association between the random location and scale effects. This can be written as:

$$y_{ij} = \beta_0 + \nu_{i} + \varepsilon_{ij}, \quad (24)$$

with

$$\sigma_{\nu_{ij}}^2 = \exp(\alpha_0), \quad (25)$$

and

$$\sigma_{\varepsilon_{ij}}^2 = \exp(\tau_0 + \omega_i). \quad (26)$$

For consistency, we include the subject and occasion subscripts i and j , respectively, on the variances in the above equations, though without covariates, the BS variance does not vary with i or j and the WS variance only varies with i (subjects). The estimates from this model

are $\hat{\beta}_0 = 2.407(\text{se} = 0.034)$, $\hat{\alpha}_0 = -0.164(\text{se} = 0.049)$, $\hat{\tau}_0 = -0.074(\text{se} = 0.049)$, and $\hat{\sigma}_\omega = 1.061(\text{se} = 0.038)$. In contrast to a null multilevel model, here the BS and WS variance parameters are estimated on the log scale ($\hat{\alpha}_0$ and $\hat{\tau}_0$, respectively), and this model additionally includes the random subject scale effect (whose contribution is estimated as the standard deviation $\hat{\sigma}_\omega$). Here, the mean negative affect is estimated as 2.407, and following Equation (21), the BS variance is estimated as $\exp(-0.164) = 0.849$, and the WS variance is estimated as $\exp(-0.074 + 0.5 \times 1.061^2) = 1.630$.¹

Using Equation (23), an estimate of the intraclass correlation is obtained as:

$$r_{ij} = 0.849 + 1.630 = 0.342, \quad (27)$$

which indicates that approximately one-third of the variation in negative affect is at the subject level. Notice that, without covariates in the variance submodels, the ICC does not vary with subjects or occasions, and so the subscripts i and j are, strictly speaking, not necessary here.

From this null model, it is interesting to examine the estimates of the random scale effects, since these random scale effects are a distinguishing feature of the MELS model relative to a standard multilevel model. As in multilevel models, the random effects are estimated using empirical Bayes estimation. Figures 5-7 provide histograms of the observed data for selected subjects with low scale, average scale, and high scale, respectively. In each of these histograms, estimated values of the location and scale random effects are listed as “Loc” and “Scale,” respectively, and the number of observations for each subject is listed as

¹ Estimates from a null multilevel are similar: $\hat{\beta}_0 = 2.435$, $\hat{\sigma}_v^2 = 0.869$, and $\hat{\sigma}_\varepsilon^2 = 1.379$.

“n.” Since the random effects are expressed as standard normals, these estimates are akin to z -values. The y-axis in these histograms represents percentages of each subject’s responses, with maximums of 100%, 60%, and 40% for the three figures, respectively. Figure 5 provides histograms of six subjects with very low scale estimates (-3 to -5.5), who also have very low location estimates (approximately -1.5). Notice that these are subjects with the bulk of their responses in the lowest level of negative affect. Thus, they are very consistently low in their negative affect responses. Figure 6 includes subjects with near-zero estimates of scale, and with various levels of location. These subjects provide average levels of variability in their negative affect responses. Finally, Figure 7 includes subjects with high scale estimates (approximately 1.7 to 1.9), and with various positive location estimates. These are subjects who are more erratic in their negative affect responses, largely providing responses across the entire range of negative affect. Note that a standard multilevel model would assume that all subjects have the same level of dispersion across the response categories, but these plots clearly indicate that subjects do differ in terms of their consistency/erraticism in negative affect.

Building from the null model, in terms of covariates, we will examine GenderF, which is a subject (level-2) covariate coded 0 for males and 1 for females, Others, which is a time-varying (level-1) covariate coded 0 if the subject was alone or 1 if with others, Age15 which is the subject’s age at baseline minus 15 (since the average age in the sample was 14.93), and COPEc, which is a grand-mean centered version of a coping scale measurement at baseline. For the prompt-level variable Others, we created both a BS and WS version (Others_BS and Others_WS) as described in the preceding section, namely $X_{ij} = \bar{X}_i + (X_{ij} - \bar{X}_i)$. Here, Others_BS, the first term on the right-hand side, equals the proportion of random prompts in which a subject was with others, and Others_WS, the latter term on the right-hand side, is the

prompt-specific deviation relative to a subject's proportion. Thus, it equals $0 - \bar{X}_i$ or $1 - \bar{X}_i$ if the subject was alone or was with others, respectively, for the given random prompt.

Finally, the time of the prompt was recorded and the following indicators were created and treated as covariates: 9am-2pm, 2pm-6pm, 6pm-10pm, and 10pm-3am; leaving 3am-9am as the reference time indicator. All covariates were included in the mean, BS and WS variance submodels.

Table 1 lists the maximum likelihood estimates, standard errors, z -, and p -values for these regressors in terms of the (a) mean submodel, (b) BS variance submodel, and (c) WS variance submodel. For the mean submodel, all of the time interval effects are positive, but only the first two are statistically significant. The estimated effects for these two intervals equal 0.054 ($z = 2.25, p < .024$) and 0.063 ($z = 2.56, p < .01$), respectively. Thus, though these effects are not large, negative affect is significantly elevated during 9am-2pm and 2pm-6pm, relative to the early morning period of 3am-9am. For the Others variable, both the BS and WS versions indicate diminished negative affect ($z = -3.08, p < 0.002$, and $z = -10.83, p < 0.001$ respectively). Thus, being with others to a greater extent (higher on Others_BS) as well as the momentary effect of being with others (Others_WS) decrease negative affect. Neither age nor gender have significant effects on negative affect, but the coping variable does ($z = -8.03, p < 0.001$), with higher coping scores leading to lower negative affect.

Turning to the effects on the BS variance, most of the variables do not have statistically significant effects. There are two notable exceptions: being with others (Others_WS) and higher scores on the coping variable decrease subject heterogeneity ($z = -6.83, p < 0.001$, and $z = -6.90, p < 0.001$, respectively).

