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Windows, Baths, and Solar Energy in the Roman Empire

JAMES W. RING

Abstract

Windows were a prominent feature of Roman architecture and were especially important in the magnificent bath buildings of the Roman Empire. A growing literature attests to the Romans' use of solar energy in heating these large buildings. Edwin Thatcher claimed in 1956 that the windows in such baths did not require glazing. In this paper I refute this claim, drawing on modern ideas about solar energy, heat transfer, human comfort, and the effect of glazed windows to analyze one room in the Forum Baths at Ostia. This analysis is compared with that of Thatcher for the same room. In window size and solar orientation, this room is typical of Roman baths in many parts of the empire. The solar science and technology of today is thus compared with that of the Romans and with that of Thatcher's day.*

INTRODUCTION

The importance of windows in architecture seems indisputable. But in its chronological development Western architecture shows striking variation in its treatment of windows. For example, medieval churches of the Romanesque style were typically dark and lit only by small windows that pierced massive masonry walls. One of the distinguishing features of later Gothic churches was the use of large windows made possible by outside buttressing. The Roman Empire prototypes, however, unlike their Romanesque successors, were often lit by magnificently large windows. The Roman public baths of the Early Empire are very good examples of this anachronism. Seneca, writing in the first century A.D., says of these baths: "Nowadays . . . people regard baths as fit only for moths if they have not been so arranged that they receive the sun all day long through the widest of windows, if men can not bathe and get a coat of tan at the same time, and if they can not look out from their bath-tubs over stretches of land and

sea."¹ One can infer from his writings that Seneca regrets this new style and, indeed, it is clear that he looks back with nostalgia on the days of the Roman Republic when baths were properly and modestly dark.

Roman baths of the Early Empire were in the forefront of developments both architecturally and technologically and thus make a very interesting study in their own right. This point has been brought out and ably developed by Yegül² and Nielsen.³ Vaults, domes, and large windows were first found in these baths, where Greek orders were also first combined with Roman vaults. Hypocausts were developed and used to heat large rooms and, indeed, to heat the imposing ensembles of large rooms that the great imperial baths represented. According to Yegül, Seneca speaks of the recent invention of *tubuli*, or hollow walls, which maintain an even temperature in the lowest as well as the highest spaces. This invention also prevents condensation on the walls and increases the area that radiates heat around the bathers. In these large evenly heated spaces, thousands of bathers could be and often were accommodated. To supply sufficient water, extensive aqueduct systems were developed. Furthermore, as Yegül maintains, and is shown in a detailed fashion by D.B. Harden, the Romans by this time had developed glassblowing and were producing flat panes of window glass.⁴ Thus, with all of these elements in hand, it is not surprising that the Romans would have utilized the radiant energy of the sun to help heat as well as light these magnificent buildings.

Indeed, so obvious is the Romans' interest in solar heating through their use of large south-facing windows that in 1956 Edwin Thatcher published a paper in which he claimed that the large windows

* I wish to thank the American Academy in Rome for help in arranging visits to various Roman sites, including Ostia, in 1985; the Corning Museum of Glass and Rakow Library for help in exploring Roman glass in 1991; Todd Moore of the Physics Department at Hamilton College for his careful reading of the manuscript; Carl Rubino of the Classics Department at Hamilton for his advice; Fikret Yegül of the University of California at Santa Barbara and Ingrid E.M. Edlund-Berry of the University of Texas at Austin for reading an earlier version of this manuscript and advising about publication; and Richard de Dear of Macquarie University, Sydney, for his advice about thermal comfort conditions in these baths. Finally, Hamilton College provided

a Faculty Fellowship in 1993 during which research leading to this paper was completed.

¹ Sen. *Ep.* 86, trans. R.M. Gummere, 1986, as quoted by F. Yegül, *Baths and Bathing in Classical Antiquity* (Cambridge, Mass. 1992) 40.

² Yegül (supra n. 1).

³ I. Nielsen, *Thermae and Balnea: The Architecture and Cultural History of Roman Public Baths* (Aarhus 1990).

⁴ Yegül (supra n. 1) 363–65; and D.B. Harden, "Domestic Window Glass: Roman, Saxon and Medieval Studies in Building History," in E.M. Jope ed., *Studies in Building History. Essays in Recognition of B.H. St. J. O'Neil* (London 1961) 39–63.

of the second-century Forum Baths at Ostia, which he took to be unglazed, provide "a striking demonstration of the potentialities of the Roman heating method and, in extension, of the principles of radiant heating. It was this method that made the open rooms possible and, to date, we have not matched them in a modern building. It is evident that the Roman engineers had a greater confidence in radiant heating than we have and a greater knowledge of what it could accomplish."⁵

This paper sets out to investigate this claim. Is Thatcher's confidence in radiant heating justified? Could there be enough heating provided by the sun and the hypocaust to allow nude bathers to be comfortable even though the large windows of the baths were open and unglazed? What do the modern principles of passive solar heating and the physics and physiology of heat transfer tell us about the Forum Baths and Thatcher's claim?