For the effects on WS variance, nearly all variables have significant effects. All time

interval effects are positive indicating increased WS variability in the mood responses relative to the early morning reference period of 3am-9am (all have $p < 0.001$). Being with others to a greater extent (higher on Others_BS) as well as the momentary effect of being with others (Others_WS) decrease WS variation on negative affect ($z = -3.13$, $p < 0.002$, and $z = -7.25$, $p < 0.001$, respectively). Females exhibit greater WS variance on negative affect ($z = 3.32$, $p < 0.001$), while higher values of coping lead to diminished WS variance ($z = -4.88$, $p < 0.001$). Finally, in terms of the random scale effect, it is highly significant ($z = 25.64$, $p < 0.001$) indicating that subjects exhibit different degrees of consistency/erraticism on negative affect, over and above the covariate effects on the WS variance of this outcome. Also, the location effect on the WS variance is positive and highly significant ($z = 16.22$, $p < 0.001$), indicating that subjects with higher levels of negative affect are more erratic, and that subjects with lower levels of negative affect are more consistent in their responses. The latter is suggestive of a floor effect of measurement in that subjects with below-average means on negative affect (say means of 1 or 2) must be consistent in their responses in order to have such a low mean.

Table 2 lists the exponentiated estimates, and 95% confidence limits, of the effects in the BS and WS variance submodels. For the intercepts in these submodels, the exponentiated values represent the reference BS and WS variance, respectively. Namely, the variances when all of the regressors equal zero. In the present case, for the BS variance, that would correspond to the 3am-9am time interval for males aged 15 with average coping values who are always alone. For the WS variance, it would also be for subjects with average levels of both the random location and scale effects. For the regressors, as mentioned above, the exponentiated estimates correspond to variance ratio estimates which indicate the ratio of variance per unit increase in the regressor. For example, for genderf it is the (estimated) BS

(or WS) variance for females divided by the same quantity for males. In this metric, if the 95% confidence limit includes a ratio of one, then the variable's effect is not statistically significant at the $\alpha = 0.05$ level. For the effects on the BS variance, only Others_WS and COPEc have intervals that do not include one. Thus, when subjects are with others the BS variance is reduced by a factor of 0.74. In percentage terms, this would represent a $100 - 74 = 26\%$ reduction in subject heterogeneity (in the mean levels of negative affect). Conversely, a unit increase in coping leads to a 58% ($100 - 42$) reduction in the BS variance. The standard deviation for the coping variable equals 0.54, so a unit change on coping represents a large change (*i.e.*, nearly a two sigma change on coping). As noted, nearly all variables had significant effects on the WS variance, and so the corresponding CIs do not include unity. For the time intervals, with 3am-9am as the reference, we see increases in subject erraticism of 19%, 28%, 35%, and 43% for the four time bins, respectively (the variance ratios are multiplied by 100 and then 100 is subtracted off to yield the percentage increases in WS variance). Subjects who are always with others are 61% ($100 - 39$) less erratic than subjects that are always alone. When a subject is with others, they are 20% ($100 - 80$) less erratic than when they are alone. Females are 34% ($134 - 100$) more erratic than males, and a unit increase in coping leads to a 32% ($100 - 68$) reduction in WS variance.

Figure 8 presents coefficient plots for the covariate effects on the mean, BS, and WS variance submodels, including 95% confidence intervals (CIs). These plots are useful as a visual representation of the regressor effects. For the effects on the mean submodel, the figure indicates the estimated regression coefficients, 95% CIs, and a reference line of zero. If the CI does not include zero for a variable, then that variable's effect is statistically significant at the two-tailed 0.05 level. For the BS and WS variance submodels, the plots provide estimates of the variance ratios associated with each variable, and their corresponding 95% confidence intervals. As mentioned, if the CI for a variance ratio includes one, then the effect of that

variable is not statistically significant at the two-tailed $\alpha = 0.05$ level. As we have already interpreted the effects of this analysis, we will not repeat that here. However, there are a few things that the plots help to reveal. For example, it is apparent that the CI for Others_BS is quite large in all submodels. The scaling of this variable plays a role here, as the effect is for a unit change on this variable. For Others_BS, this represents a comparison of a mean of 1 on Others_BS versus a mean of 0 on Others_BS. In other words, comparing subjects who are always with others to subjects that are always alone. Clearly, a different scaling of this variable might yield a more interpretable effect for this variable (e.g., a change of say .1 on this variable, rather than a unit change). The plots also reveal how a variable's effect on the three submodels might behave similarly (or not). Thus, both coping and being with others have a consistent negative effect on the mean, BS variance, and WS variance.

Software

In terms of the major statistical packages, Hedeker and Mermelstein (2012) and the supplemental materials of Hedeker et al. (2008) and Li and Hedeker (2012) provide examples of SAS PROC NLMIXED code. Unlike many software procedures, SAS PROC NLMIXED includes programming features and requires starting values for all model parameters. For this, it is advisable to estimate somewhat simpler models to get reasonable starting values for most of the model parameters. For example, using SAS PROC MIXED (or some other standard software program for multilevel models) can provide good starting values for the regression coefficients and some of the variance parameters. Another software option for the MELS model is the free stand-alone MIXREGLS software program (Hedeker & Nordgren, 2013).² It runs on the Windows operating system and the manual describes program usage, including how it can be accessed via R. Also, Leckie (2014) provides software to run the MIXREGLS

² This program is available at the website: https://voices.uchicago.edu/hedeker/mixwild_mixregls

program from within Stata. The analyses presented in this chapter were done using MIXREGLS, while the coefficient plots in Figure 8 were obtained using the `coefplot` module developed in Stata (Jann, 2014). An extension of MIXREGLS, that includes a graphical user interface (GUI), has recently been developed and is also accessible at the aforementioned website. This program, named MixWILD (Dzubar et al., 2020), allows for multiple random location effects (*i.e.*, random slopes), and is available for both the Windows and Mac OS X operating systems. SAS PROC NLMIXED, MIXREGLS, and MixWILD all use full-likelihood estimation; details are provided in the Appendix of Hedeker and Nordgren (2013).