In addition, I take up a point raised by Yegül. He suggests that Thatcher has gone too far in his claims for radiant heating:

In full admiration of the system's potential, I still doubt if the implications of radiant heating should be stretched that far. Not only is the evidence for window glass and window frames (both in wood and metal) from the heated rooms of Roman baths across the Mediterranean overwhelming, but Thatcher's thesis, despite its theoretical possibility, seems to refute the precepts of simple economic logic. It may be that by heating the floor, the walls, and the vault to a high degree, sufficient radiant energy could be released to offset the effects of low air temperature on a cold winter day, but why should fuel and energy be wasted in order to make an open-air hot bath possible when the same degree of warmth and comfort could be achieved with much lower furnace activity and fuel consumption in a glazed and well-insulated room?⁶

This question is also raised by Jordan and Perlin in an article about the use of solar energy in ancient times.⁷ They claim that by the first century B.C., Rome had to import timber from the fringes of its domains, such as the Alpine regions, in part because of the Roman love of bathhouses—there were 800 baths in Rome alone in the third century A.D.—but also because of the growth of industry and manufacture. As they point out, "prices of wood, charcoal, and small firewood rose steeply. To avoid the grow-

ing shortages and expense, the Romans, like the Greeks before them, turned to solar heat."⁸

DID THE ROMANS USE GLAZING?

The Forum Baths in Ostia were constructed early in the second century A.D. There is no controversy about the existence of large windows in several of the rooms of these baths. These windows faced the south and hence would intercept the sun's beam radiation most of the day, particularly from early afternoon to near sunset, which were the most popular hours for Romans to bathe. These rooms indeed are typical of baths built during this period in many parts of the empire.

Thatcher gives attention to the whole set of large-windowed rooms but for our purposes let us concentrate on one, room 4, which seems to have been a warm room, or *tepidarium*. Figures 1 and 2 show the southern elevation and north-south section, respectively, of this room and its window. The dimensions are those given by Thatcher.

The question is whether or not the walls, vault, and floor surrounding the nude bather on all but the window side can be maintained at a high-enough temperature to ensure comfort. Thatcher approaches the issue from the standpoint of the nude bather exposed both to the radiant energy of the sun and that given off by the surrounding heated room surfaces, holding that radiant energy, if the walls are maintained at close to skin temperature, can by itself establish a comfortable temperature. My approach, on the other hand, is to assume that comfort will be determined by conditions in the room, both the air temperature and the radiant temperature being considered along with air currents, or convective flows, and the relative humidity of the air as prescribed by the American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE).⁹ Comfortable conditions for nude subjects have been studied carefully in climate chambers. An example of one such study, which also shows the importance of heat transfer at the skin surface, is that of de Dear, Ring, and Fanger.¹⁰ Also of importance is the air flow over the skin. This too has been studied and reported on by, for example, Fanger and his colleagues.¹¹

⁵ E.D. Thatcher, "The Open Rooms of the Terme del Foro at Ostia," *MAAR* 24 (1956) 169–264.

⁶ Yegül (supra n. 1) 383.

⁷ B. Jordan and J. Perlin, "Solar Energy Use and Litigation in Ancient Times," *Solar Law Reporter* 1.3 (1979) 583–94.

⁸ Jordan and Perlin (supra n. 7) 587.

⁹ *ASHRAE Handbook, 1977 Fundamentals* (New York 1977)

ch. 8.

¹⁰ R.J. de Dear, J.W. Ring, and P.O. Fanger, "Thermal Sensations Resulting from Sudden Ambient Temperature Changes," *Indoor Air* 3 (1993) 181–92.

¹¹ P.O. Fanger, A.K. Melikov, H. Hanzawa, and J.W. Ring, "Air Turbulence and Sensation of Draught," *Energy and Buildings* 12 (1988) 21–39.

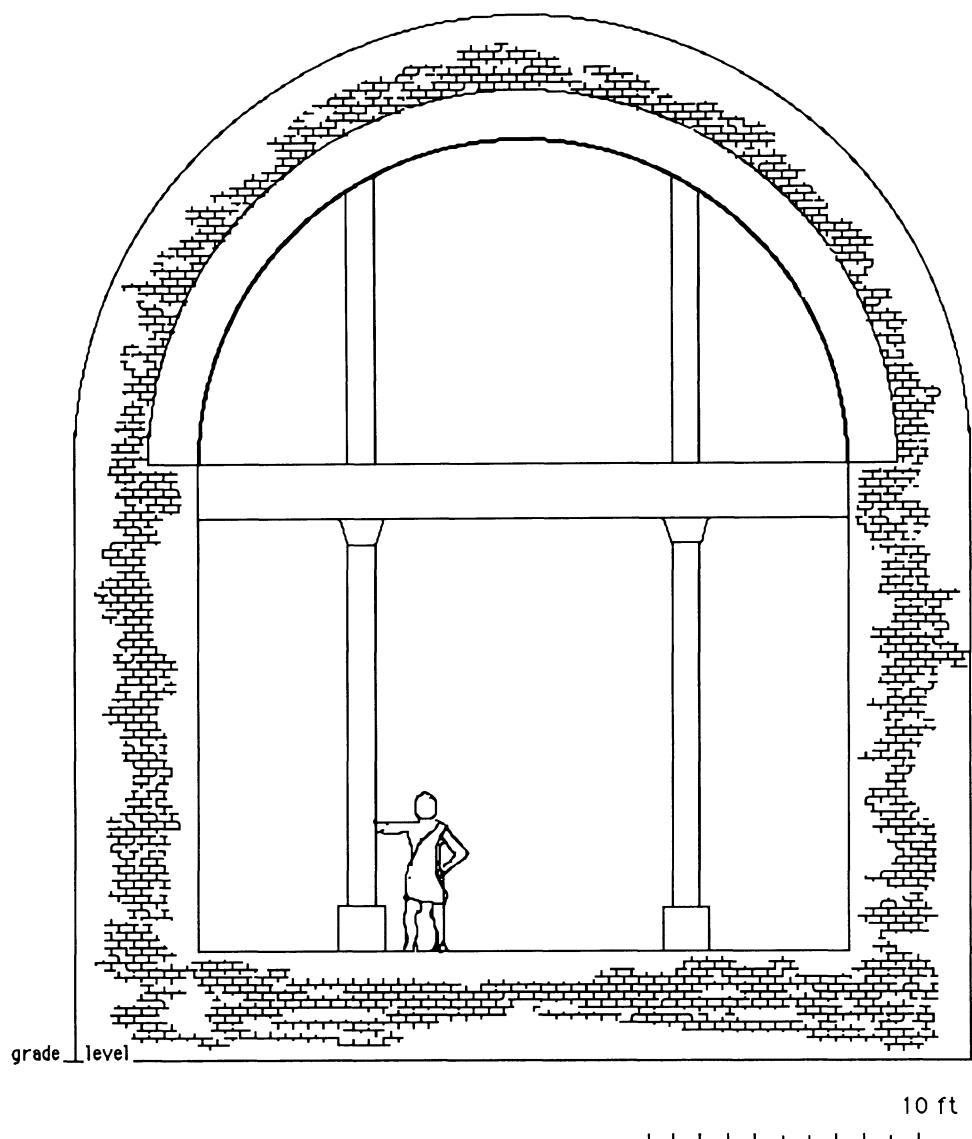


Fig. 1. Southern elevation of room 4 at the Forum Baths of Ostia

The problem then becomes one of the transfer of heat from the hypocaust and the sun to the room and from the surfaces of the room to the outside mainly through the large window. These processes determine the temperatures of the surfaces and thus the convective drafts and air temperature. Relative humidity, of course, depends on the vapor pressure of water in the space. In room 4, which apparently had no pools or baths, the relative humidity would not have been particularly high, perhaps about 50%. The temperature and convective flows were thus the primary determinants of human comfort in this room.

Naked bathers do not expect or want a thermally

neutral environment. On the contrary, they expect to feel hot. Thus, strictly speaking, we are not dealing with the usual comfort scale but rather with one biased toward the hot end. For example, the seven-point ASHRAE scale (cold, cool, slightly cool, neutral, slightly warm, warm, hot) would allow using only the upper point in the warm rooms of the baths. According to Thatcher, the temperatures of the walls and floors of these rooms were $\sim 40^{\circ}\text{C}$ (or $\sim 100^{\circ}\text{F}$).¹² In fact, this is in the range of evaporative regulation (sweating) that is adjacent to but not part of the comfort zone, i.e., these are, according to ASHRAE, “uncomfortable” conditions.

¹² Thatcher (*supra* n. 5) 190–94.