Another option for estimation is the use of Bayesian software. For this, Rast, Hofer, and Sparks (2012) describe estimation using BUGS/JAGS software. Similarly, Leckie, French, Charlton, and Browne (2014) discuss Bayesian estimation using the Stat-JR statistics package (Charlton et al., 2013) for an example with clustered data (students in neighborhoods). Lin, Hedeker, and Mermelstein (2018) provide code and details using Stan in the supplemental materials. These articles using Bayesian estimation also provide various extensions to the MELS model presented in this chapter. Similarly, the most recent version of Mplus (Muthén & Muthén, 1998-2017) also uses Bayesian estimation methods and has capability to estimate MELS models and extensions.

Discussion

This chapter has illustrated how the MELS model can be used to model differences in variances, and not just means, across subject and time-varying covariates. As such, the MELS model can help to identify predictors of both within-subjects and between-subjects variation, and to test hypotheses about these variances. Additionally, by including a random subject effect on the WS variance, this model can examine the degree to which subjects are heterogeneous in terms of their variation on the outcome variable. Our example with negative

affect clearly shows that subjects are quite heterogeneous in terms of their mood variation, as one might expect.

More applications of this class of models clearly exist. For example, many questions of both normal development and the development of psychopathology address the issue of variability or stability in emotional responses to various situations and/or contexts. Often, a concern is with the range of responses an individual gives to a variety of stimuli or situations, and not just with the overall mean level of responsivity. Intraindividual variability in mood reflects a different aspect of one's emotional life than overall mean levels, and is important in itself in predicting future behavior and psychological states (Eid & Diener, 1999; Kuppens, Van Mechelen, Nezlek, Dossche, & Timmermans, 2007). Emotion dysregulation is at the core of several psychological disorders (e.g., borderline personality disorder, bipolar disorder), and understanding more about what factors or covariates may influence affective instability may help in guiding treatment and predictability of future behavior. EMA provides a window into the affective lives of individuals and provides opportunity to both monitor and intervene, in the moment, in real-time, to help correct or prevent disruptive mood dysregulation (Ebner-Priemer & Trull, 2009). Having the analytic ability to tease apart controlling variables on individual level mood variability may help enhance the understanding and treatment of psychological disorders. The MELS model also allows us to examine hypotheses about cross-situational consistency.

Modern data collection procedures, such as EMA and/or real-time data captures, usually provide a fair amount of both WS and BS data, and so give rise to the opportunity for modeling of both WS and BS variances as a function of covariates. One might wonder about how much WS and BS data are necessary for estimation and variance modeling purposes. For random coefficient models, Longford (1993) noted the difficulty with providing general guidelines about the degree of complexity, for the variation part of a model, that a given

dataset could support. This would also seem to be true here. Nonetheless, carrying out some simulations with relatively small sample sizes (*e.g.*, 20 subjects with 5 observations each) gives the general impression that the primary issue is that the estimation algorithm does not often converge, but instead has estimation difficulties of one sort or another, in small sample situations. This improves dramatically as the number of subjects and observations increases to even modest sizes of 100 subjects and 10 observations.

Various extensions of the MELS model presented in this chapter have been developed. Li and Hedeker (2012) extends the model to three levels to allow for the nesting of observations within days within subjects. This permits one to model the within-days within-subjects variance, within-subjects between-days variance, and the between-subjects variance as functions of covariates. Pugach, Hedeker, and Mermelstein (2014) describes a bivariate MELS model that can be used to additionally model the association between the bivariate outcomes. Kapur, Li, Blood, and Hedeker (2015) builds on this work to the multivariate outcome setting using Bayesian estimation methods. Ferrer and Rast (2017) extend the MELS model for dyadic data, including multiple random location and scale effects. Leckie et al. (2014) describe MELS models with multiple random location and scale effects applied to clustered data where students are nested within schools. Lin, Hedeker, and Mermelstein (2018) extends the model to multiple waves of EMA data, and allows multiple random location and scale effects. Brunton-Smith, Sturgis, and Leckie (2016) develops a MELS model for cross-classified random effects. For longitudinal data with skewness, detection limits, and measurement errors, Lu (2017) develops a Tobit MELS model. For longitudinal human stature data, Goldstein, Leckie, Charlton, Tilling, and Browne (2017) develop a non-linear growth MELS model that includes multiple location and scale random effects. Scherer, Huang, and Shrier (2016) and Courvoisier, Walls, Cheval, and Hedeker (2019) describe approaches for MELS modeling of time-to-event data. For missing EMA data, Cursio,

Mermelstein, and Hedeker (2019) and Lin, Mermelstein, and Hedeker (2018) develop methods that jointly model the missingness and the outcomes. Hedeker, Demirtas, and Mermelstein (2009) describes an extension of the model for ordinal EMA outcome data, and Hedeker, Mermelstein, Demirtas, and Berbaum (2016) further extends this approach for cross-sectional ordinal questionnaire data. Finally, Walters, Hoffman, and Templin (2018) comprehensively evaluate and develop methods for power analysis using the MELS model.

As this is a relatively new modeling technique, certain limitations and cautions should be mentioned. The MELS model assumes that the random location effects are normally distributed and that the random scale effects are log-normally distributed. It is unclear how robust this model is to violations of these assumptions. To some extent, this can be examined empirically using the approach of Liu and Yu (2008) for estimating models with non-normal random effects. Attention should also be paid to outliers and influential observations, as these might have undue effects on estimation of the model parameters, especially the variance parameters.

Recommended Readings

In terms of technical articles, Hedeker et al. (2008) describes development and application of the MELS model to an EMA dataset including many covariates. However, this initial paper did not allow for the possibility that time-varying (level-1) covariates could influence the BS variance. As detailed in this chapter, and illustrated in Figures 1 and 2, this is clearly possible. Hedeker, Mermelstein, and Demirtas (2012) further describes application of the MELS model, including piecewise linear effects of covariates on the mean, BS variance, and WS variance submodels. As mentioned, the software program MIXREGLS is described in Hedeker and Nordgren (2013), which includes estimation details in the Appendix. Two examples are included with the program and described in the manual. In the context of educational data consisting of students nested within schools, Leckie et al. (2014)

comprehensively describe MELS models with multiple random location and scale effects.

They also include results of a simulation study that shows why inclusion of the random scale is necessary for correct inference for the covariates of the level-1 variance submodel.

Recently, several applications of the MELS model have been published in the substantive literatures. Ong, Hedeker, Wyatt, and Manber (2016) applied the MELS model to examine sleep variability in a sample of subjects with chronic insomnia. Piasecki, Hedeker, Dierker, and Mermelstein (2016) examined mood variability and nicotine dependence in adolescent smokers. From the same adolescent study, Gorke, Hedeker, Piasecki, and Mermelstein (2017) used the MELS model to test for the unique and interactive effects of alcohol coping motives and internalizing symptoms on mood changes during drinking. Gerhart et al. (2018) examined relationships between negative mood mean and variability with pain levels, comparing a sample of chronic low back pain subjects and their pain-free spouses. Geukes et al. (2017) applied the MELS model to analyze self-esteem level and variability in narcissists. Sandhu and Leckie (2016) examined orthodontic pain trajectories in adolescents. Maher, Huh, Intille, Hedeker, and Dunton (2018) examined variability in physical activity and its association with mental health outcomes among obese adults, and Maher, Dzubur, et al. (2018) further examined mood variability and its effect on physical activity and sedentary behavior. Clearly, use of the MELS model is increasing as researchers learn about its use and capabilities.

Try This with the Data

The dataset used in this chapter is available (named VarModel.dat) and includes the variables in the following order: id, PosAff, NegAff, t1, t2, t3, t4, Others_BS, Others_WS, genderf, age15, COPEc. In this chapter, we have presented analyses of the negative affect outcome NegAff. Readers are encouraged to replicate the analyses presented, as well as

running similar models on the positive affect outcome (*i.e.*, the variable PosAff).

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Table 1

Mixed-effects location scale model of negative affect, $N = 510, \Sigma n_i = 17,402$, maximum likelihood estimates, standard errors, z-values, and p-values.

Variable	Estimate	Std Error	z-value	p-value
<i>Beta (regression coefficients)</i>				
Intercept	2.869	0.175	16.355	0.001
9am-2pm	0.054	0.024	2.252	0.024
2pm-6pm	0.063	0.025	2.562	0.010
6pm-10pm	0.048	0.025	1.901	0.057
10pm-3am	0.024	0.041	0.582	0.560
Others_BS	-0.701	0.227	-3.083	0.002
Others_WS	-0.229	0.021	-10.831	0.001
genderf	0.016	0.067	0.234	0.815
age15	0.020	0.030	0.669	0.504
COPEc	-0.523	0.065	-8.032	0.001
<i>Alpha (BS variance parameters: log-linear model)</i>				
Intercept	-0.070	0.296	-0.236	0.814
9am-2pm	0.063	0.055	1.150	0.250
2pm-6pm	0.092	0.056	1.647	0.100
6pm-10pm	0.006	0.057	0.109	0.913
10pm-3am	0.047	0.090	0.526	0.599
Others_BS	-0.572	0.401	-1.426	0.154
Others_WS	-0.299	0.044	-6.833	0.001
genderf	0.054	0.132	0.409	0.682
age15	0.054	0.059	0.914	0.361
COPEc	-0.872	0.126	-6.898	0.001
<i>Tau (WS variance parameters: log-linear model)</i>				
Intercept	0.227	0.229	0.992	0.321
9am-2pm	0.174	0.042	4.110	0.001
2pm-6pm	0.248	0.043	5.797	0.001
6pm-10pm	0.297	0.043	6.918	0.001
10pm-3am	0.355	0.064	5.528	0.001
Others_BS	-0.930	0.297	-3.133	0.002
Others_WS	-0.220	0.030	-7.254	0.001
genderf	0.296	0.089	3.321	0.001
age15	-0.038	0.040	-0.938	0.348
COPEc	-0.391	0.080	-4.884	0.001
<i>Random scale standard deviation</i>				
Std Dev	0.719	0.028	25.64	0.001
<i>Random location (mean) effect on WS variance</i>				
Loc Eff	0.676	0.042	16.217	0.001

Table 2

Variance ratios and 95% confidence intervals (lower, upper).

Variable	Ratio	Lower	Upper
<i>Alpha (BS variance parameters)</i>			
Intercept	0.933	0.523	1.665
9am-2pm	1.065	0.957	1.185
2pm-6pm	1.097	0.983	1.224
6pm-10pm	1.006	0.900	1.126
10pm-3am	1.048	0.879	1.250
Others_BS	0.564	0.257	1.239
Others_WS	0.742	0.681	0.808
genderf	1.055	0.815	1.366
age15	1.055	0.940	1.184
COPEc	0.418	0.327	0.536
<i>Tau (WS variance parameters)</i>			
Intercept	1.255	0.802	1.964
9am-2pm	1.190	1.095	1.294
2pm-6pm	1.282	1.179	1.394
6pm-10pm	1.346	1.237	1.464
10pm-3am	1.426	1.257	1.617
Others_BS	0.394	0.220	0.706
Others_WS	0.803	0.756	0.852
genderf	1.344	1.129	1.600
age15	0.963	0.890	1.042
COPEc	0.677	0.578	0.791
<i>Random scale standard deviation</i>			
Location Effect	1.966	1.817	1.085
<i>Random location (mean) effect on WS variance</i>			
Std Dev	2.052	1.942	2.168