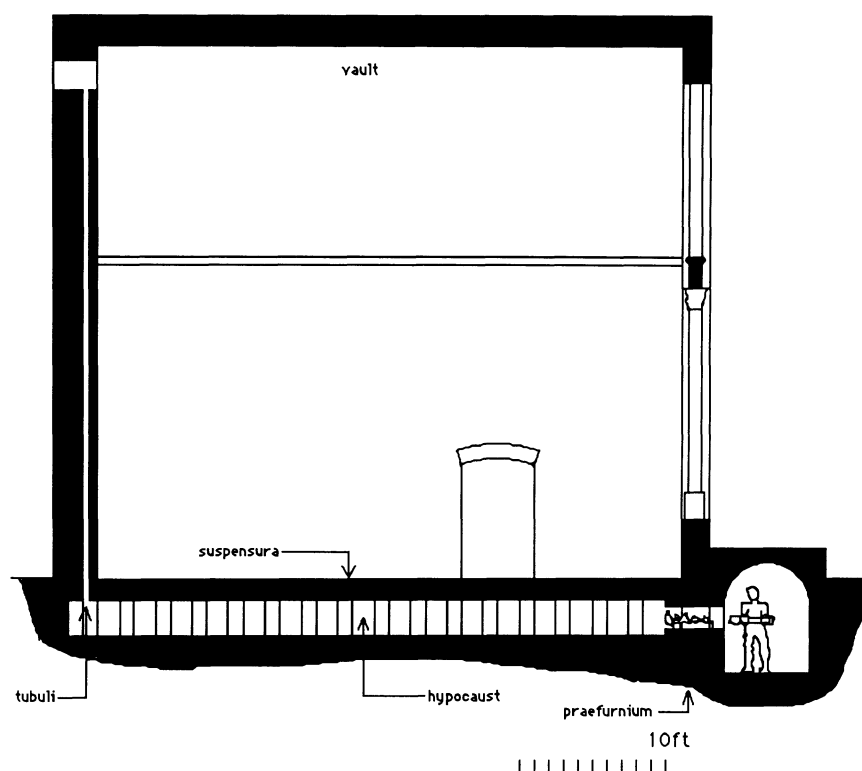


Fig. 2. North-south section of room 4 at the Forum Baths of Ostia

We take conditions to be like those assumed by Thatcher: outside temperature is just below freezing, 30°F ,¹³ the interior surfaces of the room are at 100°F , and the sun is shining in the window during December at 250 British thermal units (BTU)/ft²/hr, assuming a clear sky. Further, we take the temperature of the hot gases at the top of the hypocaust below the floor to be 400°F , which is consistent with the experiments of Rook¹⁴ with a small hypocaust in Welwyn, England, although somewhat above the temperature ($\sim 300^{\circ}\text{F}$) found by Kretzschmer¹⁵ in experiments in Saalburg, Germany. For the purposes of the argument here, any temperature up to 400°F can be posited. The crucial heat flow is that out through the open window, and if we assume 400°F we are estimating the maximum heat flow in and thus giving Thatcher the best chance of being correct. As does Thatcher, I too assume that the floor and inside wall surfaces are held at $\sim 100^{\circ}\text{F}$, including the inside of the vault, which, although unheated by tubuli, by convection and radiation will be at nearly 100°F if the walls are also at this temperature. Note

that temperatures of 70°F for these surfaces will cause the nude bather to radiate heat to them as well as losing heat to them by convection (and conduction if in contact with them). These will not be warm conditions for him but rather ones somewhat on the cool side. The thermal properties of the materials, i.e., conductivity and coefficients of heat transfer for radiative or convective flow, can be found in the appropriate part of the *ASHRAE Handbook*.¹⁶

With these parameters known, calculations can be made for the heat flows as shown below in table 1. The heat flows without and with glass are shown diagrammatically in figures 3 and 4, with figure 3 showing inflows and figure 4 outflows. Here we are concerned with the comparison of flows in versus flows out, for if they are not equal, the temperature of the walls and the room will not be constant at, or close to, the desired 100°F .

Certain caveats about these calculations ought to be made clear. None of these heat flows can be said to be precisely defined. The problems in calculation are:

¹³ This is, according to Thatcher (*supra* n. 5) 182–83, a quite possible temperature in Ostia in December or January, and in my own experience a low but not outlandish one.

¹⁴ T. Rook, "The Development and Operation of Roman Hypocausts," *JAS* 5 (1978) 269–82.

¹⁵ F. Kretzschmer, "Hypokausten," *Saalbfj* 12 (1953) 7–41.

¹⁶ *ASHRAE Handbook* (*supra* n. 9) ch. 11.

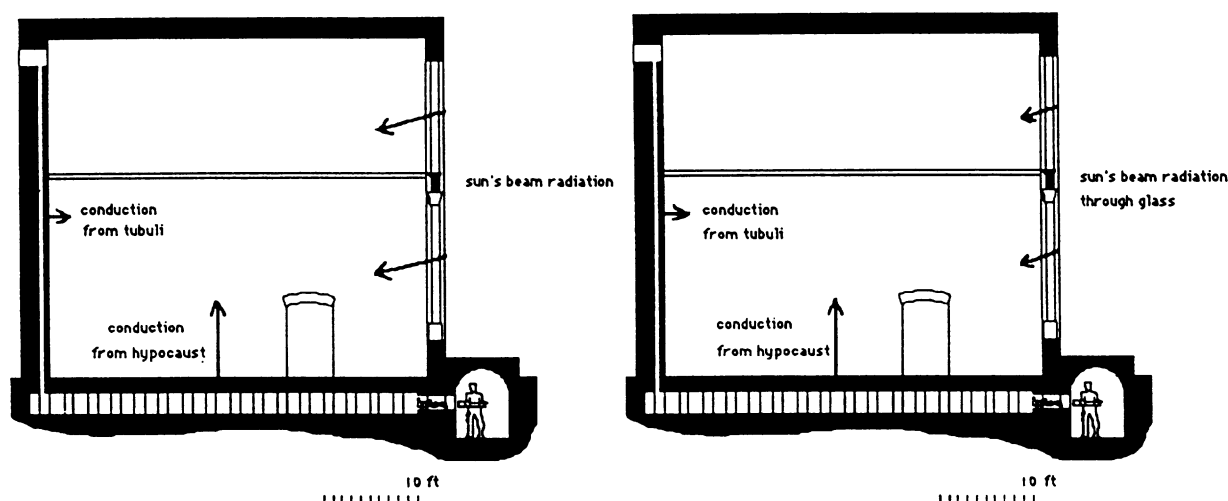


Fig. 3. Heat flows into room 4 with open windows (left) and glazed windows (right)

1) The dimensions are not always precisely known. I use those given by Thatcher.

2) In some cases the materials are only guesses as the upper part of the room has disappeared. Generally I follow Thatcher's suggestions.

3) I assume, as does Thatcher, that the vault is not heated.

4) The optical quality of Roman glass varies widely and that used at Ostia in these baths is not known. A transmission of 50%, assumed here, is probably a quite conservative estimate.

5) I estimate the temperatures of the lower *suspensura* (floor) surface and the inner surfaces of the tubuli from Rook's and Kretschmer's experiments.

6) The solar beam radiation is calculated here in the same way as it was by Thatcher, agreeing also with the method used by Ring and Hamilton¹⁷ to test the performance of a solar classroom at Hamilton College at close to the same latitude as Ostia.

7) Convective flows are notoriously hard to calculate, but since such flows do occur in solar houses we can expect to achieve order of magnitude results

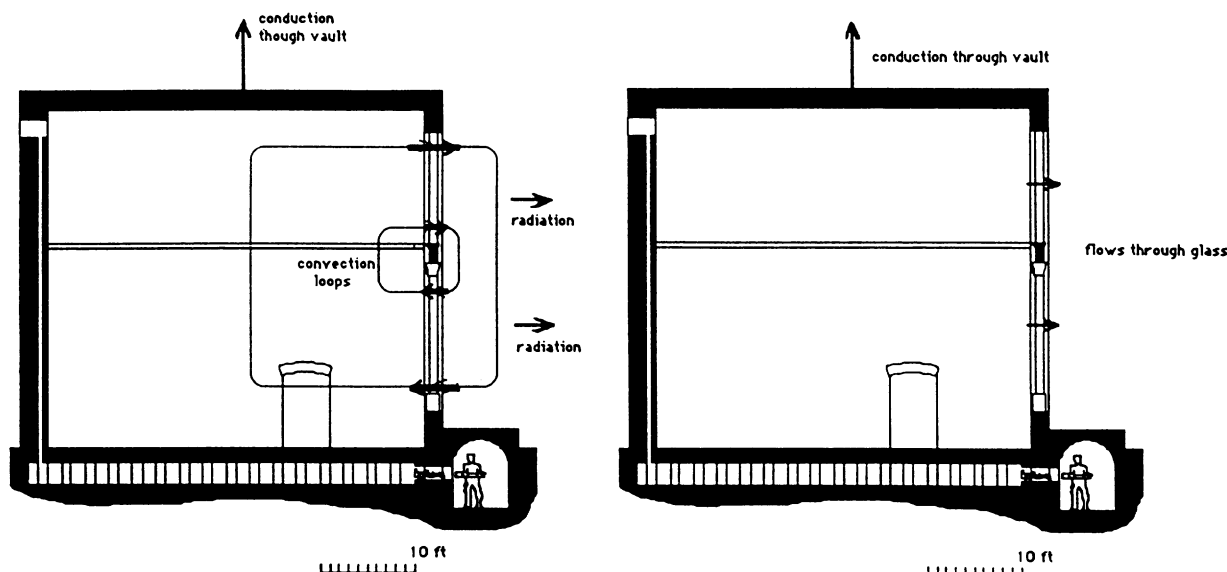


Fig. 4. Heat flows out of room 4 with open windows (left) and glazed windows (right)