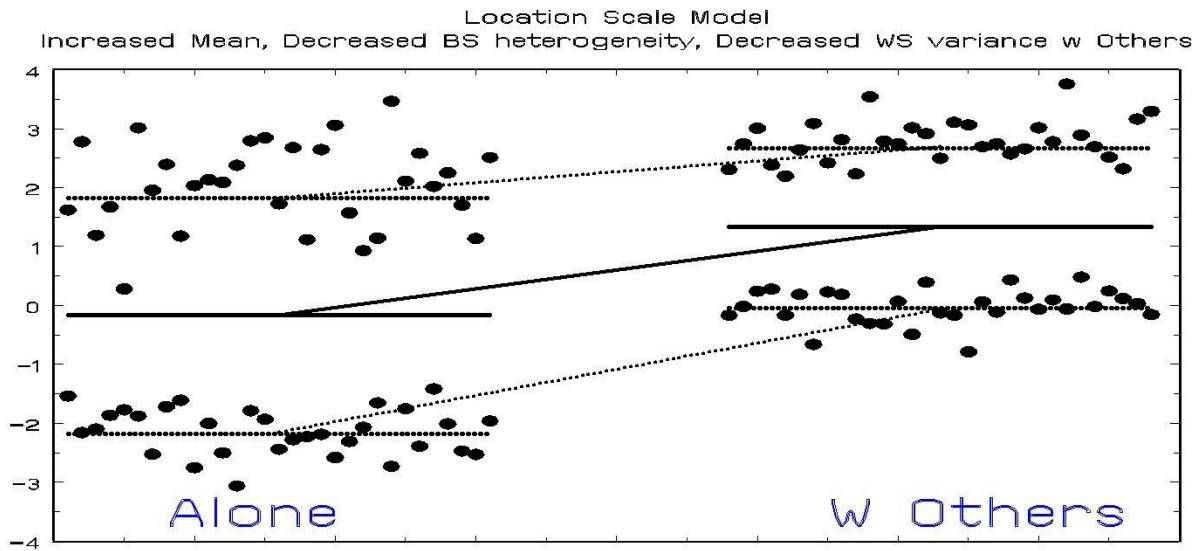


Figure 2 . Visual representation of the mean, BW and WS variance submodels.

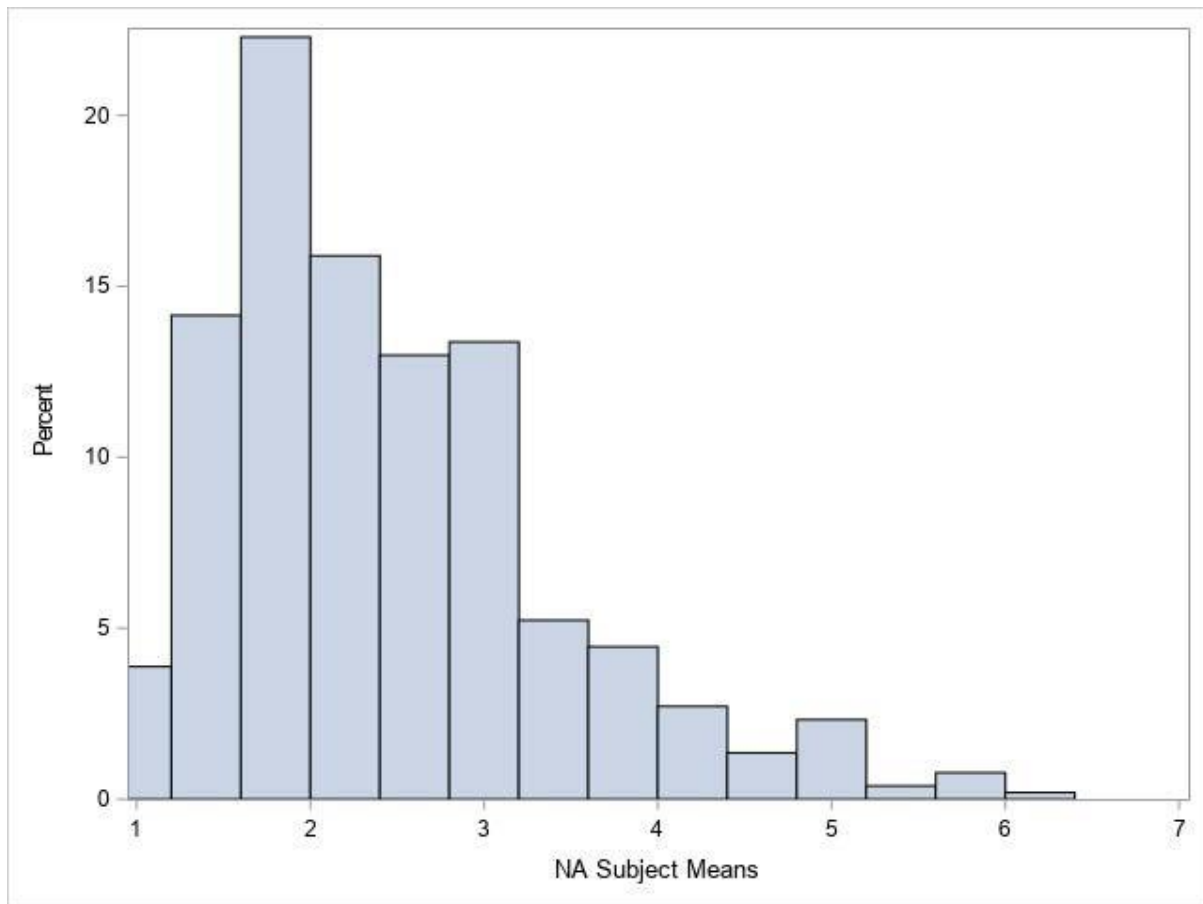


Figure 3 . Histogram of subject-level means of negative affect.