¹⁷ J.W. Ring and A. Hamilton, "The Solar Classroom at Hamilton College," *Proceedings, Conference on Solar Heating and Cooling Systems, Colorado Springs, Colorado* (Solar Energy

Research Institute TP-245-430, Washington, D.C. 1979) 107-11.

Table 1. Heat Flows for Room 4 with Glazed and Unglazed Windows

Direction and Source of Flow	Units of 100,000 BTU/hr
<i>Into room</i>	
Hypocaust (full heat)	4.0
Sun (clear day at winter solstice)	
Unglazed window	1.3
Glazed window	0.7
TOTALS	
Unglazed window	5.3
Glazed window	4.7
<i>Out of room</i> (outside temperature = 30° F)	
Conduction	0.4
Natural convection and radiation	
Unglazed window	52.5
Glazed window	0.4
TOTALS	
Unglazed window	52.9
Glazed window	0.8
Net Flow (+ = into room)	
Unglazed window	- 47.6
Glazed window	+ 3.9

using the solar designer's formulas.¹⁸ Natural rather than forced convection is assumed, i.e., there is no wind blowing. With wind the convective flow is greater, and could be much greater at high-wind velocities.

Under such circumstances we can expect only to estimate these flows. Even with these rough estimates, however, some important conclusions can be reached. The estimates for glazed and unglazed windows are given in table 1 above. The details of the calculations follow in an appendix.

RESULTS AND DISCUSSION

The results of the calculations summarized in table 1 and in the appendix indicate that with an open window the input is ~530,000 BTU/hr while the outflow is ~5,290,000 BTU/hr. In such a case it is obvious that equilibrium is not possible and that the 100° F surfaces will rapidly cool toward 30° F. The same would be true for a nude bather whose skin temperature normally should be ~93° F. Indeed the outflow would equilibrate with the inflow only when the wall and floor surfaces are within 10° F or less of the outside temperature, i.e., at ≤40° F.

On the other hand, if the window is glazed the heat from the hypocaust (400,000 BTU/hr) would be

much more than adequate to provide the outward flow (80,000 BTU/hr) in radiation, conduction, and convection through the glazed window and in conduction through the vault. The sun alone on sunny days could provide most of the energy to maintain 100° F temperatures. Indeed, even with the fires reduced on sunny days, there would probably be some thermal energy stored in the floors and walls that would maintain the temperature as the sun goes down. On days when the sun is obscured by clouds, the hypocaust with a reduced fire, or being on only part of the time, could by itself easily maintain the temperature even with the outside temperature at its coldest point of the season, i.e., the design temperature of 30° F.

The stored thermal energy, which may come from either the sun or the hypocaust or both, can be handled quite easily by the heavy masonry walls, vault, and floor of this room. With such surroundings extra heat in the room can pass readily by conduction into the masonry without heating the air in the room excessively, i.e., much above 100° F. At night, when the sun is down (even with the fire out), this stored heat will flow back into the room to offset the cooling that inevitably will occur. Note that wooden shutters closing the window area at night would enhance this storage considerably. Such heat storage is an important element in solar house design and the Romans seem to have incorporated this element into their designs as well. The thickness of the floor, or *suspensura*, is especially interesting in this regard and it may be that the thickness chosen, i.e., 15 in, is not necessary structurally but aids in long-term heat retention.

Thus, we see that Yegül is correct in his claim that the Romans would have been wise to use glass in their bath windows. Furthermore, we see that Thatcher is probably overly enthusiastic in claiming such efficiency for radiant heating. Indeed, with the windows open, the rooms in these baths could not have been maintained at 100° F and a nude bather would soon have become very chilly. Thatcher neglected to consider fully the very great heat flow out of the room due to the convective flow through the open window. He does not completely ignore the possibility of an air current but claims that such a current would only exist with no wind, and normally there would be some wind. He then says, "A wind pressure of any but the lowest magnitude would nullify the action and set in motion the various air currents already described."¹⁹ These currents, Thatcher

¹⁸ J.D. Balcomb ed., *Passive Solar Buildings* (Cambridge, Mass. 1992) 149-52.

¹⁹ Thatcher (supra n. 5) 233-34.

believed, flowed from north to south, or vice versa, across the room; in the north wall they passed through the door, cracks around the door, or the lunette at the top of the vault, and in the south wall, through the upper part of the window.

This assertion seems incorrect because a wind blowing in, or eddying through the southern window, will not nullify the convective effect. Rather it will cause a change from natural convection to forced convection, changing and distorting the geometry of the convective loop and, as a result, increasing the mixing of cold and hot air and thus increasing the heat loss above that caused by natural convection alone. The net result would be to set the 52.5×10^5 BTU/hr heat loss calculated above as a minimum value and in windy situations to expect this loss to be even greater with the concomitant effect of an even faster lowering of the bath temperature, more quickly chilling the nude bather.

Note that the natural convective flow calculated here is only a rough estimate. But it is about 10 times the inflow so that even if it is overestimated by a factor of two, it still will be many times greater than the inflow. With wind, it will be even greater than calculated here.

Finally, to return to Yegül's point, the estimates of heat flows here show not only that nude bathing in Roman baths would not have been possible without glazing but also that with glazing during sunny days, the sun with only a little help from the hypocaust and its furnaces, and hence little wood burned, could have maintained the temperature of these room surfaces at $\sim 100^\circ$ F. Furthermore, on cloudy days the hypocaust with only a low or intermittent fire would have been able to sustain this temperature. And even at night the large thermal storage capacity would have kept temperatures from dropping very fast so that by the next morning the amount of heat necessary to return to $\sim 100^\circ$ F might have been relatively small. Thus with a normal mix of sunny days, a considerable savings of fuel could be accomplished even in the depths of winter. At other seasons even more savings could be expected. The sun would therefore provide a substantial part of the heat required. This result, of course, is in accord with Jordan and Perlin's observations about the increasing cost of fuel during this period of rapid growth of Roman industry, commerce, manufacture, and population. Fuel costs would have provided a strong incentive for using glazed windows and the sun's energy.

In summary, the Romans apparently did display considerable know-how in the design of their baths when judged by the standards and practices of modern science and technology 2,000 years later.

Thatcher, it seems, was too sanguine about radiant heating but, nevertheless, the Romans deserve high praise for their use of solar energy. Even Seneca, no admirer of conspicuous consumption and easy living, might have admired the frugality that the combination of "the widest of windows" and glass panes in baths demonstrated.

Appendix

Calculations of Heat Flows for Room 4 with and without Glazing

A. Heat flows into the room:

(i) The heat flow through the suspensura: this is a heat conduction problem where Q , the heat flow per hour, is given by:

$$Q = \frac{kA\Delta T}{\Delta x} \text{ with } k \text{ the thermal conductivity of the concrete slab, } \Delta x \text{ the thickness of the slab, } A \text{ the cross-sectional area, and } \Delta T \text{ the temperature difference between the top and bottom of the slab.}$$

Using British engineering units with

$$k = 11 \text{ BTU}\cdot\text{in}/^\circ\text{F}\cdot\text{ft}^2\cdot\text{hr (value used by Thatcher)}$$

$$A = 1,200 \text{ ft}^2$$

$$\Delta T = 400 - 100 = 300^\circ \text{ F (using maximum hot gas temperature under suspensura)}$$

$$\Delta x = 15 \text{ in}$$

$$Q = 2.6 \times 10^5 \text{ BTU/hr}$$

(ii) The heat flow through the heated walls:

$$k = 7.0 \text{ BTU}\cdot\text{in}/^\circ\text{F}\cdot\text{ft}^2\cdot\text{hr (value used by Thatcher)}$$

$$A = 1,760 \text{ ft}^2 \text{ (3 vertical walls)}$$

$$\Delta T \text{ estimated to be } 200 - 100 = 100^\circ \text{ F}$$

$$\Delta x = 9 \text{ in}$$

$$Q = 1.4 \times 10^5 \text{ BTU/hr}$$

(iii) The sun's radiant energy in through the window:

In December in Rome, the sun at noon is only about 24° above the horizon and the sun's beam intensity is approximately 250 BTU/hr/ft². Note that compared with 228 at the winter solstice, the noon value on a south-facing vertical surface at equinox would be $285 \times \cos 42^\circ$, or 212 BTU/hr/ft² at this latitude of 42° N. At the summer solstice this value would be 116. At noon, regardless of the season, the sun's beam radiation on this surface would always be at its maximum for the day. Note also that these intensity values agree with those used by Thatcher:

$$A = 560 \text{ ft}^2$$

$$Q = 250 A \cos 24^\circ = 1.3 \times 10^5 \text{ BTU/hr}$$

or with Roman glass:

$$Q \cong 0.65 \times 10^5 \text{ BTU/hr}$$

Thus, the total energy input $\cong 5.3 \times 10^5$ BTU/hr, i.e., the sum of the above items, or with glass $\cong 4.65 \times 10^5$ BTU/hr.