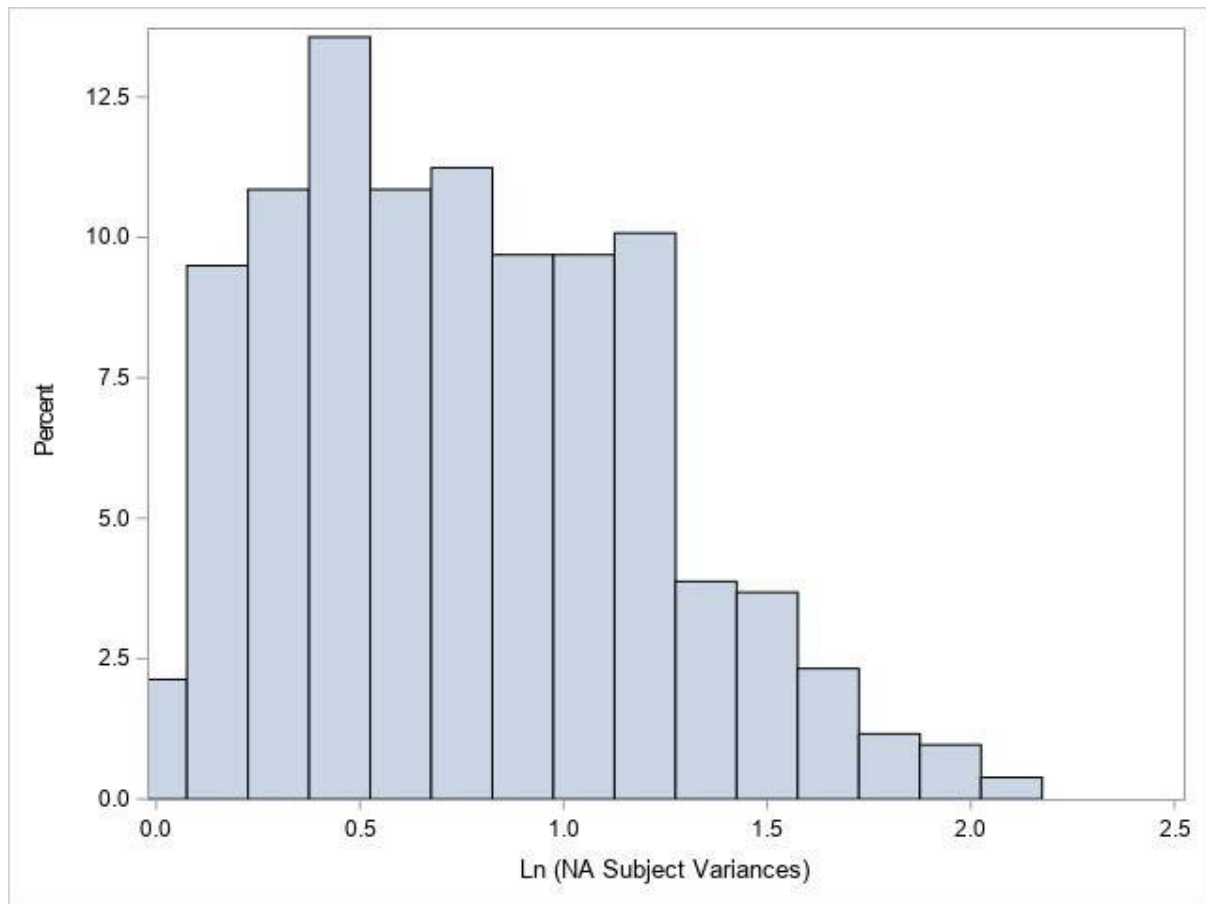


Figure 4 . Histogram of subject-level ln(variances) of negative affect.

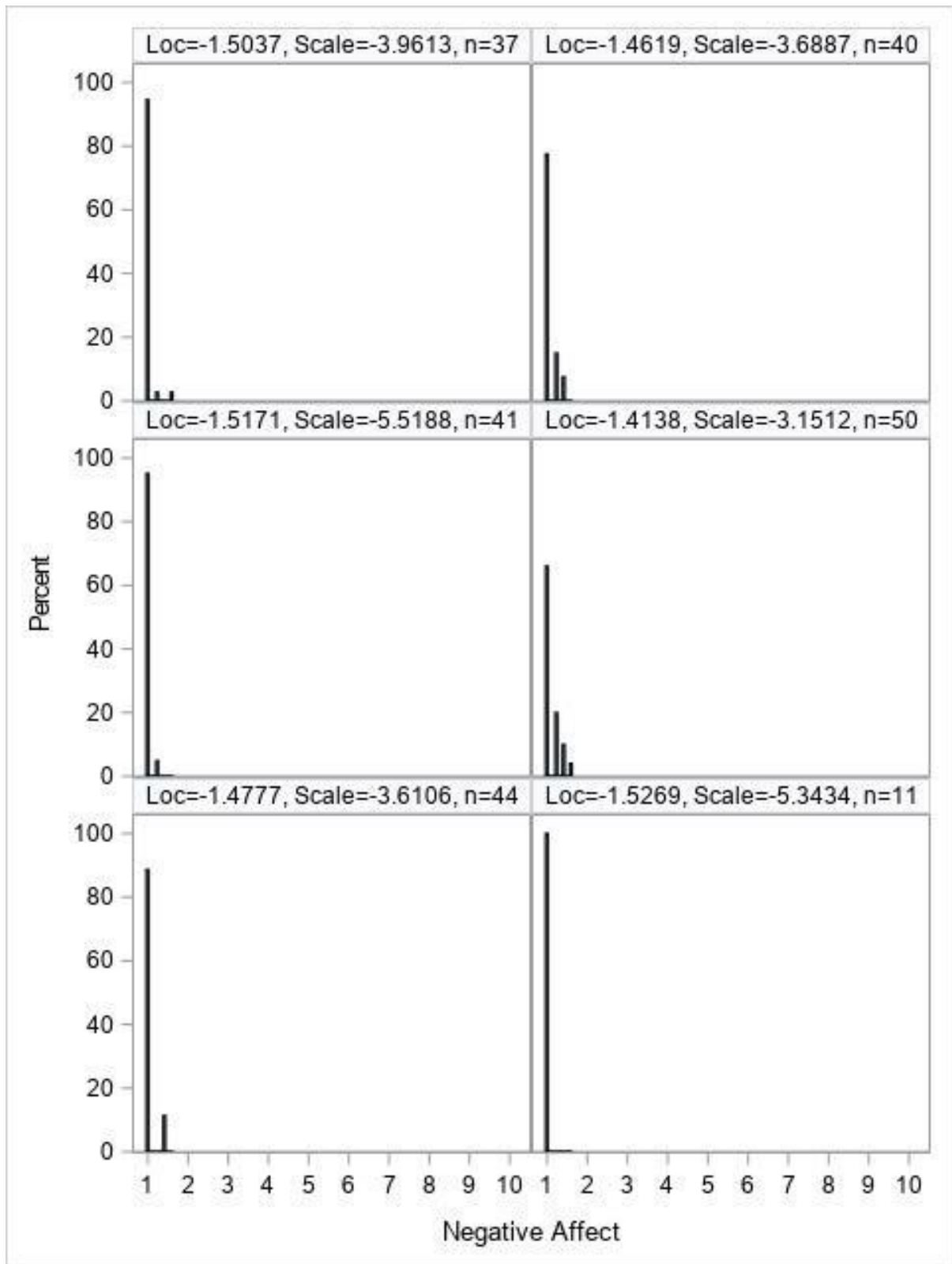


Figure 5 . Null Model of Negative Affect: Data histograms from subjects with low scale estimates.