B. Heat flows out of the room:

(iv) Conduction through vault to the outside:

$$k = 7.0 \text{ BTU}\cdot\text{in}/^\circ\text{F}\cdot\text{ft}^2\cdot\text{hr, as in the walls}$$

$$A = 1,880 \text{ ft}^2$$

$$\Delta T = 100 - 30 = 70^\circ \text{ F}$$

$$\Delta x = 24 \text{ in}$$

$$Q = 3.8 \times 10^4 \text{ BTU/hr}$$

(v) The natural convective flow through the open window:

Here the convective loop will have hot air exiting through the top of this large window and cold air coming in at the bottom. See figure 4. This flow will be caused by the stack effect in which hot, less dense gas is forced out of the room at the top of the window and, by the same effect, cold air that is more dense will flow in at the bottom. Solar house designers use the formula given below to calculate this air flow:²⁰

$$\text{CFM} = 9.4 A_{\text{eff}} \sqrt{H \Delta T}$$

where A_{eff} is the effective area (ft^2) through which the flow enters and/or leaves the space, and H is the effective height of this area. In this case the A_{eff} is half the area of the window while H is half the height of the window as half the window is used for outward flow and the other half for inward flow, or $A_{\text{eff}} \cong 260 \text{ ft}^2$, $H \cong 10 \text{ ft}$ (mean height), and $\Delta T = 100 - 30 = 70^\circ \text{ F}$ and:

$$\text{CFM} = 7.0 \times 10^4 \text{ ft}^3/\text{min}$$

Then the heat transferred will be:

$$Q = \text{CFM} \times 60 \times \Delta T \times 0.018$$

where 0.018 is the volumetric specific heat of air in BTU/ft^3 at these temperatures.

$$Q = 5.2 \times 10^6 \text{ BTU/hr}$$

Note that another equation for this convection heat transfer mentioned by Balcomb is that of Weber and Kearney,²¹ who arrived at it by similitude modeling and full-scale testing:

$$Q = 4.6 W (d \Delta T)^{3/2} \text{ where } W \text{ is width and } d \text{ is height of opening, which when converted to the variables used above becomes}$$

$$Q = 13 A \sqrt{H} (\Delta T)^{3/2}$$

Using $A = 280 \text{ ft}^2$, $H = 10 \text{ ft}$, and $\Delta T = 70^\circ \text{ F}$ as above:

$$Q = 6.7 \times 10^6 \text{ BTU/hr}$$

This is 25% more than the estimate above. I use the smaller number in table 1.

(vi) The radiant heat flow through the window:

Here the Stefan-Boltzman law governs the heat flow (the same equation used by Thatcher).

$$Q = \sigma A E (T_1^4 - T_2^4)$$

with $\sigma = 1,730 \times 10^{-12} \text{ BTU/hr-ft}^2 \cdot (^\circ\text{F absolute})^4$

A = area of body in question (the window in this case)

E = 0.9, a factor accounting for emissivities and solid angles subtended by the hot and cold bodies as seen through the window. Again this is the value used by Thatcher.

T_1 and T_2 are 560° F and 490° F absolute, respectively, for the hot inner surfaces at 100° F and outside surfaces (or air) at 30° F .

$$Q = 3.5 \times 10^4 \text{ BTU/hr}$$

If the window is glazed, a combination of convection, radiation, and conduction across boundary layers of air both on the inside and outside as well as the glass itself can be treated according to the following equation:²²

$$Q = U A \Delta T$$

where $U = 1.10 \text{ BTU/hr-ft}^2 \cdot ^\circ\text{F}$, which includes all three types of heat flow

A = area in ft^2 of window = 560

$$\Delta T = 70^\circ \text{ F}$$

$$Q = 4.3 \times 10^4 \text{ BTU/hr}$$

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²⁰ Balcomb (supra n. 18) 149–52.

²¹ D.D. Weber and R.J. Kearny, "Natural Convection Heat Transfer through an Aperture in Passive Solar Heated Buildings," *Proceedings, 5th National Passive Solar Conference*,

Amherst, Massachusetts, October 1980 (American Section of the International Solar Energy Society, Newark, Del. 1980) 1037–41.

²² ASHRAE *Handbook* (supra n. 9) ch. 11.