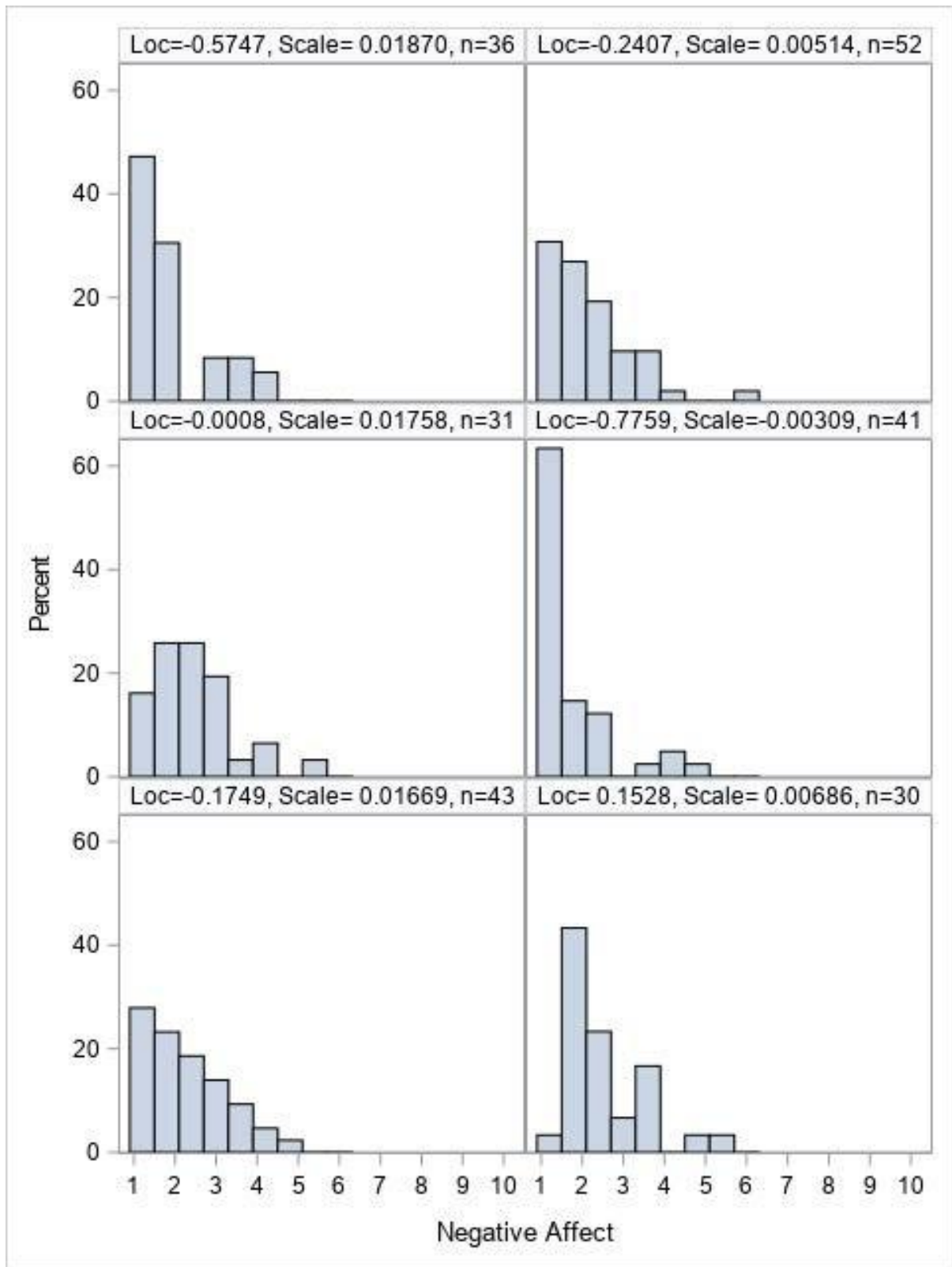


Figure 6 . Null Model of Negative Affect: Data histograms from subjects with near-zero scale estimates.

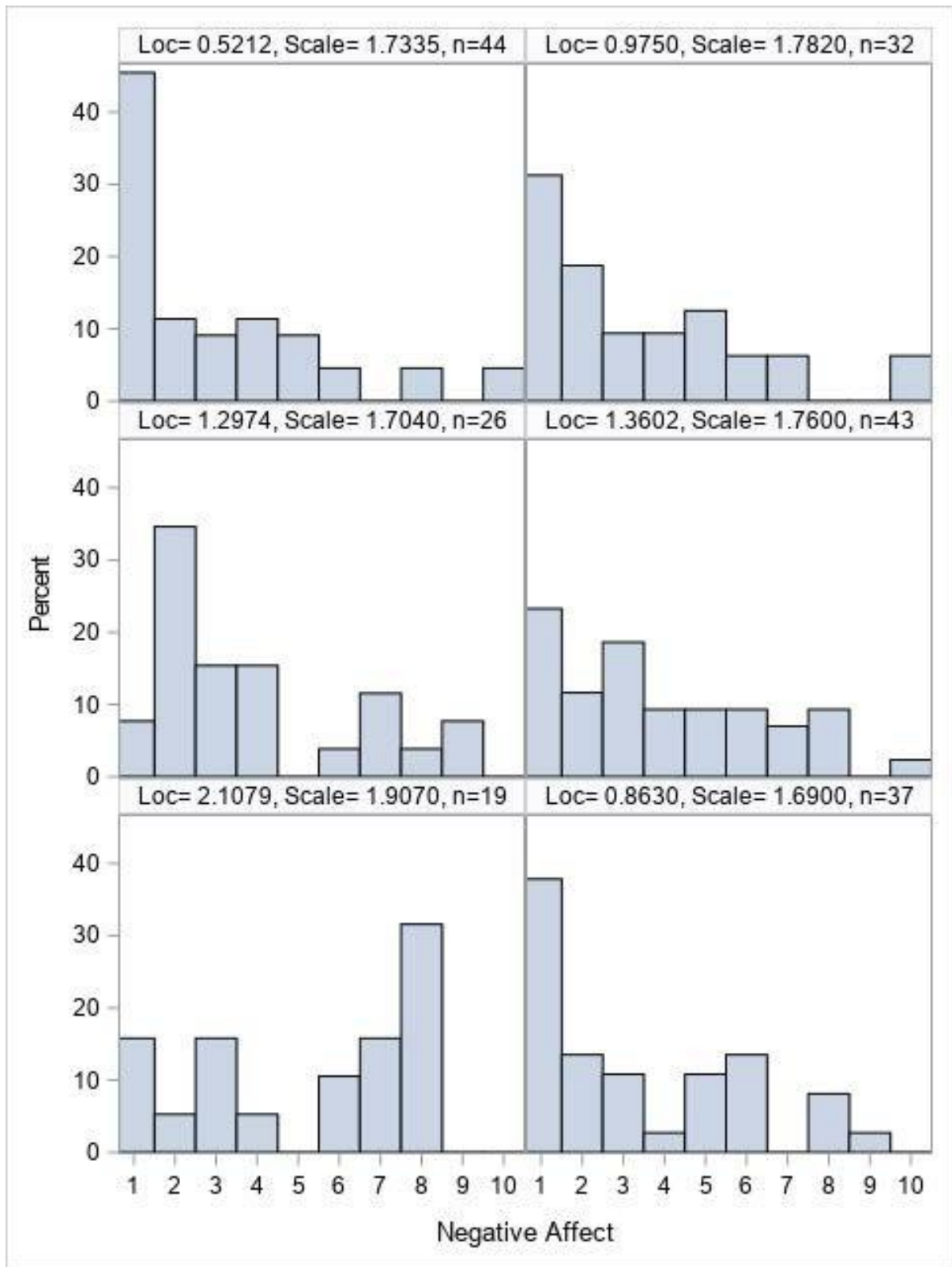


Figure 7. Null Model of Negative Affect: Data histograms from subjects with large scale estimates.

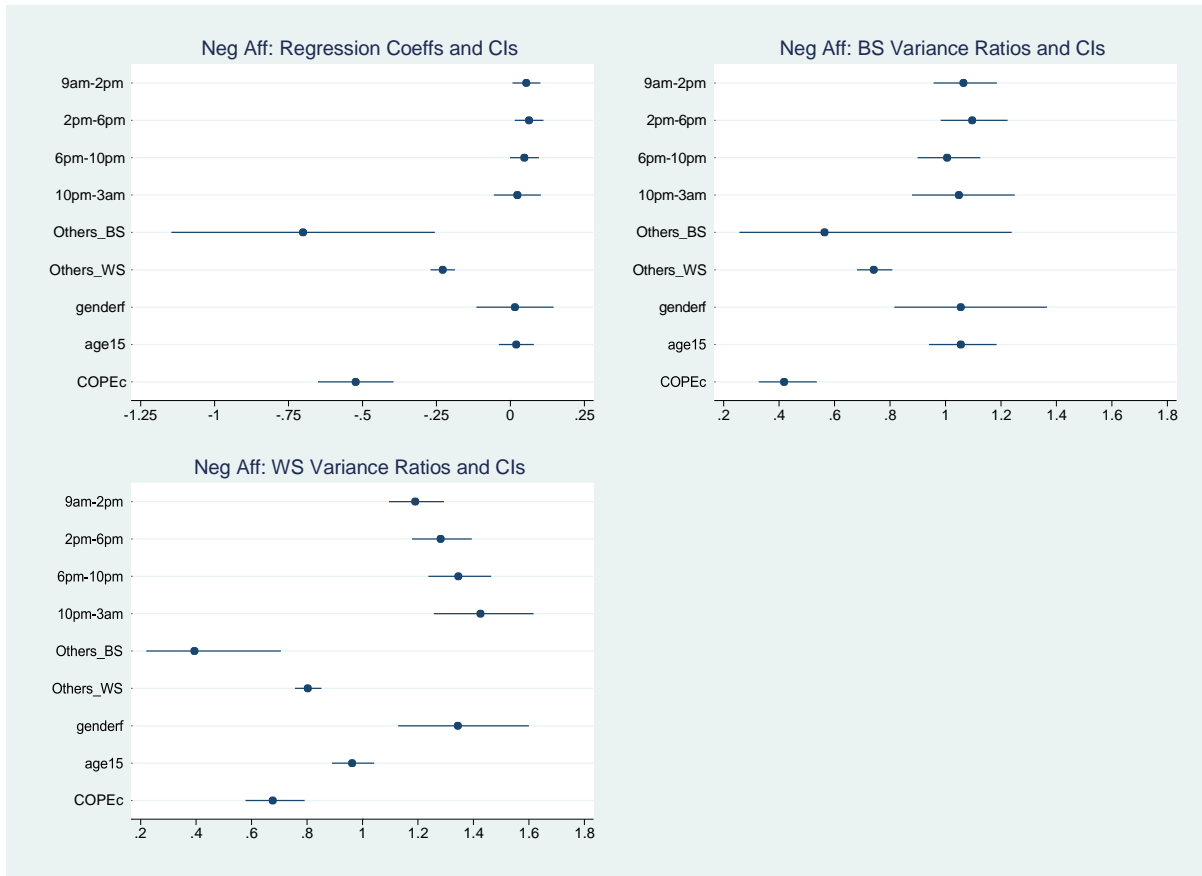


Figure 8 . Modeling of Negative Affect: (a) regression coefficients and 95% confidence intervals, (b) BS variance ratios and 95% confidence intervals, (c) WS variance ratios and 95% confidence intervals.